“Numerical modelling of acoustic loads generated by the propulsion system of space vehicles”
To whom is Love
Acknowledgements

Overall, I thank God, the giver of life, who continues providing me with everything that is convenient to me.
Special thanks to my family whose Love fuels me to go through the challenges of life.
Thanks to the project directors the invitation me to participate in this project. Their time, patience, dedicated guidance and advice were invaluable.
Thanks to all the Master´s professors for sharing part of their knowledge.
I am grateful to my classmates for their help and friendship.
Thanks to the authors of previous research in this field, whose work provided me with important understanding and guidance to successfully complete this work.
Thanks to my new friends in Gandía for their selfless help.
Abstract

At the time of lift-off of a space vehicle, severe acoustic loads are generated and can cause damage to the payload, the rocket electronics and the launcher itself. In 1957 when Sputnik 1, the first artificial satellite, was launched very little was known about rocket launch acoustics. However, soon became clear that important savings could be achieved with even a small reduction in the rocket lift-off noise level. The study of rocket noise has experienced great progress during the last decades benefiting from advancements on acoustics and computing technologies.

The European Space Agency has taken important efforts to mitigate the impact of acoustic loads on their VEGA programme rockets. This work contributes to this purpose as it aims to provide preliminary test results on noise source localization, which will be used for the process of developing tools to determine the noise sources present during the VEGA rocket lift-off. A concept 3D CAD model, beamforming numerical tools and an FDTD model were created under the scope of this work.

Keywords
VEGA Programme, rocket lift-off noise, rocket jet turbulence, microphone array, beamforming, 3D CAD model, FDTD model.
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<td>Cross-spectral matrix</td>
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<td>Delay-and-Sum</td>
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<td>Direction of arrival</td>
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<td>ESA</td>
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<td>Finite Differences in the Time Domain</td>
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<td>Perfectly Matched Layer</td>
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<td>Technology Research Programme</td>
</tr>
<tr>
<td>UCA</td>
<td>Uniform Circular Array</td>
</tr>
<tr>
<td>VEGA</td>
<td>Vettore Europeo di Generazione Avanzata</td>
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</table>
Glossary

Nomenclature
M = Total number of microphones in the antenna
G = Cross-spectral matrix
R = Microphone antenna (array) radius
f = Frequency (Hz)
f_s = Sampling frequency of measurement signals
d = Aperture of the microphone antenna
⟨ ⟩ = Expected value
c = Speed of sound in air
r = Distance from a microphone to a source point (interrogation point)
t = Time from engine ignition
λ = Wavelength
θ = Angle
k = Acoustic wave number
E = Error function
i = Imaginary unit (√−1)

Superscripts
† = Transpose complex conjugate or Hermitian transpose
* = Complex conjugate
T = Transpose

Subscripts
c = Centre of microphone antenna
j = Interrogation point index
m = Microphone index
R = Rayleigh resolution
1. Introduction

1.1 Impact of noise on space vehicles during lift-off

During the lift-off a space vehicle is subjected to severe acoustic loads and overpressure [1, 2, 3, 4, 5]. These broadband loads are generated during the engine ignition phase, by the mixing of the rocket stream turbulence with the surrounding atmosphere. The high acoustic loads cause vibration that can impact the payload, vehicle structure, propellant storage, electronics and navigational components.

Even a small reduction of the acoustic load level translates not only into considerable savings on the vehicle weight and the operation but into an increased safety of the payload carried by the vehicle.

Some of the potential problems caused by acoustics overloads are:

• Failures of the mechanical and electronic components of the vehicle.

• Failure or lost of calibration of mechanical and electronic components of the payload

• Fatigue failure of hardware supports and internal components. For instance: equipment mounting brackets, piping systems and cabling holding hardware.

• Fatigue failure of vehicle or launcher structural elements

• Unpropitious conditions for spacecraft occupants

Hence, considerations to mitigate the acoustic loads by means of different techniques, are an important aspect of the launch platform design.

Figure 1. Image of a rocket lift-off. taken from https://negativespace.co.
Figure 1 shows a picture taken during a SpaceX Falcon Heavy rocket lift-off. On this image the amount of turbulence around the rocket can be appreciated. Since the turbulence mixing with the surroundings is the origin of heavy noise, it can be understood how much impacted the rocket could be by the high acoustic loads during launch.

