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Information aggregation via midpoint theory and its applications

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Abstract

In this short note we want to explicitly state that there is a growing research activity in the field of information aggregation via midpoint theory and its applications to decision making.

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1. A recent research activity in aggregation of information and metric midpoint theory

In the last twenty years the interest in mathematical theory of aggregation and fusion of information has grown a lot owing to the wide range of applications of this

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theory to practice problems. In several realms of applied science, the scientific community has the need of using simultaneously different kinds of information coming from several sources in order to infer a conclusion or working decision. There are many used techniques for merging the information and obtaining, thus, a meaningful and useful fused data. The decision of which aggregation method must be used depends on the nature of the problem under consideration. However, in most practical cases such fusion methods are based on aggregation operators on some numerical values, i.e. the aim of the fusion process is to obtain a representative number from a finite sequence of numerical data. In the aforementioned cases, the input information presents some kind of imprecision and for this reason it is represented as fuzzy sets. Moreover, in such problems it is necessary to make comparisons between the numerical values generated by the information described by the fuzzy sets. This is done by means of a sort of similarity measured by a distance defined on fuzzy sets. Hence, numerical operators aggregating distances between fuzzy sets as incoming data play a distinguished role in applied problems. In fact, in the last years, several works have dealt with the aggregation of distances on fuzzy sets because of its applicability, among others, to medicine, multiple attribute decision problems and biology. In particular, A. Pradera, E. Trillas and E. Castiñeira have studied intensely the general problem of merging data represented by means of fuzzy relations (distances and indistinguishability operators) in [8, 9]. Several general techniques for merging a finite number of distances on fuzzy sets are introduced and studied by J. Casasnovas and F. Rosselló in [1]. Specifically they analysed the aggregation operators given by the weighted maximum, the weighted sum and by the weighted Euclidean norm in order to apply some of their properties to the comparison of biological sequences. A related work, by the same authors, with applications to diagnosis problems in medicine can be found in [2].

Recently, J.J. Nieto and A. Torres gave some applications of the aggregation of distances (using the weighted sum as numerical aggregation operator) on fuzzy sets to the study of real medical data in [7]. These applications are based on the notion of segment joining two given fuzzy sets and on the notion of set of midpoints between fuzzy sets. A few results obtained by Nieto and Torres have been generalized in turn by Casasnovas and Rosselló in [1, 2].

Asymmetric distances and other related structures provide efficient tools in some fields of Computer Science and in Bioinformatics. Metric tools based on asymmetric distances have been introduced and developed, for instance, in [11, 10, 4, 5] with the aim of providing an efficient framework in asymptotic complexity analysis of algorithms and in logic programming. In [12, 13, 14], it has proved a natural correspondence between similarity measures on biological (nucleotide or protein) sequences and asymmetric distances, giving practice applications to search in DNA and protein datasets. Nowadays, the numerous applications of asymmetric distances to the aforementioned areas of science have promoted an unceasing research activity. Motivated by such facts, on the one hand, Casasnovas and O. Valero and, on the other hand, P. Tirado and Valero, obtained in [3, 15] a version of several results by Casasnovas and Rosselló ([1, 2]) about midpoints and segments of fuzzy sets for the case of merging asymmetric distances by means of the weighted sum and weighted maximum aggregation operators. Concretely, the asymmetric upper Hamming distance and the asymmetric weighted maximum distance between fuzzy sets were defined, and then an explicit description, in the spirit of Casasnovas and Rosselló, of the set of segments and midpoints was provided. Besides, a relationship between the description of the segments and midpoint sets obtained for the classical weighted Hamming distance and the weighted maximum distance and their asymmetric counterparts was obtained, since the classical aforesaid distances can be obtained from the asymmetric ones by means of easy symmetrization techniques. In addition, an application of the developed theory obtained in the asymmetric framework was given to the asymptotic complexity analysis of algorithms. Specifically, it was proved formally that, for the Largetwo algorithm, the asymptotic complexity class of the average running time is a midpoint between the asymptotic complexity class of the running time of computing of the best and the worst case.

