Knock probability estimation through an in-cylinder temperature model with exogenous noise

P. Bares1* D. Selmanaj2 C. Guardiola1 C. Onder2

Universitat politècnica de València 1
Swiss Federal Institute of Technology (ETH) 2

Abstract

This paper presents a new knock model which combines a deterministic knock model based on the in-cylinder temperature and an exogenous noise disturbing this temperature. The autoignition of the end-gas is modelled by an Arrhenius-like function and the knock probability is estimated by propagating a virtual error probability distribution. Results show that the random nature of knock can be explained by uncertainties at the in-cylinder temperature estimation. The model only has one parameter for calibration and thus can be easily adapted online.

In order to reduce the measurement uncertainties associated with the air mass flow sensor, the trapped mass is derived from the in-cylinder pressure resonance, which improves the knock probability estimation and reduces the number of sensors needed for the model.

A four stroke SI engine was used for model validation. By varying the intake temperature, the engine speed, the injected fuel mass, and the spark advance, specific tests were conducted, which furnished data with various knock intensities and probabilities. The new model is able to predict the knock probability within a sufficient range at various operating conditions. The trapped mass obtained by the acoustical model was compared in steady conditions by using a fuel balance and a lambda sensor and differences below 1% were found.

Key words: Knock, SI engines, STFT, resonance, control

* Corresponding author:
Email address: pabamo@mot.upv.es (P. Bares1).

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1 Introduction

In spark-ignited (SI) engines the combustion is initiated by the energy released by the spark. This combustion generates a flame front which moves along all the cylinder volume in a uniform manner [1]. During the flame propagation the temperature of the unburned gas increases due to the rise of the in-cylinder pressure. In normal combustion events, the flame front reaches all of the cylinder volume in a controlled scheme; however, high unburned gas temperatures can cause the autoignition of the end-gas.

The autoignition of the end-gas (knock) is an undesirable phenomenon and is one of the main limitations in SI engines. The rapid combustion of the end-gas heavily excites cylinder head resonance, and its vibration reduces the combustion efficiency and can damage the engine [2, 3, 4].

Current approaches for knock control use the spark advance (SA) timing for modifying the combustion phasing and thus, the likelihood of knock. The control strategies can be divided into two groups: stochastic and model-based. Stochastic methods aim to directly control the knock probability by varying the SA [5, 6, 7], while model-based methodologies use the spark advance for keeping in-cylinder conditions, e.g. un-burned gas temperature, within a desired range [8, 9, 10].

Regarding the physics behind the knock phenomenon, several studies state that knock cannot be predicted in a deterministic manner [11, 12]. One one hand, when operating in steady-state conditions, the flame propagation in SI combustion has a significant cycle-to-cycle variability [13, 14], which directly influences the cycle-to-cycle knock probability. On the other hand, some research suggest that the autoignition of the end-gas could be initiated at hot spots, influenced by inhomogeneities of the mixture [15, 16, 17]. Furthermore, the autoignition of the end-gas is driven by an Arrhenius-like function [18, 19], and small errors in the gas temperature are propagated in knock models by an exponential function resulting in important errors in the prediction of knock.

The present paper proposes estimating the knock probability by adding an exogenous noise over the in-cylinder temperature model. This noise represents uncertainties such as temperature hot spots, in-cylinder pressure pegging, wall heat transfer, residual mass variations, and sensor errors. Hence, the model is developed with the assumption that the random nature of knock is due only to in-cylinder temperature uncertainties. It is beyond the scope of this paper
to prove this assumption; however, the experimental results show that it improves the prediction capabilities of a control-oriented knock model.

The model presented aims to reduce the unburned gas uncertainties estimating the trapped mass by identifying the resonant frequencies of the in-cylinder pressure waves. The method was recently applied to DI and HCCI engines [20, 21, 22], and some applications, i.e. NOx and residual gas fraction estimation, have been already explored [23, 24]. One of the most important sources of error is the estimation of the trapped mass, which is a crucial parameter for the estimation of the unburned gas temperature. Measurement errors of hot-film air mass flow sensors, which are widely employed in automotive applications, can attain up to 20% because of ageing and other non-calibrated effects [25, 26]. Furthermore, there are no sensors for estimating the residual gases, which in SI engines without negative valve overlap represent between 3% and 5% of the total mass trapped, and they exhibit important cycle-by-cycle variations [27, 28].

