Optimization of the Multi-Start Strategy of a Direct-Search Algorithm for the Calibration of Rainfall–Runoff Models for Water-Resource Assessment

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Abstract: Calibration of conceptual rainfall–runoff models (CRRM) for water-resource assessment (WRA) is a complicated task that contributes to the reliability of results obtained from catchments. In recent decades, the application of automatic calibration techniques has been frequently used because of the increasing complexity of models and the considerable time savings gained at this phase. In this work, the traditional Rosenbrock (RNB) algorithm is combined with a random sampling method and the Latin hypercube (LH) to optimize a multi-start strategy and test the efficiency in the calibration of CRRMs. Three models (the French rural-engineering-with-four-daily-parameters (GR4J) model, the Swedish Hydrological Office Water-balance Department (HBV) model and the Sacramento Soil Moisture Accounting (SAC-SMA) model) are selected for WRA at nine headwaters in Spain in zones prone to long and severe droughts. To assess the results, the University of Arizona’s shuffled complex evolution (SCE-UA) algorithm was selected as a benchmark, because, until now, it has been one of the most robust techniques used to solve calibration problems with rainfall–runoff models. This comparison shows that the traditional algorithm can find optimal solutions at least as good as the SCE-UA algorithm. In fact, with the calibration of the SAC-SMA model, the results are significantly different: The RNB algorithm found better solutions than the SCE-UA for all basins. Finally, the combination created between the LH and RNB methods is detailed thoroughly, and a sensitivity analysis of its parameters is used to define the set of optimal values for its efficient performance.

Keywords: calibration; rainfall–runoff models; multi-start; Latin hypercube; Rosenbrock; water-resource assessment

1. Introduction

Hydrological modelling is essential for water-resource assessment (WRA) [1]. Hydrologists have come to rely on hydrological models to foresee events that would otherwise be difficult to predict [2]. Estimating flows that run in a basin through a drainage system is one of the greatest tools available to address hydrological problems [3].

Currently, there are many hydrological models that can be used for WRA [4]. Conceptual rainfall–runoff models (CRRM) are often used, because they offer simplified catchment-scale representations of the transformation of precipitation into river discharge [5]. Their efficient calibration is a difficult issue, even for experienced hydrologists [6]. The models are characterised by parameters that cannot
be directly measured in the field. However, they can be inferred by the so-called calibration process [7].
During this phase/process, parameters are estimated indirectly by minimizing the discrepancy between
direct simulated and observed model outputs [8]. Moreover, the success of the application of CRRM
depends on the degree of calibration achieved and the choice of suitable calibration strategies [9,10].

To estimate the parameter values, manual calibration is one option. However, it is a time-consuming
and very laborious phase. The process is sometimes still used [11–13]. However, when complex models
are used, it is practically impossible to perform a manual calibration to find the best possible parameter
set. An alternative is to use automatic calibration algorithms [14,15], which make model calibration
easier and faster. This technique automatically enhances the quality of manual calibration, providing a
significant improvement [14]. Currently, there are many automatic calibration algorithms available.
Some are modern or recent algorithms (developed in the last decade), and others are traditional or
historical algorithms (developed from 1965 until the last decade) [16,17]. However, they do not offer
the same performance levels [2].

During the last decade, several papers comparing optimization algorithms applied to CRRM
calibration were published to prove or disprove the efficiency of a particular modern or historical
method [2,16–19]. In studies by Sörensen [20] and Piotrowski et al. [17], they contended that they
did not consider such an uncontrolled increase in the number of supposedly new algorithms to be
a positive research trend. In studies by [16,20–30], it was found that, generally, modern algorithms,
can become completely unreliable on specific problems, even though they perform reasonably on
others. Furthermore, traditional algorithms, such as the Rosenbrock (RNB) or Nelder–Mead (NM),
have proven surprisingly efficient compared to modern algorithms.

The RNB algorithm [31] is a historical technique analysed in many comparisons of optimization
algorithms since its development until the present [19]. For example, with an internal modification
made for hydrological models using synthetic error-free data, the method was found to be robust
by [32], who conducted a study comparing nine different methods for fitting hydrological models.
In the study of [33] used the technique for illustrating the application of linear rainfall–runoff models
in 14 catchments. Moreover, Liang et al. [34] demonstrated the application of two linear flow-routing
methods on three rivers in China.

Also Goswami and O’Connor [18] tried the RNB algorithm and five additional different optimization
techniques in the calibration of two catchments. They found that the direct-search methods (e.g., RNB)
were efficient in terms of fewer numbers of objective functions (OF) evaluations necessary to find
the best solution, making them very fast algorithms. However, in some cases, they were trapped in
local-optimal solutions. In that study, the RNB algorithm was started sequentially from each point of a
large sample of points in the parameter space, each aiming to reach the optimum [35–37]. By launching
it from each point, the computational time consumed was excessive, and the algorithm was unable to
come out of the local optimum in some cases. The direct-search methods use only OF values. Thus, it is
not necessary to perform derivatives [31,38–40]. Therefore, it can be applied to many cases. However,
one of the main problems in these techniques is the presence of multiple local solutions that can cause
the algorithm to be unable to find the global solution of the problem [41]. In this sense, the application
of local-search methods has been tried by leveraging the re-initialization of the algorithm at some
randomly selected points [42,43], allowing a roughly homogeneous exploration of the sample space [16].

Thirty optimization heuristics algorithms were tested by Piotrowski [22] to evaluate artificially
constructed benchmark functions. Among them were two traditional local-search algorithms (i.e., RNB
and NM) [39], which were compared with results obtained from the other 28 methods. Both algorithms
were applied with a re-initialization, depending on the number of OF calls. Interestingly, both algorithms
performed as well as the modern techniques. In fact, this study suggests that testing traditional
algorithms as RNB in other fields may be interesting.

Moreover, Piotrowski et al. [17] evaluated the performance of 33 optimization algorithms
(e.g., RNB and NM). They analysed the performance versus the speed of each algorithm in the solution
of 22 numerical real-world problems from different fields of science with the maximum number of
OF calls varying between 5000 and 500,000. They found that the historical algorithms outperformed recent algorithms, even when they were based on the first ones. Moreover, the RNB algorithm was more specialized, fast and one of the best for solving hydraulic engineering problems.

Recently, Piotrowski et al. [16] compared 26 algorithms created from 1970 to the present, including a number of modern evolutionary or swarm-intelligence methods and traditional techniques such as the RNB algorithm. These methods were used on the calibration of two different CRRMs in four catchments located in roughly similar climatic conditions on two continents. It was found that, generally, nearly all algorithms performed similarly on each calibration problem, and no method could be called superior to others in terms of final performance. It was also shown that the historical direct-search methods performed equally well in finding the best solutions as modern optimizers. These methods were implemented with a re-initialization at two points selected randomly but without any previous analysis. Thus, with the RNB method, after 100 D (where D is the dimensionality of the problem) function calls are verified; if, during the last 100 D function calls, the solution is improved by more than $1 \times 10^{-4}$, then the optimal solution is found. Otherwise, the location of the RNB point is randomly re-initialized again from a random starting point. The value of the step length is re-set to 0.1 [31], and the coordinates are re-set to the initial system. This conceptualization allows the use of randomization resources to better explore the sample space and find global solutions without previous analysis of the initial points.

