UNIVERSITAT POLITÈCNICA DE VALÈNCIA

Worked example: checking a steel beam in simple bending

Apellidos, nombre	Guardiola Víllora, Arianna (aguardio@mes.upv.es)
Departamento	Mecánica del Medio continuo y Teoría de Estructuras
Centro	Universitat Politècnica de València

1 Summary

This document shows how to check a beam with an overhang, taking into account the Ultimate Limit States conditions considered in the Spanish Code DB SE A.

The geometry of the steel frame and the bracing system shown in the design must be taken into account in the analysis and design.

2 Introduction

Given the Steel frame shown in figure 1, it is requested to check BCD beam designed with a S 275 hot rolled IPE 240 considering only Ultimate Limit States.

To understand the steel structure performance, a 3D model showing the bracing systems can be seen in figure 2.

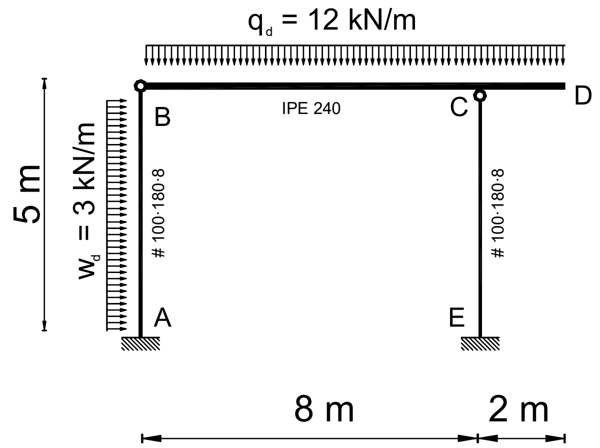


figure 1. Front view

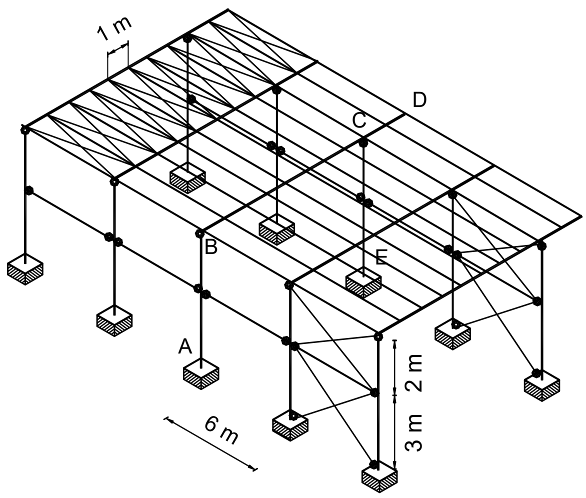


figure 2. 3D model showing the bracing Systems

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

3 Aims

At the end of this document, the student will be able to check a simple supported beam with a cantilever designed with an S 275 IPE 240 cross section.

For that purpose, the steps to be followed are:

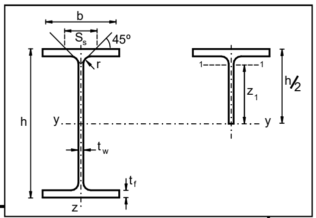
- Analyse a simple supported beam with a cantilever
- Check the designed beam in bending
- Check the designed beam in shear
- Check if the interaction Bending moments and shear forces has to be considered
- Check the lateral buckling condition
- Check the shear buckling of the web
- Check the local effects of the concentrated loads

The design will be accepted only when all the conditions have been fulfilled.

4 Worked example

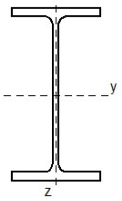
4.1 Geometric properties of the cross section

Geometric properties and resistances of the cross sections are shown in tables 1and 2 respectively.



