A REVIEW OF MODELLING AND OPTIMISATION METHODS APPLIED TO RAILWAYS ENERGY CONSUMPTION

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Abstract

Railways are a rather efficient transport mean, and yet there is increasing interest in reducing their energy consumption and making them more sustainable in the current context of climate change. Many studies try to model, analyse and optimise the energy consumed by railways, and there is a wide diversity of methods, techniques and approaches regarding how to formulate and solve this problem. This paper aims to provide insight into this topic by reviewing up to 52 papers related to railways energy consumption. Two main areas are analysed: modelling techniques used to simulate train(s) movement and energy consumption, and optimisation methods used to achieve more efficient train circulations in railway networks. The most used methods in each case are briefly described and the main trends found are analysed. Furthermore, a statistical study has been carried out to recognise relationships between methods and optimisation variables. It was found that deterministic models based on the Davis equation are by far (85% of the papers reviewed) the most common in terms of modelling. As for optimisation, meta-heuristic methods are the preferred choice (57.8%), particularly Genetic Algorithms.

Keywords: railways; energy efficiency; modelling; optimization, meta-heuristics

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1 Introduction

In the current context of climatic change and scarcity of resources, there is an increasing interest in improving efficiency in every aspect of our society. This is particularly true for transport, which is both an essential element for human activities and a heavy energy consumer. Only in Europe (EU-28), transport accounts for about 33% of final energy consumption as of 2015, which in terms of greenhouse gas emissions represents more than 1,182 million tonnes of CO_2 equivalent (European Commission, 2017).

Railways consume only about 2% of the total energy used by the transport sector, despite the fact that over 17% of freight transport and 8% of passenger transport is carried out by rail within the EU-28 (European Commission, 2017). These figures point out the remarkable efficiency of railways with regard to other transport modes, a fact that has been also analysed in different studies (García Álvarez, 2007a). This justifies the interest that the EU has on promoting railways through initiatives such as the EU 2020 Horizon R&D programme and the Shift2Rail Initiative.

However, despite their inherent efficiency, railways still have the potential for significant improvements in terms of energy consumption. In fact, in many railway networks it is still common for operators to ignore the actual, segregated energy consumption (and associated costs) of their trains: only overall consumption is known. This is especially true for diesel trains, whose energy consumption is often controlled simply by measuring the fuel level in the tank before and after each service (Baumel, 2011). In the case of electrified lines, usually the electric companies charge the energy supplied from each substation to the railway network, and thus the railway operator does not know which fraction is used for traction and which for heating, lighting, etc. (García Álvarez and Martín Cañizares, 2012). This is slowly changing and some operators are installing metering devices on their rolling stock to gather energy consumption data in real time.

In any case, this lack of accurate data, paired with the increasing need for bolstering efficiency, is the drive behind several studies and R&D projects that have been carried out over the last years. The global, long-term purpose of all these works is to reduce the carbon footprint of railways and enhance their competitiveness within the framework of a more sustainable society. Many of these studies deal with measuring the energy consumed by rolling stock during operation, hence solving the aforementioned gap of reliable data. In order to do so, trains are equipped with monitoring devices that allow measuring energy consumption in real time. This has been done for both diesel trains (Salvador Zuriaga et al., 2014) and electric trains (Martínez Fernández et al., 2015), and, as mentioned before, many railway operators have incorporated such equipment as a standard for their lines. When there is data available, is it possible to analyse not only the energetic performance of a railway network, but also the effectiveness of different measures that may be applied to improve efficiency (Douglas et al., 2015; Yamamoto, 2015), such as eco-driving, regenerative brake or new, lighter rolling stock. Moreover, once reliable data is available, it is possible to expand the boundaries of research through the development of models and the use of different techniques and algorithms to enhance the efficiency of the railway network.

Within this framework, the objective of this paper is to review the existing bibliography related to railways energy efficiency, to assess the main trends found in previous works and to identify potential gaps of knowledge that may point out new lines of research. The long-term goal of the paper is to provide researchers working in the area of railways energy efficiency with a deeper insight into the techniques and algorithms available, pointing out possible new ways of studying and applying optimisation, thus contributing to more sustainable railways.

The paper focuses on two different (although strongly related) areas and classifies all the reviewed papers accordingly. These areas are modelling techniques and optimisation studies.

The first area consists of the development of models that yield a reliable estimation of the energy consumption of a train (or a set of trains) under different operating circumstances. These models allow testing many alternative techniques and approaches without the need for actual, experimental tests, thus expanding the possibilities of research. There are different ways of modelling the energy consumed by trains, although the most common trend is to use time-step simulators (Domínguez et al., 2011; Sicre et al., 2010; Tian et al., 2015) or space-step simulators (Lu et al., 2013), both based on the Davis equation (García Álvarez, 2007b). Other alternatives include the use of neural networks (Açikbas and Soylemez, 2008) and stochastic models (Davydov et al., 2018).

The second area of interest focuses on optimisation studies, which aim to improve the efficiency of the railway network by means of a systematic analysis of several elements, ranging from driving styles to infrastructure layout and new equipment (e.g. on-board storage systems). By far the most common option is to focus on driving schemes (eco-driving) as it is usually the operative aspect that yields higher increments of efficiency (Douglas et al., 2015). Therefore, most authors have tried to optimise driving in high-speed lines (Sicre et al., 2010), metro networks (Fernández et al., 2015) and freight lines (Lukaszewicz, 2001). Some studies focus on manually driven trains (Sicre et al., 2012) and others on trains equipped with Automatic Train Operation (ATO) (Brenna et al., 2016). Many of the aforementioned studies analyse the problem of a single train, while others take into account a whole set of trains and their interaction (Komaki et al., 2016; Tian et al., 2015).