In summary, in the studies discussed, we found that modern and historical algorithms provided similar results in the solution of the same problem. Even RNB proved to be more efficient than some modern metaheuristics in finding optimal solutions with higher speed. The study [2] established that the best methods were those that converged to a better-quality minimum as rapidly as possible. However, the problem with the direct-search methods was that it could be trapped in local-optimal solutions if the random re-initialization missed. These methods have been known for many years [44], but currently the popularity of so-called metaheuristics is soaring in hydrology [21,45]. However, these algorithms have been analysed over the last decade, showing that, at least, some of them mimic the traditional methods, lack any true novelty, or are developed without scientific rigor [20,22,46,47].

Because of complications regarding local-search algorithms and because of the introduction of the University of Arizona’s shuffled complex evolution (SCE-UA) method by Duan et al. [9], used to locate the global optimum of a CRRM, many researchers in hydrology have employed this for parameter optimization [24,48–52]. Some studies have shown that it is a robust and efficient algorithm [41], whereas its efficiency has been evaluated by comparing it with many others [2,18,53]. In fact, this algorithm has become the method of choice to solve runoff model calibration problems [7]. Recently, this technique demonstrated that, in spite of being an algorithm developed in the 1990s, its results were efficient and reliable, playing a key role in analysing the relationship between calibration time and final performance of several algorithms that calibrate CRRMs [19,54].

Based on these studies, we have summarized several key points. There is no algorithm that solves all problems correctly and efficiently [21]. Therefore, it is important to explore the behaviour of some algorithms in the solution of any problem. Historical algorithms can behave the same or better than modern algorithms in solving specific problems and some authors have argued that it would be interesting to test these methods in different cases [22]. Finally, we identified that the RNB algorithm proposed during the 1960s used with a simple restart method was still competitive and, in some cases, better than some recent algorithms [17].

Moreover, Chlumecký et al. [6] suggested that a method allowing the exploration of the sample space or a selection of random values as starting points could improve results. Therefore, we decided to improve the use of RNB with the initial application of a random sampling method (Latin hypercube (LH)). This should enable the controlled exploration of the total solution space addressed in the problem and select only the best points after a previous OF analysis. The RNB algorithm will be re-initialized, but only as many times as necessary from the best sorting points, thereby saving considerable computational time. In previous studies in which RNB was used, algorithm re-initialization at uncontrolled random
points was conducted. This re-initialization depended on specific OF call numbers or a pre-defined number of re-sets (one or two), but not from a probability analysis of the solution space.

Therefore, the aim of this paper is to test the ability of a direct-search algorithm, RNB, combined with the LH method designing a controlled multi-start strategy for the calibration of CRRMs with different complexities in the WRA. We explain the mathematical relationship between the two methods, because this aspect has not been addressed in the literature. Moreover, a detailed sensitivity analysis of parameters of coupled techniques is made, setting optimal parameter values to solve problems of CRRM calibration. To assess the performance of this technique, the SCE-UA algorithm is used as a benchmark, because it has proven to be a robust technique in the calibration of CRRMs.

Finally, the focus in the application of this algorithm is oriented to the applicability of WRA for the management of complex systems and not for the analysis of flood events, which are regularly the cases that have been more frequently analysed [55,56].

2. Materials and Methods

2.1. Case Study and Data

The present study is based on data collected from nine headwaters of the Duero and Júcar River Basin Districts in Spain (Figure 1). These catchments are located in areas that commonly present severe droughts because of climatological conditions and geographical locations [57–59]. The catchment areas in the Júcar system are zones characterized by a semi-arid climate [60]. The Duero system contains zones characterized by a Mediterranean climate with temperate and rainy winters, but very dry and warm summers. Thus, the evaluation of water resources in these exploitation systems is an essential action.

Figure 1. Location of the case-study catchments.

Four basins in the Duero system have the presence of snow (Table 1) that accumulates from late autumn to spring, reaching a maximum depth and duration during the winter months. The snowmelt begins in the spring and summer when the precipitation is otherwise scarce. Thus, it is important to consider the modelling process [61].

The climatological, hydrological and morphological characteristics are different in each basin (Table 1). Every case-study catchment has a gauging station and a series of observed flows of at least
10 years of data without the need to complete or extend the data series. In Table 1 we assigned a number to each basin. From this point, we refer to them by this number and not by their name.

Table 1. Summary information about the case-study catchments.

<table>
<thead>
<tr>
<th>Case-Study Catchments</th>
<th>Surface (km²)</th>
<th>Average Annual P (mm)</th>
<th>Average Annual PE (mm)</th>
<th>Average Annual Flow (hm³)</th>
<th>Presence of Snow</th>
<th>Flow System</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Cab. del Duero</td>
<td>133.1</td>
<td>755.8</td>
<td>832.9</td>
<td>93.4</td>
<td>No</td>
<td>Duero</td>
</tr>
<tr>
<td>2 Rivanuesa</td>
<td>93.2</td>
<td>862.5</td>
<td>776.6</td>
<td>72.3</td>
<td>No</td>
<td>Duero</td>
</tr>
<tr>
<td>3 Abión</td>
<td>897.8</td>
<td>581.5</td>
<td>986.3</td>
<td>150.0</td>
<td>No</td>
<td>Duero</td>
</tr>
<tr>
<td>4 Cuenca</td>
<td>1005.6</td>
<td>601.9</td>
<td>1057.3</td>
<td>300.7</td>
<td>No</td>
<td>Júcar</td>
</tr>
<tr>
<td>5 Pajaroncillo</td>
<td>829.0</td>
<td>589.8</td>
<td>1031.3</td>
<td>161.3</td>
<td>No</td>
<td>Júcar</td>
</tr>
<tr>
<td>6 Barrios de Luna</td>
<td>492.1</td>
<td>946.7</td>
<td>760.8</td>
<td>447.6</td>
<td>Yes</td>
<td>Duero</td>
</tr>
<tr>
<td>7 Era</td>
<td>283.0</td>
<td>744.7</td>
<td>858.0</td>
<td>153.0</td>
<td>Yes</td>
<td>Duero</td>
</tr>
<tr>
<td>8 Omaña</td>
<td>403.6</td>
<td>793.4</td>
<td>820.9</td>
<td>96.1</td>
<td>Yes</td>
<td>Duero</td>
</tr>
<tr>
<td>9 Cab. del Tormes</td>
<td>627.4</td>
<td>813.7</td>
<td>973.5</td>
<td>688.0</td>
<td>Yes</td>
<td>Duero</td>
</tr>
</tbody>
</table>

To add the least amount of noise to the modelling process, only headwater basins were selected, because they are the zones where the runoff data have the minimal anthropogenic alteration, and the capacity of the algorithms can be tested homogeneously without noise resulting from the data.

