

Contents

Introduction and objectives	1
I Multi-core Optical Fiber: THEORETICAL ANALYSIS AND TRANSMISSION PERFORMANCE	5
1 Space-division multiplexing and multi-core fiber	7
1.1 Introduction	7
1.2 Space-division multiplexing in optical communications: technical approaches	8
1.3 Multi-core fiber types	11
1.4 Current and emerging applications of multi-core fiber	13
1.4.1 Backbone and access optical networks using multi-core fiber	13
1.4.2 Signal processing	15
1.4.3 Multi-core fiber lasers, amplifiers and optical sensors	16
1.4.4 Biomedical applications	17
1.4.5 Opportunities in experimental physics	18
1.5 Linear and nonlinear wave propagation in multi-core fiber: perturbation theory	20
2 Linear and nonlinear inter-core crosstalk	25
2.1 Introduction	25
2.2 Coupled-mode theory for bent and twisted multi-core fiber	26
2.2.1 Coupled-mode equations for ideal multi-core fiber	26
2.2.2 Mode-coupling coefficients	28
2.2.3 Exact solution of the coupled-mode equations and discrepancy with exper- imental measurements in real multi-core fiber	30
2.2.4 The effect of the multi-core fiber perturbations: a heuristic modification of the coupled-mode theory	32
2.3 Longitudinal evolution of inter-core crosstalk	33
2.4 Coupled-power theory for bent and twisted multi-core fiber	36
2.5 Statistical analysis of inter-core crosstalk	40
2.5.1 Single-core excitation	40
2.5.1.1 Linear regime	41
2.5.1.2 Nonlinear regime	43
2.5.2 Multi-core excitation	46
2.6 Experimental measurements	47
2.7 Conclusions	50
Appendix A2: Nonlinear wave equation	51
Appendix B2: Mode-coupling coefficients in step-index multi-core fiber	52

3 Longitudinal and temporal birefringence effects	55
3.1 Introduction	55
3.2 Coupled local-mode theory	56
3.2.1 Multi-core fiber local modes	57
3.2.2 Coupled-wave equations	58
3.2.3 Coupled local-mode equations	60
3.2.4 Mode-coupling coefficients	62
3.3 Numerical method: equivalent refractive index model	65
3.4 Numerical simulations	68
3.5 Experimental measurements	73
3.5.1 Floor vibrations	73
3.5.2 Temperature fluctuation	74
3.6 Conclusions	78
Appendix A3: Notes on the coupled-wave equations	80
Appendix B3: Coupled local-power theory	86
Appendix C3: Twisting-induced core bending effects	90
Appendix D3: Experimental characterization of first-order polarization-mode dispersion in multi-core fiber	92
4 Ultra-short pulse propagation model using local modes	95
4.1 Introduction	95
4.2 Coupled local-mode theory for ultra-short optical pulses	96
4.2.1 Multi-core fiber local modes	97
4.2.2 Coupled-wave equations	100
4.2.3 Coupled local-mode equations	102
4.3 Inter-core mode-coupling dispersion	110
4.3.1 Ideal multi-core fiber	110
4.3.2 Real multi-core fiber	112
4.4 Numerical simulations	114
4.5 Conclusions	120
Appendix A4: Multiple optical carriers	123
Appendix B4: Slowly-varying envelope approximation	127
Appendix C4: Numerical model, local split-step Fourier method	130
Appendix D4: Additional numerical examples	135
Appendix E4: Multi-mode regime	140
5 Multi-core fiber in the optical fronthaul	145
5.1 Introduction	145
5.2 Multi-wireless LTE-A and WiMAX fronthaul RoF provision using MCF	147
5.3 Pol-Mux PON extension capacity using RoF and MCF fronthaul provision	149
5.4 Fronthaul extension capacity in MCF-RoF: improvement using MIMO processing .	151
5.4.1 Tolerance of LTE-A RoF transmissions to in-band crosstalk	151
5.4.2 MCF-RoF fronthaul evaluation	155
5.5 Conclusions	158
Appendix A5: Temporal EVM fluctuations in MCF media	159
II Optical Supersymmetry: ANALYSIS AND SYNTHESIS OF FIBERS AND DEVICES	161
6 Supersymmetry	163
6.1 Introduction	163
6.2 Historical survey	164

6.3	Preliminary concepts	166
6.4	SUSY Hamiltonian factorization	171
6.5	SUSY algebra	174
6.6	Unbroken SUSY	175
6.7	Spontaneously broken SUSY	179
6.8	Explicitly broken SUSY: singular superpotentials	180
6.9	Darboux transformation	181
6.10	Continuous spectrum: spatial scattering	182
6.11	Isospectral transformations	185
6.11.1	One-parameter family	185
6.11.2	Multi-parameter family	189
6.11.3	Bound states in the continuum	190
6.12	Shape invariant potentials and SUSY	193
6.13	SUSY in optics, acoustics and thermodynamics	197
6.13.1	Optical SUSY	197
6.13.2	Extension to acoustics and thermodynamics	200
6.14	Further reading	201
6.14.1	From a general second-order ODE to a Helmholtz equation	201
6.14.2	SUSY in radially- and axially-symmetric potentials	202
6.14.2.1	Radially-symmetric potentials	202
6.14.2.2	Axially-symmetric potentials	203
6.14.2.3	Degeneracy breaking	204
6.14.3	Two-dimensional (2D) SUSY QM	204
6.14.4	Omitted topics	206
7	Supersymmetric optical fibers	209
7.1	Introduction	209
7.2	SUSY in axially-symmetric optical potentials	210
7.3	Unbroken and broken SUSY optical fibers	212
7.3.1	Unbroken SUSY fibers	213
7.3.2	Broken SUSY fibers	221
7.4	Isospectral optical fibers	223
7.5	Conclusions	227
Appendix A7:	Linear wave equation in weakly-guiding fiber	229
Appendix B7:	Unbroken and broken SUSY (discussion)	231
Appendix C7:	Isospectral transformations (discussion)	237
8	Optical supersymmetry in the time domain	243
8.1	Introduction	243
8.2	Temporal SUSY and temporal scattering	245
8.2.1	Transparent phase shifter	247
8.2.2	Optical isolator	249
8.2.3	Heterogeneous media	250
8.2.4	Methods	251
8.3	Temporal SUSY and temporal waveguide	251
8.3.1	Supersymmetric temporal waveguides	251
8.3.2	The temporal photonic lantern	254
8.3.3	Methods	255
8.4	Conclusions	255
Appendix A8:	Time-varying optical systems	256
Appendix B8:	Negative-frequency waves	261
Appendix C8:	Temporal scattering	266
Appendix D8:	Temporal waveguide	288
Appendix E8:	Temporal SUSY in acoustics	304

9 Conclusions and further work	305
9.1 Conclusions	305
9.2 Extrapolation to acoustics and quantum mechanics	307
9.3 Further work	308
Publications	311
Bybliography	315
List of Acronyms	341
List of Figures	347
List of Tables	353
List of Symbols	355