Apart from driving style, other factors may be included in the optimisation study, such as track geometry (Huang et al., 2015), on-board storage systems (Domínguez et al., 2010), regenerative brake (Tian et al., 2015) or variations of the train load (Fernández et al., 2015). Finally, many different algorithms and techniques have been used to solve these optimisation problems, including Particle Swarm Optimisation (Yang et al., 2015), Genetic Algorithms (Dündar and Şahin, 2013; Watanabe and Koseki, 2015), Fuzzy Programming (Cucala et al., 2012), Ant Colony (Lu et al., 2013) or Mixed Integer Linear Programming (Lu et al., 2016).

The review comprises 52 studies collected and analysed, of which 40 correspond to modelling techniques and 41 to optimisation studies (many of them address both subjects). However, despite the apparent equality between both areas, optimisation is broader a topic than modelling, as most papers focus mostly on it and offer an abundant bibliography related to optimisation techniques. In any case, the papers are classified and analysed, and the main research trends identified are presented. Afterwards, these results are thoroughly discussed and the main conclusions are presented.

2 Review methodology

In order to carry out the review according to the aforementioned objectives, a multi-step methodology was defined, based on the ones used by other authors for their systematic literature reviews (Alves and Mariano, 2018; Zamarrón-Mieza et al., 2017):

Step 1: Identification of the research need. In other words, a proper formulation of the study must be determined as a starting point. In this case, the review aims to answer two

questions: First, what modelling techniques have been used to model the movement and energy consumption of trains, considering these techniques as tools used in the resolution of wider optimisation problems. Second, what optimisation methods have been used to solve said optimisation problems.
Step 2: Preliminary search of papers using a set of keywords. Once the two main areas of interest were formulated, a thorough search was carried out using two renowned bibliographic databases: SCOPUS and SCIENCE DIRECT. This preliminary search was done using a set of keywords, which were defined taking into account the experience of the authors on the topic. This set of keywords included terms such as 'Railway energy consumption', 'Railway energy modelling' and 'Railway optimisation'. Because of this preliminary search, many papers apparently related to the topic were found and analysed, thus creating a framework for the review.
Step 3: Definition of rules for a refined search. From the analysis of the papers found after the preliminary search, several parameters for a more polished search were defined, including main authors, potential schools of thought regarding the two areas of interest and aspects not covered by the first set of keywords (such as certain modelling parameters). Additionally, the period to be covered (1995-present) was also fully determined after the preliminary search, as no relevant article prior to 1995 was found. In fact, optimisation problems in the field of railways efficiency prior to 2000 are rare, and either too simplistic or formulated within too wide a framework to be of interest. Only three papers published between 1995 and 2000 were chosen for review.
Step 4: Refined search. Once the set of rules was fixed (based also on the authors' experience), a second search was carried out, filtering the papers obtained taking into account publication date, authors of interest, research areas, etc. Many studies that had been previously identified were discarded as they were not closely related to the two main questions previously defined. As a result of this process, 52 papers were collected, of which 40 correspond to modelling techniques and 41 to optimisation studies. As explained before, many of them address both topics, which are strongly related.
Step 5: Systematic analysis. The 52 studies finally selected were then fully reviewed and analysed. Quantitative tables were created with the objective of classifying the papers according to the two main areas of interest defined in step 1, and the frequency of each option (i.e. modelling and/or optimisation method) was obtained.
Step 6: Finally, a statistical method known as correspondence analysis was carried out to identify underlying patterns and detect potential gaps of research.

3 Modelling and optimisation methods

3.1 Energy consumption modelling

In this section, the modelling of trains' energy consumption is analysed, and the main trends found in the literature are described.
First, in order to develop a model capable of delivering a sound estimation of the energy consumed by a train (or a set of trains) along a line or network, it is essential to decide how complex this model will be. Different factors and parameters may be taken into account or neglected depending on how accurate and close to reality the model will be, and how reliable their results. On this matter, the reviewed papers show a wide disparity in the level of complexity of their models. Certain studies only model a single train with no additional elements other than the train dynamics and basic track layout (Khmelnitsky, 2000). Others include factors such as unexpected delays (Cucala et al.,

2012), regenerative brake (Lu et al., 2016), on-board storage systems (Domínguez et al., 2012), variations of train mass (Carvajal-Carreño et al., 2014; Liu et al., 2018), or consumption of auxiliary systems (Huang et al., 2017). Some of these factors are of interest depending on the particular conditions of the line (or network) to be studied. For instance, the presence of on-board storage systems only makes sense if the train is actually fitted with regenerative brake. Other factors may be applied to almost any case. On the other hand, some factors, despite being quite influential, may be also particularly complex to add to a model. For instance, the train mass and its variation (due to the varying number of passengers) is a factor that is usually neglected as it is quite difficult to model or measure, and yet it has an evident impact on the train energy consumption. Most of the authors that have tried to add this factor to their studies have done so through fuzzy numbers (Carvajal-Carreño et al., 2014; Fernández et al., 2015). Table 1 summarises the main factors that may be incorporated into the model:

Factor	Example reference
Regenerative brake	(Lu et al., 2016)
On-board storage systems	(Domínguez et al., 2010)
Train mass variations	(Fernández et al., 2015)
Unexpected delays	(Cucala et al., 2012)
Passengers' access time during stops	(Huang et al., 2017)
Drivers' behaviour (in manual driving)	(Sicre et al., 2014)
Consumption of auxiliary systems	(Liu et al., 2018)

Table 1: Main factors that may be included in the optimisation problem

With regard to the model itself (and this election also affects which of the aforementioned factors could be included, and how), as explained before the most common choice by far in all the reviewed papers is some sort of deterministic model based on a variation of the Davis equation, which models the air drag resistance to train motion:

$$R = A + B \times v + C \times v^2 \qquad (1)$$

Where v is the train speed and A, B and C are the Davis coefficients. This equation is usually expanded to include other factors that affect the train motion, in order to better model the traction effort. (García Álvarez, 2007b) proposes the following formulation:

$$F_T = a \times M \times C_{mg} - M \times g \times p + A + B \times v + C \times T_f \times v^2 + M \times f_c \qquad (2)$$

Where F_T is the engine traction, a is the train acceleration, M is the train mass, C_{mg} is the coefficient of gyratory masses, g is gravity, p is the track slope, v is the track speed, A, B and C are the coefficients of the Davis equation proper, T_f is a tunnel factor and f_c is a curve index. This kind of equation models the train movement, and may be paired with other equations that yield energy consumption from the traction effort, thus developing a modular model. This is, as said before, the preferred option for most of the authors reviewed (Cucala et al., 2012; Domínguez et al., 2011; Huang et al., 2015; Lu et al., 2016; Sicre et al., 2010; Tian et al., 2015). The resolution of this model may be done on a time-step (Domínguez et al., 2011; Tian et al., 2015) or space-step (Lu et al., 2013) basis.

This kind of modular model allows increasing its complexity simply by adding modules that introduce some of the factors exposed in Table 1 (how each of these factors is modelled is an entirely different question). For instance, (Domínguez et al., 2008) proposes a three-module model: One part that models the selection of ATO speed profiles, another one that models the train movement (based on a variation of equation (1)) and a final one that calculates the consumed electric power. Building upon this very model, (Domínguez et al., 2010) adds two additional modules: one that calculates the ratio between required traction and maximum traction available (thus incorporating engine efficiency) and one that simulates the presence of an on-board energy storage system.

Other authors also opt for deterministic models, although based on simple mechanical equations that consider the train as a point mass and incorporate air resistance through a single coefficient (Chuang et al., 2009). Another alternative, which also considers the train as a point mass, is to formulate the problem purely in terms of energy through the Maximum Principle (Howlett et al., 2009; Khmelnitsky, 2000). This allows an exact solution from a mathematical point of view (and an exact optimisation, as explained in section 3.2) but this comes at the cost of more simplifications in order to ensure that the problem is solvable.

Only a few authors have opted for modelling approaches other than a purely deterministic one. This is likely because, generally, a deterministic approach will yield a good enough approximation of the energy consumed and, as explained, it is relatively easy to increase the complexity of the model. On the other hand, deterministic models usually require that several parameters are defined or known beforehand (e.g. train masses and damping, track layout, etc.) and, depending on their formulation, obtaining solutions may be quite time-consuming.

One alternative to deterministic models are neural networks (NN), which are computational models that have been extensively used in the past in fields as diverse as chemistry, engineering or finances. In terms of modelling energy consumption of vehicles, however, there are very few examples of NN to be found, and even fewer in the field of railway engineering. (Açikbas and Soylemez, 2008) used a NN to generate multiple driving scenarios, although it only provided simplified results (namely, the point along the line where coasting starts) and not a full simulation of the train movement. Other authors have used NN to model the energy consumption of a complete railway network (Komyakov et al., 2015) or the position of multiple trains (Chen et al., 2016). Sometimes NN are combined with a deterministic model to incorporate a particular feature to the simulation, such as braking curves (Y. Huang et al., 2016) or the behaviour of train operators (Dündar and Şahin, 2013), or to identify coasting points (Chuang et al., 2009).

The main advantage of NN is that they do not require any parameter to be calibrated or known beforehand and that, once trained, they yield many simulations very quickly as their computing requirements are quite low. On the other hand, NN require actual, reliable data for training, and the training process may be difficult.

Finally, one study used stochastic models (Davydov et al., 2018) to incorporate uncertainties such as delays and corrections made by drivers during services.