IPE	Dimensions						Geometric properties												
	h mm	b mm	t_w mm	t_f mm	r mm	s_s mm	z_1 mm	A mm²	I_T mm⁴ (x10⁴)	I_y mm⁴ (x10⁶)	$W_{el,y}$ mm³ (x10³)	$W_{pl,y}$ mm³ (x10³)	i_y mm	$A_{v,y}$ mm²	I_z mm⁴ (x10⁶)	$W_{el,z}$ mm³ (x10³)	$W_{pl,z}$ mm³ (x10³)	i_z mm	$A_{v,z}$ mm²
240	240	120	6,2	9,8	15	43,4	93,6	3910	12,88	38,9	324	366	99,7	1910	2,84	47,3	74,0	26,9	2350

Table 1. Geometric properties of the cross section



RESISTANCES

IPE cross-section

S 275 STEEL

$N_{pl,Rd}$ Design plastic resistance for axial forces; $N_{pl,Rd} = A \cdot f_{yd}$
$V_{pl,y,Rd}$ Design plastic resistance for shear forces orthogonal to y axis; $V_{pl,y,Rd} = A_{v,y} \cdot f_{yd}/\sqrt{3}$
$V_{pl,z,Rd}$ Design plastic resistance for shear forces orthogonal to z axis; $V_{pl,z,Rd} = A_{v,z} \cdot f_{yd}/\sqrt{3}$
$M_{pl,y,Rd}$ Design plastic resistance for bending moments about y axis; $M_{pl,y,Rd} = W_{pl,y} \cdot f_{yd}$
$M_{pl,z,Rd}$ Design plastic resistance for bending moments about z axis; $M_{pl,z,Rd} = W_{pl,z} \cdot f_{yd}$
f_{yd} = 275 / 1,05 N/mm²

CROSS-SECTION	axial	Bending moment about y			Bending moment about z		
	$N_{pl,Rd}$ N	$V_{pl,y,Rd}$ N	$M_{el,y,Rd}$ N·mm	$M_{pl,y,Rd}$ N·mm	$V_{pl,z,Rd}$ N	$M_{el,z,Rd}$ N·mm	$M_{pl,z,Rd}$ N·mm
IPE 240	1 024 047	288 812	84 857 142	95 857 142	355 345	12 388 095	19 354 761

Table 2. Resistances of the cross section

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

4.2 Obtaining the internal forces

In order to obtain the internal forces in the beam, and taking into account the principle of superposition, the load is divided into horizontal and vertical loads:

1. Considering vertical loads

Vertical reactions are obtained isolating the beam (see figure 3) and taking into account the vertical forces and bending moments equilibrium equations (Equation 3 and 4 respectively):

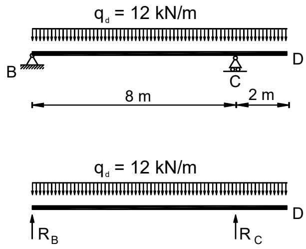


Figure 3. Vertical loads

$$\sum F_V = 0 \rightarrow R_B + R_C = 12 \cdot (8+2) = 120\ kN$$

Equation 1. Vertical loads Equilibrium equation

$$\sum M_B = 0 \rightarrow R_C \cdot 8 - 12 \cdot (8+2) \cdot 5 = 0$$

Equation 2. Bending moments equilibrium equation

Calculating:

$$R_C = \frac{12 \cdot (8+2) \cdot 5}{8} = 75\ kN; \qquad R_B = 120 - 75 = 45\ kN$$

2. Considering horizontal loads (see figure 4)

Horizontal reactions are obtained taking into account an additional compatibility condition in equation 3 (beams compressive strain has been neglected)

$\delta_{h1} = \delta_{h2}$ *Equation 3. Compatibility condition*

Being $\delta_{h1} = \dfrac{W_d \cdot h_1^4}{8 \cdot E \cdot I_1} + \dfrac{(-P) \cdot h_1^3}{3 \cdot E \cdot I_1}$

$$\delta_{h2} = \frac{P \cdot h_2^3}{3 \cdot E \cdot I_2}$$

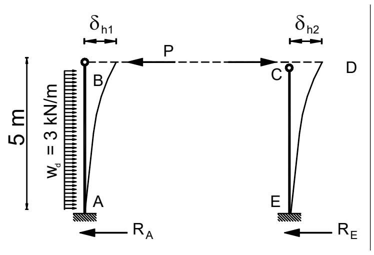


Figure 4. Horizontal loads

As in this case $h_1 = h_2$ and $I_1 = I_2$: $\dfrac{W_d \cdot h^4}{8 \cdot E \cdot I} = \dfrac{2 \cdot P \cdot h^3}{3 \cdot E \cdot I} \rightarrow P = \dfrac{3 \cdot W_d \cdot h}{2 \cdot 8}$

substituting and calculating: $P = \dfrac{3 \cdot 3 \cdot 5}{2 \cdot 8} = 2{,}81\ kN$

Taking into account former values for reactions, the internal forces functions and diagrams are obtained in the following epigraph.

