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Additional Information

1 **Experimental analysis of longitudinal shear between the web**
2 **and flanges of T-beams made of fibre-reinforced concrete.**

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15 **ABSTRACT.**

16 The longitudinal shear between the web and flanges of T-beams is an Ultimate Limit
17 State contemplated by technical codes. For this reason, the longitudinal shear must be
18 compared with the longitudinal shear resistance of the flange. Longitudinal shear
19 strength can be increased by including steel fibres in the concrete mass. This article
20 shows the experimental results of 13 T-beams mounted on two supports subjected to
21 two central loads. Four of these beams were made with conventional concrete and nine
22 with fibre-reinforced concrete. The direct instrumentation results are discussed and the
23 failure process is described. **Longitudinal shear cracking load** is studied on the basis of
24 both a theoretical approach and experimental results. An analysis is performed to
25 evaluate each specimen's longitudinal shear, not only in the ultimate state, but also
26 throughout the loading process evolution, based on load and strain records. This
27 process involves determining each beam's effective width. The experimental data
28 confirm an increase in longitudinal shear strength caused by adding steel fibres to
29 concrete.

30

1 **KEY WORDS:**

2 Longitudinal shear strength; longitudinal shear cracking load; web-flange junctions;
3 fibre-reinforced concrete; T-beams.

4 **HIGHLIGHTS:**

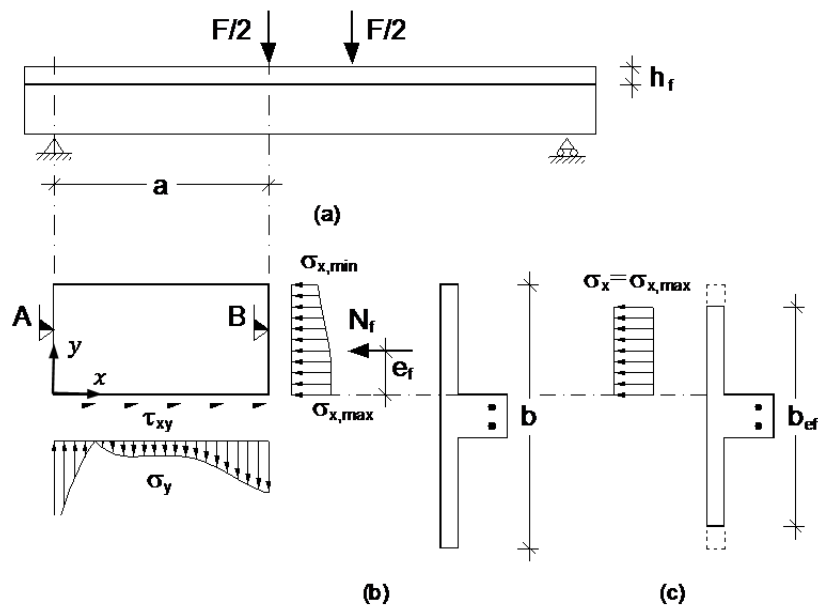
- 5 • 13 T-beams were tested to study the longitudinal shear strength of web-flange
6 junctions.
- 7 • The inclusion of steel fibres improves longitudinal shear strength.
- 8 • Effective width depends on transversal reinforcement and steel fibre content.
- 9 • A procedure was developed to calculate longitudinal shear cracking load.
- 10 • Maximum web-flange longitudinal shear and bending moment situations may not
11 match.

12

1. Introduction.

The use of reinforced concrete T-beams or double T-beams is widespread both in buildings and bridge decks due to their high flexural strength / weight ratio. The design codes [1,2] contemplate the longitudinal shear analysis between the web and flanges as a Ultimate Limit State in order to ensure the integral behaviour of the compressed wings against normal or tangential stresses. The web-flange tangential stress τ_{xy} is produced by the variation of the axial force N_f in the flange along a beam zone [1,2]. This tangential stress τ_{xy} is simultaneous with a flange normal stress σ_x and with a web-flange normal stress σ_y (Figure 1.a-b). The web-flange stress σ_y is produced as a consequence of the flange stress equilibrium, because of the eccentricity e_f of the axial force N_f with respect to the web-flange junction. The resultant of stresses σ_y is null. The stress combination of σ_x , σ_y and τ_{xy} , can lead to the formation of cracks in the flange of a reinforced concrete T-beam [3]. The presence of transverse reinforcement or fibres in the concrete mass is necessary to guarantee the integrity of the section. This cracking adversely affects the service and ultimate performance of the beams. If the transverse reinforcement yields, a plastic redistribution of the tangential stresses along a beam zone can be assumed [1,2] (the tangential stress τ_{xy} distribution can be assumed uniform). Otherwise, if the transverse reinforcement does not yield, the web-flange tangential stress τ_{xy} should be calculated by elastic theory. In this case, the distribution of tangential stresses τ_{xy} is not uniform. In general, all codes coincide in attaching importance to guarantee the web-flange junction by a high enough minimum reinforcement ratio to have a tough ductile mechanism with either strut-and-tie model-based codes [1,2,4] or shear-friction model-based codes [5–9]. An adequate web-flange junction behaviour allows to take into account the contribution of the compressed flanges both for the flexural strength and for the vertical shear strength of beams with T-shaped sections. Although the present design codes ignore the

1 contribution of flanges for shear strength, recent theoretical [10–13] and experimental
2 studies [14–16] show that it is not negligible.”



3

4 Figure 1: Flange stress distribution. (a) Test arrangement; (b) Actual flange stress distribution;
5 (c) Flange effective width

6

7 1.1. On the evaluation of longitudinal shear in reinforced concrete members.

8 The beam model is imposed to evaluate longitudinal shear between the flange and web

9 given its simplicity and wide acceptance. The flange effective width is needed to apply

10 this model. It is a reduced flange width in which the normal stresses are supposed

11 uniformly distributed in all flange points located at the same distance from the neutral

12 fibre of the section (Figure 1.b and c). This uniform normal stress distribution replaces

13 the actual non-uniform stress distribution that takes place as a result of the flexibility of

14 the flanges in their plane. However, this phenomenon cannot be appreciated in T-

15 beams with narrow flanges. As flange width increases, the normal stress is greater in

16 the web-flange junction than in the end of the flange. The design codes [1,2,4,5]

17 propose different expressions for the calculation of the flange effective width in the

18 absence of a more precise determination (for example, FEM). The resultant of normal

19 stresses in the effective flange width is calculated with the beam model and, based on

1 this result, longitudinal shear is calculated along flange length. Other authors like
2 Badawy and Bachmann [17] have proposed a global strut-and-tie model to study their
3 five tested T-beams. The effective flange width is needed to apply this method as well.

4 Razaqpur and Ghali [3] applied a linear elastic analysis by finite elements to a variety
5 of beams with different loading setups, with one span and two spans. They evaluated
6 not only the longitudinal shear between the flange and web, but also the concomitant
7 transverse axial load. The difference in the results between the beam model and the
8 FEM model was not significant. Therefore, these authors concluded that more detail in
9 evaluating longitudinal shear is not necessary, especially in beams with isostatic
10 schemes. In addition, longitudinal shear is redistributed in the ultimate state [17–20].

11 The transverse axial load concomitant with the longitudinal shear between the flange
12 and web is a force that the beam model is unable to provide, but can be evaluated with
13 more complex models. Razaqpur and Ghali [3] and Páez and Díaz del Valle [21] used
14 their elastic finite element model results to consider the concomitant transverse axial
15 load to design flange transverse reinforcement. The problem is that using elastic
16 results is not appropriate for proposing the design criteria of transverse reinforcement
17 in the ultimate state. In this case, the plastic redistribution of internal forces is expected
18 [18,20,22].

19 Morley and Rajendran [22] theoretically determined the effective width of beams by the
20 so-called limit curves. This model was applied only to estimate the strength capacity of
21 four beams previously tested by Davies [23]. These beams were made with not enough
22 transverse reinforcement. Subsequently, Fiorito [19] and Tizatto [20] especially
23 instrumented the flange to experimentally deduce the effective width value. Once the
24 effective width is known, longitudinal shear can be more accurately assessed.

25 More sophistication can be applied to evaluate longitudinal shear by applying either
26 finite elements with a non-linear analysis or the compression field theory. The
27 advantage of these analyses is that they include the longitudinal shear evaluation and

1 the response of the resistant mechanism in the same calculation. Hence they are able
2 to provide the beam's strength capacity. Their disadvantages are their calculation cost
3 and having to adjust many parameters.

4 **1.2. On the longitudinal shear strength in reinforced concrete.**

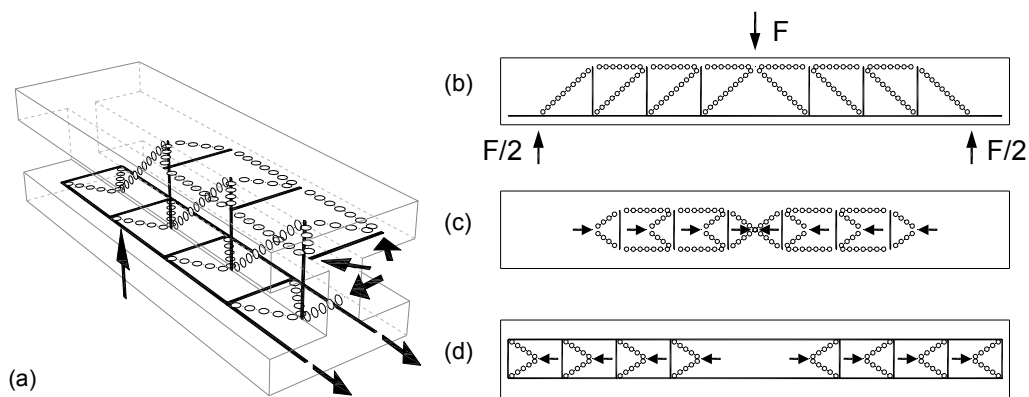
5 Longitudinal shear strength in T and I shape cross section RC beams is difficult to
6 predict due to complex phenomena involved, such as the cracking induced anisotropy,
7 the interaction with others forces as vertical shear or bending moment, the localization
8 of the critical diagonal crack, the confinement effects of the flanges, the 3D flow of
9 forces from the web to the flanges (shear lag effect), the interaction between concrete
10 and reinforced or the size effect. The critical diagonal crack in T-beam has two
11 branches. The first branch is more vertical and it is in the web. The second branch is
12 more inclined and it is in the flange. The critical crack crosses the web, continues along
13 the flange-web interface, until it propagates inclined inside the flange towards the load
14 application point [24]. The crack inclination determines the strength capacity of the
15 beam.

16 Several empirical and rational methods were developed taking as reference
17 experimental results and theoretical studies to evaluate the longitudinal shear strength.
18 Among these methods there is the Generalized Stress Field Approach (GSFA) [25–27],
19 Modified Compressive Fields Theory (MCFT) [26], the strut-and-tie method (STM)
20 [1,2,4], and the shear-friction method [5–9], as in a particular case of shear
21 transference models.

22 Modified Compressive Fields Theory (MCFT) uses compatibility equations and non-
23 linear constitutive laws for materials that consider the biaxial state of concrete strains.
24 This rotating crack model provides accurate results compared with experimental tests.
25 This method uses an iterative procedure to determine the inclination of the
26 compressive struts. It can be employed for membrane elements subjected to in-plane
27 normal and shear stresses, like the web and flanges of T-beam. Minelli and Vecchio

1 [28] validated MCFT-based numerical model with experimental results obtained on full-
2 scale fibre-reinforced concrete (FRC). This numerical model adequately simulate the
3 strength, stiffness, ductility, crack pattern development, and failure modes of all
4 specimens tested. Other authors [29–31] applied MCFT-based numerical model to
5 analyse the behaviour of elements made with steel fibres.

6 The strut-and-tie method allows analysing the beam behaviour as pinned statically
7 determined strut-and-tie structure (Figure 2). The struts are the elements resistant to
8 compression (concrete and compressed reinforcements), while the ties are the
9 elements resistant to tension (tensioned reinforcements).” This method is used by
10 European codes [1,2,4]. Badawy and Bachmann [17] resorted to the strut-and-tie
11 method and tentatively proposed an angle of inclination of struts between 22° and 26.6°
12 in compressed flanges. EC2 [2] includes the value of 26.6° as the minimum angle to be
13 considered and EHE-08 [1] uses an angle of 45° by default, which doubles the required
14 reinforcement. Tizatto and Shehata [32] recommended an angle that depends on
15 flange dimensions.



