

## Characterization and assessment of composite materials via inverse finite element modeling

Carlos Llopis-Albert<sup>1</sup>, Francisco Rubio<sup>1</sup>, Francisco Valero<sup>1</sup>

<sup>1</sup>Centro de Investigación en Ingeniería Mecánica (CIIM), Universitat Politècnica de València,  
Camino de Vera s/n, 46022 Valencia, Spain

Emails: cllopisa@upvnet.upv.es, frubio@mcm.upv.es, fvalero@mcm.upv.es

Received: 13 January 2019; Accepted: 26 August 2019

### Abstract

Characterizing mechanical properties play a major role in several fields such as biomedical and manufacturing sectors. In this study, a stochastic inverse model is combined with a finite element (FE) approach to infer full-field mechanical properties from scarce experimental data. This is achieved by means of non-linear combinations of material property realizations, with a certain spatial structure, for constraining stochastic simulations to data within a non-multiGaussian framework. This approach can be applied to the design of highly heterogenous materials, the uncertainty assessment of unknown mechanical properties or to provide accurate medical diagnosis of hard and soft tissues. The developed methodology has been successfully applied to a complex case study.

### Keywords

Inverse modeling; finite element; mechanical properties; heterogeneity characterization; biomedical; uncertainty assessment.

## 1. Introduction.

Recently, there has been increased interest in highly heterogeneous materials and composite materials due to its wide engineering applications. Nevertheless, heterogenous materials show a large spatial and temporal variability, which lead to significant uncertainties in the estimation of the full-field material properties (Wu and Zhu, 2017).

For instance, composite material encompasses three components: discontinuous multiphases, the matrix as the continuous phase and the fine interface area. Examples of such materials can be found in bioengineering such as musculoskeletal tissue and bone or implants in orthopedics, porous ceramics, and metal-composite joints in automotive and aerospace applications (Ni et al., 2007). As a result, there are composite materials with mechanical properties changes within the vicinity with up to five orders of magnitude (Wu and Zhu, 2017). A comprehensive review on the uncertainty representation of material properties can be found in Charmpis et al., 2007; Sriramula and Chryssanthopoulos 2009. There are also attempts in the literature for modelling the effects of the heterogeneity on the mechanical response (e.g., Zottis et al., 2018; Zhang et al., 2018).

The material property characterization is hampered by the problem of obtaining reliable experimental data either by the financial cost or by technical impediments. Experimental tests for determining material properties cover a wide range of techniques, including non-destructive methods with the ability to access internal variables such as strains (e.g., Mortazavi et al., 2014).

Examples of experimental techniques that can be used in conjunction with this methodology comprise extensometers, photoelasticity, X-ray techniques, thermography, Digital Image Correlation (DIC) and Digital Volume Correlation (DVC) (Li et al., 2014).

This paper makes use of the experimental information obtained with these approaches within an inverse model framework to provide accurate predictions of the effects of heterogeneous material properties. The reliability in numerical models strongly depends on the properties of the underlying material, with justifies the use of inverse models to reduce the uncertainty of highly heterogeneous materials. The methodology has been successfully applied to a complex case study, while providing an uncertainty assessment of the results.

## **2. Material and methods.**

Inverse problems are often ill-posed although there are new computational schemes to properly overcome this drawback. Inverse methods are intended to determine the input and the characteristics of a system from some of the output from the same system.

The methodology encompasses a finite element (FE) approach embedded into a stochastic inverse framework for calculating effective material properties for heterogeneous materials.

The stochastic correlation structure of the material properties relies on an indicator conditional simulation technique (Gómez-Hernández and Srivastava, 1990). As a first step, this technique generates a set of material property realizations, named as seed fields, that honours the material property measurements within a non-multiGaussian framework. Then seed

parameter fields are conditional to measurements, and also to secondary data, for example, from expert judgement.

The a priori stochastic structure of seed parameter fields is defined through the conditional probability distribution function (cpdf) and the indicator variograms, which allow to estimate variables at any unsampled location using indicator kriging algorithms (e.g., Goovaerts, 1997). This enables to adopt any Random Function (RF) model and to reproduce the coalescence and connectivity among phases and existing crack patterns, which are of vital importance to provide reliable safety factors and fatigue life predictions.

The second step entails a numerical approximation of the mechanical stress ( $\sigma$ ) and displacement ( $u$ ) field by means of the Finite Element Method (FEM). The ANSYS software is used for that purpose ([www.ansys.com](http://www.ansys.com)).

