



Microarticle

3D sound wave focusing by 2D internal periodic structure of 3D external cuboid shape

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A B S T R A C T

In this work, we report the focusing effect of a 2D sonic crystal cuboid. The proposed sonic composite lens is vertically extended from a 2-D flat Phononic Crystal Structure, but it is found to be able to focus waves in a three-dimensional manner. By varying the cuboid size, beam dimensions change and transverse beam width values smaller than the classical diffraction limit (~ 0.3 of wavelength) can be obtained. Numerical results have been obtained by Finite Element Method.

As it well known [1] 2D Phononic Crystal (PhC) possesses periodicity of the permittivity along two directions, while in the third direction the medium is uniform instead of 3D PhC in which permittivity modulation along all three directions. The focusing effect of two-dimensional (2D) [2–8] sonic crystals have been studied intensively during the last decades. It was pointed out that lattice geometry, scatterers shape; size and orientation are parameters that play an important role in sound focusing. By modifying one of these parameters, it is possible to tune sound beam properties. In this paper, we show a sound crystal structure, the three-dimensional focusing properties of which are determined by both the 2D internal structure and the 3D external dimensions. Therefore, the structure proposed is a 2D internal sonic crystal of 3D external cuboid shape consisted of square solid rods distributed on a square lattice immersed in air. Hereinafter referred as metamaterial cuboid. The side dimensions of the metamaterial cuboid (L_{cuboid}) are function of the incident wavelength, λ , as shown in Fig. 1.

Such structure is working below the first band gap, which is below the first Brillouin Zone as it can be seen in Fig. 1e. Due to the square geometry of the scatterers, the shape of the equifrequency contours of the structure for the selected frequency is elliptical [9]. This one, together to the anisotropy of the metamaterial produce a refraction phenomenon, which causes the focalization [10]. For a given refractive index structure of $n = 1.2$ which is determined from the filling fraction (ff) through the expression [2] $n = \sqrt{1 + ff}$, the focusing performance of the structure is evaluate by means the Finite Element Method (FEM) realized in the commercial software COMSOL Multiphysics. In order to simplify the model, the results are presented considering the rigid elements. However, it has been verified that considering fluid-structure interaction with thermoviscous losses, there are no differences for the

frequencies of interest. Because of the lattice constant value ($a = 0.073\lambda$) the selection of the refractive index is a tradeoff between the computational time and the physical requirements of the phenomenon. In our case, the wavelength is 0.686 m that corresponds to 500 Hz in air, which is marked in blue solid line in Fig. 1e.

Fig. 2 shows the results obtained by illuminating with a plane wave a rigid cuboid (Fig. 2a) and a metamaterial cuboid (Fig. 2b) with the same dimensions. It can be observed that for cuboid sizes from 2λ to 3.5λ , while the rigid cuboid is not able to focus, the metamaterial is, thus a focusing effect is observed. This implies that the cuboid's total length, i.e. the diffraction (edge effects), does not determine the main role for the focusing effect. As can be seen from Fig. 2b, by varying the cuboid dimensions (3λ and 3.5λ) maintaining the refractive index constant, a variation in the beam occurs. It is noted that as the cuboid size is increased, the length of the beam decreases as it narrows and the sound gain increases. That means that as higher cuboid size a better quality beam is obtained.

The quality of the beam is usually evaluated by means the Full-Width at Half-Maximum (FWHM) and the Full-Length at Half-Maximum (FLHM) values (see Table 1). Fig. 3 represents a cross section at the point of higher gain. From this cut the FWHM can be evaluated. It can be observed that FWHM also depends on the cuboid dimensions, as expected. Accomplishing that as the size of the cuboid increases, the FLHM decreases and the FWHM (see Table 1). Thus, FWHM values lower than the diffraction limit (0.5λ) are obtained for cuboid sizes greater than 3λ . Therefore, a subwavelength beamwidth can be obtained by increasing the size of the cuboid. Fig. 3c shows the curvature of the pressure wavefront due to the anisotropic relation of the elements.