16 (a)
17 Figure 2: Strut-and-tie model of a pinned T-beam with a single load (a) 3D model; (b) Front view;
18 (c) Flange top side plant view; (d) Flange bottom side plant view.

19 American standard ACI-318 [5] adopts the shear-friction model as a general method to
20 study shear through a plane of weakness. Other American standards [6,7,9] and the
21 Japanese JSCE/SSCS [8] adopt a linear model of friction plus cohesion (modified
22 shear friction model).

1.3. On fibre-reinforced concrete (FRC).

There are experimental studies of reinforced concrete T-beams made of steel fibres [33–37]. These studies analyse the contribution of steel fibres in flexural strength or vertical shear. The relationship between the width and the thickness of the analysed T-beam flanges is moderate and, therefore, the flexibility of the flanges in their plane is reduced. In addition, these beams have been designed in order for the longitudinal shear not to be a failure mode. Consequently, no experimental results have been found for T-beams with wide flanges manufactured with FRC and tested to analyse the contribution of the fibres to the web-flange longitudinal shear resistance. Nevertheless, the vertical shear strength or the flexural strength of FRC has received plenty of attention and is included in design codes. Very few documents can be found on how to extend general models, such as strut-and-tie or shear-friction, to consider the effect of fibres on T-beams subjected to longitudinal web-flange shear. These models should consider the effectiveness of shear reinforcement mechanisms of fibres crossing the web-flange interface since it depends significantly on the fibre orientation. Fibres oriented with a certain angle to this interface do not provide any contribution. The fibre orientation and distribution is associated to the cast-in-place system.

The extension of the strut-and-tie method has received little attention in the scientific literature. There are some publications about simple structural elements, such as corbels [38–43], dapped-end beams [44–46], deep beams [47–50], bottle-shaped struts [51] or beams with span-depth ratio no more than 2.5 [52], which analyse how to include the effect of steel fibres. Fehling et al. [53] developed a proposal for the reduction of the compressive strength of cracked reinforced concrete. These authors considered the influence of fibres in addition to bar reinforcement. Campione [47] provided the only practical solution to determine strut compressive strength by considering the effect of steel fibres. Other studies have proposed using a term of residual tensile strength to be added to strength of reinforcement [54], as well as the

1 possibility of using ties that derive exclusively from fibres [55]. López-Juárez, J A [56]
2 describes more in detail strut-and-tie model applications to fibre reinforced concrete
3 elements. Nothing was found about the shear friction model extension.

4 Empirical models [57–62] are the most widespread and contrasted, and are basically
5 an image of the modified shear-friction models used in reinforced concrete, where
6 fibres contribute as a term that is added to the reinforcement ratio. These empirical
7 models are based on the results of push-off and prismatic tests.

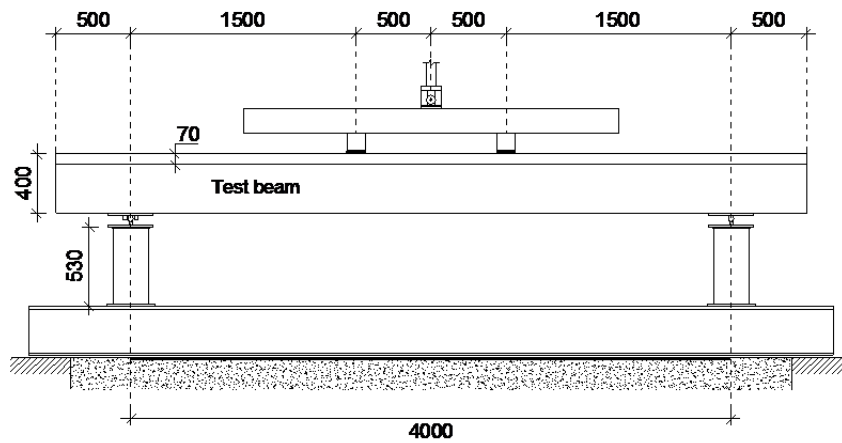
8 After analysing the scientific literature, the objective of this research is to provide
9 experimental results and to study the longitudinal shear between the flange and web in
10 T-beams made with conventional concrete and FRC. The longitudinal shear of each
11 tested beam and longitudinal shear cracking load were evaluated based on both a
12 theoretical approach and the experimental results.

13 **2. Experimental programme.**

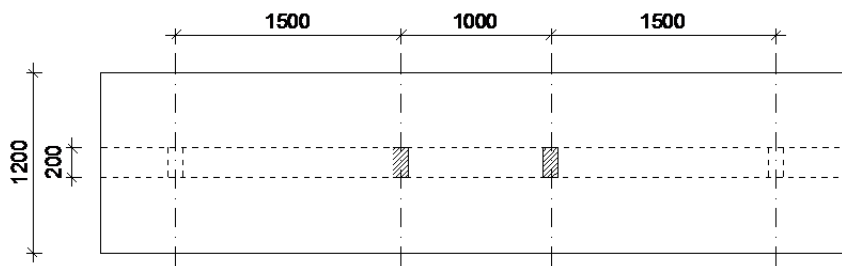
14 The objective of the experimental programme was to study 13 T-beams, of which four
15 were made of conventional concrete and nine of FRC.

16 **2.1. Test programme.**

17 Thirteen isostatic 5-metre-long T-beams were tested. They were mounted onto two
18 symmetric supports. The loading of these beams was applied at two symmetric points.
19 Figure 3 depicts the loading configuration. Beams laid on two supports that allowed the
20 rotation perpendicular to the bending plane. The horizontal movement was prevented
21 in one of the supports, while the horizontal movement is allowed in the other end with
22 the objective of avoiding the axial strain of the element. A steel beam was designed to
23 divide the load of the actuator into two loading points, connected to a joint for keeping
24 the load vertical all the time. This steel beam laid on two solid steel profiles placed on
25 the beam. A neoprene layer was placed between the beam and each of the two solid
26 steel profiles in order to regularize contact surface.



(a)



(b)

1

2

Figure 3: Loading configuration: (a) Front view, (b) Plan view. Units: mm.

3

The separation between loading points was slightly wider than twice the beam depth to

4

ensure that the central section underwent the appropriate conditions to apply the

5

classic beam theory hypotheses according to the Saint-Venant principle. Figure 6

6

shows the layout of a beam test.

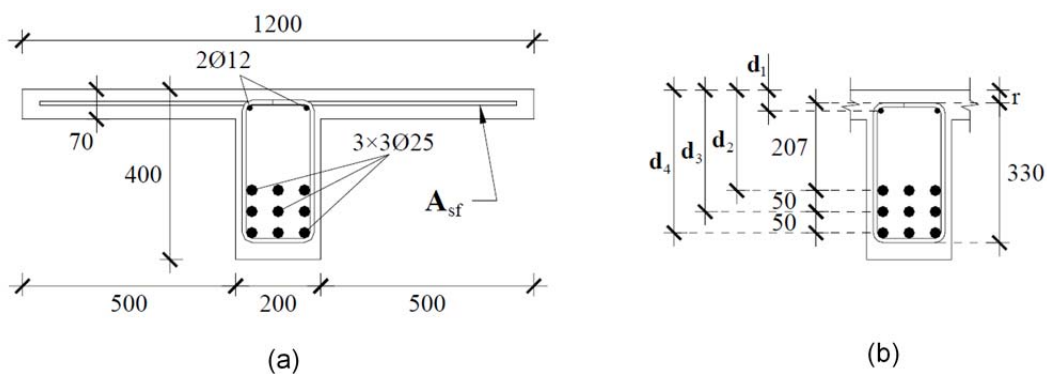


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8

Figure 4: Experimental test layout.

1 The cross section had flanges that were 1200 mm wide and 70 mm thick. Flange size
 2 is similar than the one used by other authors [17–20,63]. A flange size (width and
 3 thickness) was considered with the objective of studying longitudinal shear behaviour
 4 at web-flange junction (strength and failure mode). Flange size was defined so as the
 5 vertical load that causes web-flange junction cracking by longitudinal shear was half
 6 the vertical load that theoretically causes bending failure. The total cross section depth
 7 was 400 mm with a web thickness of 200 mm (Figure 5). The distribution of the
 8 longitudinal and transverse reinforcements in the web was the same in all specimens.
 9 Initially, geometrical concrete cover r (Figure 5.b) was established at 30 mm, but was
 10 modified during the experimental campaign (Table 1) to change the longitudinal shear
 11 failure plane. Figure 5 displays the cross section geometry and the longitudinal
 12 reinforcement arrangement. A longitudinal reinforcement ratio was arranged in order
 13 for the entire flange to be compressed in ultimate state. In this way, the maximum web-
 14 flange longitudinal shear was obtained. Table 1 shows the depth values at which the
 15 longitudinal reinforcements were located in each specimen.



16

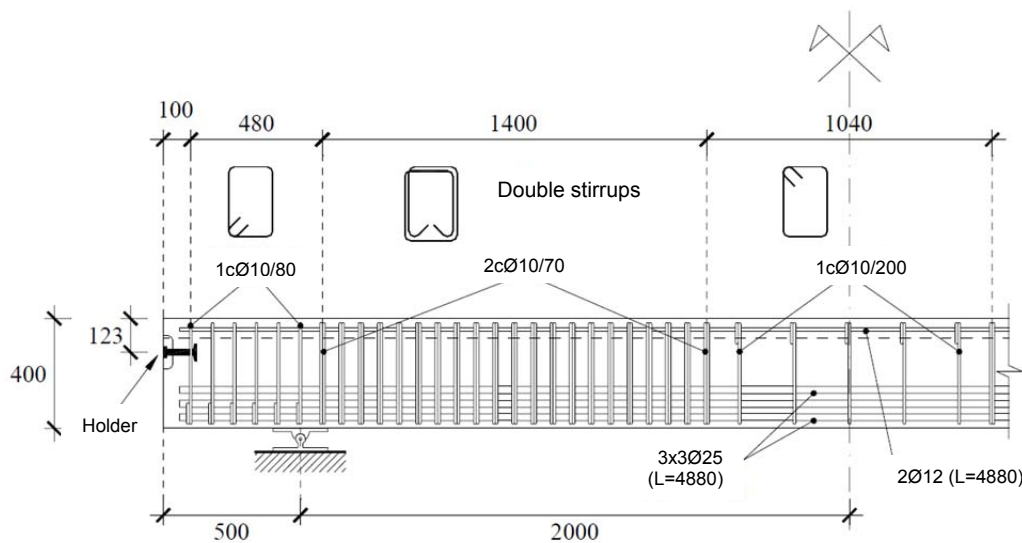
17 Figure 5: Cross section: (a) Geometry and longitudinal reinforcement arrangement; (b)
 18 Longitudinal reinforcement locations. Units: mm.

r	d_1	d_2	d_3	d_4	Specimens
30	49	247	297	347	V1-0, V2-0
25	44	242	292	342	V3-0, V4-0, V2-20, V3-20, V1-40
12	31	229	279	329	V1-20, V1-30, V2-30, V3-30, V2-40, V3-40

19

Table 1: Geometric concrete cover and reinforcement depths (units: mm)

1 The web transverse reinforcements consisted of either single or double $\text{Ø}10$ stirrups,
 2 with the distribution indicated in Figure 6. The transverse reinforcement ratio is
 3 conditioned by the longitudinal reinforcement required ratio to obtain the maximum
 4 web-flange longitudinal shear. Transverse reinforcement is provided in the web so that
 5 the theoretical load of the beam that causes vertical shear failure is much greater than
 6 the load that causes bending failure. A holder was provided at the ends of specimens
 7 to lift them.