Table 2 shows a summary of the main options regarding the modelling of train energy consumption according to the literature reviewed:

Basic approach	Formulation	References
Deterministic	Maximum Principle	(Khmelnitsky, 2000) (Liu and Golovitcher, 2003) (Howlett et al., 2009) (Su et al., 2013) (Wang and Zhu, 2014) (Albrecht et al., 2016a)
	Davis equation	(Jong and Chang, 2005) (Domínguez et al., 2008) (Sicre et al., 2010) (Domínguez et al., 2010) (Domínguez et al., 2011) (Kang, 2011) (Cucala et al., 2012) (Domínguez et al., 2012) (Sicre et al., 2012) (Lu et al., 2013) (Domínguez et al., 2014) (Carvajal-Carreño et al., 2014) (Sicre et al., 2014) (Huang et al., 2015) (Tian et al., 2015) (Shangguan et al., 2015) (Yang et al., 2015) (Zhao et al., 2015) (Fernández et al., 2015) (Lu et al., 2016) (Salvador Zuriaga et al., 2017) (Wang and Rakha, 2017) (Ahmadi and Dastfan, 2016)
	Other	(Kim et al., 2010) (Yang et al., 2012) (Brenna et al., 2016) (He and Xiong, 2018) (Liu et al., 2018)
Stochastic models		(Davydov et al., 2018)
Neural network (NN)		(Açikbas and Soylemez, 2008) (Komyakov et al., 2015) (Chen et al., 2016)
Combinations	Deterministic + NN	(Chuang et al., 2009) (Y. Huang et al., 2016)

Table 2: Modelling methods

3.2 Optimisation studies

This section focuses on optimisation studies, assessing how different authors have formulated the problem, which factors have been taken into account and what algorithms and methods have been used to obtain a solution. As said in the introduction, many authors have worked on this aspect of the studies related to railways efficiency, and thus there is an abundant bibliography to review.

3.2.1. Optimisation problem

The first and foremost step to carry out an optimisation study is to formulate the problem, i.e. to accurately define what to optimise and which factors are taken into account. The latter relates to the level of complexity of the problem to be addressed, and a compromise should be achieved to ensure that the formulated problem is close enough to reality to be of use while being also simple enough to be solvable.
Optimisation problems may be one-dimensional or multi-dimensional depending on the pre-defined objectives. In the case of railway efficiency, although some studies only aim at optimising running time (Dündar and Şahin, 2013), the vast majority of the reviewed works takes into account at least two objectives: time and energy. However, there are differences as to how these two objectives are formulated (i.e. what variables are actually optimised), particularly with regard to time. Some authors simply try to optimise the running time between two consecutive stations, while others take into account the whole line and instead aim at optimising the distribution of available slack time (Cucala et al., 2012), or the energy consumption at substation level (Domínguez et al., 2012). Another option found is to optimise both time and energy indirectly by focusing on the speed at certain points of the route (Lu et al., 2013), the definition of coasting points (Brenna et al., 2016; Howlett et al., 2009) or the optimisation of ATO speed commands in automated lines (Carvajal-Carreño et al., 2014). Table 3 summarises the main trends with regard to the definition of variables to be optimised.

Objective	Variable	Example reference
Time	Interstation running time	(Domínguez et al., 2008)
	Slack time redistribution	(Sicre et al., 2010)
Energy	Interstation train energy consumption	(Domínguez et al., 2010)
	Substation energy consumption	(Domínguez et al., 2012)
Combined time and energy	ATO speed commands	(Shangguan et al., 2015)
	Coasting points	(Howlett et al., 2009)
	Full speed profile	(Sicre et al., 2014)

Table 3: Main optimisation variables and their formulation

Another aspect to be considered in the problem definition is whether only a single train is optimised or the system as a whole is taken into account. This choice (one train vs. multiple trains) is strongly related to how the variables are defined, as explained before: time defined as the time lapse between two consecutive stations is a suitable approach when the motion of a single train is optimised, while the optimal redistribution of slack time is a more appropriate definition when studying multiple trains.
Finally, with regard to the problem complexity, different factors and parameters may be taken into account in order to bring the study closer to reality and obtain results that are more useful to achieve an efficient railway system. The same factors that were described for modelling (see Table 1 in section 3.1) also apply to the formulation and resolution of an optimisation problem.

3.2.2. Optimisation method

With regard to which optimisation method is applied, several techniques have been used over the last years to optimise railways energy consumption with varying results. The first, simplest way to carry out the optimisation of a given problem is through Direct Search i.e. the exhaustive analysis of all possible solutions in order to choose the optimum one(s) (Domínguez et al., 2008). Obviously, this is only feasible when the solution space is relatively small, for instance, in some automated metro networks when the ATO speed profile is chosen at each stop among a limited set of possibilities. Another alternative option for a small solution space is Decision Theory (Domínguez et al., 2010), which consist on the definition of rules that help choosing between competing solutions.

Another approach is to perform an analytical optimisation, i.e. to minimise a predefined target function (e.g. total energy per interstation trip) through analytical methods (Howlett et al., 2009; Khmelnitsky, 2000). The advantage of this approach is that an exact solution for the problem may be obtained, if it exists. On the other hand, usually important simplifications and assumptions are inevitable in order for the problem to be solvable, and even then, the resolution of the algorithm may be too complex and/or time consuming. In terms of railways energy efficiency, most authors that choose an analytical approach use the Maximum Principle Analysis.

However, despite the variety of optimisation techniques, the most used algorithms belong to the category of meta-heuristics, which are techniques that do not guarantee to obtain a globally optimal solution, but rather may reach a sufficiently good solution when there is incomplete or imperfect information and/or limited computation capacity. These methods were developed to sample a set of solutions in optimisation problems where the whole solution space is too large to be completely sampled, and they have become widely used in multiple fields (Martínez et al., 2010).