Daily series of precipitation (P) were obtained from the Spain02 database [62] at a grid resolution of 0.2° (approx. 20 km). The State Meteorological Agency and the Meteorology Group of Santander in Spain prepared this dataset jointly. Potential evapotranspiration (PE) was calculated using the Hargreaves and Samani method [63] and the temperature series obtained from the aforementioned database. The observed daily flows were obtained from the Information System of the Discharges Yearbook of the Ministry of Agriculture and Fisheries, Food and Environment of the Government of Spain [64].

2.2. Conceptual Rainfall–Runoff Models

In this section, we describe three lumped conceptual models (i.e., the French rural-engineering-with-four-daily-parameters (GR4J) model, the Swedish Hydrological Office Water-Balance Department (HBV) model and the Sacramento Soil Moisture Accounting (SAC-SMA) model) used to test the design multi-start strategy of a direct-search algorithm during the calibration stage. The models involve a configuration of interconnected stores with mathematical transfer functions used to direct the movement of water in a basin. Each has a different structural complexity, a different number of parameters (4, 8 and 16) and a specific mathematical formulation. Their selection was based on their successful application in WRA studies [50,65–68]. The parameter ranges for each model are based on experience and the literature review [69–72]. We applied these models using the open and freely available software ‘EVALHID’ Water Resources Assessment, version 1.3, [73] developed by the Universitat Politècnica de València in Valencia, Spain.

2.2.1. French Rural-Engineering-with-Four-Daily-Parameters (GR4J) Model

The GR4J model is a lumped, conceptual, daily model developed by [72], belonging to the family of soil-moisture accounting models. It performs well even for data within short time intervals ref. [74] or practically ungauged catchments [75]. The GR4J model is represented by two tanks: production and routing. It is controlled by four parameters (description in Appendix A Table A1). For a detailed mathematical explanation see [72].

2.2.2. Swedish Hydrological Office Water-Balance Department (HBV) Model (with the Snow Component)

The HBV model [70] is a lumped, conceptual and daily model that simulates discharge in the catchment using rainfall, temperature (if applicable) and estimates of potential evaporation. It has been
used in more than 30 countries under different climatic conditions. The model simplicity of the input data, and the robust model structure have demonstrated reliable performance in solving water-resource problems [76].

The model comprises three primary routines: first is snow accumulation and melt, which is computed by a simple degree-day method, where a threshold temperature (usually close to 0°C) is defined at the location above where snow melt occurs [77]. Second is a soil moisture accounting routine that computes an index of the wetness and soil moisture storage in a catchment. Third is a response routine that transforms excess water from the soil moisture routine to discharge to each sub-basin. There are two reservoirs connected in series by a constant percolation rate. Generally, the input variables are daily P, estimated PE and mean-air temperature when the snow module is applied. The model is controlled by eight parameters for rain and two for snow (see parameter description in Appendix A Table A1). The model equations of HBV (including snow routine) can be found in other works [70,77,78].

The snow routine described here is used as a model for the four basins with this component. It uses, simultaneously, the reduction in the dimensionality of the problem.

2.2.3. Sacramento Soil Moisture Accounting (SAC-SMA) Model

The SAC-SMA model [71] is a conceptual water-balance physical model based on the principles of water movement in a catchment. It is one of the most widely used rainfall–runoff model [79]. It works by using a system of water reservoirs (zones). The basic design incorporates two soil-layer structures. Each comprises tension and free-water reservoirs that interact to generate soil moisture, and there are five runoff components. The total streamflow is the sum of all partial runoffs. The model was configured with 16 calibration parameters described in Appendix A Table A1. A detailed mathematical analysis of the model can be found in [71].

2.3. Optimization Methods

In this section, the optimization methods used to calibrate CRRMs are described. First, we explain in detail the linking of the RNB algorithm to the random sampling method, LH, to create a multi-start strategy of a direct-search algorithm. Next, we describe the SCE-UA algorithm that was selected as a benchmark method.

2.3.1. Latin Hypercube (LH) and Rosenbrock (RNB) Combined Algorithm

LH Sampling Method

The LH sampling method [80] has been recognized as one of the most efficient, effective and strategic techniques to homogeneously reduce a large sample space and to randomly generate a set of values with the same probability of occurrence and to reduce the number of simulations of the optimization algorithms and computational demand [80–82]. A ‘Latin square’ is a grid containing a set of possible solutions in different positions. However, only one sample is possible in each row and column. There cannot be rows or columns with more than one sample (Figure 2a). An LH is the generalization of this concept to an arbitrary number of dimensions. Thus, only one sample is possible in each hyperplane containing it. The sides of the hypercube are parallel to the reference axes, and the value range of the distribution of each random variable is divided into n non-overlapping segments of equal occurrence probability. Generally, the purpose is to create a convenient division of the sample space to consider all possible events proportionally to the real probability of their occurrence. LH allows for well-distributed sampling based on the problem dimension. It has been used successfully in several cases. LH was used by [81] to generate input samples for a model, improving computational efficiency of their methodology. Additionally, it was used to sample the full feasible parameter space for a multi-objective evolutionary algorithm [23] and as a generator of 10⁶ sets of parameters for the Markov chain Monte Carlo algorithm by [83]. Recently, [16] also used this technique to generate the values of the initial calibration parameters of the HBV and GR4J models using the DREAM_(ZS) algorithm [7].
obtained from the RNB launches is stored and the set of parameters corresponding to the iteration are shown as the optimal solution. RNB is launched at least once in any case, but the number of RNB launches is conditioned by an RLAUNCH (Rosenbrock’s launches) parameter from the LHR algorithm. An important advantage of this first stage is that, to solve the problem, the OF evaluation number depends on the size of the LH and not on the number of parameters.

Figure 2. Joint operation of the Rosenbrock (RNB) algorithm and the method of statistical design of experiments (Latin hypercube, LH). It shows the first RNB launch, where (a) relates the functioning of the LH and (b) indicates the RNB objective.

The RNB algorithm [31] is a local-search technique, which does not require derivatives from the OF. This algorithm represents an improvement in the axial direction method and if it starts from a non-optimal coordinate system, the method can spend a lot of time until convergence. It avoids monodimensional optimization in the directions of the coordinate axis and uses a time step for each search direction. The search is initiated by using the original coordinate system for the first iteration and the system axes that are the parameters to be optimized. Depending on the results, the increments are multiplied by the coefficients of advance or setback (i.e., algorithm parameters). If the direction is good, a success occurs, and the step size is multiplied by the advance coefficient, typically 3 [31]. However, if the search direction is wrong, a failure occurs, and the step size is reduced by half. Therefore, the sign changes. This algorithm has the peculiarity of changing the axis when failures occur in every search direction tested, and at least a positive result is found, such that the search axis is redirected
reducing the OF evaluations and efficiently reaching the optimal value. The criteria for changing search directions is usually based on successes followed by failures in all directions tested (not necessarily consecutively). During the changeover, new axes are selected to coincide with the direction between start point (or the previous axes’ optimization point change) and the last best OF result (Figure 2b). The change axes are calculated using the Gram–Schmidt process [84].