8
 9 *Figure 6: Web's transverse reinforcements. Units: mm.*

10 The study parameters of the experimental programme were the transverse
 11 reinforcement ratio in flanges and the steel fibre content. The beams were
 12 manufactured with an insufficient theoretical transverse reinforcement ratio to force the
 13 web-flange longitudinal shear failure and thus analyse the contribution of the fibres.
 14 The EC2 [2] was applied to determine the minimum required transverse reinforcement
 15 in the flanges, by choosing the maximum possible plastic redistribution length and the
 16 lowest strut inclination angle according to EC-2 [2]. According to section 6.2.4 (3) of
 17 EC2 [2], the maximum value of the plastic length that can be considered is half the
 18 distance between the zero bending moment and the maximum bending moment (750
 19 mm). This value cannot be greater than the distance between loading points. If a
 20 greater plastic redistribution length was chosen, the mean web-flange longitudinal

1 shear would be smaller. In the absence of more rigorous studies, section 6.2.4 (4) of
2 EC2 [2] provides a range for the strut inclination angle (θ_f) in the compressed flanges: 1
3 $\leq \cot \theta_f \leq 2$. If $\cot \theta_f = 2$ ($\theta_f = 26.5^\circ$) is adopted, the minimum value for the flange
4 transverse reinforcement is obtained. The theoretical ultimate moment of the T-beam
5 cross section was calculated according to the EC2 [2] for plain concrete beams. This
6 moment was determined by adopting the parabola-rectangle law for concrete, the
7 elasto-plastic law for reinforcement and the classic hypothesis of sectional analysis: (1)
8 plane sections remain plane; (2) the strain in the bonded reinforcement is the same as
9 that in the surrounding concrete; (3) limit strains are defined by EC2 [2] for the ultimate
10 limit state: reinforcing steel tension strain limit $\varepsilon_{ud} = 10\text{‰}$ taken from EHE-08 [1];
11 concrete compression strain limit $\varepsilon_{cu2} = 3.5\text{‰}$ concrete pure compression strain limit
12 $\varepsilon_{c2} = 2.0\text{‰}$. The flange effective width according to EC2 [2] was considered to
13 calculate the ultimate bending moment. The flange effective width coincides with the
14 actual width of 1200 mm for the T-beam cross section geometry and the distance
15 between points of zero moment of 4000 mm. A minimum amount of 568 mm²/m was
16 determined for the transverse reinforcement ratio. The following four amounts of
17 transverse reinforcement (A_{sf} in Figure 5.a) were established based on the minimum
18 amount: 87.5 mm²/m (7Ø8 in 4 m), 179.5 mm²/m (Ø8/280 mm), 359 mm²/m (Ø8/140
19 mm) and 561 mm²/m (Ø10/140 mm). These amounts represent 15.5, 31.6, 63.2 and
20 98.8% of the minimum reinforcement amount. No beam with zero transverse
21 reinforcement was analysed as there was a minimum of 7Ø8 bars, which would be
22 used to stick on strain gauges to measure flange transverse strains. The minimum fibre
23 content was established as the minimum for the concrete to be considered FRC for
24 structural purposes. The minimum content was 20 kg/m³ according to EHE-08 [1] and
25 approximately matched the 0.3% of CNR-DT 204 [31]. A maximum steel fibre content
26 of the 40 kg/m³ was established by manufacturer's recommendations, and an
27 intermediate value corresponding to 30 kg/m³ was also chosen. According to the

1 recommendations of the Model Code 2010 [4], a FRC was designed for a toughness
2 class 1.5d.

3 Table 2 shows the details of the 13 specimens. The difference between them lay in the
4 steel fibre content (4 made with conventional concrete and 9 with FRC) and in the
5 flange transversal reinforcement area (A_{sf}). Four families of beams were established.
6 The steel fibre content varied within each family, except for specimen V4-0, which
7 consisted of a single reinforced concrete beam that was adopted as a reference beam.

Family	Flange transverse reinforcement area (A_{sf})	Steel fibre content kg/m ³			
		0	20	30	40
V1	87.5 mm ² /m [7Ø8 in 4 m]	V1-0	V1-20	V1-30	V1-40
V2	179,5 mm ² /m [Ø8/280mm]	V2-0	V2-20	V2-30	V2-40
V3	359,0 mm ² /m [Ø8/140mm]	V3-0	V3-20	V3-30	V3-40
V4	561,0 mm ² /m [Ø10/140mm]	V4-0	-	-	-

8 *Table 2: Specimen notation.*

9 Figure 7 displays the distribution of the flange transverse reinforcements. This
10 distribution corresponded to the assembly indicated in Table 2. The web-flange
11 longitudinal shear is theoretically zero in the central part of the beam, between the
12 loading points. Nevertheless, a quantity of flange transverse reinforcement slightly
13 lower than that at the beam ends was arranged.

14 2.2. Material characterisation.

15 A nominal compressive concrete strength of 25 MPa was set. This strength is
16 representative of most buildings and construction members. Dose consisted of 1100
17 kg/m³ of sand 0/4, 880 kg/m³ of gravel 6/12, 300 kg/m³ of cement 42.5R, 1.8 l/m³ of
18 superplasticiser additive ACE 425 of BASF and 150 l/m³ of water. As mentioned above,
19 the steel fibre contents were 0, 20, 30 and 40 kg/m³. The type of steel fibres was BASF
20 Masterfiber 530, whose yield stress was higher than 3000 MPa, 30 mm long, 0.35 mm
21 in diameter, with a slenderness of 86 and an elastic modulus of 190 GPa.

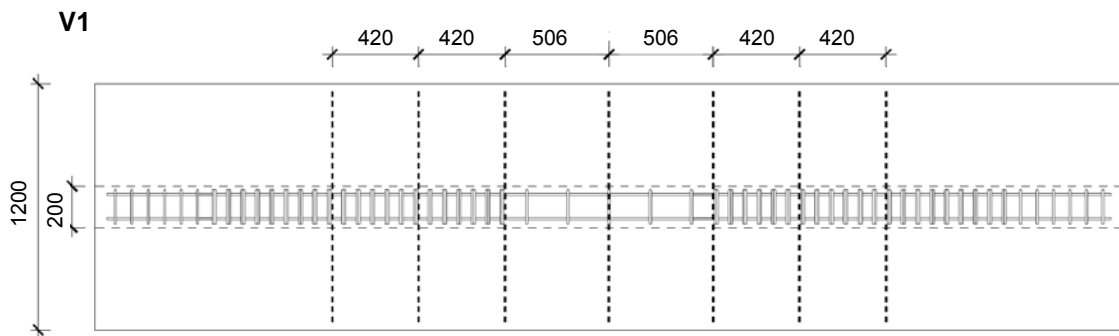
22 Concrete compressive strength was obtained by testing three cylindrical 150×300 mm
23 test pieces of each specimen according to UNE-EN 12390-3:2009 [64]. The average
24 value ($\bar{\sigma}$) and coefficient of variation (CV) of the concrete compressive strength are

1 shown in Table 3, where f_{cm} is the mean compressive strength. The time from
2 manufacturing to testing is also depicted in Table 3 (concrete age in days). These
3 values were lower than expected because the concrete mixer had a minimum volume
4 of 1.5 m³ to guarantee correct mixing, but only 1 m³, corresponding to each beam, was
5 used.

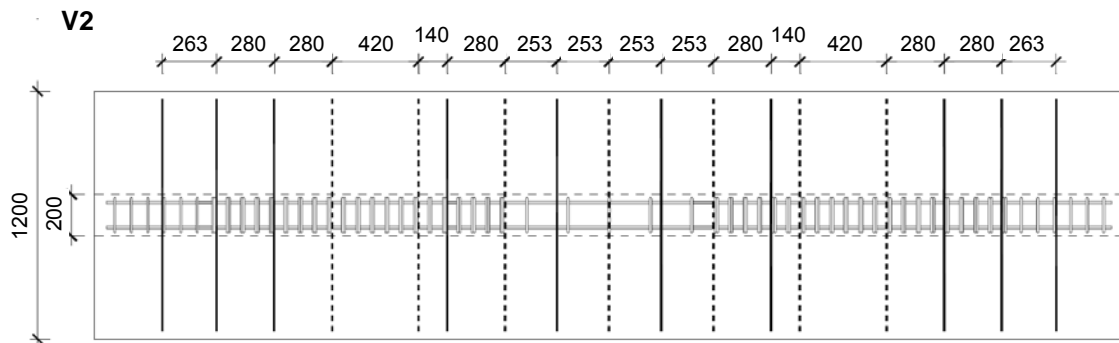
6 Apart from the cylindrical specimens, three prismatic specimens, measuring
7 150×150×580 mm, were obtained from each concrete mixture to test flexural tensile
8 strength according to UNE-EN 14651 [65]. The average value ($\bar{\sigma}$) and coefficient of
9 variation (CV) of the flexural tensile strength are shown in Table 3, where f_{LOP} is the
10 limit of proportionality in the flexural tensile strength test and $f_{R,j}$ (for j =1–4)
11 corresponds to crack mouth opening displacements (CMOD) of 0.5, 1.5, 2.5 and 3.5
12 mm, respectively. The distance between the bottom of the specimen and the
13 displacement transducer used to measure the CMOD was 6 mm. This value was
14 slightly higher than the maximum value recommended by UNE-EN 14651 [65]. To
15 correct the error, the proposal of Ferreira et al. [66] valid for distances between 0 and
16 10 mm was employed. These authors proposed correction factors that depended on
17 the relative height of the crack. A more detailed description can be found in [56] about
18 the used correction and the obtained results (load-CMOD diagrams).

Specimen	Age (days)	f_{cm} (MPa)		f_{LOP} (MPa)		$f_{R,1}$ (MPa)		$f_{R,2}$ (MPa)		$f_{R,3}$ (MPa)		$f_{R,4}$ (MPa)	
		$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)
V1-0	29	19.7	2.0	3.3	6.5	-	-	-	-	-	-	-	v
V2-0	28	20.7	9.1	3.1	8.0	-	-	-	-	-	-	-	-
V3-0	28	20.6	14.8	3.1	3.2	-	-	-	-	-	-	-	-
V4-0	27	20.2	2.8	3.1	3.8	-	-	-	-	-	-	-	-
V1-20	36	19.0	9.2	3.1	6.5	1.0	20.0	1.1	23.6	1.1	18.4	1.1	18.4
V2-20	28	19.9	1.0	3.1	1.6	1.0	32.9	1.1	31.0	1.1	31.0	1.1	36.4
V3-20	33	21.0	11.7	3.5	1.6	1.0	10.0	1.1	5.4	1.1	5.4	1.1	5.4
V1-30	30	23.6	9.2	3.5	4.7	1.9	14.9	2.1	14.8	2.2	12.0	2.1	9.5
V2-30	29	20.5	1.7	3.3	0.0	1.9	30.0	2.0	33.1	2.0	33.1	1.9	31.6
V3-30	34	22.4	4.2	2.9	6.1	1.5	4.9	1.6	9.4	1.6	13.7	1.5	18.9
V1-40	39	21.2	3.4	3.3	2.9	2.1	20.1	2.4	20.8	2.5	20.0	2.4	18.2
V2-40	41	22.0	5.1	3.5	8.8	2.8	11.0	2.9	8.8	2.8	7.3	2.8	5.5
V3-40	40	19.0	8.5	3.2	6.5	2.5	11.3	2.9	9.8	2.8	12.4	2.7	10.5

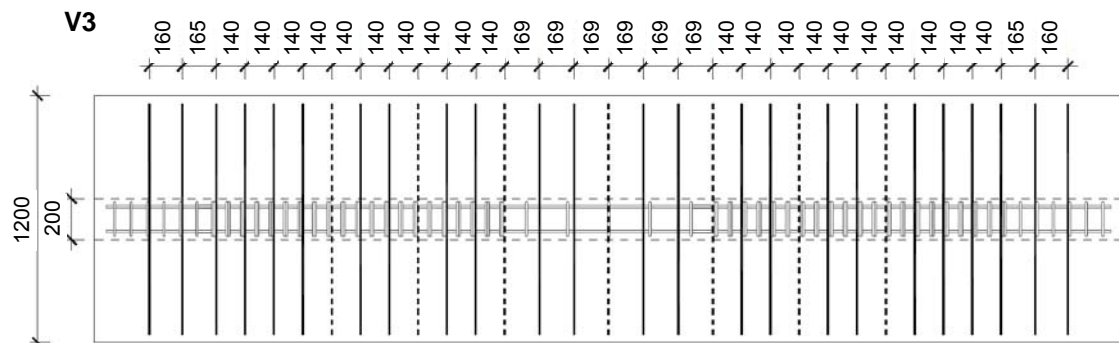
Table 3: Mechanical properties of concrete.