There are different families within the meta-heuristic category of algorithms, but considering the ones used in the papers reviewed, the most noteworthy with regard to railways efficiency are:

- Evolutionary algorithms: These are population-based algorithms inspired by biological evolution, where candidate solutions equal to individuals in a population and a fitness function is defined to determine their quality. This information is then passed onto the next generation (evolution). Noteworthy examples belonging to this family are Genetic Algorithms (Chang and Sim, 1997; Dündar and Şahin, 2013) and Hybrid Evolutionary Algorithms (Shangguan et al., 2015).
- Swarm intelligence algorithms: These are also population-based algorithms inspired by the collective behaviour of self-organised systems (such as some insect colonies). They consist of simple individuals who interact locally with one another as well as with their environment. Noteworthy examples belonging to this family are Ant Colony (Eaton et al., 2017) and Particle Swam (Domínguez et al., 2014).
- Teaching Learning Based Optimisation: Another population-based method, in this case inspired by the influence of a teacher on learners. It is a two-phase algorithm, with a first stage that represents the individuals learning from the teacher (teacher phase) and a second stage where the individuals learn by interacting between each other (Huang et al., 2015). Its main advantage is that it

does not require specific parameters to be tuned, only common control parameters such as population size and number of generations.

Other possible algorithms that have been used to solve optimisation problems related to railways efficiency belong to the categories of Integer Linear Programming (Liu et al., 2015; Lu et al., 2016), Dynamic Programming (Liu et al., 2018) and Fuzzy Linear Programming (Cucala et al., 2012), although these are much less used than those belonging to the meta-heuristics category.
As a summary, the main algorithms found in the literature are shown in Table 4, grouped into families depending on their basic formulation. This classification is partially based on (Nguyen et al., 2014).

Family	Algorithms	References
Analytical	Maximum Principle Analysis (MPA)	(Khmelnitsky, 2000) (Liu and Golovitcher, 2003) (Howlett et al., 2009) (Su et al., 2013) (Wang and Zhu, 2014) (Albrecht et al., 2016b)
Direct search (DS)	Exhaustive Search	(Domínguez et al., 2008)
Decision theory (DT)	Definition of systematic rules	(Domínguez et al., 2008) (Domínguez et al., 2010) (Domínguez et al., 2011) (Domínguez et al., 2012) (Y. Huang et al., 2016)
Dynamic Programming (DP)		(Lu et al., 2013) (Liu et al., 2018)
Integer programming family	Mixed Integer Linear Programming (MILP)	(Kim et al., 2010) (Lu et al., 2016)
	Tabu Search (TS)	(Liu et al., 2015) (Huang et al., 2017)
Fuzzy Linear Programming (FLP)		(Cucala et al., 2012)
Meta-heuristics	Genetic algorithms (GA)	(Salim and Cai, 1995) (Chang and Sim, 1997) (Wong and Ho, 2004) (Bocharnikov et al., 2007) (Açikbas and Soylemez, 2008) (Kang, 2011) (Sicre et al., 2012) (Yang et al., 2012) (Lu et al., 2013) (Dündar and Şahin, 2013) (Carvajal-Carreño et al., 2014) (Domínguez et al., 2014) (Sicre et al., 2014) (Zhao et al., 2015)

		(Watanabe and Koseki, 2015) (J. Huang et al., 2016) (Brenna et al., 2016) (He and Xiong, 2018) (Ahmadi et al., 2018)
	Hybrid Evolutionary Algorithm (HEA)	(Shangguan et al., 2015)
	Ant colony (AC)	(Lu et al., 2013) (Zhao et al., 2015) (Eaton et al., 2017)
	Particle Swarm (PS)	(Domínguez et al., 2014) (Yang et al., 2015) (Fernández et al., 2015)
	Teaching Learning Based Optimisation (TLBO)	(Huang et al., 2015)

Table 4: Optimisation methods

4 Discussion

4.1 Overview

As explained before, up to 52 papers focused on modelling and solving optimisation problems related to railways efficiency have been reviewed in order to assess the main trends in both areas. With regard to modelling (Figure 1), deterministic models account for the vast majority (85%), of which those based on variations of the Davis equation are the most common (57.5% of all papers reviewed). Neural networks (NN) are only used in 12.5% of the cases, but usually to a limited extent, either by modelling only a simpler part of the problem (Açikbas and Soylemez, 2008) or as an auxiliary tool combined with deterministic equations (Chuang et al., 2009), with only 3 papers belonging to former case and 2 to the latter. It is evident that most authors choose the same kind of approach, and other alternatives have not been thoroughly tested.