1.2 Efforts to mitigate the impact of noise on the rocket

Over the last decades extensive work has been done by researchers and engineers who are looking for ways of mitigating the impact of noise a rocket during lift-off. Different models have been developed as part of these efforts. Some theoretical models are based the theory that is commonly considered the foundation of modern aeroacoustics which was established by M. J. Lighthill through his papers from 1952 and 1954 [6, 7]. Those documents describe a precise theory regarding the generation of sound by a turbulence within a larger fluid volume.

On the other side, the starting point of empirical models [3] was marked by K. M. Eldred from NASA in 1971. However, a problem with this methodology is that the sound power model is based on a simple acoustic efficiency factor, which is not consistent with Lighthill’s well established jet noise theory. Additionally, the empirical model does not consider the sound reflections on the platform structure. Nevertheless, multiple authors have successively modified, combined with numerical models to improve this model and continue using it obtaining results that better fit the experimental data.

Eldred’s monograph indicates that, during a space vehicle lift-off, although the noise is radiated in all directions, the magnitude of the acoustic field is highly directional, being found 50º from the jet flow direction to be the angle of maximum radiation level for a standard chemical rocket [1]. This means, the acoustic loads on the vehicle are higher during the initial lift-off seconds since, at that time, the jet flow is perpendicular to the vehicle’s axis. Figure 1 shows a conceptual representation of the direction of maximum radiation.

![Figure 2. Concept Sketch of rocket jet flow and approximate equal SPL level contours during lift-off](image)
The monograph also describes in detail the empirical model allocating the noise sources along the jet stream. Some sources are distributed and other are compact. The sources are assumed to be completely uncorrelated, so the results for each of them can be added.

The concepts presented on Eldred’s document have been considered relevant by many researchers and engineers, who have subsequently taken it as a reference for their works in this field.

As part of the efforts to mitigate the impact of the acoustic load on the rocket during lift-off, multiple means have been proposed and utilized including [1, 8, 9] the geometry of deflectors, the geometry of launch pad, use of a tunnel for the flow to travel, the injection of water near the nozzle, the use of sound attenuators (e.g. sound crystals) and the use of sound absorptive padding on sensitive areas, among others. For a specific rocket launch platform, decisions need to be taken on which technique or combination of techniques would be appropriate to reduce the impact of noise.

1.3 Noise source determination

In order to determine which techniques would be appropriate for a specific rocket and launch pad, it is important to first determine the location and characteristics of the noise sources present during the rocket lift-off. Once the noise sources have been identified, educated decisions can be taken to implement the appropriate techniques to reduce the noise impact.

With the information regarding noise sources, simulation models can be implemented to evaluate the benefits of different techniques and configurations before they are applied.

Hence, determining the noise sources, is an important step on the path to implementing a satisfactory solution to mitigate the impact of noise on the rocket during launch.

The process to determine the noise sources have benefited from the technological advancements of the last decades. In particular, the progress on computational tools (both hardware and software) and the development of microphone arrays and beamforming algorithms have greatly benefited the studies on rocket launch acoustics.

1.4 Microphone Arrays

Probably the first known acoustic array was implemented by Sergeant Jean Perrin [10], from the French military, during World War I to detect enemy aircraft. It consisted of two sensors, with 6 sub-sensors each, mounted on a hexagonal shape.

Later in the 1930’s with the invention of directional microphones, some of the fundamental principles of microphone array processing were published [11]. However, the formal term of “microphone array” was used for the first time in the 1960’s, and their beginnings were influenced by radar and sonar sensor arrays.

Later, in 1974, John Billingsley developed the first system based on a microphone array. In 1976 John Billingsley and Roger Kinns implemented and array of fourteen ¼” condenser microphones for localization of noise source on full-size jet engines in real time [12].
Probably the first book containing a formal presentation of the theory and fundamentals of sensor arrays was “Array Signal Processing: Concepts and Techniques” by Don H. Johnson and Dan E. Dudgeon published in 1993 [13].

Since that time a lot of research and engineering efforts have been invested in the field of microphones arrays which are now found in multiple applications.