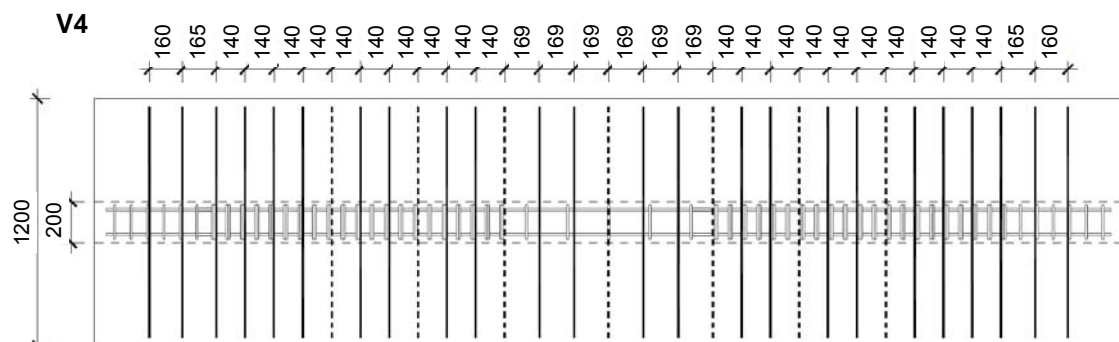
(a)



(b)



(c)



(d)

1

2 **Figure 7: Flange transverse reinforcement arrangement: (a) Family V1; (b) Family V2; (c) Family**
 3 **V3; (d) Family V4. Units: mm.**

1 The steel used in all the beam reinforcements was B500SD. Three bar samples were
 2 selected from each diameter before being tested according to the procedure
 3 established in UNE-EN 10002-1 [67]. The average value ($\bar{\sigma}$) and coefficient of variation
 4 (CV) of the mechanical properties of reinforcements are shown in Table 4. In this table,
 5 f_y , ε_y , f_{sh} , ε_{sh} , f_u , ε_u , E_s are respectively the yield stress, the strain that corresponded
 6 to the yield stress, the stress at which the hardening branch began, the strain
 7 associated with f_{sh} , the maximum stress, the strain associated with the maximum
 8 stress and the elasticity modulus.

9 The beams were horizontally in inverted position and on an externally-vibrated form.

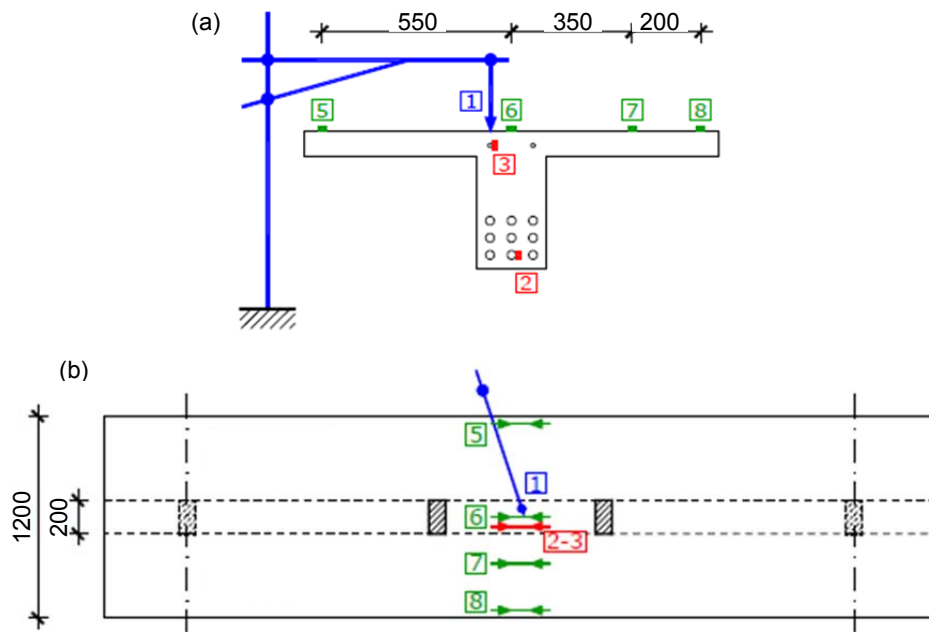
	Longitudinal				Transverse			
	Ø12		Ø25		Ø8		Ø10	
	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)	$\bar{\sigma}$ (MPa)	CV (%)
f_y (MPa)	552	1.3	552	1.1	592	1.4	531	1.2
ε_y	0.0026	3.8	0.0028	3.7	0.0027	3.6	0.0026	4.0
ε_{sh}	0.0300	3.2	0.0235	4.1	0.0200	2.3	0.0375	5.3
f_u (MPa)	658	0.6	660	0.7	662	0.7	638	0.7
ε_u	0.1220	2.7	0.1041	3.1	0.0412	0.1	0.1671	5.2
E_s (MPa)	209685	4.1	195414	5.3	223320	7.0	206180	4.2

10 Table 4: Mechanical properties of reinforcements.

11 2.3. Instrumentation.

12 The vertical load was applied by the hydraulic actuator and was controlled by a 1000kN
 13 load cell. A LVDT was arranged in the central section of each specimen to measure
 14 vertical displacement (LVDT-1, see Figure 8). Six strain gauges were arranged in the
 15 central section of the specimens: two on the longitudinal steel reinforcements (channel
 16 2 and 3, Figure 8) located on the most tensioned and most compressed bars of the
 17 section, and four of them were placed on concrete in the longitudinal direction of the
 18 beam, across the flanges (channel 5-8, Figure 8). In addition, seven strain gauges
 19 were arranged on the transverse reinforcements located on the flanges at the web-
 20 flange junction (channel 9-15, Figure 9).

21
 22
 23

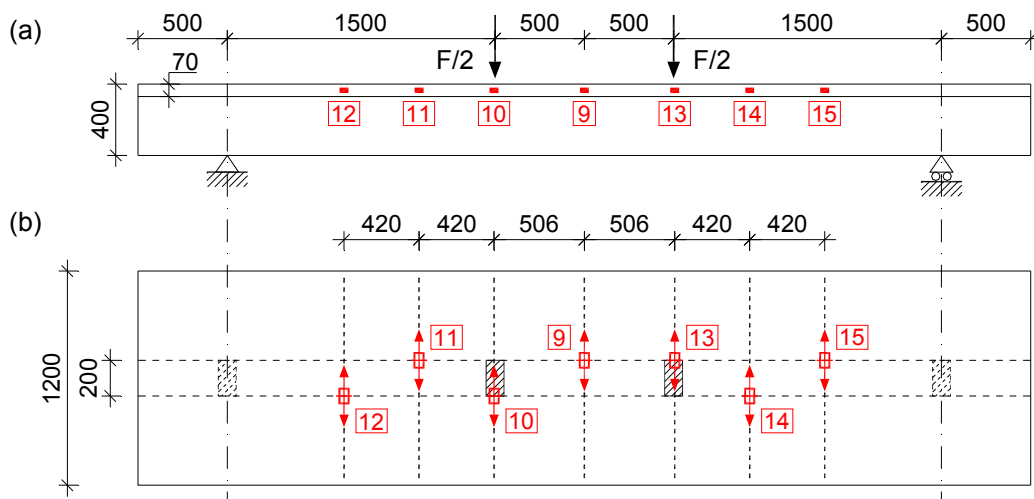


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Figure 8: LVDT to measure displacement at midspan, strain gauges on concrete top side and strain gauges on longitudinal reinforcements. (a) Front view, (b) Plan view. Units: mm.



4

5

Figure 9: Strain gauges on flange transverse reinforcements: (a) Front view, (b) Plan view. Units: mm.

6

Web-flange junction is subjected to both tangential stresses (τ_{xy}) and transverse axial

7

stresses (σ_y), see Figure 1.b. These stresses act simultaneously with the longitudinal

8

sectional forces (bending moment and vertical shear). According to Razaqpur and

9

Ghali [3], the transverse axial stress (σ_y) in the flange is compressive in the vicinity of

10

the supports, at a distance of less than 10% of span length, for the loading scheme of

11

tests. This compressive axial stress is resisted by concrete [3]. According to these

1 authors [3], the transverse axial stress in the flange (σ_y) is tensile in the rest of the
2 span, and is maximum under the loading point. Consequently, the strain gauges
3 numbered from 9 to 15 (see Figure 9) were arranged along the beam in the transverse
4 reinforcements, without covering the length around the supports. This was achieved by
5 using 7Ø8 in all the beams, except in reference beam V4-0 which had 7Ø10.

6 **3. Experimental results.**

7 The experimental results corresponding to the 13 tested T-beams are described in this
8 section.

9 **3.1. Strength capacity and failure modes.**

10 Table 5 summarises the main results. It shows the maximum load F_u and the maximum
11 experimental bending moment in central section $M_{E,max}$ of all the tested specimens. As
12 expected, the higher the transverse reinforcement ratio, the greater the strength
13 capacity of the beams. In general, a higher steel fibre content in the concrete mass
14 improved the strength capacity of beams, except for the VN-40 type beams. This was
15 due to possible errors while manufacturing beams, perhaps because the concrete
16 mixture was not homogeneous.

17 **3.2. Strain plane of the central section.**

18 Table 5 includes the midspan longitudinal bar strains called ε_{s2} for the most tensioned
19 bars (from channel 2 in Figure 8), and ε_{s3} for the most compressed bars (from channel
20 3 in Figure 8) for the maximum load situation. Table 5 also includes the strain on the
21 most compressed edge of section ε_{c6} (from channel 6 in Figure 8) and the depth of
22 neutral fibre x calculated from strains ε_{s2} and ε_{s3} . It was not possible to determine
23 depth x in four beams (V2-20, V3-20, V2-30 and V3-30) as gauges failed before
24 reaching the maximum load. As seen, depth x was greater than flange thickness (70
25 mm) in all cases and, consequently, the entire flange was compressed. In addition, the
26 most tensioned reinforcements were yielded in six of the 13 beams (strain greater than
27 2.8 ‰). The most tensioned reinforcements came close to the yield in the other beams.

Beam	F_u (kN)	$M_{E,max}$ (kNm)	$\varepsilon_{s2}[^*]$ (‰)	$\varepsilon_{s3}[^*]$ (‰)	$\varepsilon_{c6}[^*]$ (‰)	x (mm)	Failure mode
V1-0	375.0	281.2	-1.986	1.385	0.828	171.4	Web-flange shear V
V2-0	506.2	379.7	-3.552	2.215	2.918	163.5	Web-flange shear H
V3-0	533.0	399.7	-2.857	1.223	1.826	133.3	Web-flange shear H
V4-0	536.2	402.2	-4.235	1.453	2.293	120.1	Vertical shear H
V1-20	455.9	341.9	-2.533	2.378	1.605	175.3	Web-flange shear V
V2-20	589.1	441.8	-1.240 [**] ($M_E=194.9$)	3.848	1.871	-	Web-flange shear H
V3-20	691.2	518.4	-6.372 [**] ($M_E=480.6$)	5.736 [**] ($M_E=515.4$)	2.919	-	Bending with longitudinal shear cracking
V1-30	546.9	410.2	-5.120	3.565	2.841	153.3	Web-flange shear V
V2-30	634.6	475.9	-3.636	2.201 [**] ($M_E=428.4$)	2.322	-	Bending
V3-30	663.5	497.6	-4.493	1.611 [**] ($M_E=465.0$)	[*]	-	Bending with longitudinal shear cracking
V1-40	494.9	371.1	-2.421	1.084	1.440	136.2	Vertical shear H
V2-40	530.9	398.2	-2.749	1.520	1.671	137.1	Bending
V3-40	496.3	372.2	-2.671	1.446	1.844	135.7	Bending

[*] Compression strain positive.

[**] Failure of strain gauges. The last recorded value for the moment M_E is displayed.

1 *Table 5: General results in the ultimate state.*

2 3.3. Failure modes.

3 Table 5 shows the five failure types identified in this experimental programme: the
4 longitudinal shear between the flange and web according to the vertical plane of the
5 web-flange junction (Web-flange shear V in Table 5) (Figure 10.a-b); longitudinal shear
6 according to a horizontal surface tangent to stirrups (Web-flange shear H in Table 5)
7 (Figure 10.a-c); bending with concrete crushing and the local buckling of the
8 compressed reinforcements (Bending in Table 5) (Figure 10.d-f); bending with concrete
9 crushing with an advanced state of longitudinal cracking due to the shear between the
10 flange and web (Bending with longitudinal shear cracking in Table 5) (Figure 10.g-h);
11 shear in the web because of concrete crushing due to the presence of hollows in
12 concrete (Vertical shear H in Table 5) (Figure 10.i).

13 In general, longitudinal cracking occurred at the web-flange junctions. This cracking
14 first appeared on the bottom edge of the encounter. Diagonal cracking was also
15 produced with an inclination angle of 45° in relation to the beam's longitudinal axis
16 (Figure 10.h). Cracking formation followed a double transversal and longitudinal

1 symmetry pattern, but failure was not symmetrical. Only one of the four possible flange
2 parts separated from the core.