This technique was found to be robust by [32] who compared nine different methods for fitting hydrological models. In addition [33] used it for the optimization of linear techniques in modelling rainfall–runoff transformations in 14 catchments. Additionally, [34] used it for demonstrating the application of two linear-flow routing methods on three rivers in China. Moreover, [85] successfully used this method to estimate the parameters of a CRRM and to discuss aspects of the calibration process. In 2007, [18] compared the efficiency of this technique with five different algorithms and found that, in some cases, this algorithm presents some problems for finding the optimal solution. However, it is a fast technique. Also, [22] evaluated the performance of this technique to analyse sets of artificially-constructed benchmark problems. The algorithm was found to have weaknesses with very large search-space problems. Surprisingly for small sampling spaces, RNB found better results than the other 30 algorithms tested. Finally, [16,22] applying an algorithm reset, found that it could find results as good as the rest of the 26 tested algorithms. An interesting aspect of these studies was the speed of convergence of the algorithm with acceptable results.

Leveraging the antecedents of previous studies, the selected methods, Latin hypercube and Rosenbrock (LHR), were used together to ascertain their efficiency in the calibration of CRRMs. A description of this connection is described below.

Combination Methods

To use local-search algorithms for global optimization, they are applied as multi-start algorithms. Once an algorithm runs, converges and stops, another run is started at a different randomly selected initial point [14]. The LH method is used to randomly sample the solution space of each parameter to be optimised, creating sets of parameters at each point of the hypercube. These points are evaluated and ranked according to the value obtained from the OF. The best points are selected to run the RNB algorithm that carries out the parameter optimization process.

The coupled algorithm begins with the generation of a pseudo-randomized sample conditioned by the LH. The uncertainty of the search domain is reduced as the domain size decreases. However, to select the LH size it is necessary to consider the dimensionality of the problem. Each point of the initial sample is evaluated by the OF and arranged in descending order. At this point, a certain number of elements (the best ones of this first step) are selected for use as starting points to launch the RNB algorithm (Figure 2). This subroutine searches around the point, evaluating close solutions in all directions to improve the OF value. Not every point is tested in every possible direction. Rather, it is tested sequentially at each point until it is found that the axes change condition. The best value obtained from the RNB launches is stored and the set of parameters corresponding to the iteration are shown as the optimal solution. RNB is launched at least once in any case, but the number of RNB launches is conditioned by an RLAUNCH (Rosenbrock’s launches) parameter from the LHR algorithm. An important advantage of this first stage is that, to solve the problem, the OF evaluation number depends on the size of the LH and not on the number of parameters.

A detailed description of the equations and the steps of the proposed method are shown below.

1. Define the dimension of the LH ($LHDIV$), number of parameters or variables, $X_i$, the minimum value ($X_{i\min}$) and the maximum value ($X_{i\max}$) of the interval defined in each parameter.

2. Divide the sample space $F(x)$, for each $X_i$, into $n$ intervals with the same probability of occurrence to plot a grid (Figure 2a). The increase between intervals ($\Delta n$) is calculated in Equation (1).

$$\Delta n = \frac{X_{i\max} - X_{i\min}}{LHDIV}$$

(1)
where \( n \) coincides with \( LHDIV \).

3. For each parameter, \( X_i \), a vector is generated comprising the points, \( x_j \), at each intersection of the dividing lines of the grid generated by the intervals with axes, where \( j = 1, \ldots, (n + 1) \). Each \( x_j \) is associated with a random number, \( x_k [0,1] \) to form a vector with the same dimension. The vector, step 11, is organized in descending order as a function of \( x_k \) for each \( X_i \).

4. The ordered vectors, \( X_{i-ord} \), allow determining the random sampling of points in the sample space, \( F(x) \), combining the positions of \( x_j \) and \( x_k \) to select a point, \( x_{jk} \). Thus, there is only one point in each row and column in \( F(x) \) (Figure 2a).

5. The maximum number of combinations for an LH of \( n \) intervals and \( X_i \) parameters is calculated in Equation (2).

\[
\text{Number of LH} = (n!)^{X_i-1}
\]  

(2)

6. At each point, \( x_{jk} \), determined in \( F(x) \), the OF value is calculated, and the points are ranked in descending order according to the OF value obtained.

7. The best points found \( (x_{jk,n}) \) in the previous step are the input to the RNB’s algorithm. The number of input points for the algorithm is defined by the \( RLAUNCH \) parameter, which defines how many times RNB will be launched to find the optimal solution of the problem.

8. The RNB algorithm must define a starting point \( (X_{ini}) \) with coordinates \( (x_1(0), x_2(0)) \), a step for each direction \( (h_1(0), h_2(0)) \) and the number of RNB launches \( (RLAUNCH) \).

9. It starts from the first set of iterations in the axial search directions, coinciding with the coordinate axes of the \( X_{ini} \) point:

\[
\begin{bmatrix}
1 \\
0 \\
0 \\
\vdots \\
0 \\
1 \\
0 \\
\end{bmatrix} , \quad
\begin{bmatrix}
0 \\
1 \\
0 \\
\vdots \\
0 \\
0 \\
1 \\
\end{bmatrix} , \quad \ldots, \quad
\begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
\vdots \\
0 \\
1 \\
\end{bmatrix} 
\]  

(3)

10. The criterion for changing search directions is usually taken when there have been successes (at least one) followed by failures in all directions tested, not necessarily consecutively. In the change, a new axis is selected, coinciding with the direction in which the greatest success occurs. It is complemented with an orthonormal set for the other axis (Figure 2b).

11. If it starts from \( x_{ini} \), and, after a number of iterations, it is determined that a change of direction must be realized with \( x \) in the last successful point, the greatest successful direction will be determined with the vector showed in Equation (4).

\[
r_1 = \Delta x = x - x_{ini} = [\Delta x_1 \Delta x_2 \ldots \Delta x_n]^T
\]  

(4)

The remainder of the auxiliary vectors is calculated from Equations (5)–(7).

\[
r_2 = [\Delta x_1 \Delta x_2 \ldots \Delta x_{n-1} 0]^T
\]  

(5)

\[
r_n = [\Delta x_1 0 \ldots 0]^T
\]  

(6)

\[
r_i = [\Delta x_{1-n+i-1}^T\ldots 0^T_{1x i-1}]^T
\]  

(7)

12. The new calculated directions have the disadvantage of not being orthonormal. Therefore, this characteristic is achieved using the Gram–Schmidt orthogonalization method, which entails obtaining a new set of orthonormal vectors. Thus, the first vector, \( r_1 \), is simply normalized (Equation (8)). For the rest of the vectors, the corresponding part rendering them non-orthonormal
to each other (i.e., the projection of one vector on another) is annulled. Then, they are normalised. The steps 8 to 12 are repeated.

\[ e_1 = \frac{r_1}{|r_1|}, \quad e_i = \frac{\bar{w}_i}{|w_i|}, \quad w_i = r_i - \sum_{j=1}^{i-1} \left( r_j^T e_j \right) e_j \]  

(8)

13. The algorithm stops when one of the convergence criteria established in ERR or MAXN is satisfied. These criteria are explained in more detailed below.