The microphone array is a sound acquisition system sing multiple microphones that spatiotemporally measure the field of the evaluated signal [13]. These measurements are processed and merged by the processing algorithm to accomplish some or all these goals:

- Augment SNR above that of a single sensor’s output
- Describe the field by determining the number of sources, their locations and the waveforms they are emitting.
- Track the sources as they may move in space.

The microphone array is organised into a specific geometry in which each Microphone’s position relative to a reference point is known to the subsequent processing system. The array’s geometry plays an important role on the performance of the processing algorithm and depends deeply on the application characteristics.

As in this work a circular microphone array was utilized to record the acoustic pressure during the VEGA rocket launch, we will dedicate the following paragraphs to describe the basic concepts of this type of microphone arrays.

Circular microphone arrays have advantages in applications requiring similar or equal response to signals coming from one side or the other, like those where digital steering is necessary. Also, its symmetry property substantially simplifies the processing method.

A conceptual representation of a circular microphone array is shown in figure 2 [11].

![Figure 3. Representation of a uniform circular microphone array (UCA)](image)

Figure 3. Representation of a uniform circular microphone array (UCA)
The array shown on figure 2 is centred to the coordinates’ origin, it consists of $M$ microphones, has radius $r$ and the $m^{th}$ microphone is located at an angular position $\psi_m$, where:

$$\psi_m = \frac{2\pi(m - 1)}{M}$$

When the array is in the farfield, the time delay between the $m^{th}$ microphone and the array centre is:

$$\tau_m = \frac{r}{c} \cos(\theta - \psi_m), \ m = 1, 2, 3 ... M$$

With $c$ being the speed of sound in air.

And the interelement separation $\delta$ in the UCA is:

$$\delta \approx \frac{2\pi r}{M}$$

1.5 Beamforming

Beamforming is one of the simplest and most robust means of spatial filtering, it is the name given to a large variety of array processing algorithms that focus the array’s signal-capturing capabilities to a specific direction. Beamforming basically provides a directional map of contributions at the array position [13, 14, 15].

For the creation of the beamforming map of the sources, the interrogation region, which is the region where sources are expected to be located, is divided into a set of point as a grid, those points are called the interrogation points [2].

For a planar microphone phased array, the steering vector is a column vector with its elements defined by:

$$W_{jm} = \frac{r_{jm}}{r_{jc}} e^{-jk\tau_{jm}}$$

Where $j$ is the interrogation point index, $m$ is the microphone index, $r_{jm}$ is the distance from each interrogation point to a microphone, $r_{jc}$ is the distance from each interrogation point to the array centre, $k=2\pi f/c$ is the acoustic wave number.

This equation is valid if the source is equivalent to a monopole, which is a basic assumption of beamforming that has been under scrutiny since the beginnings of microphone arrays. It has been justified by the small changes shown on the beamforming map when other source types are considered.

In the case of rocket noise during lift-off, the types of source poles are not known beforehand and, as mentioned before, some sources are distributed and other are compact. Hence, to minimize the chances to introduce a considerable error, the sources are assumed to correspond to a collection of numerous monopoles.
1.6 Beamforming frequency validity range

The frequency validity range for the results of the beamforming process depends on the geometrical conditions and dimensions of the array of microphones: The highest validity frequency is dependant on the interelement separation, based on the criteria of avoiding spatial aliasing. On the other hand, the minimum frequency of validity for the UCA is related to the aperture (diameter) of the array, based on the application of the Rayleigh criterion as a reference [6], initially developed for optics but applied in acoustics [16].

In first place, with purpose of avoiding spatial aliasing, the interelement spacing should be less than $\lambda/2$, where $\lambda=c/f$ is the wavelength [11]. This means, for a UCA:

$$M > \frac{4nf}{c} \quad (1)$$

Where, $f$ is the maximum validity frequency for a given microphone array.

Regarding the lower end of frequencies, evaluating the Rayleigh criterion combined with the distance from microphone array to the interrogation plane, provides an indicator of the minimum separation between interrogation points required for the noise sources to be resolvable.

$$\sin \theta_R = 1.22 \frac{\lambda}{d} \quad (2)$$

Where, $d$ is the diameter of the circular aperture, $\theta_R$ is the angular separation in radians and $\lambda$ is the wavelength corresponding to the minimum intended frequency.

Even though several authors have claimed higher resolution using different processing methods and others have reported acceptable results using interrogation point separations well below the Rayleigh criterion, it is still considered a guide [2, 3].