3 The web-flange longitudinal shear failure occurred in 6 out of 13 beams. The failure
4 plane was vertical in three specimens (V1-0, V1-20 and V1-30) according to the vertical
5 plane (1-2) in Figure 10.a and b. The failure plane was horizontal in the other 3
6 specimens (V2-0, V3-0 and V2-20) according to the horizontal plane (1-3) in Figure
7 10.a and c. The horizontal failure plane occurred in the first beams that were tested,
8 because the employed geometrical concrete cover was excessive in comparison with
9 the flange thickness (25/30 mm vs 70 mm). When using 30 mm separators on the
10 stirrups, the web transverse reinforcement remained in the lower half of the flange
11 thickness, which was 70 mm. The legs of the transverse reinforcement of the web
12 introduced an abnormal failure plane, disturbing significantly the development of the
13 other failure mechanism. The concrete cover was reduced to 12 mm, which resulted in
14 two more web-flange longitudinal shear failures (V1-20 and V1-30) according to a
15 vertical surface type 1-2 (Figure 10.a). By reducing the coating, web-flange longitudinal
16 shear in plane 1-3 decreased.

17 Three beams failed due to the longitudinal shear between the flange and web
18 according to the vertical plane of the web-flange junction (web-flange shear V in Table
19 5). This failure mode was observed in beams V1-0, V1-20 and V1-30, which had the
20 lowest flange transverse reinforcement ratio (15.5% of the minimum transverse
21 reinforcement ratio) and had a steel fibre content lower or equal to 30 kg/m³. In ultimate
22 situation, the flange longitudinal and inclined cracking were very visible and the relative
23 displacement between the flange and the web was important (Figure 10.b). The
24 separation between the flange and the web took place in beam V1-0, which has no
25 steel fibres, and some transverse bars broke. The insufficient fibre content together
26 with a very small flange transverse reinforcement ratio resulted in the loss of integrity of
27 the section.

1 The failure mode due to a longitudinal shear according to horizontal surface tangent to
2 stirrups (Web-flange shear H in Table 5) occurred in three specimens: beams V2-0 and
3 V3-0, which had a geometrical concrete cover of 25 or 30 mm, a flange transverse
4 reinforcement ratio lower than required (31.6% and 63.2%) and no steel fibres and in
5 beam V2-0 with a 31.6% of the required transverse reinforcement ratio and with a low
6 fibre content (20 kg/m^3). In these cases, a relative web-flange displacement was
7 observed. The entire flange separated from the web in the case of beam V2-0 with no
8 steel fibres (Figure 10.c). The insufficient combination of the flange transverse
9 reinforcement and the steel fibres content in addition to the excessive geometrical
10 concrete cover meant the integrity loss of the cross section.

11 The bending failure mode (Bending in Table 5) occurred in three specimens: V2-40,
12 V3-40 and V2-30. The beams V2-40 and V3-40 had higher fibre content (40 kg/m^3) and
13 a transverse reinforcement ratio in the flanges lower than required (31.6% and 63.2%).
14 The beam V2-30 had 31.6% of the required transverse reinforcement ratio and 30
15 kg/m^3 of fibre content. The longitudinal and inclined cracking network in the flanges
16 was difficult to observe. In these specimens, a combination of fibre content (greater
17 than or equal to 30 kg/m^3) and a flange transverse reinforcement ratio were sufficient to
18 control the crack width and ensure the integrity of the section. In general, the failure
19 mode was brittle due to concrete crushing in the loading section (Figure 10.d). The
20 tensioned longitudinal reinforcement remained in the elastic range, except in beam V2-
21 30. In addition, the compressed reinforcement buckled when the concrete cover
22 spalled (Figure 10.e and f).

23 Two beams underwent bending failure with longitudinal shear cracking (Bending with
24 longitudinal shear cracking in Table 5). This failure occurred in beams V3-20 and V3-30,
25 which had 60% of the required flange transverse reinforcement ratio and a fibre content
26 of 20 kg/m^3 and 30 kg/m^3 . This failure corresponded to an intermediate situation
27 between bending failure and web-flange longitudinal shear failure. In this case, the

1 longitudinal and inclined cracking network in the flanges was visible (Figure 10.g). The
2 combination of transverse reinforcement with the steel fibres was sufficient to control
3 the crack width and the integrity of the section. There was not any relative web-flange
4 displacement either. The failure was due to concrete crushing in the loading section
5 (Figure 10.g), with longitudinal reinforcement yielding and with a significant web-flange
6 longitudinal shear cracking (Figure 10.h).

7 Finally, an unexpected failure by vertical shear occurred in beams V1-40 and V4-0 due
8 to the presence of the hollows in the web. The cause of these hollows was a poor
9 concrete compaction (Vertical shear H in Table 5). The presence of hollows reduced
10 both the strength capacity of the struts and the adherence of the longitudinal
11 reinforcements. This combined effect can be observed in the web in Figure 10.i.

12 In short, the longitudinal shear between flanges and web failure took place in the
13 beams with less transverse reinforcement (V1-0, V1-20 and V1-30), except in beam
14 V1-40. This exception was due to lack of concrete compaction and the consequent
15 vertical shear failure. The bending failure mode went from longitudinal shear to bending
16 in two situations: (1) in the specimens with a 31.6% transverse reinforcement ratio of
17 the required one and a fibre content of 30 kg/m³ or higher; (2) in the specimens with a
18 63.2% transverse reinforcement ratio of the required one and a minimum fibre content
19 of 20 kg/m³. The proper combination of transverse reinforcement and fibre content in
20 these beams was sufficient to control the longitudinal cracking in the web-flange
21 junction and ensure the integrity of the section. In these beams, there was a failure
22 mode by concrete crushing (Figure 10.d and g), with or without visible web-flange
23 junction cracking. The incorporation of steel fibres into concrete not only increased
24 strength capacity, but also modified the failure mode, from web-flange longitudinal
25 shear to bending.

26

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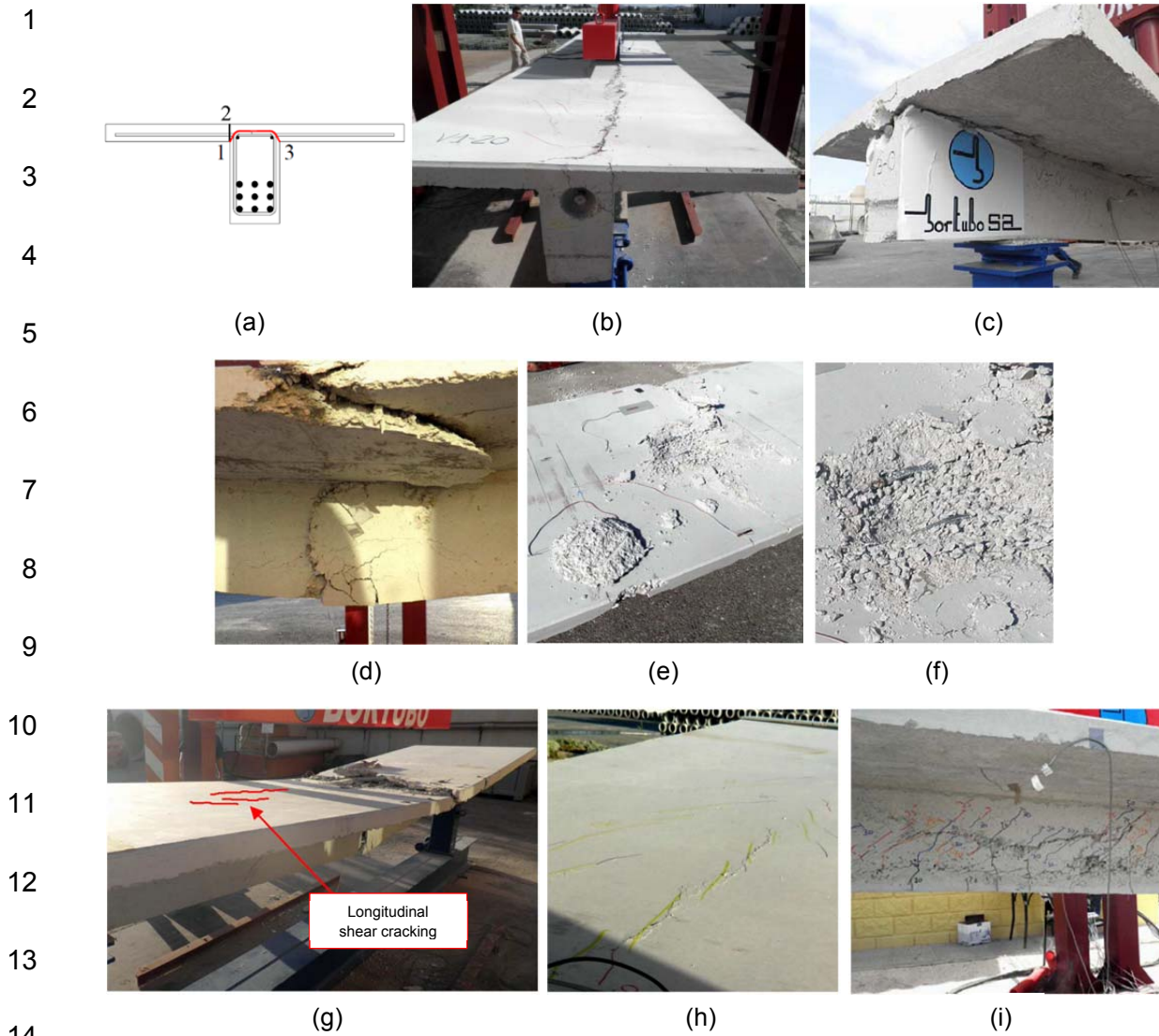


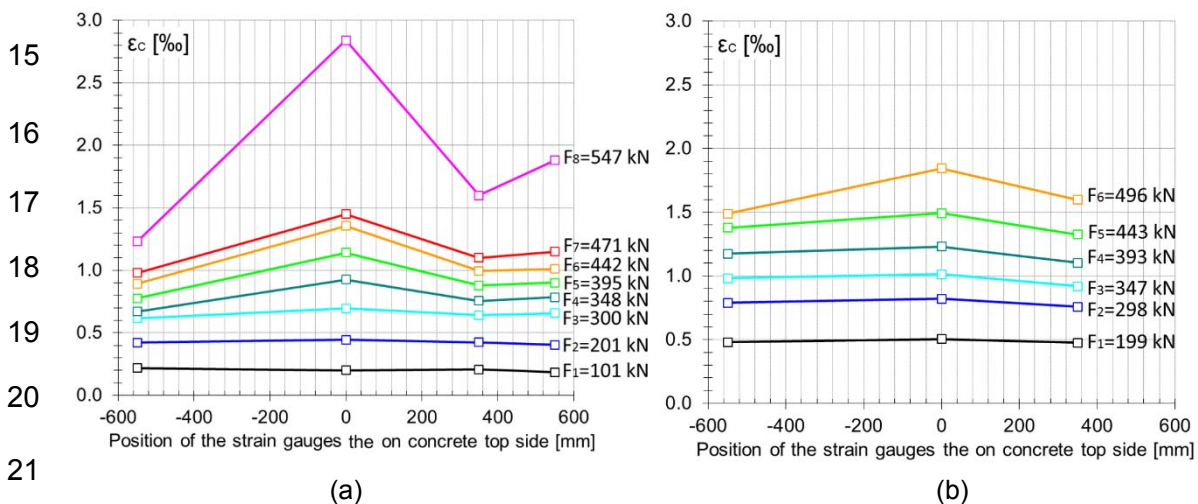
Figure 10: Specimen failure modes: (a) Possible web-flange longitudinal shear failure planes: 1-2 vertical plane, 1-3 horizontal plane; (b) Web-flange longitudinal shear H (V3-0); (c) Web-flange longitudinal shear V (V1-20); (d) Bending failure (V2-30): web and flange bottom part view; (e) Bending failure (V2-30): flange top side view (concrete crushing); (f) Bending failure (V2-30): compressed reinforcement buckling detail; (g) Bending with longitudinal shear cracking (V3-20), general post-test specimen view; (h) Bending with longitudinal shear cracking (V3-20), detail of shear cracks on the flange top side; (i) Vertical shear H, due to the presence of hollows in concrete (V1-40).

3.4. Flange longitudinal strains.

Flange compressive longitudinal strains (ϵ_{ci} , where i is the number of the channel) could only be measured at four points (channel 5-8, Figure 8). This information was useful in order to verify if the whole flange width was effective to validate the classic sectional analysis. According to the formulation of EC2 [2], the theoretical effective width coincided exactly with the actual width. However, the tested beams had the

1 particularity of possessing a deficient transverse reinforcement area to bear
 2 longitudinal shear, which can be partially compensated with steel fibres.