In addition, past works on the method, as tested in cylinders with an in-piston bowl, do include the engine speed fluctuations with the bowl effect by a crank angle dependent parameter [29]. In this paper, a spark-ignited engine was used and the instantaneous engine speed was measured and modelled by the camshaft model given by Li and Stone in [30]. Herein, the frequency is demonstrated to follow Draper’s equation [31] with no need for any calibration effort. A fuel balance, a lambda sensor, and a residual mass model were used for validation in steady conditions.

The resonance methodology is specially suitable for the knock modelling of SI engines for three reasons:

1. SI engines normally do not have a bowl. Theory based on a cylindrical combustion chamber thus fits the physics better.
2. Knocking conditions are produced at high loads, where combustion heavily excites the resonant frequencies, even in the absence of knock.
3. SI combustion is performed in stoichiometric conditions and the resulting end-gas properties can be easily identified [32].

In order to evaluate the improvement associated with the methodology based on in-cylinder pressure for estimating the trapped mass, the knock model was run by using the trapped mass from the resonance method and by using the air mass flow sensor.

The paper is organized as follows: The next section introduces the experimental set-up and the tests performed. Section three describes the methodology for
determining the knock autoignition and Section four describes the procedure for estimating the knock probability by the addition of an exogenous noise. Section five shows the performance of the knock model and illustrates the effect of sensor errors by comparing the air mass flow sensor and the trapped mass obtained from resonance. Finally, the last section highlights the main contributions of the method and points out the future work required for using the model in a control scheme.

2 Experimental set-up

Tests were performed in a turbocharged four-stroke SI engine with 0.5 l of unitary displacement. The air-to-fuel ratio was maintained at stoichiometric conditions by controlling in a closed-loop the amount of fuel injected with a lambda sensor placed at the exhaust line. The air mass flow was measured by a hot-film anemometer and controlled by a waste-gate valve at the turbocharger. The main characteristics of the engine are summarized in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Main engine characteristics</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinders</td>
<td>[-]</td>
<td>3</td>
</tr>
<tr>
<td>Combustion type</td>
<td>[-]</td>
<td>SI</td>
</tr>
<tr>
<td>Unitary displacement</td>
<td>[cc]</td>
<td>499.6</td>
</tr>
<tr>
<td>Bore</td>
<td>[mm]</td>
<td>82</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>[-]</td>
<td>10.1:1</td>
</tr>
</tbody>
</table>

The engine was equipped with a variable valve timing (VVT) system and direct gasoline injection. The timing of the gasoline injection was set at 270 CAD before top dead center (TDC), the intake valve opening and closing (IVO and IVC) were set at 387 and 180 CAD before TDC, respectively, and the exhaust valve opening and closing (EVO and EVC) were set at 146 and 357 CAD after TDC. The two valve lift profiles are shown in Figure 1.

A fuel balance was used for fuel mass flow metering, which implies accurate measurements in steady-state conditions. Used in conjunction with a lambda sensor, this measurement provides an air mass flow estimation, as follows:

\[ m_{\text{air}} = \lambda 14.7 m_{\text{fuel}} \] (1)
The residual gas fraction (RGF) was estimated from in-cylinder conditions at the EVO and EVC, such as several authors proposed [33, 34], as follows:

\[ RGF = \left( \frac{V_{EVC}}{V_{EVO}} \right) \left( \frac{p_{EVC}}{p_{EVO}} \right)^{1/\gamma} \]  \hspace{1cm} (2)

To analyse the pressure signals a short time Fourier transform (STFT) and Wigner distribution (WD) were used. The former uses the Fourier transform and a time-varying window to obtain the frequency spectrum with time resolution, while the latter multiplies the signal at a past time by the signal at a future time [35, 36, 37]. Each method has some advantages and disadvantages: On one hand, STFT is the most commonly used methodology as it is a robust method and does not require much computational time. However, STFT dilutes the signal by using a window function, which means that the frequency content obtained at instant \( t \) is affected by the surroundings. On the other hand, WD has a precise frequency spectrum, but ghost terms caused by artificial frequencies mask the real components. In this work Blackman-Harris window of 2.8 ms was used for STFT, while WD was implemented over the full signal.