Parameter Description of the Latin Hypercube and Rosenbrock (LHR) Algorithm

The LHR-coupled algorithm is controlled by a set of five parameters and two convergence criteria (Table 2). \( LHDIV \) controls the OF evaluation number executed by the LH of the initial sample space. The \( RLAUNCH \) parameter corresponds to the number of RNB algorithm launches using the best results of the LH as initial points. The \( ALPHA \) coefficient represents the advance magnitude in one direction when a positive result occurs. The \( BETA \) coefficient represents a direction change and a setback or step-reduction coefficient when a search direction failure has occurred. The \( STEPROS \) parameter defines the initial increments, \( \Delta_i \), for each search direction, calculated according to Equation (9).

\[ \Delta_i = \frac{\text{Max} \_ \text{ParameterValue}_i - \text{Min} \_ \text{ParameterValue}_i}{\text{STEPROS}} \]  

(9)

ERR stores the increment in each time step. If many failures occur (with no successes) in all possible search directions, the increment is multiplied by the advance or setback coefficient to change the search direction. Nevertheless, if the increment is very low, the advance or setback step will reach zero, indicating that the point is an optimal value. Then, when this increment is less than or equal to ERR, the algorithm will converge.

Generally, the algorithm stop criteria are defined when the algorithm achieves the value defined in ERR or when the maximum iteration number defined as MAXN is reached. Finally, the seed value marks the randomness of the process.

The success in the automatic calibration of CRRMs depends largely on the proper selection of the algorithm and its assigned parameters. In this paper, to define the value of the algorithm parameters, a sensitivity analysis is performed (see Section 3.6). The optimal parameter set for solving calibration problems of CRRMs are shown in Table 2.

Table 2. Set of parameter values for the Latin hypercube and Rosenbrock (LHR) method.

<table>
<thead>
<tr>
<th>Number</th>
<th>Parameter Name</th>
<th>Description</th>
<th>Set Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ALPHA</td>
<td>Advance or progress coefficient</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>BETA</td>
<td>Setback coefficient</td>
<td>-0.5</td>
</tr>
<tr>
<td>3</td>
<td>RLAUNCH</td>
<td>RNB launches number</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>LHDIV</td>
<td>LH dimension</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>STEPROS</td>
<td>Range subdivision parameter</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>ERR</td>
<td>Increment in each time step</td>
<td>0.001</td>
</tr>
<tr>
<td>7</td>
<td>MAXN</td>
<td>Maximum number of iterations</td>
<td>3000</td>
</tr>
</tbody>
</table>

2.3.2. University of Arizona’s Shuffled Complex Evolution (SCE-UA) Algorithm

To benchmark the results obtained from the LHR algorithm, the SCE-UA algorithm developed by [9] at the University of Arizona was selected. Its efficiency has been widely recognized worldwide in the context of hydrological model calibration. It has a high number of parameters for solving optimization problems in water-resource systems [30,50,86].

Derived from studies by [9,86] and [51], it was found that the efficiency of the SCE-UA algorithm has depended largely on the selection of the algorithm execution parameters. However, a detailed
analysis of the SCE-UA parameter set is beyond the scope of this study. Therefore, we refer to previous studies [9,30,86] to select the parameter set values, where exhaustive analyses were performed.

This algorithm displays certain weaknesses when it is applied to very complex (generally distributed) models with a high number of parameters [7,8]. However, this does not affect the scope of this analysis. In fact, the National Weather Service River Forecast System’s SAC-SMA model [51,86] demonstrated that SCE-UA was an effective, consistent and efficient algorithm for global optimization of CRRM parameters [8,9,86,87]. This technique was widely applied for CRRM calibration [50,51,67,68,86].

2.4. Objective Functions (OF)

Automatic optimization techniques require an OF to maximize (or minimize) the given value and find the optimal solution. Heretofore, it has not been possible to prove that a specific OF is better than others for the calibration of a certain model. However, it is possible to use some specific indices for specific problems [88,89]. In this study, to measure the similarity achieved between model output and observed data, a combined criterion is applied as an OF [90] (Equation (10)). Both algorithms maximize this OF [66,77], trying to fit the simulated flows with the observed data through the variation of model parameters.

Maximize $OF(\theta) = (w_1\text{NSE}(\theta) + w_2\ln\text{NSE}(\theta) + w_3 r(\theta) + w_4 MS(\theta))$  

where $\theta$ represents the parameter set of the CRRM, and $w_1 \ldots w_4$ are the weights for each criteria. In this case $w_1 = 0.25$, $w_2 = 0.25$, $w_3 = 0.25$ and $w_4 = 0.25$. $\text{NSE}$ is the Nash–Sutcliffe efficiency [91], $\ln\text{NSE}$ is a log-transformation of the $\text{NSE}$ coefficient [66], $r$ is the Pearson’s correlation coefficient and $MS$ is the mean symmetry measure. Equations (11)–(14) show the mathematical formulation of these indices, which were used in similar cases [92,93] with acceptable results.

We assigned the weights in the OF according to the characteristics evaluated by each index in the flow simulation. For WRA, the low flows and the conservation of the mean in the simulated series are important parameters for calibration and validation of CRRM [94]. Therefore, some selected indices are oriented to the simulation of these values (e.g., in $\text{NSE}$ and $MS$). The $\text{NSE}$ criterion tries to adjust the high flows in the simulated series with the observed data, and the $r$ index measures the co-variability of simulated flows with observed data without a bias penalty.

$$\text{NSE} = 1 - \frac{\sum_{i=1}^{N} (Q_{\text{sim} i} - Q_{\text{obs} i})^2}{\sum_{i=1}^{N} (Q_{\text{obs} i} - \bar{Q}_{\text{obs}})^2} = f_1$$  

$$\ln\text{NSE} = 1 - \frac{\sum_{i=1}^{N} (\ln Q_{\text{sim} i} - \ln Q_{\text{obs} i})^2}{\sum_{i=1}^{N} (\ln Q_{\text{obs} i} - \ln \bar{Q}_{\text{obs}})^2} = f_2$$  

$$r = \frac{\sum_{i=1}^{N} (Q_{\text{sim} i} - \bar{Q}_{\text{sim}}) \times (Q_{\text{obs} i} - \bar{Q}_{\text{obs}})}{\sqrt{\sum_{i=1}^{N} (Q_{\text{sim} i} - \bar{Q}_{\text{sim}})^2 \times \sum_{i=1}^{N} (Q_{\text{obs} i} - \bar{Q}_{\text{obs}})^2}} = f_3$$  

$$MS = 1 - \max \left(\frac{Q_{\text{sim}}}{\bar{Q}_{\text{obs}}}, \frac{Q_{\text{obs}}}{\bar{Q}_{\text{sim}}}\right) - 1^2 = f_4$$

where $Q_{\text{obs} i}$ and $Q_{\text{sim} i}$ represent the observed and simulated flow values for the calibration period, respectively. $\bar{Q}_{\text{obs}}$ is the average of the observed flow values for the same period. The best adjustment value for each index is the unit (1). The variation range can start from $-\infty$ for $\text{NSE}$, $\ln\text{NSE}$ and $MS$. It can start, from $-1$ for $r$.