1.7 Aim of this work

The European Space Agency (ESA) created the VEGA programme with the purpose of getting the continent a share on the market of small satellite launch [21]. The initial stages of the programme took place in Italy in 1988 and was later adopted by ESA in June 1998 to complement the Ariane family with a small launch vehicle. The maiden VEGA voyage took place on 13 February 2012 at the Europe’s Spaceport in Kouru, French Guiana, South America [22].

VEGA stands for Vettore Europeo di Generazione Avanzata, which could translate as European carrier rocket of advanced generation.

ESA has taken important efforts to mitigate the acoustic loads generated during the VEGA rockets lift-off. For this purpose, was created the Technology Research Programme (TRP) “ESA AO/1-9479/18/NL/LvH. Launch Sound Level Reduction”

This work aims to execute preliminary tests on localization of sound sources utilizing beamforming techniques and time domain simulations. The results of this work are expected to become a contribution to the TRP’s objectives.
2. Methodology

The tools developed under the scope of work of this project will be described in this section. The tools include a 3DCAD model, beamforming program codes and an FDTD model. A brief description of these tools and some specific information on their configuration will be provided.

The preliminary tests of this work included the generation of time-domain acoustic recordings using the FDTD model, those controlled recordings where then introduced into the beamforming codes to verify correctness of the process and the confirm the conditions and results of acoustic source identification.

2.1 Creation of 3D CAD model

As a first step, and in order to provide a better understanding of the launch platform configuration including the locations of the microphone array (antenna), a 3D CAD model was created.

The model was developed using Autodesk AutoCAD 2019 software. Some dimensional information of the launch platform and a circular microphone array (or antenna) including its dimensions and orientation, as well as dimensional data published by ESA on their website [22] were the source of information to construct the 3D model.

2.1.1 Location of microphone array and evaluation plane on 3D model

The microphone antenna location is fixed, while the evaluation plane can be located anywhere within the model where the noise source estimation needs to be executed.

Figures 4 and 5 show views of the 3D CAD model, figure 6 shows a concept front view of the circular microphone array used on this work and figure 7 shows an example of locating the evaluation plane in the model.

With the purpose of recording acoustic pressure measurements during the rocket lift-off, a 32-microphone uniform circular array (UCA) was installed on one of the lightning masts of the launch platform. The UCA was aimed towards the exit of the exhaust tunnel, which is the point where the rocket jet starts mixing with the surrounding atmosphere, creating the noise sources that need to be studied.
Figure 4. North-west view of 3D CAD concept model of VEGA launch platform

Figure 5. South-west view of 3D CAD concept model of VEGA launch platform
Figure 6. Concept model of microphone array (antenna) used for rocket noise recording

Figure 7. Example of evaluation plane added to 3D CAD model
Using the 3D model provides the flexibility of easily relocating the evaluation plane to a position and orientation as needed, with the advantage of visualizing the area to be evaluated in reference to other elements of the rocket launch platform.

2.2 Coordinates extraction

Other benefit of using the 3D CAD model is the possibility of obtaining the interrogation points coordinates directly from the model once the evaluation plane is located at the desired position.

Using the function ‘Extract Data’ from AutoCAD, the coordinates of the selected objects can be exported as a table in a format readable to other programs like Microsoft Excel. This data extraction process from the 3D model was utilized in this work to obtain the cartesian coordinates of the microphone array and the evaluation plane for further processing. In this work the cartesian origin was located at north-west corner of the base of the lightning mast where the microphone antenna is mounted. The microphone coordinates extracted from the 3D model are shown on table 1.
Table 1. Coordinates of the microphone on the antenna (in mm)

<table>
<thead>
<tr>
<th>Center X</th>
<th>Center Y</th>
<th>Center Z</th>
<th>MIC ID</th>
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</table>

As acoustic recordings from some microphones were not available, table 1 only contains coordinate information of the microphones from which recordings were available and the centre of the array.

2.3 Calculation of frequency range of validity of the microphone array

Utilizing the equation (1) described in section 1.6, the frequency range of validity was calculated for the microphone antenna used to capture the noise.