Knock was detected by analysing the frequency content of the in-cylinder pressure signal such as in [38]. This methodology does detect low-knocking cycles by comparing the resonance excitation near the maximum heat release rate (normal combustion) and next to the maximum unburned temperature (autoignition).

Various SA settings, ranging from 0 to 13 CAD before TDC, were applied by performing steps at eight operating conditions. Table 2 summarizes the operating conditions tested. Here, the engine speed, the air mass flow, and the coolant temperature (controlled by varying the coolant flow) were changed to
create diverse knocking conditions. Figure 2 shows the spark advance evolution during an SA step test.

![Spark Advance Evolution during an Example of the Steady-State SA Step Test](image)

**Fig. 2.** SA evolution during an example of the steady-state SA steps test

<table>
<thead>
<tr>
<th>OP</th>
<th>( m_{air}^{ref} ) [g/s]</th>
<th>( n ) [rpm]</th>
<th>( T_{cool} ) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25</td>
<td>1500</td>
<td>85</td>
</tr>
<tr>
<td>B</td>
<td>22</td>
<td>1500</td>
<td>85</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>1500</td>
<td>90</td>
</tr>
<tr>
<td>D</td>
<td>22</td>
<td>1500</td>
<td>90</td>
</tr>
<tr>
<td>E</td>
<td>19</td>
<td>1250</td>
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<td>F</td>
<td>16</td>
<td>1250</td>
<td>85</td>
</tr>
<tr>
<td>G</td>
<td>16</td>
<td>1250</td>
<td>90</td>
</tr>
<tr>
<td>H</td>
<td>19</td>
<td>1250</td>
<td>90</td>
</tr>
</tbody>
</table>

### 3 Arrhenius-based knock model

As stated above, the autoignition of the end-gas is characterized by the incylinder chamber conditions and it can be modelled by an Arrhenius-like function, such as:

\[
\tau = C_1 e^{\frac{C_3}{p}} p^{C_2}
\]  

(3)

where \( \tau \) is the ignition delay and \( T_{ub} \) are the pressure and temperature of the unburned gasses, respectively, and \( C_1, C_2 \) and \( C_3 \) are constants defining the autoignition process.
In the present work, the values suggested by Douaud and Eyzat were used [39], namely \( C_2 = -1.7 \) and \( C_3 = 3800 \). The model is easily adaptable for a model-based knock control scheme and the only parameter that needs calibration is \( C_1 \).

Autoignition is produced when

\[
AI = \int_{IVC}^{EOC} \frac{1}{\tau} d\alpha > 1
\]

and the unburned temperature can be calculated by assuming a polytropic process after the spark is produced.

\[
T_{ub} = \begin{cases} 
\frac{pV}{mR} & \text{if } \alpha < SA \\
T_{ub}^{SA} \left( \frac{p}{p^{SA}} \right)^{\frac{\kappa}{\kappa - 1}} & \text{otherwise}
\end{cases}
\]

where \( \kappa \) is the polytropic exponent. In this work, a value of 1.3 was used.

Note that the trapped mass calculated is used in an exponential function, such that, if there was an error in the trapped mass input, it would become in a high deviation at the autoignition delay output.

Figure 3 shows the probability functions of the AI index defined in (4) for four values of SA steps and for all the cycles together, i.e. the steps illustrated in Figure 2. The AI index increases as the spark is advanced because combustion starts closer to TDC provoking higher temperatures of the unburned gas.