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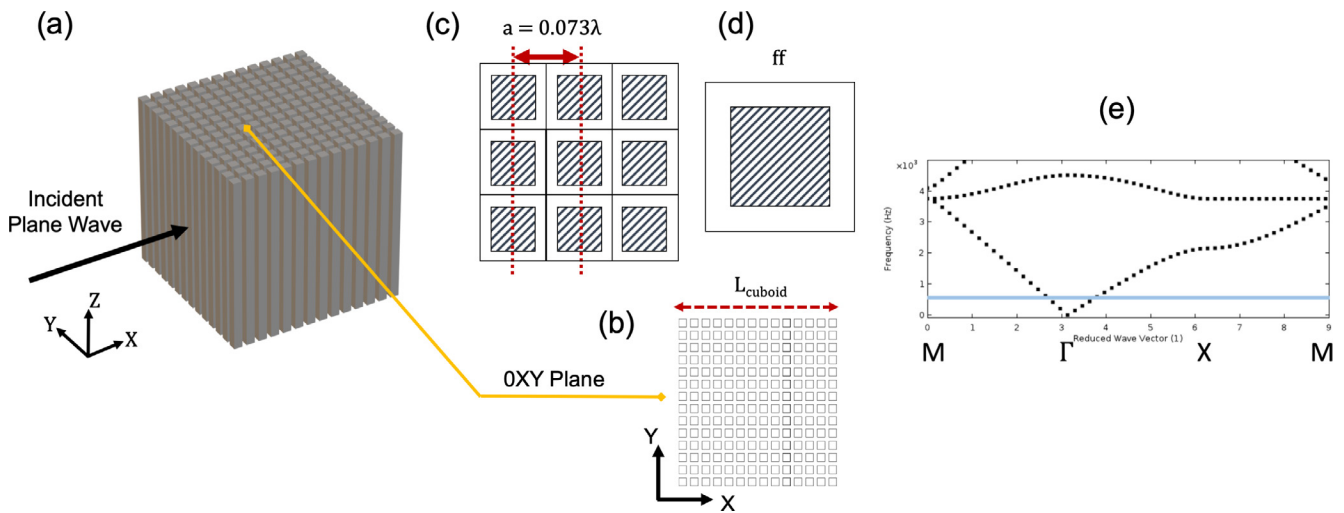


Fig. 1. (a) 3D scheme of the metamaterial cuboid structure the plane wave travels a long X direction, (b) Plane XY view of the crystal, (c) Zoom of XY plane, where a is the lattice constant and (d) the filling fraction is the relation between the area occupied by the material and the area of the unite cell. (e) Band structure of the unit cell along M-Γ-X-M.