From equation (1), with \( M=32 \) microphones and \( r=1m \), the spatial aliasing limit, which determines the highest frequency, provided a result of:

\[
 f_{\text{max}} = 873 \, \text{Hz} \quad \quad (3)
\]
This result indicates the installed microphone array would not provide reliable measurements above this frequency. The frequency evaluation range on this work was limited considering this parameter.

2.4 Steering tensor calculation

Once the microphone and interrogation points coordinates are available and the frequency operation range is determined, the steering tensor can be calculated. This tensor will contain the steering vector each microphone and for each frequency to be evaluated. This means it is a three-dimensional tensor.

Using the formula mentioned in section 1.5

\[ W_{j,m} = \frac{r_{jm}}{r_{jc}} e^{-jk_{1}r_{jm}} \]  

(4)

Where \( j \) is the interrogation point index, \( m \) is the microphone index, \( r_{jm} \) is the distance from each interrogation point to a microphone, \( r_{jc} \) is the distance from each interrogation point to the array centre, \( k=2\pi f/c \) is the acoustic wave number.

A frequency vector was obtained to cover the evaluation range of interest, the frequencies were used to generate the vector of wave numbers which generated the third dimension of the tensor. As a result, the dimensions of the steering tensor are \( M \times J \times NF \), where \( M \) is the number of microphones, \( J \) is the number of interrogation points and \( NF \) the number of evaluation frequencies.

A computing code for the MATLAB platform was created to calculate the steering vectors, compose steering matrices and build with them the steering tensor \( W_{j,m}(f) \).

2.5 Cross-Spectral tensor (CST) calculation

The most important part of the process is the calculation of the cross-spectral tensor, as it generates the spectral data to be used for source. This element is tensor to MATLAB containing a group of frequency-dependant cross-spectral matrices.

For this step the acoustic recordings from the VEGA rocket launch were used. These were loaded from a spreadsheet into a MATLAB code to be processed. There were in total more the 85,000 signal samples for each microphone.

The elements of a the cross-spectral matrix are calculated as [2]:

\[ G_{m,m'} = \langle P^*_m P_{m'} \rangle \]  

(5)

Where \( P_m \) corresponds to the Fourier transform of the \( m^{th} \) microphone signal. When a specific portion of the recordings was processed, the selected segment was windowed using a Hamming window.
As the elements of the diagonal of the matrices contain the auto-spectra which is characterized for carrying spurious and noisy content, from external, electronic and environmental sources, their values were zeroed to reduce their negative influence on the calculations.

The dimensions of the cross-spectral tensor where then $M \times M \times NF$, where $N$ is the number of microphones included in the calculations, 25 in our case, and $NF$ the number of evaluation frequencies, that is, the length of the frequency vector.

### 2.6 Conventional beamforming map generation

Once the CST is calculated, it can be utilized to apply almost any of the beamforming algorithms.

In this work the initial step was to implement the conventional beamforming (CB) also known as delay-and-sum (DaS) beamforming.

The CB method has two assumptions: The complex acoustic sources can be represented by the addition of multiple monopoles and, the propagation of sound is totally linear [2].

To obtain the monopole noise source levels, corresponding to each interrogation point, the first step of the CB algorithm is to calculate the Hermitian conjugate or conjugate transpose of each matrix within the steering tensor.

With this operation a new tensor called $H$ is obtained. The dimensions of this tensor are $J \times M \times NF$, where $J$ is the number of interrogation points, $M$ is the number of microphones and $NF$ the number of evaluation frequencies.

The next step consists of multiplying each matrix in tensor $H$, with the matrix from the CST corresponding to each evaluation frequency. From this product $H \times CST$, a new tensor called $Q$ results. The dimensions of this new tensor are $J \times M \times NF$.

The following step consists in multiplying tensor $Q$ with the steering tensor. For this each matrix from tensor $Q$ is multiplied with the matrix from the steering tensor that corresponds to the same evaluation frequency. The result is a new tensor called $B$, with dimensions $J \times J \times NF$, where $J$ is the number of interrogation points and $NF$ the number of evaluation frequencies.

The diagonal values are extracted from tensor $B$, as they corresponding to the acoustic strength identified at each interrogation point.

The results are presented on the interrogation plane as a 2D plot and can also be integrated to the 3D model, as the examples shown on figures 8 and 9.
2.7 FDTD model implementation

To generate in a controlled manner the signals to be used on the preliminary tests for acoustic source localization, an FDTD model was constructed.
The FDTD model was a 2D tool with a configurable 5m x 3m map, surrounded by a perfectly matched layer (PML) to minimize reflections, with a 0.25m thickness by default but adjustable if required to obtain appropriate results.