![Fig. 3. AI probability functions of various SA steps at point H](image)

Figure 4 shows the knock probability found for the cycles at point H. The
knock probability was calculated by dividing the data in groups of similar AI values, with a resolution of 0.025. Clearly, the knock events are more frequent when the AI index increases, nevertheless, the autoignition hypothesis stated at (4) is not always accomplished, as some cycles with AI values below 1 present knock and others above do not.

![Fig. 4. Knock probability measured as a function of the AI index at point H](image)

### 4 Exogenous noise and knock probability estimation

Errors at the temperature estimation of the unburned gas preclude a deterministic prediction of knock. These errors can be caused by unobservable effects, e.g. temperature hot spots or residual mass variations, but also by measurement and modelling errors, such as pegging errors or errors at the occurring estimations of the trapped mass and $\gamma$.

The present paper proposes the use of an error probability distribution $\varphi_T$ which is added in the computed unburned gas temperature and which takes into account all the uncertainties in the measurement. This probability distribution is used to calculate, at each cycle, a probability distribution for the autoignition parameter defined in Equation (4), as follows:

$$\varphi_{AI} = AI + \frac{dAI}{dT} \varphi_T$$ (6)

where $\frac{dAI}{dT}$ is numerically calculated by:

$$\frac{dAI}{dT} = \frac{AI(T_{ub} + \Delta T, p) - AI(T_{ub}, p)}{\Delta T}$$ (7)
Finally, the knock probability can be obtained by integrating the probability that knock occurs:

\[ p_{KE} = \int_{AI>1} \varphi_{AI} \]  

(8)

Following these steps, if the exogenous noise was set to 0, the function of the knock probability would be a digital function. If the noise was a bias in any measurement, this would result in an offset of the final probability function. And if the error was a random function, it would smooth the probability function by having the 50% probability at \( AI = 1 \). Figure 5 exemplifies the effect of various exogenous noises at the knock probability function by using a constant temperature sensitivity \( dAI/dT \) of 0.04.

Fig. 5. Effect of various types of errors to the knock probability function

The exogenous noise was chosen to be a normal distribution with 0 mean and constant standard deviation. The standard deviation was calibrated at point H by using the least squares method. The value found was 5.29% and the result of the model is shown in Figure 6.

Fig. 6. Effect of various type of errors on the knock probability function
Note that the knock probability shown in Figure 6 is calculated cycle-by-cycle by varying the temperature sensitivity $dAI/dT$, while the example shown in Figure 5 is computed with a constant temperature sensitivity of 0.04. Figure 7 plots the autoignition parameters for 50 consecutive cycles for each SA value at the point H. It must be noticed that the SA not only strongly influences the autoignition time ($AI$) calculated, but also the temperature sensitivity.

Fig. 7. Autoignition parameter and temperature sensitivity for different SA settings at point H

Figure 8 illustrates the knock probability calculation by plotting the probability function of the AI for four individual cycles with different SA values. The knocking probability is computed by integrating the area above the knocking criteria, which is represented by a dashed line. Note that, the autoignition parameter (AI) determines whether a cycle is knocking or not, i.e. above or below 50% of knock probability, while the temperature sensitivity determines the trust worthiness of this estimation.

Fig. 8. AI probability distribution computed for four cycles at point H
5 Model Validation

As stated above, the knock model was validated at eight operating conditions, as summarized in Table 2. Each cycle was analysed independently and an exogenous noise with $\sigma = 5.29\%$ was used for all of them. Figure 9 shows the probability density function of AI for each test. Clearly, points G and H have higher values of AI, while points B and D have lower values.

![Fig. 9. Probability density functions for each OP](image)

Left plot of Figure 10 shows the results of the knock model when $T_{ub}$ is computed by using the air mass flow sensor for all the operating conditions. The model shows an offset when the operating conditions are varied. This offset is caused by air mass flow sensor errors. If traditional sensors were employed, each operating condition would need a different value of $C_1$ for dealing with the sensor bias, and the model would require a constant adaptation when transients are performed. Nevertheless, this offset can be corrected by using the pressure resonance of the cylinder for detecting the total mass trapped.