4.2.1 Axial forces

Axial function and diagram is (figure 5)

From B to C: $N_{(x)} = 2{,}81\ kN$

From C to D $N_{(x)} = 0\ kN$

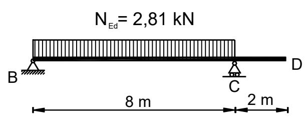


Figure 5. Axial forces diagram

4.2.2 Shear forces

Shear forces functions and diagram is (see figure 6)

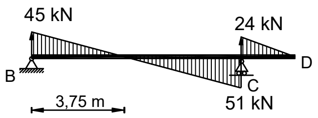


Figure 6. Shear forces diagram

From B to C: $V_{(x)} = 45 - 12 \cdot x$

Shear force on the left of C: $V_{Ed} = 51\ kN$

Shear force on the right of C: $V_{Ed} = 24\ kN$

Being the point where the shear force is zero: $45 - 12 \cdot x = 0 \rightarrow x = 3{,}75\ m$ from B

From C to D: $V_{(x)} = 24 - 12 \cdot x$

4.2.3 Bending moments

The maximum positive bending moment B is obtained considering the equilibrium of bending moments equation at 3,75 m form B (equation 4)

$$M_{max+} - 45 \cdot 3{,}75 + 12 \cdot \frac{3{,}75^2}{2} = 0$$

equation 4. Maximum bending moment

Calculating: $M_{max+} = 84{,}375\ kN \cdot m$.

The bending moments diagram is sketched in figure 7, being the maximum negative bending moment at C equal to:

$$M_{max-} = -12 \cdot 2 \cdot 1 = -24\ kN \cdot m$$

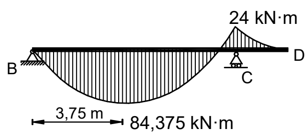


Figure 7. Bending moments diagram

4.3 Ultimate Limit States Conditions

4.3.1 Resistance

• **Bending moment condition**: In the absence of shear force, the design value of the bending moment M_{Ed} at each cross-section will satisfy equation 5

$$M_{Ed} \leq M_{c,Rd}$$

equation 5. Bending moment condition

with $M_{Ed} = 84.375.000\ N \cdot mm$ (maximum value at 3,75 m from B)

And $M_{c,Rd} \equiv M_{pl,Rd} = \frac{W_{pl} \cdot f_y}{\gamma_{M0}}$ because S 275 IPE 240 in bending is Class 1

As $M_{pl,y,Rd} = 95.857.142\ N \cdot mm$ (see table 2)

And $M_{Ed} = 84.375.000\ N \cdot mm < M_{pl,y,Rd} = 95.857.142\ N \cdot mm \rightarrow$ the beam fulfils the bending condition.

• **Shear forces condition:** The design value of the shear force V_{Ed} at each cross-section will satisfy equation 6.

$$V_{Ed} \leq V_{pl,Rd}$$ equation 6. Shear force condition

with $V_{Ed} = 51.000 \ \ N \cdot mm$ (maximum value on the left of B)

And $V_{pl,y,Rd} = 288.812 \ N$ (see table 1)

As $V_{Ed} = 51.000 < V_{pl,y,Rd} = 288.812 \ N$ (see table 1) → complies shear condition

• **Shear and bending moments interaction:**

The theoretical plastic bending moment resistance of a cross-section is reduced by the presence of shear. For small values of the shear force this reduction is so small that may be neglected. Provided that the design value for the shear force V_{Ed} does not exceed the 50% of the design plastic shear resistance $V_{pl,Rd}$ no reduction needs to be made in the value of the former plastic bending moment resistance.

The point to analyse if shear and bending interaction must be taken into account is C, where $V_{Ed} = 51.000 \ N$ and $M_{Ed} = 24 \ kN \cdot m$

Provided that $V_{Ed} = 51.000 < 50\% \cdot V_{pl,y,Rd} = 144.406 \ N$ the shear and bending interaction is not considered.