Figure 10 shows the FDTD model map with a specific configuration. The dashed red line around indicates the border with the surrounding PML. The locations of sources and microphones are indicated with white dots.

The sources on the FDTD model emit a Ricker wavelet which travels omnidirectionally across the model map. The dominant frequency of the wavelet can be modified to provide flexibility to the tests.

The sources and microphones could be changed in quantities and locations as needed throughout the test execution.

The simulation time was set to 15ms as this time allowed the signal emitted to travel through the entire model map.

The signal sensed by microphones were recorded as registered signals to be processed by the beamforming codes. Figure 11 shows an example of signals registered by a configuration of 7 microphones. Those graphics were useful for immediate verification of signal integrity.
2.8 Test Procedure

Once the beamforming code and the FDTD model were complete, a procedure for testing these tools was conceived.

The procedure was very straightforward and consisted of the following steps:
- configuring the FDTD model on a desired manner with sources and microphones as required.
- Run the model code to get the emitted signal traverse the map for 15ms
- Obtain the registered signals plot to verify integrity
- Generate an evaluation line including the known location of the sources
- Fed the beamforming codes with microphone coordinates, microphone registered signals and evaluation line coordinates.
- Execute the beamforming codes
- Verify signals delivered by each step of the processing to confirm correctness (i.e. Spectrum, FFT results, cross-spectra)
- Obtain signal strength curves at the wavelet’s dominant frequency along the evaluation line. The maximum of the curve should coincide with the location of the source(s) on the y axis, indicating a sound source was detected at that position.

Running 10 cases was considered enough to complete the test. The parameters used on each case are shown on table 2.
<table>
<thead>
<tr>
<th>Case ID</th>
<th>Number of microphones</th>
<th>Source(s) qty and location(s)</th>
<th>Ricker’s dominant frequency</th>
<th>Number of interrogation points</th>
<th>Number of evaluation frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1 - Top</td>
<td>2 kHz</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1 – Bottom</td>
<td>2 kHz</td>
<td>7</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>1 – Bottom</td>
<td>2 kHz</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>1 – Bottom</td>
<td>2 kHz</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>1 – Bottom</td>
<td>2 kHz</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>1 – Bottom</td>
<td>2 kHz</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>1 – Bottom</td>
<td>2 kHz</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>17</td>
<td>1 – Bottom</td>
<td>1 kHz</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>1 – Bottom</td>
<td>1 kHz</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 – Centre</td>
<td>1 kHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 - Top</td>
<td>1 kHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>17</td>
<td>1 – Bottom</td>
<td>1 kHz</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 – Centre</td>
<td>2 kHz</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 - Top</td>
<td>3 kHz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Test procedure cases

The parameters modified on each case with respect to the previous are shown in **bold**.

The objective for running all these cases was to test the robustness of the beamforming programs. By progressively changing each of the parameters on the FDTD model execution, the ability of the beamforming codes to locate a noise source under different scenarios was tested. This series of test cases provided information on strengths and weaknesses of the beamforming codes, for orientation of future work to improve them.
3 Results

In this section will be presented the results of this work with the execution of test cases previously described. Each test case corresponded to a specific configuration of the FDTD model which generated information to be entered to the beamforming codes to try their robustness by gradually changing different configuration parameters.

The results of the test should indicate to what parameter changes are more sensitive the beamforming codes. This information is useful to identify any improvements that might be required.

3.1 Case 1

This was the starting point of the test procedure. The configuration used is shown on table 3

<table>
<thead>
<tr>
<th>Number of microphones</th>
<th>Source(s) qty and location(s)</th>
<th>Ricker’s dominant frequency</th>
<th>Number of interrogation points</th>
<th>Number of evaluation frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1 - Top</td>
<td>2 kHz</td>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 3. Test procedure case 1 configuration

Initially were verified the conditions of results along the signal processing. Figure 12 shows the signal as registered by the first microphone in the beamforming code.

Figure 12. Evaluation case 1. Signal registered by microphone 1
Another important partial result to verify was the spectrum obtain for the signals. Figure 13 show an example of the plots obtained.