2.5. Calibration and Validation Periods

Time series from each catchment are divided into the calibration and the validation periods according to data availability. The calibration set, roughly 80% of available data, is used during model
optimization. The validation set is used only to verify the quality of calibrated models on independent data. However, until 1980, calibration data were considered a ‘warm-up’ item and were not used to compute the OF value. For catchments 1, 2, 3, 4, 5 and 7, the calibration period used was from 1 October 1980 to 1 September 2002 and the validation period was from 1 October 2002 to 1 September 2007 (22 years for calibration and 5 for validation). For catchment number 6, the periods spanned from 1 October 1995 to 1 September 2005 and 1 October 2005 to 1 September 2007 (10 years for calibration and 5 years for validation). For catchment number 8, the periods spanned 1 October 1969 to 1 September 1988 and 1 October 1988 to 1 September 1993 (19 years for calibration and 5 for validation). Finally, for catchment number 9, the periods were 1 October 1950 to 1 September 1970 and 1 October 1970 to 1 September 1975 (20 years for calibration and 5 for validation).

3. Discussion and Analysis of Results

In this section, we discuss the results obtained from the application of a multi-start strategy of a direct-search algorithm using SCE-UA as a benchmark algorithm to calibrate CRRMs.

3.1. Comparison of the OF Evaluations Number versus Model Complexity

Measuring the efficiency of an algorithm using the OF evaluations number (iterations) required to reach the optimum solution as a comparison parameter facilitates the analysis of the performance of two or more algorithms in the solution of the same problem [21]. It is also important to evaluate the speed of the algorithms to find an optimal solution, because the best methods are those that converge with the better-quality solutions in the minimum possible time [2].

To analyse the number of iterations performed by the algorithms, each of the nine case studies was conceptualized using three different CRRMs (i.e., GR4J, HBV and SAC-SMA) and calibrated with two different optimization techniques (i.e., LHR and SCE-UA), producing 54 different calibration scenarios. All experiments were repeated 30 times to consider the randomness of the seed production after 1620 calibrations. We did not obviate or set model parameters in the calibration stage to analyse the behaviour of the algorithms in front of problems with different numbers of parameters. The calibration performed was completely automatic, and the initial parameter values of each model corresponded to the mean value of the variation range in every case (see Appendix A Table A1). From this value, both algorithms were launched under the same modelling conditions and only once in each experiment.

To represent the differences in the number of iterations required for algorithm convergence, we show the results in Figure 3. We observe that the performance of LHR improves with the rise in the complexity of the CRRM (4, 8 and 16 parameters).

![Figure 3](image_url). Dispersion and median values in the iteration number required per model type and per algorithm to convergence.
For the 4-parameter model, GR4J, in six of the nine case studies, the LHR displayed better behaviour than the SCE-UA by performing fewer iterations to find a solution. In the case of HBV, having eight calibration parameters, the LHR was clearly better in six cases. However, for the SAC-SMA model, having 16 parameters and more complex equations, the LHR performed fewer iterations in all cases. As the complexity of the model increases, the LHR algorithm finds the optimal solution with fewer iterations than the SCE-UA. Therefore, it is faster. These analyses are necessary because as found, studies on the optimization model parameters are recommended to ensure the most accurate determination of runoff in the catchments. However, the following section discusses the quality of the solutions found by the LHR and SCE-UA algorithms.

3.2. Comparison of the OF Values

The OF value reached by an algorithm allows evaluation of the quality of a solution and comparing it to others found by different methods to solve the same problem. In this case, we compared the OF values reached by the LHR with the SCE-UA algorithm as a benchmark. Figure 4 shows the median OF values obtained from the calibration stage (for each case-study by hydrological model and for both algorithms), whereas Figure 5 shows the results for the validation stage. Figure 4 shows that the median OF values achieved by both algorithms during the calibration stage are generally the same in all the study cases for GR4J and HBV. Furthermore, the differences in the OF values for the SAC–SMA model are significant, because the LHR found better OF values than the SCE-UA in the nine study cases. Thus, the variability of the series is better represented by the set of parameters found by the LHR algorithm that that of the SCE-UA. The analysis of the differences in the set of parameters found by both algorithms is shown in Section 3.3.

Figure 4. Comparison of the median indices’ values used to calculate the objective functions (OF) during the calibration stage for each case-study and for three rainfall–runoff models of different complexities. The figure shows Nash–Sutcliffe efficiency (NSE) (a), ln NSE (log-transformation of the NSE coefficient) (b), r (c) and MS (mean symmetry measure) (d). Every index is the result of the median of 30 simulations in each case-study.
The calibrations of the nine case studies and the three CRRMs were ‘very good’ or ‘good’, according to [96], except in some cases of SAC-SMA, where the calibration was ‘satisfactory’. There, lower values of the indices were obtained for the rest of the catchments. We attribute this mainly to the complexity of the model. However, it was the best calibration achieved with the conditions established to evaluate the performance of both algorithms under the same circumstances. Moreover, the results from the validation stage are shown in Figure 5 to demonstrate the capacity of the models to represent the processes present in the basins. Generally, we observed that the OF values in the validation stage were satisfactory for both algorithms. In basins 2 and 8, the results from SCE-UA were slightly better than the results from LHR.

![Figure 5. Comparison of the median indices' values used in the validation stage for each case-study and for three rainfall–runoff models with different complexities. The figure shows NSE (a), ln NSE (b), r (c) and MS (d). Every index is the result of the median of 30 simulations of each case-study.](image)

An analysis of algorithm efficiency was also performed when a perturbation in the initial parameters of the CRRM was added. According to [85], this is an important characteristic in the calibration stage. Thus, it was concluded that perturbations in the initial CRRM parameter values did not influence the OF value achieved by the algorithms.

3.3. Comparison of the Effective Parameter Values

Rarely is there a single solution for a specific problem. ‘Equifinality’ of parameter sets is a concept introduced by [97] and [98], who formulated that many different parameter combinations gave an acceptable solution of the same problem. Thus, model simulations should be analysed to generate a set of confidence parameter into a range determined by the model.

To establish the valid range of the parameter values for each model, we launched 30 simulations for each CRRM and basin with both algorithms. Thus, we considered the randomness of the calibration process and each algorithm when searching the optimal solution from different initial points in the sample space. If the algorithm converged at the same set of parameters many times from different points (seeds), it indicates that it could be an optimal parameter set.
Figures 6–8 show the optimal solutions found by each algorithm for basin number 5. This analysis was performed for the nine case studies. However, the most representative case is shown below.

The snow parameter values for the four basins are given in Appendix A Table A2. Note that, in the calibration process described in this paper, the snow parameter was set. The parameters were calibrated independently of the 30 simulations for each model, and the snow component was considered an independent variable of the rest of the parameters. Therefore, we reduced the dimensionality of the calibration problem and tested the algorithms homogeneously in all cases.

In the histograms of Figures 6 and 7 (GR4J and HBV, respectively), we observed that the variability of the solutions found by the LHR algorithm is more widespread than that found by the SCE-UA. However, there are also cases where LHR found parameter solutions with minor variations from the benchmark algorithm (Figure 6d). We found cases where the parameter variation was concentrated in a very short range for 30 repetitions with both algorithms (Figure 7g).