In the third step, the method carries out an iterative optimization procedure based on successive non-linear combinations of two seed realizations and the previous optimized parameter fields (the elastic modulus ( $E$ ) and Poisson's ratio ( $\nu$ )). The procedure is based on an iterative minimization of a penalty function which expresses the discrepancy between the experimentally measured and the numerically computed response of the underlying physical system. In this sense, the optimization technique is based on data assimilation to identify stochastic structures of uncertain mechanical parameters. This way of proceeding has been widely used in the literature in many research fields (Llopis-Albert et al., 2015; 2018; 2018a; Llopis-Albert and Pulido-Velazquez, 2015; Rubio et al., 2015; 2016; 2019). Further information about the inverse

method used in this paper can be found in Llopis-Albert and Capilla, 2009; 2010; 2010a; Llopis-Albert et al., 2014; 2016.

### **3. Case study.**

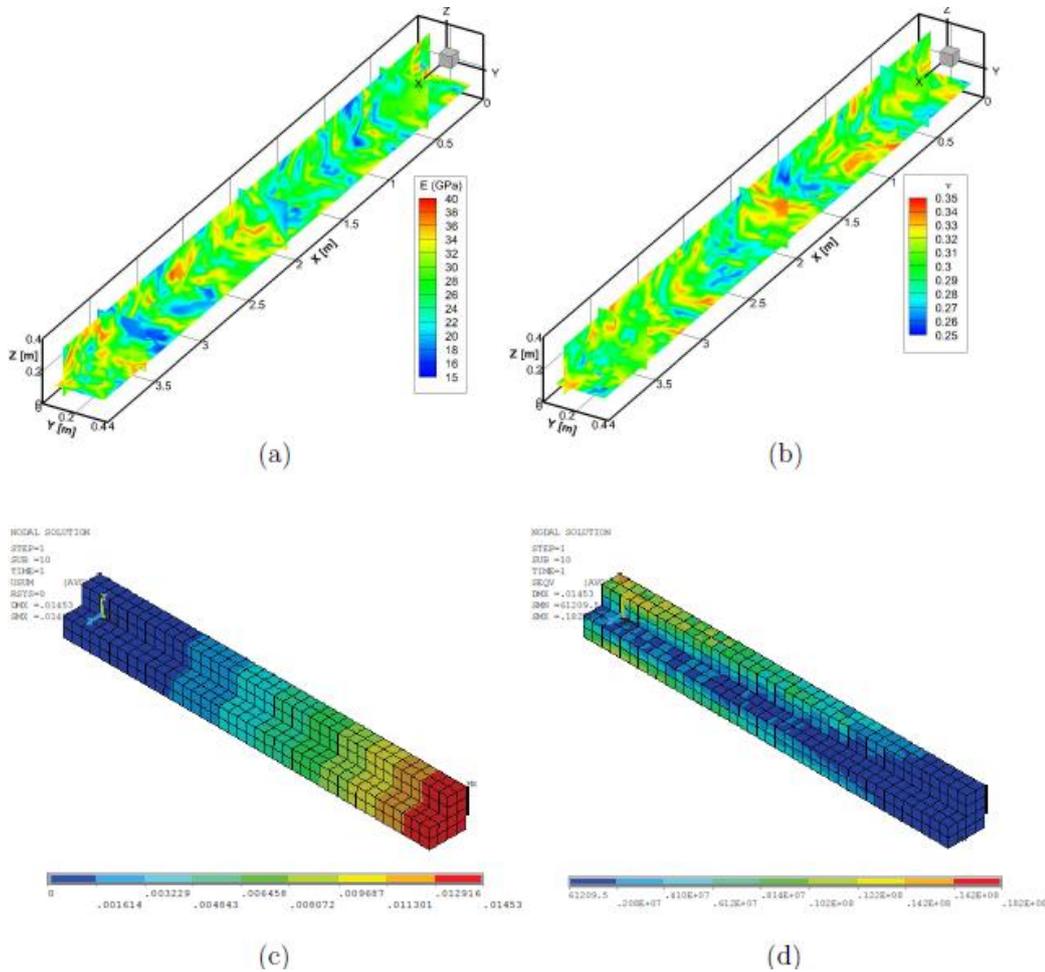
The methodology is used for the simulation of uncertain material properties (elastic modulus and Poisson's ratio) conditional to measurements of those parameters and also to stress and strain data. The case study deals with the bending of a composite beam. The beam has a length of 4 m, and a height and width of 0.4 m. It has been discretized using blocks of 0.1 m, thus leading to 640 blocks. Several boundary conditions are applied. The boundary conditions applied is that the beam is fixed at one end in all degrees of freedom and the blocks belonging to the free-end are subjected to a bending moment as a result of a pressure of 1 MPa at the top face.