• **Axial forces and bending moments interaction:**

In I cross-sections, the reduction of the theoretical plastic moment resistance by the presence of small axial forces is balanced by strain hardening and may be neglected. When the axial force is smaller than the axial resistance of the web (equation 7) bending and axial force interaction can be neglected.

$$N_{Ed} < 0{,}5 \cdot A_v \cdot \frac{f_y}{\gamma_{M0}}$$ equation 7

Provided that $N_{Ed} = 2810 \ N < 0{,}5 \cdot 1910 \cdot \frac{275}{1{,}05} = 250.119 \ N$ the axial and bending interaction is not considered.

4.3.2 Lateral buckling condition

Lateral-torsional buckling condition is shown in equation 8.

$$M_{Ed} \leq M_{b,Rd}$$ equation 8. Lateral buckling condition

with $M_{Ed} = 84.375.000 \ \ N \cdot mm$ (maximum value at 3,75 m from B)

and $M_{b,Rd} = \frac{\chi_{LT} \cdot W_y \cdot f_y}{\gamma_{M1}}$ equation 9. Lateral-torsional buckling resistance for beams

as the cross-section is Class 1, equation 9 es equivalent to $M_{b,Rd} = \chi_{LT} \cdot M_{pl,y,Rd}$

being χ_{LT} the reduction factor for lateral-torsional buckling, which value can be obtained from the buckling curves in figure 8, considering the non-dimensional buckling slenderness $\overline{\lambda}_{LT}$, calculated in equation 9, while M_{cr} value can be obtained following equation 10, once $M_{LT,v}$ and $M_{LT,w}$ (equations 12 and 12 respectively) are known. For that purpose, values of $b_{LT,v}$, $b_{LT,w}$ coefficients are shown in figure 8

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}}$$ equation 10. non-dimensional buckling slenderness

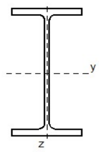
$$M_{cr} = \sqrt{M_{LT,v}^2 + M_{LT,w}^2}$$ equation 11. Elastic critical bending moment

$$M_{LT,v} = b_{LT,v} \cdot \frac{C_1}{L_c}$$ equation 12. Uniform torsional rigidity component

$$M_{LT,w} = b_{LT,w} \cdot \frac{C_1}{L_c^2}$$ equation 13. Non-uniform torsional rigidity component

LATERAL BUCKLING
IPE and S 275

Non-dimensional lateral buckling slenderness $\overline{\lambda}_{LT}$



$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} \quad \text{and} \quad M_{cr} = \sqrt{M_{LT,v}^2 + M_{LT,w}^2}$$

$$b_{LT,v} = \pi \cdot \sqrt{G \cdot I_t \cdot E \cdot I_z} \quad \text{to obtain} \quad M_{LT,v} = b_{LT,v} \cdot \frac{C_1}{L_c}$$

$$b_{LT,w} = W_{el,y} \cdot \pi^2 \cdot E \cdot i_{f,z}^{\ 2} \quad \text{to obtain} \quad M_{LT,w} = b_{LT,w} \cdot \frac{C_1}{L_c^{\ 2}}$$

$i_{f,z}$ radius of gyration of the compressed flange + 1/3 web, along the z axis

Cross-section	$i_{f,z}$ mm	$b_{LT,v}$ x 10^6 N · mm²	$b_{LT,w}$ x 10^9 N · mm²	$W_{el,y} \cdot f_y$ N·mm	$W_{pl,y} \cdot f_y$ N·mm
IPE 240	29,4	247 810	580 441	89 100 000	100 650 000