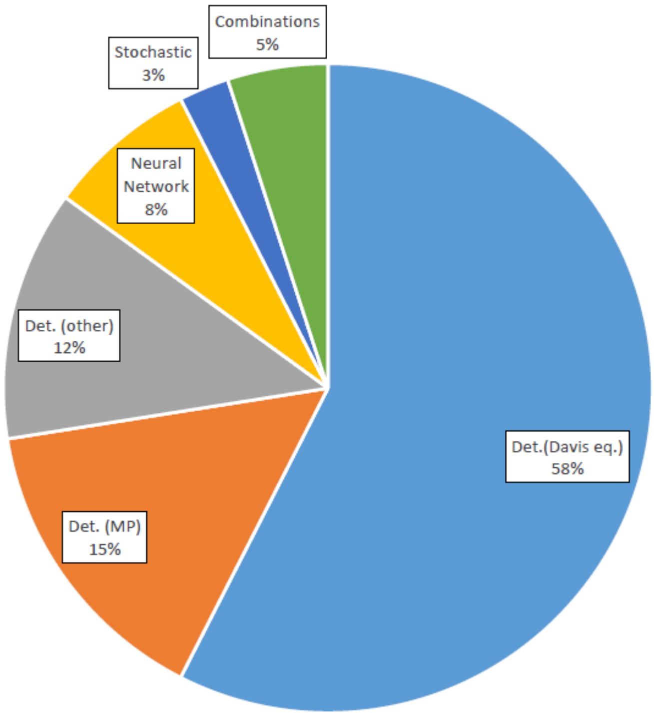


Figure 1: Frequency for modelling methods

Deterministic models are the preferred choice because they are based on known physical principles and, as explained before, they provide good enough solutions (as long as they are properly formulated and their parameters are well defined). For instance, (Domínguez et al., 2011) claims to achieve an error lower than 4.2% when comparing their model's output with actual, measured data, whereas (Sicre et al., 2010) indicates an even smaller error (0.45%). These results are rather good and help explaining the overwhelming prevalence of deterministic models. Moreover, it is relatively easy to increase the complexity of such models by adding equations or modules. How each particular item is formulated, however, is an entirely different problem, as certain aspects (such as varying train mass or unexpected delays) may be quite tricky. This opens the way for combined solutions, but, as seen in Figure 1, this is still a rather unexplored option that probably deserves further consideration.

Another aspect that may hamper the performance of deterministic models is their dependence on predefined parameters. As an example, the model described by (Domínguez et al., 2011), which is the base of many successive works (Carvajal-Carreño et al., 2014; Domínguez et al., 2014, 2012; López-López et al., 2014) requires no less than 12 train parameters, including some that may be difficult to determine, such as train running resistance or rotational mass. The effect that many of these parameters have on the model output may considerably affect its reliability.

In any case, it is evident that most authors have chosen to base their studies on deterministic models based on their aforementioned advantages. This does not preclude the possibility of exploring different alternatives, as a few authors have already tried. The combination of a core deterministic model with auxiliary tools that help adding complexity (and accuracy) to the model is likely the most promising option for future research, according to the literature reviewed.

With regard to optimisation techniques (Figure 2), by far the most common algorithms used belong to the meta-heuristics category (57.8%), and more specifically to the family of Genetic Algorithms (GA). As figure 2 shows, GA has been applied by 40% of the authors reviewed, sometimes as a benchmark to compare with other algorithms, as in (Domínguez et al., 2014; Lu et al., 2013). Comparatively, Particle Swarm (PS) and Ant Colony (AC) have been each used only by 7% of the authors reviewed.

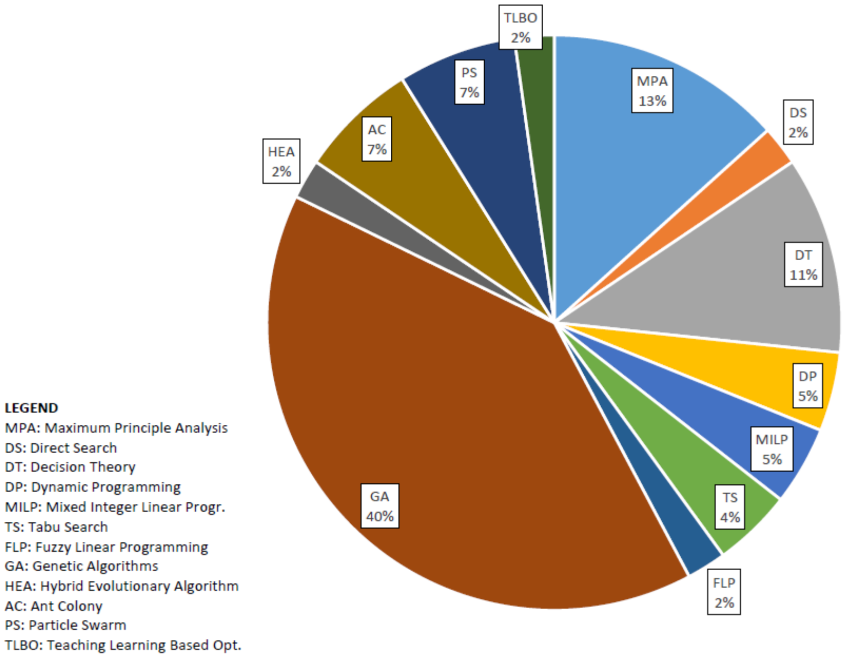


Figure 2: Frequency for optimization methods

The second most used approaches are Decision Theory (DT) and Maximum Principle Analysis (MPA), which account for 11% and 14% of the papers, respectively. However, these are mostly limited to simpler problems with a small solution space (optimisation of ATO speed profiles among limited possibilities) or as an auxiliary device combined with other algorithms (J. Huang et al., 2016).

Overall, it is clear that meta-heuristic algorithms seem to be the best option when there is a large solution space, as they were originally designed to cope with such situations and solve optimisation problems that are unsolvable by means of purely analytical methods. Many of the optimisation problems reviewed fall within this category, either because of their complexity (multi-objective, many factors to consider) or because an exact solution does not exist (and thus approximate solutions are the only attainable ones).