Recently, in [6] S. Massanet and Valero, inspired by the fact that the aforementioned study of segments and midpoint sets was done considering (asymmetric) distances obtained via the aggregation of a collection of a finite family of another (asymmetric) distances, provided a general framework for the study of midpoint sets for asymmetric distances through aggregation theory. In particular, they gave a description of those properties that an aggregation function must satisfy to characterize the segment and midpoint set for an asymmetric distance generated by means of the fusion of a collection of asymmetric distances in terms of the segments and midpoint sets for each of the asymmetric distances that are merged.

Instead of the exposed research activity, nowadays there are many challenges and questions that can be addressed in order to improve the mathematical methods for decision making in the problems that arise in a natural way, among others fields, in Engeenering, Medicine and Economics. Of course in this research line, Mathematics and the aforesaid applied sciences continually feedback each other in such a way that the former provides formal methods for solving the practical problems under consideration and the latter inspire the development of new mathematical theories. Therefore, it would be very positive that multidisciplinary research groups will focus their efforts on combining generalized metric structures and information fusion to solve practical problems and thus to maximize the profit of both, Science and Society.

References

[1] J. Casasnovas, F. Roselló, Averaging fuzzy biopolymers, Fuzzy Sets Syst. 152 (2005), 139–158.

[2] J. Casasnovas, F. Roselló, Midpoints as average representations of pairs of descriptions by means of fuzzy subsets, Proc. Information Processing and Management of Uncertainty in Knowledge-based Systems international Conference, B. Bouchon-Meunier et al. (eds), pp. 2157–2164. Editrice Universita La Sapienza, Roma, 2004.

[3] J. Casasnovas, O. Valero, On midpoints for the weighted upper Hamming distance betwen fuzzy sets, Proc. Information Processing and Management of Uncertainty in Knowledge-based Systems international Conference, B. Bouchon-Meunie et al. (eds), pp. 114–120. Éditions EDK, Paris, 2006.

[4] P. Hitzler, A. K. Seda, Generalized distance functions in the theory of computation, Comput. J. 53 (2010), 443–464.

[5] P. Hitzler, A. K. Seda, Mathematical Aspects of Logic Programming Semantics, Boca Raton, CRC Press, 2011.

[6] S. Massanet, O. Valero, A general approach to midpoint theory and aggregation of quasi-metrics, Int. J. Intell. Syst. 29 (2014), 279–306.

[7] J. J. Nieto, A. Torres, Midpoints for fuzzy sets and their application in medicine, Artif. Intell. Med. 27 (2003), 81–101.

[8] A. Pradera, E. Trillas, E. Castiñeira, On distances aggregation, Proc. Information Processing and Management of Uncertainty in Knowledge-based Systems international Conference, B. Bouchon-Meunie et al. (eds), pp. 693–700. Univ. Politecnica Madrid Press, Madrid,2000.

[9] A. Pradera, E. Trillas, E. Castiñeira, On the aggregation of some classes of fuzzy relations, Technologies for constructing intelligent systems, B. Bouchon-Meunier et al. (eds), pp. 125–147. Springer-Verlag, Berlin, 2002.

[10] S. Romaguera, O. Valero, Asymptotic complexity analysis and denotational semantics for recursive programs baed on complexity analysis, Semantic-Adavances in Theories and Mathematical Models, Edited by M. Tanvir Afzal, Volume 1, pp. 99–120, InTech Open Science, Rijeka (2012).

[11] M. Schellekens, The Smyth completion: a common foundation for denotational semantics and complexity analysis, Electron. Notes Theor. Comput. Sci. 1 (1995), 211–232.

[12] A. Stojmirović, Quasi-metric spaces with measure, Topology Proc. 28 (2004), 655–671.

[13] A. Stojmirović, Quasi-metrics, similarities and searches: aspects of geometry of protein datasets, Ph. D. Th., Victoria University of Wellington, Wellington, 2005.

[14] A. Stojmirović, Y.-K. Yu, Geometric aspects of biological sequence comparison, J. Comput. Biol. 16 (2009), 579–610.

[15] P. Tirado, O. Valero, The average running time of computing as a midpoint between fuzzy sets, Math. Comput. Modell. 49 (2009), 1852–1868.