3 **Figure 11** represents the flange longitudinal strain profile for different load levels (F_j)
 4 across flanges. The recorded values are only displayed until the beam's maximum load.
 5 The two cases in **Figure 11** reflect the general behaviour of the 13 tested beams. A
 6 reduction in the effective width value was observed as the load level increased in the
 7 beams with either a smaller transverse reinforcement area or a lower steel fibre content.
 8 However, codes like **EC2 [68]** affirm that effective width is the same in both the service
 9 limit and ultimate limit states, as explained by Hendy and Smith [69]. This is no
 10 contradiction as the explanation lies in the deficient transverse reinforcement area. In
 11 general, a decrease in effective width began when longitudinal web-flange cracking
 12 appeared, just as the increase in the strain at the web-flange junction showed (**Figure**
 13 **11**). In general, the family of VN-40 beams were the least affected, as were the beams
 14 with the biggest transverse reinforcement area of each family.



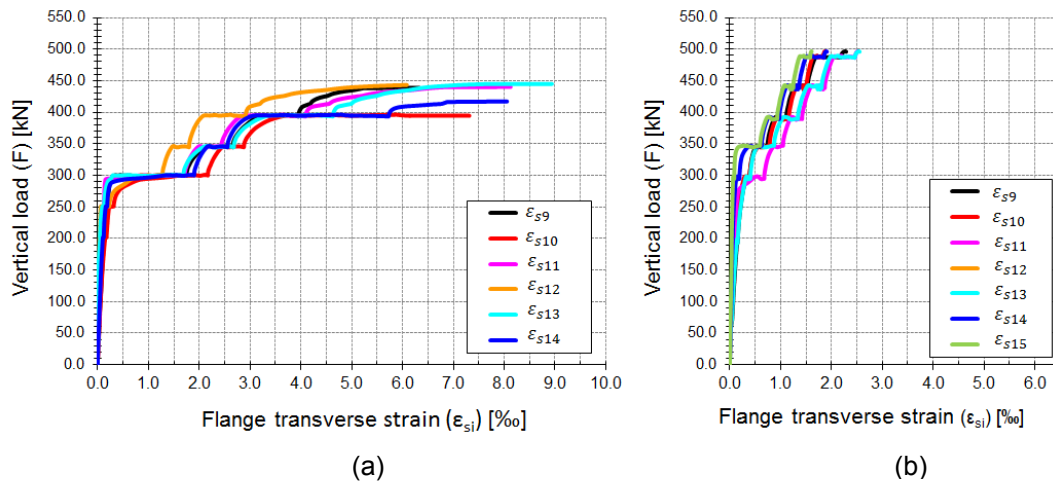
22 *Figure 11: Longitudinal strains across the flange: (a) Web-flange longitudinal shear V failure*
 23 *(beam V1-30), (b) Bending failure (beam V3-40). [Compressive strain ϵ_c (> 0)].*

24 3.5. Flange transverse strains.

25 Flange transverse strains were measured thanks to the arrangement of the seven
 26 strain gauges in the transverse reinforcements (channel 9-15, **Figure 9**). **Figure 12**
 27 represents the strain evolution of each gauge in relation to the total load applied to the

1 beam, where ε_{si} is the strain recorded by the strain gauge corresponding to channel-i
2 (see Figure 9). Two beams are shown as representative examples of flange-web
3 longitudinal shear failure according to a vertical plane and bending failures: beams V1-
4 20 (Figure 12.a) and V3-40 (Figure 12.b), respectively. The leaps that are seen in the
5 strains are due to the load increase was stopped to observe the cracking. The first
6 zone of load – flange transverse strain diagram is lineal in both cases. A jump was
7 perceived in most of the gauges of beam V1-20 at around a load value of 300 kN
8 (Figure 12.a), while this was perceived for only one gauge in beam V3-40 and to a
9 lesser extent (Figure 12.b). The jump was followed by a higher growth rate in the strain
10 value. This jump could only be justified by the increase in the transverse reinforcement
11 tension level when cracking took place, which was more marked in those flanges with
12 either a smaller transverse reinforcement area or fewer steel fibre content. Figure 12
13 shows how in beam V1-20, which has a smaller amount of transversal reinforcement
14 and fibre content, undergoes a greater strain increase than beam V3-40, which has a
15 higher amount of transversal reinforcement and fibre content.

16 Figure 12.a also shows how the gauge recordings of the V1-20 transverse
17 reinforcements shot up at values close to the maximum load. This was due to
18 transverse reinforcement yielding. Gauges ε_{s10} , ε_{s14} and ε_{s15} failed before the
19 maximum load was reached. Figure 12.b displays how the transverse reinforcement of
20 specimen V1-30 did not yield because the higher recorded strain was 2.5‰, which was
21 less than the yield strain (2.7‰). Figure 13 shows the state of one of the flange
22 transverse reinforcements of beam V1-0.



1 **Figure 12: Vertical load – flange transverse strain diagrams: (a) Beam V1-20 (web-flange**
 2 **longitudinal shear V failure); (b) Beam V3-40 (bending failure). [Tensile strain ϵ_{si} (> 0)].**



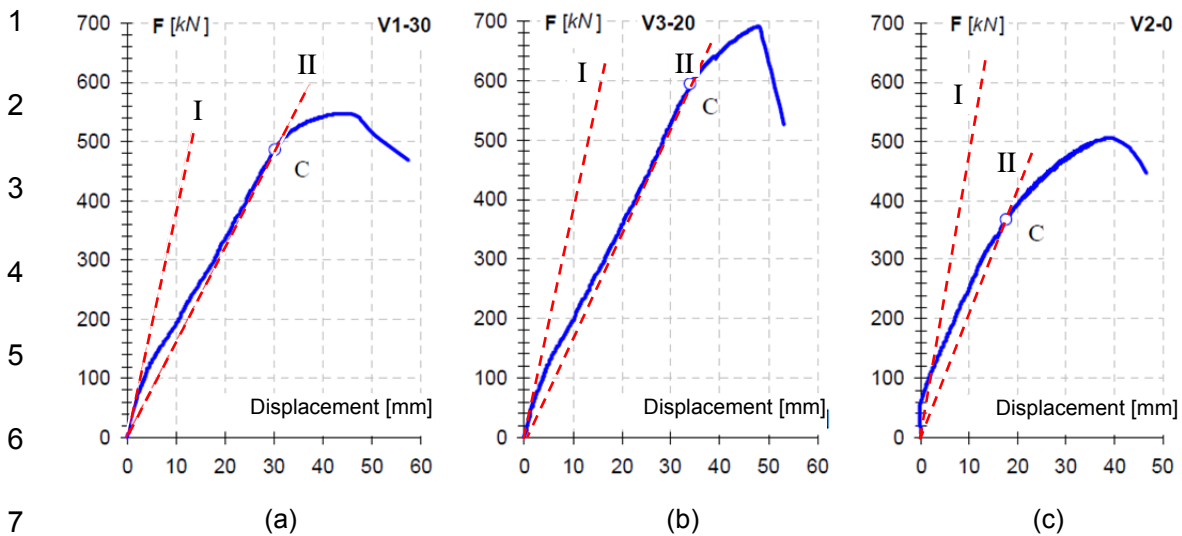
3 **Figure 13: Final appearance of a flange transverse reinforcement, beam V1-0: (a) The**
 4 **remaining longest part of a reinforcement that broke, approximately 200 mm over the web plus**
 5 **450 mm corresponding to the flange at one-side; (b) Detail of the strain gauge location zone.**

8 3.6. Vertical load – displacement.

9 **Figure 14** shows three vertical load - midspan displacement diagrams corresponding to
 10 three beams with different failure modes, which are representative of the other beams.

11 **More detailed information of the experimental results can be found in [56].**

12 These diagrams have a first initial region with a more marked slope (**Figure 14.a, slope**
 13 **I**), which corresponds to the uncracked elastic state, followed by another region that
 14 was almost rectilinear with a less pronounced slope, which corresponded to the
 15 cracked elastic state (**Figure 14.a, slope II**). The completion of this second rectilinear
 16 region occurred when one of the failure modes started, as noted in **Table 5**. **In all cases,**
 17 **the failure mode is brittle, since there is no plastic deformation before reaching the**
 18 **ultimate load.**



8 *Figure 14: Vertical load-displacement in the midspan diagrams: (a) Specimen V1-30, the web-*
 9 *flange longitudinal shear failure according to a vertical plane; (b) Specimen V3-20, bending*
 10 *failure with longitudinal shear cracking; (c) Specimen V2-0, web-flange longitudinal shear failure*
 11 *according to a horizontal plane.*

12 Point C is indicated in Figure 14 and corresponds to loss of linearity. Point C is
 13 identified with two phenomena for beam V1-30 (Figure 14.a), which failed by
 14 longitudinal shear between the flange and web along a vertical plane: the beginning of
 15 the tensile longitudinal reinforcement yielding and the beginning of the flange
 16 transverse reinforcement yield. Point C is identified with tensile longitudinal
 17 reinforcement yielding in beam V3-20 (Figure 14.b), which failed by bending with
 18 concrete crushing, while the flange transverse reinforcement strains continued to grow
 19 (no sudden growth was recorded, as in the previous case). Finally, point C corresponds
 20 to a sudden strain jump of more than 1‰ in four of the seven strain gauges located at
 21 the flange reinforcements in beam V2-0 (Figure 14.c), which failed by longitudinal
 22 shear between flanges and the web along a horizontal plane. In this case, the most
 23 tensioned longitudinal reinforcements were not yielded.

24 4. Analysis of the experimental results.

25 Both the web-flange longitudinal shear cracking vertical load and the mean web-flange
 26 longitudinal shear in the maximum load situation are determined in this section

27 4.1. Web-flange longitudinal shear cracking vertical load.

1 In order to theoretically calculate the cracking vertical load ($F_{f,crack}$) corresponding to
 2 longitudinal shear of the web-flange junction, flange effective width was determined
 3 with the equations of the following codes: EC2 [2], EHE-08 [1] and EH-91[70], based
 4 on the Brendel [71] proposal (Table 6). The EC2 [2] equation, which coincides with CM
 5 2010 [4], provides the highest value. The effective width b_{eff} proposed by the EC2 [2]
 6 (1200 mm) for the determination of the vertical load $F_{f,crack}$ has been adopted for the
 7 following reasons: (1) According to Hendy and Smith [72], the value of the effective
 8 width proposed by the EC-2 [2] approximates the elastic values; (2) The distribution of
 9 the longitudinal strain across the flange is uniform for loads lower than the vertical load
 10 of experimental cracking $F_{f,crack}$ (Figure 11). No equation contemplated the
 11 dependence of the effective width with the flange transverse reinforcement ratio.

Effective width b_{eff} (mm)	All limit states
EC2 [2] and CM 2010 [4]	1200
EHE-08 [1]	1000
EH-91 [70] (Brendel [71])	880

12 *Table 6: Flange effective width according to several codes and authors.*

13 Longitudinal shear cracking occurred for a vertical load value that came close to 300
 14 kN in all the experimental tests. This load was determined from a jump in the strains
 15 recorded by the gauges located in the flange transverse reinforcements and also
 16 visually (Section 3.5).

17 The cracking vertical load $F_{f,crack}$ of the web-flange junction is assessed in a similar
 18 way to Regan and Placas [18] and Tizatto [20]. Unlike these authors, the formulation
 19 proposed by Razaqpur and Ghali [3] was used to calculate the transverse axial force
 20 N_y and the variation of the normal stresses was considered according to the flange
 21 thickness. The approach to the problem is illustrated in Figure 15. The problem was
 22 simplified by symmetry (Figure 15.a). The ABCD flange region was delimited (Figure
 23 15.b). The membrane stresses applied to the flange are represented (Figure 15.c-e),
 24 based on the applied load.