![Figure 6. Frequency histogram of parameters of the French rural-engineering-with-four-daily-parameters (GR4J) model for basin 5.](image)

![Figure 7. Frequency histogram of parameters of the Swedish Hydrological Office Water-balance Department (HBV) model for basin 5.](image)

However, for all the parameter of the GR4J and HBV models, the 30 repetitions were concentrated in similar ranges with different frequencies. Thus, whereas the mathematical formulation was different in both methods, the variability was represented similarly by the algorithms.

Overall, the solutions found by both algorithms were different in the three CRRMs. However, in the results for the SAC-SMA model, the parameters had significant differences. The LHR algorithm had less dispersion in the solutions from the 30 simulations than did the SCE-UA for the nine study cases.
Figure 8 shows the frequency histogram of the solutions found by the algorithms in the calibration for the SAC-SMA model. It is clear that in most of the parameters, the variability found by the LHR algorithm was less than that obtained by solutions with SCE-UA. The LHR plotted 30 repetitions within a unique range, and there have been cases where it was not possible to identify the range of valid solutions using the SCE-UA algorithm (Figure 8i). This behaviour was observed in at least 10 of the 16 parameters calibrated in the SAC-SMA model. Similar conclusions to those described in this section were obtained for the other eight.

![Frequency histograms of parameters](image.png)

**Figure 8.** Frequency histogram of parameters of the SAC-SMA model for basin 5.

The ranges plotted in Figures 6–8 represent the range where the algorithms found solutions, not the total search range of the algorithm, which is given in the Appendix A Table A1.

### 3.4. Comparison of GR4J, HBV and SAC-SMA Performance

This analysis was performed to compare the behaviour of different CRRMs in the simulation of flows in the same catchment. One may note that in many cases GR4J outperforms HBV and SAC-SMA. This variability in the behavior of the models was also found in studies such as [99], relating this performance also to the basin areas.

According to the index values in Figure 5, the GR4J model is capable of better simulating the low flows in the series, because the value obtained from the $\ln NSE$ index is almost always slightly higher.
The HBV model reflects a high ability to simulate high values better than low flows, because the NSE index has better values than does ln NSE. We also identified that the GR4J model had a higher capacity to simulate the flows following an extreme rainfall event. Regarding the SAC-SMA model, it behaved satisfactorily in all the catchments but has not proved to be the best in any of the cases. This again may be caused by the number of model parameters. Such differences in results are not surprising, because, over a decade ago, [100] showed that various CRRMs performed very differently for different catchments, even those located in zones with similar climatic conditions [101].

3.5. Comparison of the Estimated Runoffs for the Calibration and Validation Periods

Figures 9 and 10 show the calibration and validation plots per month for catchment 5, respectively. The results were analysed for each case-study, using the effective parameter values selected in the previous section. We compared the observed flows for each basin with the simulated flows estimated from the effective parameters for the validation and calibration stages. Generally, the scenario with the least capacity to simulate the measured flows is the SAC-SMA model calibrated with the SCE-UA algorithm. It is observed that, for months where flows were high, this scenario overestimated values and, when flows were low, it did not have the capacity to represent them. The wrong interpretation of the flows measured by this model can also be verified in the OF values calculated in Section 3.2. However, when a comparison is made between the scenario of this model but calibrated with the LHR algorithm, better performance was observed over the previous scenario in both low and high flows. GR4J and HBV represented the variability of measured flows in nearly every month independently of the algorithm used to calibrate the model. These results could be verified with the validation results (Figure 10). Similar conclusions were obtained for the other eight case studies.
3.6. Sensitivity Analysis of the Parameters of the LHR Algorithm

One of the fundamental problems in the application of optimization algorithms in the calibration of CRRMs consists of the determination of a correct and effective set of parameters to search the OF in a reliable and efficient way [102]. A sensitivity analysis of the parameters of the LHR algorithm was conducted to determine their impact on the search for the OF.

The influence evaluation of each parameter on the OF calculation was analysed using 80 combinations of different parameter sets. Each parameter was attempted with variations between the permissible ranges. In each simulation, a unique parameter was variated, whereas the rest retained the main value. The analysis was performed for nine catchments. Figure 11 shows the most representative cases of this analysis.

The ALPHA parameter was analysed in a range of values between 0.5 and 10. Figure 11 shows that, for very high values of ALPHA (from 7.5–10), high OF values were obtained with a high number of iterations. According to previous analyses, this translates into a higher consumption of time.

Commented [M22]: Please check if there is Figure 11 caption in this manuscript.

Figure 9. Comparison of runoff (average year) obtained from the calibration period for catchment 5 for the three conceptual rainfall-runoff models (CRRMs) and both algorithms.

Figure 10. Comparison of runoff (average year) obtained from the validation period for catchment 5 for the three CRRMs and both algorithms.
3.6. Sensitivity Analysis of the Parameters of the LHR Algorithm

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Figure 11. Parameter sensitivity analysis of LHR from 80 algorithm runs.

The ALPH A parameter was analysed in a range of values between 0.5 and 10. Figure 11 shows that, for very high values of ALPH A (from 7.5–10), high OF values were obtained with a high number of iterations. According to previous analyses, this translates into a higher consumption of time. However, it can be observed that, for very low values of ALPH A (from 0.5–3.5), high OF values are reached. In this last range, the OF evaluation number is lower. For average ALPH A values (from 3.5–7.5), the
number of iterations is not very high, but the OF value reached is lower than in the previous intervals. The study [31] suggested that this parameter had a value of around 3.0. Based on the sensitivity analysis specifically performed for an \textit{ALPHA} value equal to 3.0, a high OF value, (no the highest of all analyses, but acceptable) and a very low number of iterations (the second lowest of all analyses) was achieved.

Similarly, the \textit{BETA} parameter was analysed in a range from $-0.9$–$0.1$. According to [31], the suggested value for this parameter was $-0.3$. The significant differences shown with the variation of this parameter are reflected in the iterations number, rather than in the OF value reached, because the latter remains more or less constant according to Figure 11. For values between $-0.9$ and $-0.6$, the number of iterations increased considerably, and the OF value varied by less than 0.01. For values between $-0.5$ and $-0.1$, the number of iterations decreased considerably, whereas the OF value reached remained about the same, except for two cases. In the analysis, it was determined that the most suitable value for \textit{BETA} was $-0.3$, requiring a low number of iterations for reaching a high OF value.

The number of RNB launches was determined by \textit{RLAUNCH}. There is a directly proportional relationship between this parameter and the number of iterations. The higher the number of launches, the greater number of iterations. According to Figure 11, it can be established that for \textit{RLAUNCH} values from 3–5, we get the same OF value, with a different number of iterations. For \textit{RLAUNCH} values greater than 5, the number of iterations increases considerably, but the improvement in the OF value achieved does not. For \textit{RLAUNCH} values less than 1 and 2, the number of iterations was smaller but the OF value reached was very low. This determines that the appropriated value of \textit{RLAUNCH} is equal to 3 in cases of WRA.