Table 4 – Reduction factor χ

$\overline{\lambda}$	Buckling curve				
	a_0	a	b	c	d
≤ 0,2	1,00	1,00	1,00	1,00	1,00
0,3	0,99	0,98	0,96	0,95	0,92
0,4	0,97	0,95	0,93	0,90	0,85
0,5	0,95	0,92	0,88	0,84	0,78
0,6	0,93	0,89	0,84	0,79	0,71
0,7	0,90	0,85	0,78	0,72	0,64
0,8	0,85	0,80	0,72	0,66	0,58
0,9	0,80	0,73	0,66	0,60	0,52
1,0	0,73	0,67	0,60	0,54	0,47
1,1	0,65	0,60	0,54	0,48	0,42
1,2	0,57	0,53	0,48	0,43	0,38
1,3	0,51	0,47	0,43	0,39	0,34
1,4	0,45	0,42	0,38	0,35	0,31
1,5	0,40	0,37	0,34	0,31	0,28
1,6	0,35	0,32	0,31	0,28	0,25
1,7	0,31	0,30	0,28	0,26	0,23
1,8	0,28	0,27	0,25	0,23	0,21
1,9	0,25	0,24	0,23	0,21	0,19
2,0 (1)	0,23	0,22	0,21	0,20	0,18
2,1 (1)	0,21	0,20	0,19	0,18	0,16
2,2 (1)	0,19	0,19	0,18	0,17	0,15
2,3 (1)	0,18	0,17	0,16	0,15	0,14
2,4 (1)	0,16	0,16	0,15	0,14	0,13
2,5 (2)	0,15	0,15	0,14	0,13	0,12
2,6 (2)	0,14	0,14	0,13	0,12	0,11
2,7 (2)	0,13	0,13	0,12	0,12	0,11
2,8 (2)	0,12	0,12	0,11	0,11	0,10
2,9 (2)	0,11	0,11	0,11	0,10	0,09
3,0 (2)	0,11	0,10	0,10	0,10	0,09

(1) non-acceptable slenderness ratio for principal members
(2) non- acceptable slenderness ratio, including bracing systems elements

Figure 8. Lateral-torsional buckling coefficients and buckling curves

The coefficient C_1 depends on the load case and the effective length factor k. It takes into account the bending moment distribution along the beam.
General cases are shown in table 3

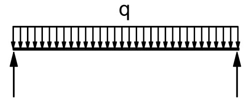
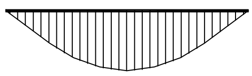
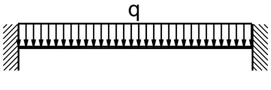
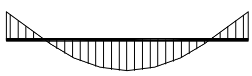
Loads and support conditions	**Bending moment diagram**	***k***	***C₁***
 		1,0 0,5	1,13 0,97
 		1,0 0,5	1,28 0,71

Table 4. Transverse loading cases

Considering the bending moment diagram of the beam and the position of the points where the out of plane movement is restrained is shown in figure 9, it can be assumed that L_c = 1000 mm, k= 1, and therefore C_1 = 1,13 and

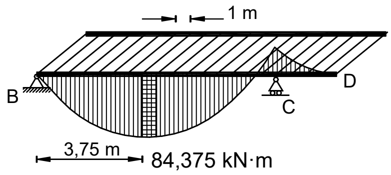


Figure 9. Bending moments and bracing system

Operating $$M_{LT,v} = 247.810 \cdot 10^6 \cdot \frac{1,13}{1.000^2} = 280.025.300\ N \cdot mm$$

$$M_{LT,w} = 580.441 \cdot 10^9 \cdot \frac{1,13}{1.000^2} = 655.898.330\ N \cdot mm$$

$$M_{cr} = \sqrt{280.025.300^2 + 655.898.330^2} = 713.173.743\ N \cdot mm$$

And $$\bar{\lambda}_{LT} = \sqrt{\frac{100.650.000}{713.173.743}} = 0,3756 < 0,40$$ (lateral buckling cannot occur)

4.3.3 Shear buckling of the web

Considering that the web is unstiffened, shear buckling of the web cannot occur if:

$$\frac{d}{t_w} \le 70 \cdot \varepsilon$$ equation 14. Shear buckling of the web condition

Being in this case $$\frac{240 - 2 \cdot 9,8 - 2 \cdot 15}{6,2} = 34,7 \le 70 \cdot \sqrt{\frac{235}{275}} = 64,7$$

therefore, the shear buckling of the web cannot occur.