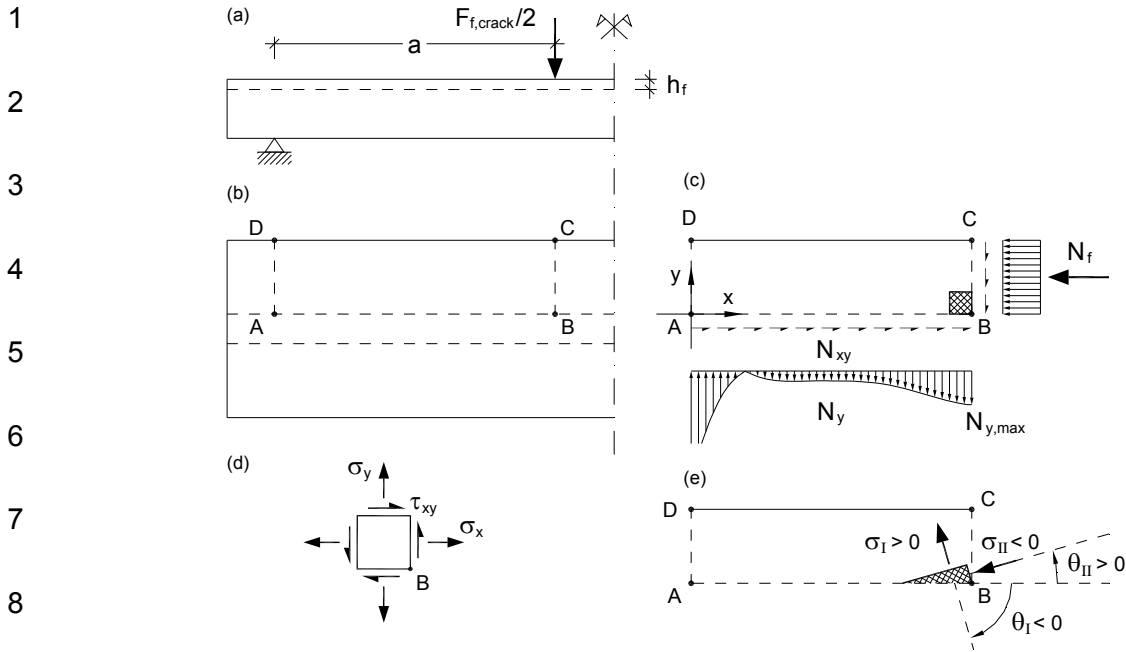


Figure 15: Scheme to determine the web-flange longitudinal shear cracking vertical load ($F_{f,crack}$): (a) Frontal view; (b) Plan view; (c) Flange stresses on ABCD; (d) Stresses on B; (e) Principal stresses on B.

Point B, located under the load application point, was the point at which the maximum transverse tensile force per unit length $N_{y,max}$ was produced together with a maximum value for N_f . The sign criterion for stresses is schematised in Figure 15.d. The principal directions are illustrated in Figure 15.e, where σ_I and σ_{II} are the principal tensile and compressive stresses, respectively.

The criterion for evaluating load $F_{f,crack}$ was to establish the principal tensile stress at point B (Figure 15) that equalled the mean concrete tensile strength (f_{ctm}):

$$\sigma_I = f_{ctm} \quad (1)$$

The principal tensile stress (σ_I) at point B is calculated from the following expression

$$\sigma_I = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (2)$$

where:

$\sigma_x = -|N_f| / (b_{eff} \cdot h_f)$, longitudinal stress in the flange plane in x direction

$\sigma_y = |N_{y,max}| / h_f$, transverse stress in the flange plane in y direction

$\tau_{xy} = -|N_{xy}|/h_f$, tangential stress in the flange plane

h_f : Flange thickness

b_{eff} : Effective width of flange

N_f : Axial force in the flange

$N_{y,max}$: Maximum transverse tensile force per unit length

N_{xy} : Longitudinal shear load per unit length

- 1 There are no direct data available on tensile strength f_{ctm} . Therefore, the mean value
- 2 of tensile strength was estimated from the compressive strength f_c according to
- 3 Section 5.1.5.1. of Model Code 2010 [4] (see Table 7).

	f_c (MPa)	f_{LOP} (MPa)	f_{ctm} (MPa)	$M_{b,crack}$ (kNm)	$F_{b,crack}$ (kN)
V1-0	19.7	3.3	2.11	40.3	53.8
V2-0	20.7	3.1	1.98	37.6	50.2
V3-0	20.6	3.1	1.98	36.9	49.2
V4-0	20.2	3.1	1.98	37.0	49.4
V1-20	19.0	3.1	1.98	35.6	47.4
V2-20	19.9	3.1	1.98	37.1	49.4
V3-20	21.0	3.5	2.23	41.6	55.4
V1-30	23.0	3.5	2.23	39.3	52.4
V2-30	20.5	3.3	2.11	37.2	49.6
V3-30	22.4	2.9	1.85	32.6	43.4
V1-40	21.2	3.3	2.11	39.2	52.2
V2-40	22.0	3.5	2.23	39.5	52.6
V3-40	19.0	3.2	2.04	36.7	49.0

4 Table 7: The mean concrete tensile strength (f_{ctm}), cracking bending moment ($M_{b,crack}$) and the
5 load ($F_{b,crack}$) that caused the cracking bending moment.

- 6 Table 7 also shows both the bending moment ($M_{b,crack}$) and load ($F_{b,crack}$) of each
- 7 beam which produced bending cracking. The own-weight bending moment (6.562
- 8 kNm), and the flexural tensile strength f_{LOP} determined in the prismatic specimens
- 9 tests were considered to calculate the cracking bending moment. The mechanical
- 10 characteristics of the homogenised section and the maximum effective width provided
- 11 by the regulations (Table 6) were used to calculate them. Beam V3-20 presented the
- 12 highest cracking moment $M_{b,crack}$ (41.6 kNm), which corresponded to a load $F_{b,crack}$ of
- 13 approximately 55.5 kN (Table 7). This value was much lower than the load that led the
- 14 longitudinal cracks to appear at the web-flange junction observed in the tests, which

1 was around 300 kN for all the specimens (Section 3.5). Therefore, when longitudinal
 2 cracks appeared at the vertical web-flange junctions, bending cracks previously
 3 appeared. Hence membrane stresses must be determined using the sectional
 4 parameters by assuming that the section is cracked.

5 The axial force in flange N_f can be obtained approximately with Equation (3), which
 6 arises from integrating the sectional stresses at point B (Figure 15):

$$N_f \cong \frac{S_{f,crack}}{I_{crack}} (F_{f,crack}/2 \cdot a + M_g) \quad (3)$$

7 where:

8 $S_{f,crack}$: Static moment of the flange in the cracked state in relation to the cracked
 9 cross section's centre of gravity in plain concrete

10 I_{crack} : The cross section's inertia moment in the cracked state in relation to the
 11 cracked cross section's centre of gravity in plain concrete

12 a : Length between the support and loading point (Figure 15.a).

13 M_g : Bending moment due to own weight in the load applying section ($M_g =$
 14 6.562 kNm).

15 The contribution of the fibres has been neglected for the calculation of the mechanical
 16 characteristics of the section, $S_{f,crack}$ and I_{crack} , due to the important amount of
 17 longitudinal reinforcement arranged in the section. This approach allows both
 18 calculating the mechanical characteristics of the section independently of the applied
 19 sectional forces and simplifying the expressions to determine the forces in the flange
 20 plane (N_f , N_y and N_{xy}).

21 Longitudinal shear load per unit length N_{xy} was assessed at point B (Figure 15) with
 22 the following equation:

$$N_{xy} \cong \frac{S_{f,crack}}{I_{crack}} (F_{f,crack}/2 + V_g) \quad (4)$$

23 where:

1 V_g : Vertical shear due to own weight in the load applying section ($V_g=1.875$
2 kN).

3 The formulation of Razaqpur and Ghali [3] was used to determine the transverse
4 tensile force per unit length $N_{y,max}$ at point B (Figure 15), which is the stress that the
5 classic beam model is unable to provide. The result, considering own weight, gave this
6 equation (5):

$$N_{y,max} \cong \frac{S_{f,crack}}{I_{crack}} \left[(b - b_w) \cdot 0.4 \cdot q + \frac{(F_{f,crack}/2)}{4} \right] \quad (5)$$

7 where: $b = 1.2$ m, $b_w = 0.2$ m, $q = 3.75$ kN/m.

8 Point B in Figure 15 underwent the maximum value of σ_I by considering how the
9 membrane stresses were distributed according to line AB of the flange, and by
10 observing the equation of principal tensile stress σ_I (1). Tensile stress σ_y (>0)
11 decreased rapidly from point B to point A. The magnitude of compression σ_x (<0) also
12 displayed the same behaviour, but more smoothly. Web-flange tangential stress τ_{xy}
13 remained constant because vertical shear remained constant before the onset of
14 cracking at point B. Thus, the principal tensile stress descended from point B to point A.
15 When the study point moved from B toward the midspan, tensile stress σ_y remained
16 constant, and practically the same happened with compressive stress σ_x as the effect
17 of own weight was minimal. On the contrary, tangential stress τ_{xy} became null
18 theoretically. For this reason, the principal tensile stress value was also lower in the
19 central region of the beam between loading points.

20 The stresses formulated in (1) - (2) represent average values along flange thickness at
21 point B (Figure 15). To contemplate the fact that longitudinal compressive stress σ_x
22 could considerably vary along thickness, two more options were added to the
23 determination of the web-flange longitudinal cracking load. They consisted in
24 formulating the normal stresses on the top ($\sigma_{x,top}$) and bottom ($\sigma_{x,bot}$) flange sides. For
25 the longitudinal stresses at point B, the equations below were used:

1

$$\sigma_{x,top} \cong -\frac{F_{f,crack} \cdot a + M_g}{I_{crack}} x_{crack} \quad (6)$$

$$\sigma_{x,bot} \cong -\frac{F_{f,crack} \cdot a + M_g}{I_{crack}} (x_{crack} - h_f) \quad (7)$$

2 where x_{crack} is the depth of the cracked section's centre of gravity, measured from the
3 top side; and $M_g = 6.562$ kNm is the bending moment due to own weight.

4 For transverse stresses σ_y at point B, which tend to be less important than longitudinal
5 stresses, the variation caused by transverse bending due to the flange's own weight
6 was considered in the top ($\sigma_{y,top}$) and bottom ($\sigma_{y,bot}$) flange sides, and resulted in:

$$\sigma_{y,top} \cong \sigma_y + 0.27 \text{ (MPa)} \quad (8)$$

$$\sigma_{y,bot} \cong \sigma_y - 0.27 \text{ (MPa)} \quad (9)$$

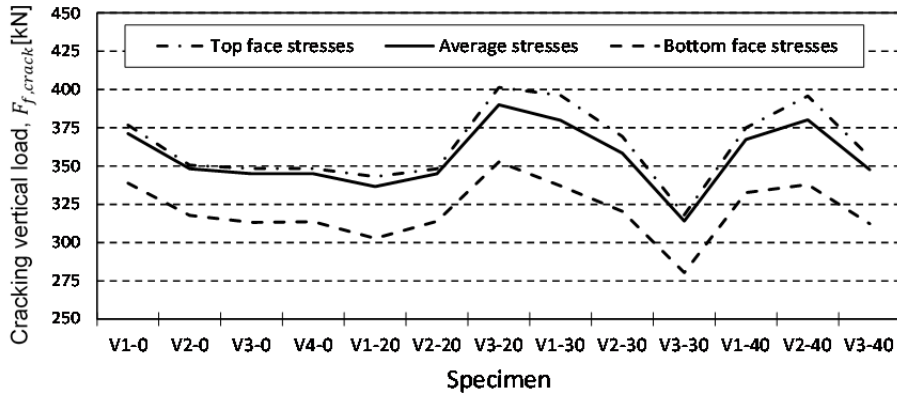
7 where σ_y is the transverse tensile stress, equal to $|N_{y,max}|/h_f$.

8 Figure 16 shows the web-flange longitudinal shear cracking vertical load ($F_{f,crack}$). That
9 is, the load that satisfied Equation (1) calculated with average stresses, top side
10 stresses and bottom side stresses at point B. The cracking vertical load $F_{f,crack}$
11 needed to cause the principal stress to reach the concrete tensile strength value was
12 lower when the condition was applied to the flange bottom side, which indicates that
13 longitudinal cracking started on the bottom side to finally propagate through the
14 thickness toward the top side.

15 Table 8 shows the detailed results for the bottom side stresses condition at point B. An
16 effective width of the flange $b_{eff} = 1200$ mm was considered to obtain the results
17 shown in Figure 16 and Table 8. As further data of interest, the main compression
18 stress σ_{II} , whose equation was analogous to (2), is included in the tables, but with a
19 minus sign accompanying the root term. The principal directions, as illustrated in Figure
20 15, were defined with the following equations:

$$\theta_{II} = \frac{1}{2} \cdot \arctan\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \quad (10)$$

$$\theta_I = \theta_{II} - 90^\circ \quad (11)$$



1
2

Figure 16: The vertical load $F_{f,crack}$ that caused web-flange cracking.