This methodology for inferring the mass trapped from in-cylinder pressure resonance oscillations was recently developed and it is the first time that it is used in conventional SI engines. A more detailed description of the procedure can be found in Appendix A. The right plot of Figure 10 shows the results obtained when the unburned temperature is detected by using the resonant frequencies. Note that all the experimental lines collapse around the predicted value and only a single value of $C_1$ was required for running the model. This model could be used for knock control by maintaining the AI index at a desired level, e.g. if the AI index was maintained at 0.8, the percentage of knocking cycles would remain below 20%.

The complete model uses the in-cylinder pressure and the geometrical param-
Fig. 10. Knock probability calculation

eters of the engine to estimate the probability of knock. The model can be summarized by the following steps:

1. The content of the in-cylinder pressure compressed on the frequency band of resonance (between 4 kHz and 20 kHz) is used for characterizing the acoustical waves and estimating the in-cylinder trapped mass.
2. The low-frequency band of the pressure signal is used to analyse the combustion, compute the temperature of the unburned gases, and estimate the autoignition delay.
3. An exogenous noise is added to the unburned gas temperature and it is propagated along the model. This yields a probability distribution of the autoignition delay (not a single value).
4. The knock probability is obtained by integrating the autoignition delay probability distribution which predicts an autoignition of the end-gas before the combustion ends.

Figure 11 shows a scheme of the complete model. Note that the in-cylinder pressure signal is the only input required for computing the knock probability.

6 Conclusions

A knock model was developed for predicting knock probability. The knock probability is calculated by assuming an error probability distribution on the temperature of the unburned gases and propagating this error along a simple knock model based on an Arrhenius-like function. The model uses in-cylinder pressure resonant frequencies to determine the unburned gas temperature. Doing so, sensor bias is avoided and the model does not need to be constantly adapted in transient operation.
The most important contributions of the model developed are:

- The in-cylinder conditions of the SI engines when knock must be controlled are specially suitable for detecting the resonance phenomenon: High loads excite in-cylinder pressure resonance, the absence of a bowl in SI engines ensures a constant Bessel factor and there is no air after combustion, facilitating the estimation of the gas properties.
- Only the pressure signal is required, which significantly reduces the sources of errors.
- The trapped mass is directly computed, with no need for a residual gas fraction model.
- The model can be adapted by actualizing a single parameter. This is essential for knock models as they must be continuously adapted to deal with any variation, e.g. caused by ageing.

The model was validated in a four-stroke SI engine by varying several knocking conditions; namely spark advance, engine speed, load, and coolant temperature. Results show an important bias when using current sensors for computing the trapped mass. However, the model accurately predicts the knock probability when the trapped mass is obtained from the resonant content of the pressure signal.

A  Estimation of the trapped mass

It is possible to determinate the in-cylinder trapped mass from the acoustic characteristics of the pressure waves, caused by the combustion [20, 21]. As known from [31], the evolution of the resonant frequency depends on the
cylinder geometry and the in-cylinder speed of sound, as follows:

\[ f_{res}(\alpha) = \frac{a(\alpha)B}{\pi D} \]  

(A.1)

where \( a \) is the speed of sound, \( f_{res} \) is the resonant frequency, \( B \) is the Bessel coefficient for the given resonant mode, \( D \) is the cylinder bore and \( \alpha \) represents the crank angle evolution.