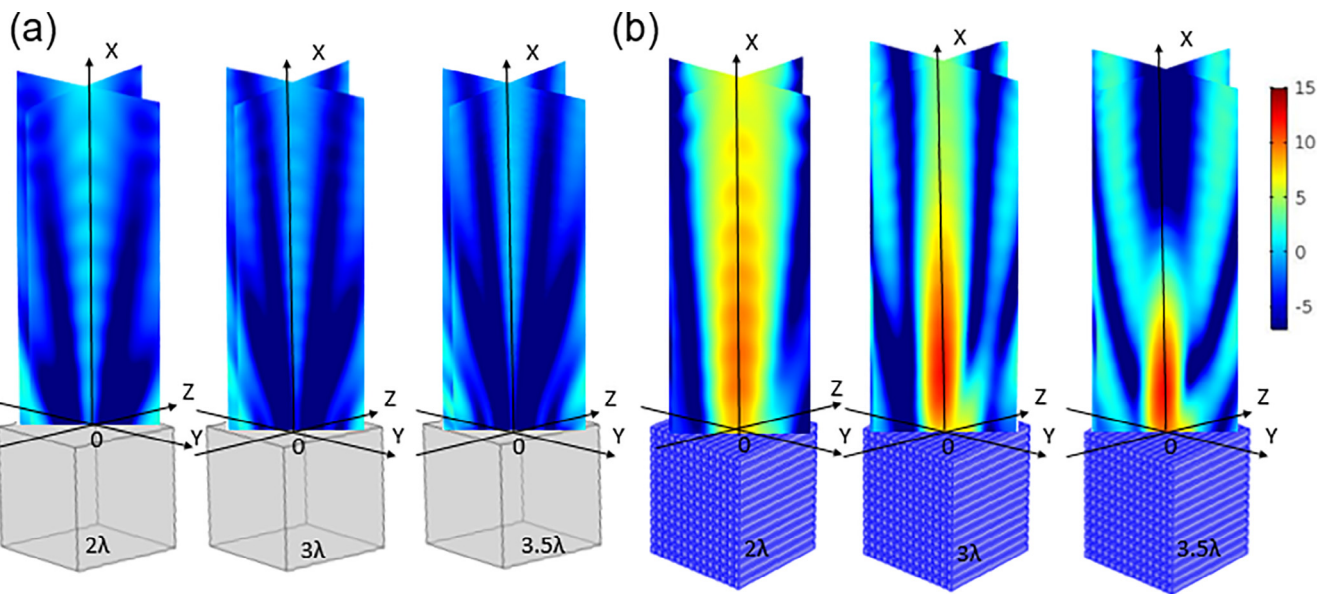


Fig. 2. Gain pressure level distribution for different cuboid size and nature (a) rigid cuboid, (b) metamaterial cuboid. The plane wave travels along X axes from bottom to top in the figures.

Table 1
FLHM and FWHM for rigid and metamaterial cuboid a function of cuboid size.

L_{cuboid}	FLHM		FWHM	
	Rigid	Metamaterial	Rigid	Metamaterial
2λ	-	$> 2.9\lambda$	-	0.56λ
3λ	-	1.17λ	-	0.31λ
3.5λ	-	0.61λ	-	0.29λ

Fig. 4 shows different equispaced planes for three sizes of the cuboid. It is interesting to note that with a plane symmetry lens a three dimensional focal spot can be obtained. It is interesting to note that with a plane symmetry lens a three dimensional focal spot can be obtained. This phenomenon of focalization is possible, as it has been already explain, thanks to the characteristics of the equifrequency contour. The elliptic shape of these contours and the dispersion relation (which defines the group velocity of the wave, v_g) which imposes that

v_g is normal to these contour, indicates the direction of the pressure field. In this sense, the v_g is perpendicular to the wavefront and therefore the focalization is achieved. In conclusion, we have presented a sonic crystal cuboid with 2D internal PhC structure whose focusing properties depend on its entire 3D dimensions. It was found that it is possible to obtained 3D sonic wave focusing by 2D periodic internal structure of 3-D external cuboid shape. It has been shown that by changing the cuboid size, both the sound pressure enhancement and FWHM at focus can be modified. FWHM below the diffraction limit can be achieved. These results could have significant applications in medical ultrasound.