![Microphone 2 spectrum](image1)

*Figure 13. Evaluation Case 1. Microphone 2 spectrum.*

The next partial result to be verified was the FFT results, as the example shown on figure 14.

![FFT values for mics 1 and 2 within band of interest](image2)

*Figure 14. Evaluation Case 1. FFT for microphones 1 and 2 signals.*
The last partial result to be confirmed was the cross-spectra, an example is shown on figure 15.

![Cross Spectra Graph](image)

Figure 15. Evaluation Case 1. Cross spectra: Microphones 1,1 and 1,2

To this point all the partial results were acceptable and provided initial confidence on the signal processing method of the beamforming codes.

As the actual purpose of this work and the codes was the localization of sound sources, the most relevant results are the curves of the signal strength level along the evaluation line. The maximum strength level indicates the interrogation point where the sound sources have been identified. For case 1, the curve is shown on figure 16. The horizontal axis corresponds to the interrogation line located along the y axis of the FDTD model. The vertical axis corresponds to the signal strength level.
Figure 16: Evaluation Case 1. Levels detected near 2kHz along evaluation line.

The blue dot indicates the source location

Figure 16 shows an acceptable performance. Even though the source and the maximum do not coincide exactly, they are only one interrogation point apart, which makes this a good result specially considering this configuration was the staring point of the test with the smaller number of microphones and interrogation points.
3.2 Case 2

Parameter change from previous case: Source moved lower on FDTD map

The first parameter change was to move the source lower on the FDTD map, to evaluate how symmetrical is the source identification on the beamforming tools. The result is shown on figure 17.

![Conv Beamforming. frequency= 2178.9474Hz, time 0 to 0.01425s](image)

*Figure 17. Evaluation Case 2. Levels detected near 2kHz along evaluation line.*

The blue dot indicates the source location.

In this case can be observed how the curved shifted to the left following the source location. Additionally, in this case the maximum and the source location coincide perfectly. This was a good indicator of symmetrical response and accuracy for the beamforming tools. The results obtained was good.
3.3 Case 3

Parameter change from previous case: Increment interrogation points to 15

For this case the number of interrogation points was incremented. The expectation was a higher resolution providing a better representation of the coincidence of actual source location and the detected position.

![Conv Beamforming. frequency= 2178.9474Hz, time 0 to 0.01425s](image)

Figure 18. Evaluation Case 3. Levels detected near 2kHz along evaluation line.

The blue dot indicates the source location

Having more interrogation points curve 18 provided a more precise confirmation of the coincidence of the detected source and the actual location of source on the simulation map. This was a good result.
3.4 Case 4

Parameter change from previous case: Increment interrogation points to 20

Figure 19. Evaluation Case 4. Levels detected near 2kHz along evaluation line.

The blue dot indicates the source location

With an even larger number of interrogation points, the higher resolution confirms the exact coincidence of the maximum amplitude and the source location is confirmed. This is a good indicator of the precise result obtained with the beamforming tools to this point.
3.5 Case 5

Parameter change from previous case: Increase number of microphones to 12

Figure 20. Evaluation Case 5. Levels detected near 2kHz along evaluation line.

The blue dot indicates the source location

In this case the curve appeared slightly shifted to the right. Although incrementing the number of microphones is expected to provide more accurate localization. However, this is still considered a good result as the maximum and the source location are only one interrogation point apart.
3.6 Case 6

Parameter change from previous case: Increase number of microphones to 17

![Graph](image)

*Figure 21. Evaluation Case 6. Levels detected near 2kHz along evaluation line.*

The blue dot indicates the source location.

In comparison to the previous case, the curved appeared shifted one interrogation point to the right, making the separation from maximum to source location equal to 2 interrogation points. Still a good result but called the attention the fact that increasing the number of microphones did not represent higher accuracy. The reason could be related to the fact that the microphones were added to the array mostly at the top, that means distant from the source which was placed lower on the simulation map.
3.7 Case 7

Parameter change from previous case: Increase number of evaluation frequencies to 100

*Figure 22. Evaluation Case 7. Levels detected near 2kHz along evaluation line.*

The blue dot indicates the source location

Incrementing the number of evaluation sources brings higher resolution to the results allowing evaluating the performance at a frequency closer to the dominant frequency of the Ricker wavelet of the source. In this case the evaluation frequency was closer to 2kHz, but the results were essentially equal to case 6.
3.8 Case 8