Regarding the prevalence of GA over other meta-heuristic algorithms (even to the point of being a benchmark option to some authors), this is likely due to it being one of the first evolutionary algorithms developed (Domínguez et al., 2014). In fact, it is possible to find applications of GA to railway traffic optimisation by mid 1990s (Salim and Cai, 1995), more than 20 years ago. Therefore, GA is a well-established technique with multiple variants. GA are also the most common option in other fields were meta-heuristic algorithms are used for optimisation, such as sustainable building design (Evins, 2013; Nguyen et al., 2014) or industrial processes (Rana et al., 2019). However, some authors have found that other, more recent meta-heuristic methods may offer a better performance. For example, (Lu et al., 2013) found that AC tends to converge more easily than GA, while (Domínguez et al., 2014) claims that PS covers the full Pareto front of potential solutions, and also converges faster than conventional GA. In any case, (Lu et al., 2013) recommends to use more than one optimisation technique so as to better cover the full space solution, as each algorithm may have problems to detect a particular area of potential solutions. This option is likely to become more common in future applications in order to tackle the shortcomings of each particular algorithm.

4.2 Statistical Analysis

In order to gain a deeper insight into the main trends regarding optimisation of railways energy consumption, a statistical analysis was carried out. A correspondence analysis was used to identify existing patterns between how the optimisation problem is formulated and the method used to solve it. This kind of tool (which is a variation of the Principal Components Analysis with qualitative variables) has been used by other authors to expand their reviews on different engineering areas (Jato-Espino et al., 2014; Penadés-Plà et al., 2016), and aims to detect subjacent relationships between two (or more) variables.

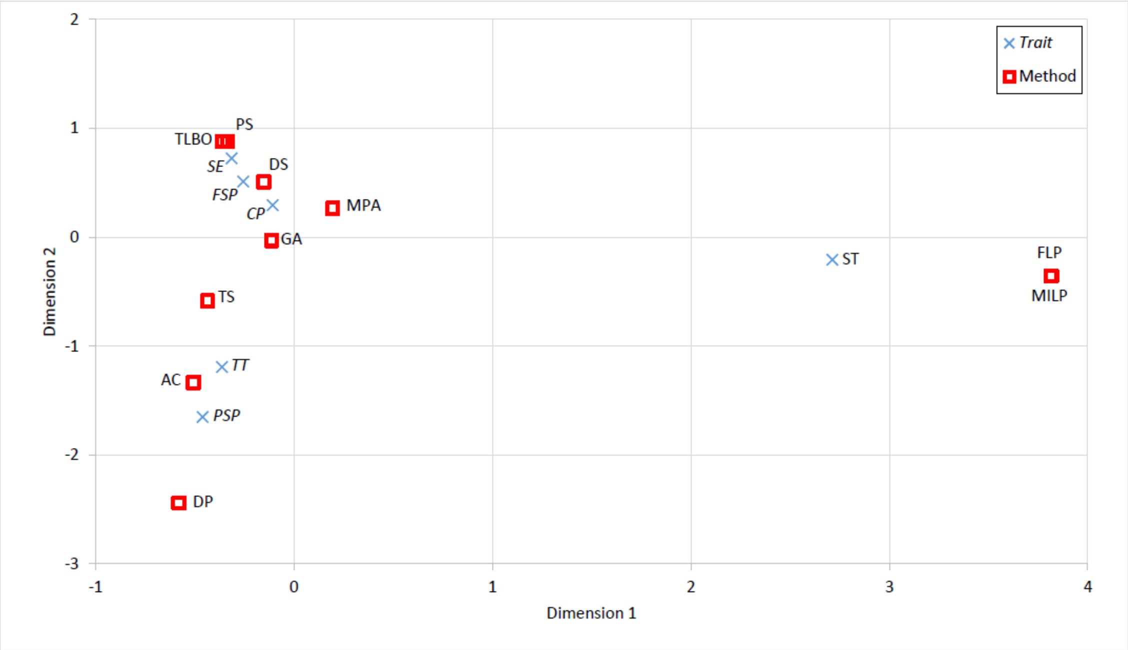
The information obtained from the reviewed papers was organized in the form of a contingency table where each row is a defining aspect of the optimisation problem (namely the variable to be optimised), each column is an optimisation method, and each cell represents the frequency of each method associated to each aspect. Twelve optimisation methods were considered (the same shown in Table 4), and the following five variables:

- Slack time redistribution (ST).
- Full timetable optimisation (TT).
- Coasting points (CP).
- Partial speed profile (PSP).
- Full speed profile (FSP).
- Substation energy consumption (SE).

These variables have been defined to encompass the different cases found in the reviewer papers. The first two cover the optimisation of the running time, be it the redistribution of a given time margin (ST) or a full timetable optimisation (TT). The next three cover the optimisation of the speed, where CP considers only the coasting points, FSP considers optimising the full profile, and PSP covers the wide range of possibilities between the other two (e.g. ATO speed commands, or optimising the speed at certain points of the journey). Finally, the sixth variable represents the optimisation of the energy consumed at the substation level.

The analysis takes the contingency table aforementioned and calculates, for each item of the table (be it an optimisation method or a variable) a set of scores related to orthogonal components. The main purpose of this method is to explore underlying correlations between qualitative variables that may be difficult to identify from the data itself. For a more detailed description of the methodology, see (Greenacre, 2016). The correspondence analysis was performed on SPSS Statistics 16.0 (IBM Corp., Armonk, NY, USA).