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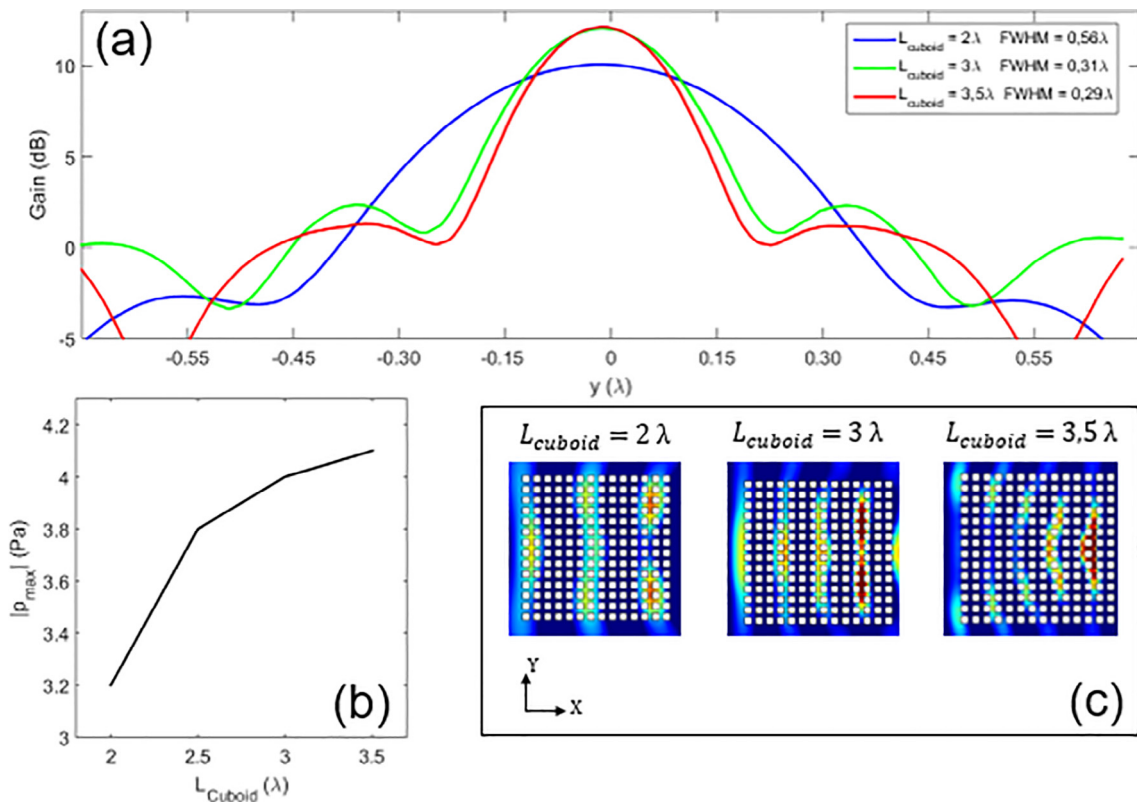


Fig. 3. (a) Transverse Gain (dB) distributions in the focus and FWHM for different metamaterials cuboid dimensions. (b) Absolute maximum pressure value as a function of the metamaterial dimension. (c) Sound pressure wavefront inside the metamaterial for the three different cuboid dimensions.