A set of 76 data are used as conditioning data for each variable. They are uniformly distributed along the domain.

### **4. Results and discussion.**

The gradual deformation process for constraining simulations to measurements leads to significant differences between unconditional and conditional fields, i.e., important changes in the seed parameter fields are induced. In this sense, results show a good agreement between the estimated effective Young's modulus and Poisson' ratio in the conditional simulations regarding their corresponding values in the reference field.

After the iterative optimization procedure, Fig. 1 represents for a given conditional realization the material property fields of the elastic modulus and Poisson's ratio, together with its corresponding displacement and stress fields. It depicts how the heterogeneity in the material properties is also present in the displacement and stress fields. The conditional fields also show the presence of non-Gaussian features. The perturbation field is able to partially change the stochastic structure of seed fields to come close to data, thus reducing the uncertainty in the results. To better analyse the results a performance measure is defined as the square root of a weighted mean of the square departures of computed values from the measured values after a certain iteration. The conditional field presents, after fifty iterations of the inverse model, a performance measurement of  $1.34\text{E-}09$  m for the displacement field ( $u$ ), and 16.47 MPa for the von Mises stress field ( $\sigma$ ). This entails a reduction in the performance measurement, regarding the unconditional field, of around 63% for the ( $u$ ) field and 44% for the ( $\sigma$ ) field.



**Figure 1.** Material property fields for a given conditional realization: elastic modulus (a), Poisson's ratio (b); and its corresponding displacement (c) and stress fields (d).

## 5. Conclusions.

A stochastic inverse model combined with a finite element (FE) approach is presented to characterize heterogeneous mechanical properties within a non-multiGaussian framework. The FE method allows solving the problem in hand and obtaining the mechanical stress and

displacement fields. The iterative optimization process allows constraining simulations to available data by minimizing an objective function that penalizes the difference between measured and computed data. The methodology allows characterizing the structural parameter fields of the effective modulus of elasticity and the Poisson's ratio and their corresponding mechanical response. Finally, it has been successfully applied to a complex case study.

## References

- Charpis, D. C., G. I. Schueller, M. F. Pellissetti (2007). The need for linking micromechanics of materials with stochastic finite elements: A challenge for materials science. *Computational Materials Science* 41(1), 27–37. <https://doi.org/10.1016/j.commatsci.2007.02.014>
- Gómez-Hernández, J.J., R.M. Srivastava, (1990). ISIM3D: an ANSI-C three-dimensional multiple indicator conditional simulation program. *Computer Geoscience* 16(4), 395–440. [https://doi.org/10.1016/0098-3004\(90\)90010-Q](https://doi.org/10.1016/0098-3004(90)90010-Q)
- Goovaerts, P. (1997). *Geostatistics for Natural Resources Evaluation*. Oxford University Press, NY, NY, 483 pp.
- Ni, Y., M. Y.M. Chiang (2007). Prediction of elastic properties of heterogeneous materials with complex microstructures. *Journal of the Mechanics and Physics of Solids* 55, 517–532. <https://doi.org/10.1016/j.jmps.2006.09.001>
- Li, G., F. Xu, G. Sun, Q. Li (2014). Identification of mechanical properties of the weld line by combining 3D digital image correlation with inverse modelling procedure. *International Journal of Advanced Manufacturing Technology* 74, 893–905. <https://doi.org/10.1007/s00170-014-6034-x>
- Llopis-Albert, C., Capilla, J.E. (2009). Stochastic inverse modeling of non multiGaussian transmissivity fields conditional to flow, mass transport and secondary data. 2 Demonstration on a synthetic aquifer. *Journal of Hydrology*, 371, 53–65. <https://doi.org/10.1016/j.jhydrol.2009.03.014>
- Llopis-Albert, C., Capilla, J.E. (2010). Stochastic simulation of non-Gaussian 3D conductivity fields in a fractured medium with multiple statistical populations: case study. *Journal of Hydrologic Engineering*, 15, 554-566. [https://doi.org/10.1061/\(ASCE\)HE.1943-5584.0000214](https://doi.org/10.1061/(ASCE)HE.1943-5584.0000214)