Figure 3 shows the results obtained from this statistical analysis. This figure aims to explore the tendency of using each optimisation method when the problem is formulated using a particular trait. This tendency is higher when each point is closer to each other. In other words, the closer two points of different categories are to each other, the more common is for that method to be used for that particular variable. Moreover, the relationship is the more exclusive the greater the distance between the related points and the origin. Please note that both axes represent the orthogonal components calculated and are thus dimensionless.



Methods: MPA: Maximum Principle Analysis DS: Direct Search DT: Decision Theory DP: Dynamic Programming MILP: Mixed Integer Linear Progr. TS: Tabu Search FLP: Fuzzy Linear Programming GA: Genetic Algorithms HEA: Hybrid Evolutionary Algorithm AC: Ant Colony PS: Particle Swarm TLBO: Teaching Learning
Traits: ST: Slack time redistribution TT: Full timetable optimisation CP: Coasting points PSP: Partial Speed Profile FSP: Full Speed Profile SE: Substation energy

Figure 3: Row and column points

As the figure shows, the most common algorithms irrespective of the optimised variable are GA, MPA and DS. This is demonstrated by their closeness to the origin. This is a coherent result because these three methods seem to be the preferred choices for three distinctive problem-solving patterns: MPA when looking for mathematically exact solutions, DS for problems with a limited solution space (where all solutions are evaluated) and GA as the preferred meta-heuristics method for problems with a large solution space. Moreover, these three methods are strongly related to problems solved through the optimisation of coasting points (CP) or the full speed profile (FSP).
Full timetable optimisation (TT) or a partial description of the speed profile (PSP) is a less conventional choice and seems to be related to AC algorithms. FLP and MILP are clearly separated from the rest and thus are rather unconventional optimisation methods not associated with a particular problem formulation. The same can be said for DP.
Finally, optimising energy consumption at the substation level (SE) represents a rather different approach (associated with full network modelling instead of single train) which seems to be addressed mainly using PS and TLBO.
The most interesting result from this analysis is the comparison between methods belonging to the same family (i.e. meta-heuristics) which are based on similar principles and thus could potentially yield analogous results. The prevalence of GA while other algorithms such as AC or PS are apparently limited to specific optimisation variables is not fully explained by any of the authors reviewed. In fact, as stated before, some authors even point out that PS and AC offer certain advantages with respect to the more traditional GA. This highlights the need for further research in the application of meta-heuristic algorithms other than GA for optimisation problems with diverse formulations and variables.

5 Conclusions

This paper reviews the existing literature concerning energy efficiency in the railways sector. More specifically, two main areas have been addressed: Modelling techniques used to simulate the train movement and energy consumption (as a necessary tool for further study of efficiency), and optimisation methods used to solve optimisation problems (including single and multi-objective problems as well as single train and full network approaches). Up to 52 studies published since late 1990's have been collected and analysed, of which 40 deal with modelling techniques and 41 deal with optimisation problems (many addresses both topics). Main trends related to each of the two aforementioned areas have been identified, and a statistical study has been carried out to detect underlying relationships between optimisation methods and variables.

Concerning modelling techniques, by far the preferred choice for most authors is a deterministic model based on the Davis equation for the train running resistance. Variations of this choice have been found in 58% of the papers analysed. If other deterministic models are included (e.g. formulation based on the Maximum Principle Analysis) the percentage rises up to 85%. It is clear that other alternatives (such as neural networks or stochastic models) are marginal at best. This is likely due to deterministic models being flexible, reliable and physically sound, although they also present some disadvantages (such as high dependence on several parameters) which might be avoided in some cases through alternative modelling approaches. A promising course for future works may be to combine deterministic models with other auxiliary tools so as to better include problematic aspects that are usually neglected or simplified in most of the reviewed papers. This could contribute to a more accurate modelling of the energy consumed by railways.

With regard to optimisation methods, the most common ones are those belonging to the meta-heuristics family, as these are well suited to solve problems with a large solution space, which is the most widespread situation in railways optimisation problems. Meta-heuristics account for about 58% of the papers reviewed, of which a large share (40% of the total) are Genetic Algorithms (GA). Other common options are Decision Theory (related to problems with a smaller space solution where all solutions may be analysed) and MPA (related to problems where an exact solution is achievable through purely mathematical formulation).

The statistical study carried out further identified these trends, and pointed out a strong relation between the three most common methods (GA, MPA and DS) and the optimisation of both coasting points and the full speed profile. On the other hand, the Ant Colony method (AC) seems more related to problems that aim to optimise timetables and partial speed profiles. Finally, the statistical study also indicated that optimising the energy consumed at the substation level (a particular case of optimisation of a complete railway network) is mainly done through TLBO and Particle Swarm (PS) methods.

From these results it is clear that GA are the preferred choice of researchers due to their rather good performance and well-established tradition, but other meta-heuristic algorithms may be better suited to solve particular problems and deserve further consideration. Considering all the literature reviewed, an advisable option for future works would be to combine different meta-heuristic algorithms in order to tackle the deficiencies of each one and to better cover the whole solution space. This may contribute to improve optimisation methodology, thus increasing energy savings and enhancing railways sustainability.

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