Parameter change from previous case: Change the dominant frequency of the Ricker wavelet to 1 kHz

The blue dot indicates the source location

Changing the wavelet’s dominant frequency had the intention of evaluating the ability of the beamformer to work on a different part of the spectrum. As shown on figure 23, the maximum and the source location are very close to each other, which made this a good result and gave points in favour of the robustness of the beamformer.
3.9 Case 9

Parameter change from previous case: Increase the number of sources to 3

![Figure 24. Evaluation Case 9. Levels detected near 980Hz along evaluation line.](image)

The blue dots indicate the sources locations.

To test the ability of the beamforming tools to locate multiple sources was implemented case 9. The results are not acceptable which makes this an aspect to improve on the beamformer.
3.9.1 Variant 9A

A variant of case 9 was tested to check if separating the sources could bring any changes to the results. In this case 45 interrogation points were necessary to cover the locations of sources.

However, no improvement was observed on the results by separating the sources as shown on figure 25.

![Conv Beamforming. frequency= 977.7778Hz, time 0 to 0.01425s](image)

*Figure 25. Evaluation Variant 9A. Levels detected near 980Hz along evaluation line.*

The blue dots indicate the sources locations

This result provides more elements regarding the poor ability of the beamformer to identify multiple sources.
3.10 Case 10

Parameter change from previous case: Assign different dominant frequency to the Ricker wavelet of each source: 1kHz, 2kHz and 3kHz. 

The last case consisted in getting a different dominant frequency to each source to test the multi-source configuration and check how the deficiency noted on the previous case was related to the operation frequency.

Unfortunately, the source localization results were not satisfactory in this case either as shown on figure 26, 27 and 28.

![Conv Beamforming. frequency= 977.7778Hz, time 0 to 0.01425s](image)

*Figure 26. Evaluation Case 10. Levels detected near 980Hz along evaluation line.*

The blue dot indicates the source location

Figure 26 shows deficient identification at 1kHz in the multi-source scenario. This is information to be considered on the process of improving the beamformer.

Low levels were observed at the dominant frequencies as shown on figures 63, 64 and 65. These results do not correspond to the expectations.
Figure 27. Evaluation Case 10. Levels detected near 2kHz along evaluation line.

The blue dot indicates the source location

The case at 2kHz is not better. The source location is not acceptable and the beamformer requires improvement in this area.

Figure 28. Evaluation Case 10. Levels detected near 3kHz along evaluation line.

The blue dot indicates the source location

The low performance on a multi-source scenario is confirmed at 3kHz by figure 28. This is important information that indicates where the beamformer performance requires enhancement.
In general, for the test cases using a single source, good results at localizing it on the evaluation line were obtained.

It was noted how the increment on the number of microphones could brought benefits to the accuracy of the source identification, however, the area where the microphones where added with respect to the source location may not have favoured the work, since the microphones were added to the most distant end of the array.

It was also noted that a higher number of interrogation points does not help directly the accuracy of the signal processing but may allow a better presentation of results for its higher resolution, in that sense it brings a benefit. Though, it must be considered that a larger number of interrogation points also means a higher computational cost.

On the other hand, the results obtained in the cases where three sources were used, did not satisfy the expectations. These results are also useful, for they have revealed aspects of the beamforming codes needing improvement.
4 Conclusions

As the objective of this work was to develop tools that could contribute to the greater project of noise source identification during the VEGA rocket launch and to do preliminary test of those tools, and considering the process executed and the results obtained, it can be concluded that the beamforming tools created under the scope of this work are useful and the objective is considered achieved.

In general, the test results were acceptable showing robustness of the tools in several aspects. However, further research and test are required to enable the beamformer to manage a multi-source scenario as required by applications.

The use of time domain simulations was beneficial to ensure the tests were done under controlled conditions, and the parameters to be evaluated could be adjusted as demanded for the tests. Using other simulation methods could be explored for further testing of the beamforming tools and/or for other stages of the project.

Additionally, the use of a 3D CAD model brings important benefit to the process of identifying noise sources and this is another contribution of this work to the greater project to mitigate the noise impact during the VEGA rocket lift-off.
References


[21] https://www.esa.int/