-Llopis-Albert, C., Capilla, J.E. (2010a). Stochastic inverse modelling of hydraulic conductivity fields taking into account independent stochastic structures: A 3D case study. *Journal of Hydrology* 391, 277–288. <https://doi.org/10.1016/j.jhydrol.2010.07.028>

-Llopis-Albert, C., Palacios-Marqués, D., Merigó, J.M. (2014). A coupled stochastic inverse-management framework for dealing with nonpoint agriculture pollution under groundwater parameter uncertainty. *Journal of Hydrology* 511, 10–16. <https://doi.org/10.1016/j.jhydrol.2014.01.021>

-Llopis-Albert, C., Rubio, F., Valero, F. (2015). Improving productivity using a multi-objective optimization of robotic trajectory planning. *Journal of Business Research* 68, 1429–1431. <https://doi.org/10.1016/j.jbusres.2015.01.027>

-Llopis-Albert, C., Pulido-Velazquez, D. (2015). Using MODFLOW code to approach transient hydraulic head with a sharp-interface solution. *Hydrological processes* 29(8), 2052-2064. <https://doi.org/10.1002/hyp.10354>

-Llopis-Albert, C., Merigó, J.M., Xu, Y.J. (2016). A coupled stochastic inverse/sharp interface seawater intrusion approach for coastal aquifers under groundwater parameter uncertainty. *Journal of Hydrology* 540, 774-783. <https://doi.org/10.1016/j.jhydrol.2016.06.065>

-Llopis-Albert, C. Rubio, F., Valero, F. (2018). Optimization approaches for robot trajectory planning. *Multidisciplinary Journal for Education, Social and Technological Sciences* 5(1), 1-16. <https://doi.org/10.4995/muse.2018.9867>

-Llopis-Albert, C. Rubio, F., Valero, F. (2018a). Designing Efficient Material Handling Systems Via Automated Guided Vehicles (AGVs). *Multidisciplinary Journal for Education, Social and Technological Sciences*, 5(2), 97-105. <https://doi.org/10.4995/muse.2018.10722>

-Mortazavi, F., E. Ghossein, M. Lévesque, I. Villemure (2014). High resolution measurement of internal full-field displacements and strains using global spectral digital volume correlation. *Optics and Lasers in Engineering* 55, 44–52. <https://doi.org/10.1016/j.optlaseng.2013.10.007>

-Rubio, F., Llopis-Albert, C., Valero, F., Suñer, J.L. (2015). Assembly line productivity assessment by comparing optimization-simulation algorithms of trajectory planning for industrial robots. *Mathematical Problems in Engineering*, vol. 2015, Article ID 931048, 10 pages, 2015. <https://doi.org/10.1155/2015/931048>

-Rubio, F., Llopis-Albert, C., Valero, F., Suñer, J.L. (2016). Industrial robot efficient trajectory generation without collision through the evolution of the optimal trajectory. *Robotics and Autonomous Systems* 86, 106-112. <https://doi.org/10.1016/j.robot.2016.09.008>



-Rubio, F., Valero, F., Llopis-Albert, C. (2019). A review of mobile robots: Concepts, methods, theoretical framework, and applications. *International Journal of Advanced Robotic Systems*, 16(2). <https://doi.org/10.1177/1729881419839596>

-Sriramula, S., M. K. Chryssanthopoulos (2009). Quantification of uncertainty modelling in stochastic analysis of FRP composites. *Composites Part A: Applied Science and Manufacturing* 40(11), 1673–1684. <https://doi.org/10.1016/j.compositesa.2009.08.020>

-Wu, X., Y. Zhu (2017). Heterogeneous materials: a new class of materials with unprecedented mechanical properties. *Materials Research Letters* 5:8, 527-532. <https://doi.org/10.1080/21663831.2017.1343208>

-Zhang, Z., C. Zhan, K. Shankar, E.V. Morozov, H. Kumar, T. Ray (2017). Sensitivity analysis of inverse algorithms for damage detection in composites. *Composite Structures* 176, 844-859. <https://doi.org/10.1016/j.compstruct.2017.06.019>

-Zottis, J., Theis, C.A., da Silva, A (2018). Evaluation of experimentally observed asymmetric distributions of hardness, strain and residual stress in cold drawn bars by FEM-simulation. *Journal of Materials Research and Technology*. <https://doi.org/10.1016/j.jmrt.2018.01.004>