4.3.4 Local effects, concentrated loads

Considering that the reactions are those shown in figure 10, The condition will be that every punctual load:

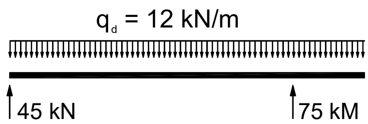


Figure 10. Reactions

$R < R_{b,Rd}$ equation 15. Punctual loads condition

where $R_{b,Rd}$ is the design buckling resistance of the web of the beam considered as a short column:

$R_{b,Rd} = N_{b,Rd},$ with $$N_{b,Rd} = \frac{\chi_{min} \cdot A \cdot f_y}{\gamma_{M1}}$$

Being $N_{b,Rd}$ the buckling resistance of the portion of the web bearing the punctual load.

Where $A = (20 \cdot t_w \cdot \varepsilon) \cdot t_w = 20 \cdot 6,2 \cdot \sqrt{\frac{235}{275}} \cdot 6,2 = 710\ mm^2$

$$I = \frac{(20 \cdot t_w \cdot \varepsilon) \cdot t_w^3}{12} = \frac{20 \cdot 6,2 \cdot \sqrt{\frac{235}{275}} \cdot 6,2^3}{12} = 2.265\ mm^4$$

$$i_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.265}{710}} = 1,78\ mm$$

$$\lambda = \frac{0,8 \cdot d}{i_{min}} = \frac{0,8 \cdot (240 - 2 \cdot 9,8 - 2 \cdot 15)}{1,78} = 85,57$$

being $\bar{\lambda} = \frac{\lambda}{\lambda_R}$ and considering that the steel is S 275, $\lambda_R = 86,8$

$$\bar{\lambda} = \frac{85,57}{86,8} = 0,98 \simeq 1 \xrightarrow{\text{curve c}} \chi_{min} = 0,54$$

Then $N_{b,Rd} = \frac{0,54 \cdot 710 \cdot 275}{1,05} = 100.414\ N$

As $N_{Ed} = R_c = 75.000\ N < N_{b,Rd} = 100.414\ N$ it is not necessary to stiffen the beam.

4.4 Final design of the beam

As the S 275 hot rolled IPE 240 cross section fulfils all the Ultimate Limit States conditions, it can be designed with that cross section.

5 Conclusion

This document explains, with a worked example, how to check Ultimate Limit States conditions of a simple beam with a cantilever loaded with a uniform load

With this purpose, the student has to follow the following steps:

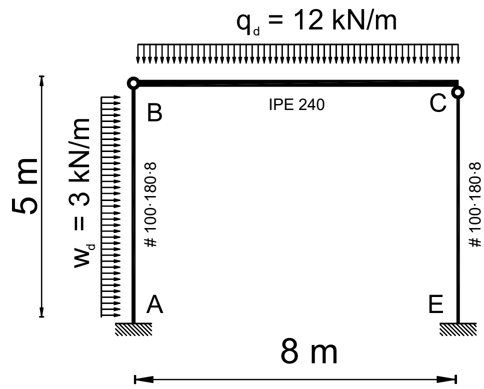
1. Obtain the internal forces in the analysed member
2. Check the bending moments resistance condition
3. Check the shear forces resistance condition

4. Check the bending moments-shear forces interaction
5. Check the axial forces-bending moments interaction
6. Check the lateral-torsional condition
7. Check the shear buckling of the web
8. Check the local effects of the concentrated loads.
9. Finally, once al the conditions have been fulfilled, the initial design has been accepted.

Finally, it is important to point out that the initial design of the cross section is not always going to fulfil all the conditions. In those cases, the cross section must be redesigned and checked until all the conditions are fulfilled.

6 Proposed exercise

Considering the frame in the figure, where the overhanging has been eliminated from the initial design, it is requested to check if the same cross section, with the same bracing conditions, will fulfil the Ultimate Limit State conditions.



7 Bibliography

7.1 Books:

Guardiola-Víllora, A; Pérez-García, A: "Steel structures. Worked examples according to the Spanish Code", Ed. Universitat Politècnica de València, 2017, chapter 4

7.2 Codes:

Steel structures Spanish code "Documento Básico, Seguridad Estructural, Acero". Ministerio de Fomento 2006. It can be downloaded from:

https://www.codigotecnico.org/

8 Solution

Being the maximum bending moment, at the middle of the span equal to:

$M_{Ed} = \frac{q \cdot L^2}{8} = 96\ kNm$, and $M_{pl,Rd}$ equal to 95857142 Nmm (table 2) the cross section fails the first condition, so, BC beam cannot be designed with an S 275 IPE 240