Hickling et al. were the first authors trying to take advantage of resonance by measuring the oscillation period and estimating the bulk temperature [40]. The approach was adapted by Guardiola et al. for obtaining the trapped mass by using STFT [20, 21]:

\[ m = \gamma pV \left( \frac{B}{f_{res} \pi D} \right)^2 \]  

(A.2)

where \( p \) is the in-cylinder pressure, \( V \) the instantaneous cylinder volume and \( \gamma \) is the ratio between constant pressure and constant volume heat capacities \((c_p/c_v)\), as shown in Figure A.1:

\[ S(m) = R[p(t)] = \int_{-\infty}^{\infty} p(t) e^{-j2\pi \int_{-\infty}^{t} \frac{B\sqrt{\gamma V(\tau)} p(\tau) V(\tau)}{\pi D \sqrt{m}}} d\tau \]  

(A.3)

The actual trapped mass makes this transformation maximum, and the value of the peak provides information about the strength of that resonant mode. A scheme of the trapped mass estimation procedure that uses the direct transformation is shown in Figure A.2.
Equation A.3 was discretized and computed from 30 to 80 CAD after TDC, which yields:

$$S(m) = \sum_{\alpha=30}^{\alpha=80} T(\alpha)p(\alpha)e^{-j2\pi \sum_{\phi=0}^{\phi=m} \frac{T(\phi)B\sqrt{V(\phi)}V(\phi)}{\pi D\sqrt{m}}}$$  (A.4)

As the engine is closed-loop controlled by a lambda sensor to have stoichiometric conditions, all the gases after combustion are burned products and the value of $\gamma$ can be computed by a single polynomial equation, such as in [41].

In past works [20, 21, 22], the method was applied to CI engines and a crank-angle-dependent parameter was included to take into account the disturbance associated with the existence of the bowl in the proximity of the TDC and the engine speed fluctuations. Herein, the method is applied for the first time in a conventional SI engine and it was improved by modelling the instantaneous engine speed by using the model proposed by Li and Stone [30]. This model predicts the engine speed fluctuations by modelling the inertial forces of the piston movement. Figure A.3 shows the measured and modelled engine speed for 50 consecutive cycles, at the operating conditions of point A.

![Engine Speed Graph](image)

Fig. A.3. Measured and modelled engine speed for 50 consecutive cycles in steady conditions

The inclusion of the engine speed fluctuations into the trapped mass estimations is needed for relying on Draper’s equation. Figure A.4 shows one cycle pressure signal (top plot), analysed by WD (medium plot) and by STFT (bottom plot). Herein, the resonance frequency evolution, computed by Equation (A.3), is represented by a dashed line and was calculated for the first radial mode using a constant Bessel factor ($B = 1.842$) and including the engine speed fluctuations in the sampling period ($T = 1/F_s = 1/30n$).

If a constant engine speed is assumed, the frequency shift does not correspond
with theory of a cylindrical combustion chamber and a variable Bessel factor is needed. Figure A.5 shows the band-passed pressure signal of Figure A.4 and the Bessel factor evolution identified with and without including the engine speed fluctuations. This coefficient was computed by measuring the trapped mass, computing the speed of sound, and measuring the resonant frequencies \( f_{\text{res}} \), as follows:

\[
B(\alpha) = \frac{f_{\text{res}}(\alpha) \pi D}{a(\alpha)} \tag{A.5}
\]

The resonant frequencies \( f_{\text{res}} \) were identified by using the WD and by selecting the most excited frequency at each crank angle location.
There are two benefits of including the engine speed fluctuations in the calculation. First, there is no need for calibration and the method can be directly used. Second, a crank-angle-dependent calibration, as suggested in previous works, does not take into account the effect of the load at the engine speed fluctuations. The differences at the engine speed between high load and low load can cause errors of up to 2% in the trapped mass calculation for the crank angle range considered.

A.1 Trapped mass results over steady-state tests

The trapped mass was computed from the fuel balance, from the air mass flow sensor, and by using the resonance methodology for all the steady operating conditions listed in Table 2. The RGF was estimated between 2.9% and 3.5% and lambda was maintained near stoichiometric conditions ($m_{air} \approx 14.7m_{fuel}$). Figure A.6 compares all three methodologies. Measurement data from resonance better fits the measurement results obtained by the fuel balance, and the small discrepancies between them can also be attributed to the $\lambda$ sensor or to the RGF estimation.

![Graph of trapped mass computations over steady operating points](image)

Fig. A.6. Results of trapped mass computations over steady operating points

References


