Comments on "nonlinear H_∞ output feedback control with integrator for polynomial discrete-time systems".

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SUMMARY

KEY WORDS: integrator approach; polynomial discrete-time systems; SOS

The above-cited paper proposes an integrator-based dynamic output feedback setup in order to achieve some $\mathcal{H}_\infty$ performance bounds for discrete-time polynomial systems. This note will show that, even if theoretical results are correct, they do not improve over the open-loop performance bound. Indeed, the chosen structure of a matrix decision variable (in order to render the problem convex) make the proposed synthesis conditions imply standard SOS performance analysis ones for the open-loop system, so there will not be improvement over a controller $u(k) = 0$, as follows[†].

Consider the closed-loop system (12), the triangular matrix $\hat{S}(\hat{x}(k))$ in (17) and a full Lyapunov function for such extended system (12):

$$\hat{V}(\hat{x}(k)) = \hat{x}^T(k) \begin{bmatrix} P_1(x(k)) & P_2(x(k)) \\ (*)^T & P_3(x(k)) \end{bmatrix} \hat{x}(k)$$

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[†] In these comments, non-bold equation numbers refer to the original paper being commented upon.

Then, if we write condition (16) (discrete increment of the Lyapunov function), showing all its block elements, it leads to the following polynomial matrix inequality:

$$\begin{bmatrix} P_1(x(k)) & (*)^T & (*)^T & (*)^T \\ P_2(x(k))^T & P_3(x(k)) & (*)^T & (*)^T \\ S_1(\hat{x}(k))A(x(k)) & S_1(\hat{x}(k))B(x(k)) & S_1(\hat{x}(k)) + S_1(\hat{x}(k))^T - P_1(x(k+1)) & (*)^T \\ S_2(\hat{x}(k))A(x(k)) + S_3(y,x_c)\hat{K}(y,x_c)C_y(x(k)) & S_2(\hat{x}(k))B(x(k)) + S_3(y,x_c) & S_2(\hat{x}(k)) + S_2(\hat{x}(k))^T - P_2(x(k+1))^T & S_3(y,x_c) + S_3(y,x_c)^T - P_3(x(k+1)) \end{bmatrix} > 0 \quad \textbf{(16b)}$$

Note that the (conservative) special form of matrix $\hat{S}(\hat{x}(k))$ in (17) is required to avoid the appearance of nonconvex terms in **(16b)**. Then, taking the submatrix formed with the first and third row and column blocks, a necessary condition to fulfill **(16b)** is the following:

$$\begin{bmatrix} P_1(x(k)) & (*)^T \\ S_1(\hat{x}(k))A(x(k)) & S_1(\hat{x}(k)) + S_1(\hat{x}(k))^T - P_1(x(k+1)) \end{bmatrix} > 0 \qquad \textbf{(16c)}$$

Then, as the above matrix must be SOS for all $\hat{x} = (x, x_c)$, it includes the particular case $x_c = 0$, i.e., $\hat{x} = (x, 0)$. Note now that, actually, inequality **(16c)** is a sufficient condition to check stability of the open-loop system (1) with $u(k) = 0$, $\omega(k) = 0$ and a candidate Lyapunov function $V = x(k)^T P_1(x(k))x(k)$. Therefore, Corollary 3.1 will never obtain a controller (11) which stabilizes an open-loop unstable system. Note that this issue is related to the special structure chosen for matrix $\hat{S}(\hat{x}(k))$. A full structure for $\hat{S}(\hat{x}(k))$ would add extra elements in **(16c)** allowing improvements over $u(k) = 0$, but products of decision variables would appear in **(16b)**, breaking convexity.

Moreover, as results presented in the paper are claimed to be *global*, see Remark 3.6 in the paper, the controller (61) in Example 1 is invalid. Indeed, in system (58), the term $x_1(k+1) = -0.5Tx_1(k)^3$ easily allows obtaining some open-loop simulations where trajectories diverge to infinity, for instance, $x(0) = (15, 0)^T$, $\omega = 0$. Hence, from the above discussion, the SOS solver should have failed in finding a feasible solution. Actually testing the provided controller, simulating the undisturbed closed-loop system for the same initial conditions $x(0) = (15, 0)^T$, the trajectory tends to infinity, too[‡].

Following a similar argument, extracting suitable rows and columns from (31), a necessary condition for (30) being positive is the matrix-SOS condition

$$\begin{bmatrix} P_1(x(k)) & 0 & (*)^T & (*)^T \\ 0 & \gamma^2 I & (*)^T & 0 \\ S_1(\hat{x}(k))A(x(k)) & B_w(x(k)) & S_1(\hat{x}(k)) + S_1(\hat{x}(k))^T - P_1(x(k+1)) & 0 \\ C_z(x(k)) & 0 & 0 & I \end{bmatrix} > 0$$

[‡]Stable simulation results depicted in Figure 2 are obtained starting from $x(0) = 0$ (usual H_∞ assumption): by sheer luck, the controller is mild enough to keep the system stable around the open-loop (locally) stable equilibrium $x = 0$.

which gives, in fact, an $\mathcal{H}_\infty$ bound for the open-loop system (1) under nonzero external disturbances.

Conclusions: From the above developments, the main results (Corollary 3.1, for stabilization, Theorem 3.1 and Corollary 3.2 for $\mathcal{H}_\infty$ performance) are useless to obtain a controller providing better performance than the plain open-loop $u(k) = 0$.

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