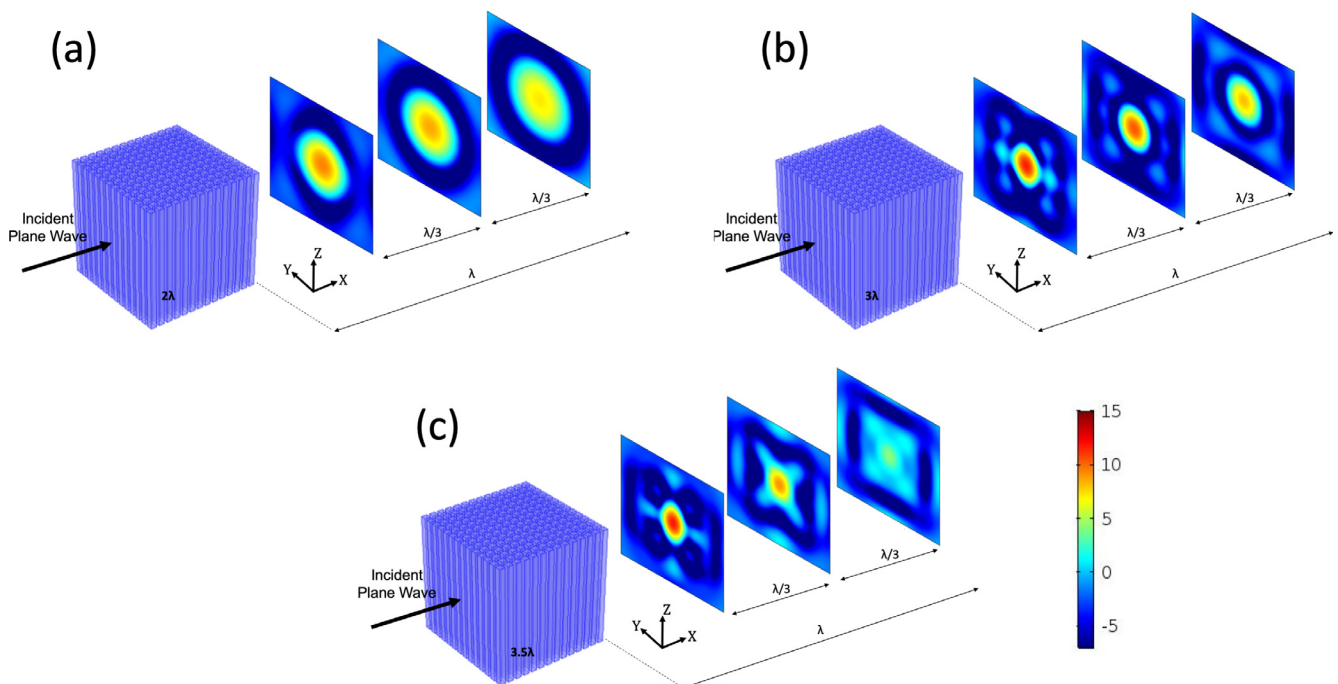


Fig. 4. Transverse Gain (dB) planes obtained in the Y-Z plane along X-axis for (a) 2λ , (b) 3λ and (c) 3.5λ cuboid.

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References

[1] Sukhoivanov IA, Guryev IV. Photonic crystals: physics and practical modeling Springer-Verlag Berlin Heidelberg; 2009. <https://doi.org/10.1007/978-3-642-02646-1>.

[2] Cervera F, Sánchez-Pérez JV, Martínez-Sala R, Rubio C, Meseguer F, Lopez C, et al. Refractive acoustic device for airborne sound. Phys Rev Lett 2001;88:023902.
 [3] Kuo CH, Ye Z. Sonic crystal lenses that obey the lensmaker's formula. J Phys D: Appl Phys 2004;37:2155–21592.
 [4] Feng S, Li Z-Y, Feng Z-F, Ren K, Cheng B-Y, Zhang D-Z. Focusing properties of a rectangular-rod photonic-crystal slab. J Appl Phys 2005;98:063102.
 [5] Feng L, Liu X-P, Lu M-H, Chen Y-B, Chen Y-F, Mao Y-W, et al. Refraction control of acoustic waves in a square-rod-constructed tunable sonic crystal. Phys Rev B

- 2006;73:193101.
- [6] Alagoz S, Alagoz BB. Frequency-controlled wave focusing by a sonic crystal lens. *Appl Acoust* 2009;70:1400–5.
- [7] Climente A, Torrent D, Sánchez-Dehesa J. Sound focusing by gradient index sonic lenses. *Appl Phys Lett* 2010;97:104103.
- [8] Minin IV, Minin OV. Brief review of acoustical (sonic) artificial lenses. Proc. of the 13th Int. Scientific-technical conf. On actual problems of electronic instrument Engineering (APEIE)-39281, Novosibirsk. 2016. p. 136–7.
- [9] Luo J, Yang Y, Yao Z, Lu W, Hou B, Hang ZH, et al. Ultratransparent media and transformation optics with shifted spatial dispersions. *Phys Rev Lett* 2016;117(22):223901.
- [10] Lock EH. The properties of isofrequency dependences and the laws of geometrical optics, *UFN*, 178:4 (2008), 397–417. *Phys Usp* 2008;51(4):375–93.