\textit{LHDIV} represents the LH size for initial random sampling. Based on Figure 11, for divisions of LH between 40 and 50, the number of iterations was the lowest of all analyses, and the OF value achieved was among the highest recorded values. For greater \textit{LHDIV}, the OF evaluation number increased considerably, and the OF value obtained was not necessarily the best. Therefore, the value of the selected \textit{LHDIV} parameter was 50.

The graph of Figure 11 (corresponding to the \textit{STEPROS} parameter) shows that the lowest number of iterations evaluations is found for a \textit{STEPROS} equal to 40, whereas the highest OF values are found for values between 5.0 and 30. However, the number of iterations increases. For values greater than 45 to 100, the number of iterations decreased, compared to the previous interval. Still, the OF value is not the best. It has been suggested to use parameter values between 40 and 80.

\textit{MAXN} and \textit{ERR} are the stop criteria of the algorithm, determined according to the dimension of the problem to ensure that these do not affect the calculation of the optimal solution.

This analysis allowed determination of the effective parameters of the LHR algorithm for its application in the solution of problems in the calibration of CRRMs for the evaluation of water resources shown in Table 2. These parameters are used in the calibration of models used in Section 3.5.

4. Conclusions

In this study, a traditional local-search algorithm coupled with a random sampling method as a designer and optimizer of a multi-start strategy was tested. The combination was compared to an evolutionary algorithm for the calibration of three CRRMs with different complexities (HBV, GR4J and SAC-SMA) for nine headwater basins located in potential drought areas of Spain, with different climatic conditions for the WRA.

We observed that the combination of the local search algorithm, RNB, coupled with an efficient random sampling method, LH, which allows prior inspection and evaluation of the launch points of the RNB algorithm, works efficiently in the estimation of the optimal parameters of different CRRMs. When comparing the results of the LHR algorithm with those obtained by the evolutionary algorithm, SCE-UA, we observed that, for models with high complexities, such as the SAC-SMA model, the LHR algorithm found significantly better solutions than the SCE-UA, an algorithm widely used in the solution of this type of problems. For models like GR4J and HBV, having relatively inferior complexities than those of SAC-SMA, the algorithm achieved solutions at least equal to those found by the reference algorithm.
We joined the conclusions of studies of [6,16,17,19], in which it was argued that algorithms catalogued as traditional or historical, could work efficiently to solve actual problems if they are used correctly. The evaluation of the behaviour of a local-search algorithm improved with the random sampling method, in the calibration of the main rainfall–runoff models used in the WRA, was considered. Surprisingly, this algorithm gave positive results and, in the nine cases of the SAC-SMA model, LHR was the best.

It was also observed that the selection of the type of model for simulation of flows in a basin played an important role, because the same model can behave very differently in each case. In fact, the complexity of hydrological models affected the results obtained, depending on the optimization technique selected. Finally, we confirmed that for a flow simulation with a given model, distinct optimization methods perform differently for different catchments. Hence, it is recommended that a series of repetitive tests with different starting values of parameters by each optimization method be undertaken for deciding the most appropriate optimization method. In this case, after a sensitivity analysis of parameters of the LHR algorithm, it was possible to establish the set of optimal values to use for the calibration of CRRMs.

Author Contributions: This manuscript is a result of the Doctoral research of L.G.-R. Formal analysis, L.G.-R.; Investigation, L.G.-R.; Methodology, J.P.-A. and E.B.; Supervision, J.P.-A.; Writing—original draft, L.G.-R.; Writing—review and editing, A.S., J.A. and S.T.S.-Q.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Description of the parameters for each CRRM and ranges for the optimization algorithms.

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameter (Units)</th>
<th>Description</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X1 (mm)</td>
<td>Maximum capacity of the production store</td>
<td>100</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>X2 (mm/day)</td>
<td>Groundwater exchange coefficient</td>
<td>−5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>X3 (mm)</td>
<td>One day ahead maximum capacity of the routing store</td>
<td>20</td>
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</tr>
<tr>
<td></td>
<td>X4 (days)</td>
<td>Time base of unit hydrograph UH1</td>
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<td>5.8</td>
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<tr>
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<td>Shape coefficient of recharge function</td>
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<td>6</td>
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<tr>
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<td>FC (mm)</td>
<td>Maximum water storage in the unsaturated zone store</td>
<td>30</td>
<td>650</td>
</tr>
<tr>
<td></td>
<td>PWP (mm)</td>
<td>Soil moisture value above which actual evaporation reaches potential evaporation</td>
<td>30</td>
<td>650</td>
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<td>Groundwater near the surface parameters</td>
<td>Lmax (mm)</td>
<td>Threshold parameter for extra outflow from upper zone</td>
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</tr>
<tr>
<td></td>
<td>K0 (day(^{-1}))</td>
<td>Additional recession coefficient of upper groundwater store</td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td>K1 (day(^{-1}))</td>
<td>Recession coefficient of upper groundwater store</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>Groundwater parameters</td>
<td>K2 (day(^{-1}))</td>
<td>Recession coefficient of lower groundwater store</td>
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</tr>
<tr>
<td>Deep groundwater parameters</td>
<td>Kperc (mm d(^{-1}))</td>
<td>Maximum percolation to lower zone</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>Snow parameters</td>
<td>TT (°C)</td>
<td>Threshold temperature</td>
<td>−1.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>DD (mm °C(^{-1}); day(^{-1}))</td>
<td>Degree-day factor</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Surface parameters</td>
<td>PCTIM (decimal fraction)</td>
<td>Impervious fraction of the watershed area</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>ADIMP (decimal fraction)</td>
<td>Additional impervious area (decimal fraction)</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>RIVA (decimal fraction)</td>
<td>Riparian vegetation area</td>
<td>0</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Table A1. Cont.

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameter (Units)</th>
<th>Description</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil parameters</td>
<td>UZTWM (mm)</td>
<td>Upper zone tension water maximum storage</td>
<td>10</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>UZFWM (mm)</td>
<td>Upper zone free water maximum storage</td>
<td>10</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>UZK (day⁻¹)</td>
<td>Upper zone free water lateral depletion rate</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>REXP (dimensionless)</td>
<td>Exponent of the percolation equation</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>ZPERC (dimensionless)</td>
<td>Maximum percolation rate</td>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>Groundwater parameters</td>
<td>PFREE (decimal fraction)</td>
<td>Fraction of water percolating from upper zone directly to lower zone free water storage</td>
<td>0</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>LZTWM (mm)</td>
<td>Lower zone tension water maximum storage</td>
<td>5</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>LZFPM (mm)</td>
<td>Lower zone free water primary maximum storage</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>LZFSM (mm)</td>
<td>Lower zone free water supplemental maximum storage</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>RSERV (decimal fraction)</td>
<td>Fraction of lower zone free water not transferrable to lower zone tension water</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>LZPK (day⁻¹)</td>
<td>Lower zone primary free water depletion rate</td>
<td>0.0001</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table A2. Snow parameter values resulting from the independent calibration process for the basins with snow influence.

<table>
<thead>
<tr>
<th>Case-Study Catchments</th>
<th>Threshold Temperature TT (°C)</th>
<th>Degree-Day Factor DD (mm °C⁻¹ Day⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>10.05</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>11.72</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>4.49</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>15.22</td>
</tr>
</tbody>
</table>

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