Beam	$F_{f,crack}$ (kN)	N_f (kN)	N_{xy} (kN)	$N_{y,max}$ (kN)	$\sigma_{x,bot}$ (MPa)	τ_{xy} (MPa)	$\sigma_{y,bot}$ (MPa)	$\sigma_{I,bot}$ (MPa)	$\sigma_{II,bot}$ (MPa)	$\theta_{I,bot}$ (°)	$\theta_{II,bot}$ (°)
V1-0	338.80	378.10	248.40	65.40	-6.64	-3.55	0.66	2.11	-8.08	-67.90	22.10
V2-0	317.70	355.50	233.30	61.50	-6.14	-3.33	0.61	1.98	-7.51	-67.70	22.30
V3-0	313.20	356.60	234.00	60.70	-6.11	-3.34	0.60	1.98	-7.49	-67.60	22.40
V4-0	313.60	356.80	234.20	60.70	-6.13	-3.35	0.60	1.98	-7.51	-67.60	22.40
V1-20	302.70	360.00	236.10	58.70	-6.10	-3.37	0.57	1.98	-7.51	-67.30	22.70
V2-20	313.90	357.00	234.30	60.80	-6.14	-3.35	0.60	1.98	-7.52	-67.60	22.40
V3-20	352.60	400.20	263.10	68.00	-6.99	-3.76	0.70	2.23	-8.53	-67.80	22.20
V1-30	337.00	400.90	263.40	65.10	-6.77	-3.76	0.66	2.23	-8.34	-67.30	22.70
V2-30	320.40	380.90	250.10	62.00	-6.46	-3.57	0.62	2.11	-7.95	-67.40	22.60
V3-30	280.40	335.30	219.60	54.50	-5.49	-3.14	0.51	1.85	-6.83	-66.90	23.10
V1-40	332.60	378.10	248.40	64.30	-6.54	-3.55	0.65	2.11	-7.99	-67.70	22.30
V2-40	337.90	401.80	264.00	65.30	-6.82	-3.77	0.66	2.23	-8.39	-67.40	22.60
V3-40	312.30	371.10	243.60	60.50	-6.32	-3.48	0.59	2.04	-7.77	-67.40	22.60

3 Table 8: Web-flange longitudinal shear cracking vertical load. Stress condition applied to the low
4 flange side at point B: $\sigma_{I,inf} = f_{ctm}$ for effective width $b_{eff} = 1200$ mm.

5 The average analytical force $F_{f,crack}$ value that produced longitudinal cracking was
6 321.00 kN, with a standard deviation of 14.16 kN, for the case of effective width $b_{eff} =$
7 1200 mm and by assuming that cracking started on the bottom side. The variations in
8 the values between specimens were due to the differences in both the values of the
9 concrete tensile strength and the effective depth which modified the sectional
10 parameters. The average force $F_{f,crack}$ value that produced longitudinal cracking, in
11 accordance with the experimental results (Section 3.5), was approximately 300 kN.
12 Consequently, when comparing the theoretical and experimental force $F_{f,crack}$ values
13 that produced longitudinal cracking, effective width $b_{eff} = 1200$ mm was suitable for
14 the service state before longitudinal web-flange cracking took place. This behaviour

1 was also observed in the tests performed by Tizatto [20]. The analytical process shown
2 in this section also corroborated that onset of cracking occurred on the bottom side,
3 which coincides with the experimental observations made by other authors [32].

4 The methodology proposed in this section to calculate the vertical cracking load $F_{f,crack}$
5 can be applied to determine a minimum flange transverse reinforcement ratio that
6 avoids a brittle flange failure. For this purpose, the amount of transverse reinforcement
7 needed to resist the load $F_{f,crack}$ will be determined e.g. using a strut-and-tie model or
8 a shear friction model. A minimum flange transverse reinforcement ratio is required if
9 the load $F_{f,crack}$ is lower than the theoretical load that would produce beam failure
10 because of either bending or vertical shear.

11 **4.2. The mean web-flange longitudinal shear assessment.**

12 This section shows the procedure to obtain the mean web-flange longitudinal shear
13 stress during the loading process until failure based on the experimental results.

14 **4.2.1. Analytical approach.**

15 An analytical approach is proposed to evaluate the mean longitudinal shear stress
16 τ_{xy} at the web-flange junction in a region 1.5-metre length [1,2] (from the support to the
17 loading point, see Figure 3). This mean stress τ_{xy} is $\Delta F_f / (h_f \cdot \Delta x)$, where ΔF_f is the
18 change of the normal (longitudinal) force in the flange in this region, h_f is the flange
19 thickness and Δx is the length under consideration equal to 1.5 metre. According to the
20 loading scheme of the specimens (Figure 3), only one of these sections (over support
21 or under the loading point) was studied due to symmetry. The supporting section
22 presented a null bending moment and the section under the loading point presented a
23 maximum bending moment if own weight was neglected. Thus, the force ΔF_f is equal
24 to compressive axial load in the flange N_f at the section under the loading point.

25 The compressive resultant force N_f , which was determined by integrating compressive
26 stresses into the flange, was calculated in the maximum bending moment section for

1 each load level. It was firstly necessary to transform the moment – curvature diagram
 2 ($M-\chi$) into a moment – flange compressive load ($M-N_f$).

3 Curvatures were obtained from the data provided by the strain gauges located in both
 4 the longitudinal reinforcements (ε_{s2} and ε_{s3}) (Figure 8) and the most compressed side
 5 of concrete (ε_{c6}) at the midspan section (Figure 8). Thus the strain plane of the section
 6 could be determined at each load level if experimental curvature χ_E and experimental
 7 bending moment M_E were known because axial force was zero. To do this, a system of
 8 two non-linear equations, which corresponded to the two equilibrium equations of the
 9 section (Equations (12) - (13)), was solved. The unknowns of the system were the
 10 strain at the reference centre of sectional forces ε_0 and the effective section width b_{eff}
 11 (Figure 17).

$$N_t(\varepsilon_0; b_{eff}) = 0 \quad (12)$$

$$M_t(\varepsilon_0; b_{eff}) = M_E \quad (13)$$

12 where:

13 N_t : The resultant axial force determined by integrating the normal stresses of the
 14 section.

15 M_t : The resultant bending moment determined by integrating the normal stresses
 16 of the section.

17 Therefore, internal stresses (N_t and M_t) were determined by integrating sectional
 18 stresses based on both the strain plane of the section (ε_0 and χ_E) and the constitutive
 19 equations of the materials, which relate stresses with strains. The constitutive
 20 equations selected for concrete and steel are indicated below.

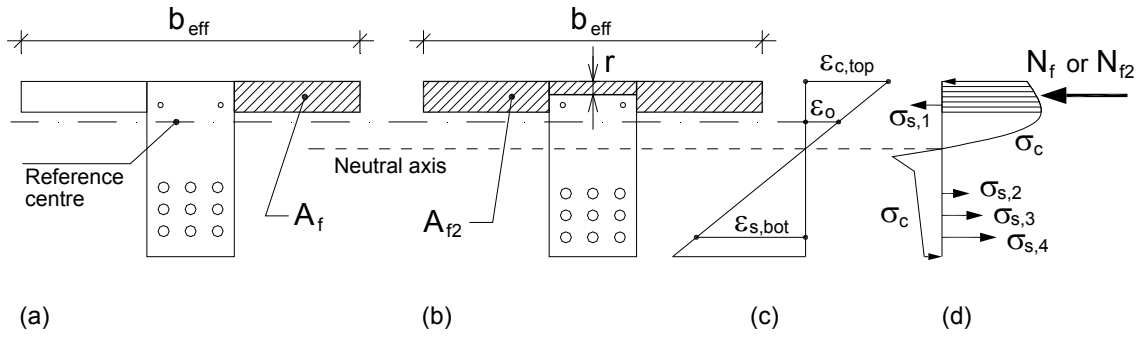


Figure 17: Compressive axial load in the flange: (a) Web-flange longitudinal shear failure according to a vertical plane; (b) Web-flange longitudinal shear failure according to a horizontal plane (specimens V2-0, V3-0 and V2-20); (c) Generic strain plane of the section; (d) Generic stress diagram of a section.

4.2.2. Material constitutive models.

The constitutive equation proposed by Campione [73] (Equation (14)) was used for modelling concrete behaviour under compression (Figure 18.a). The input data of Campione's expression are fibre factor F_V and concrete compressive strength without considering steel fibres f_c . In the present study, however, the compressive strength of the concrete with fibres f_{cf} was known. Therefore, this compressive strength was used directly. This law is also useful for non-FRC by simply imposing $F_V = 0$.

$$\sigma_c(\epsilon_c) = f_{cf} \cdot \frac{\left(\frac{\epsilon_c}{\epsilon_{f0}}\right) \cdot \beta}{\beta - 1 + \left(\frac{\epsilon_c}{\epsilon_{f0}}\right)^\beta} \quad (14)$$

where:

f_{cf} : Maximum compressive stress of the σ - ϵ curve.

$$f_{cf} = f_c + 6.913 \cdot F_V \text{ [MPa]}$$

ϵ_{f0} : Strain corresponding to f_{cf} .

$$\epsilon_{f0} = 0.0016 + 0.00002 \cdot f_{cf} + 0.00178 \cdot F_V \text{ [MPa]}$$

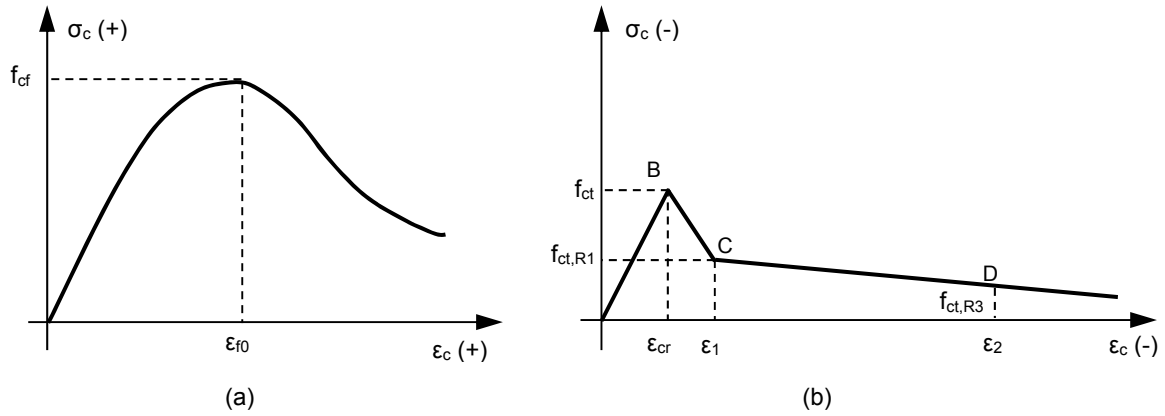
$$\beta = \begin{cases} 1.4276 \cdot e^{0.0247 \cdot (f_{cf} - 6.913 \cdot F_V)} & \text{for } 0 \leq \epsilon \leq \epsilon_{f0} \\ 1.4276 \cdot e^{0.0247 \cdot (f_{cf} - 6.913 \cdot F_V)} + 0.175 \cdot F_V & \text{for } \epsilon_{f0} < \epsilon < \epsilon_{cu} \end{cases} \text{ [MPa]}$$

F_V : Fibre factor.

$$F_V = V_f \left(\frac{L_f}{D_f}\right)$$

V_f : Ratio between steel fibre volume and concrete volume.

- 1 L_f : Steel fibre length.
 2 D_f : Steel fibre diameter.



3 Figure 18: Complete concrete constitutive law: (a) Behaviour under compression (Campione
 4 [73], positive compression); (b) Behaviour under tension (EHE-08 [1], negative tension).

5 Regarding tension behaviour, the trilinear constitutive law for concrete under tension
 6 proposed by EHE-08 [1] (Equation (15)) (Figure 18.b) was used.

$$\begin{aligned}
 \text{Point B: } f_{ct} &= 0.6 \cdot f_{LOP} & \text{for } \epsilon_{cr} &= 1000 \cdot \frac{f_{ct}}{E_{c0}} \\
 \text{Point C: } f_{ct,R1} &= 0.45 \cdot f_{R,1} & \text{for } \epsilon_1 &= \epsilon_{cr} + 0.1\text{‰} \\
 \text{Point D: } f_{ct,R3} &= k_1 \cdot (0.5f_{R,3} - 0.2f_{R,1}) & \text{for } \epsilon_2 &= \frac{2.5}{l_{cs}}
 \end{aligned} \tag{15}$$

7 where ϵ_{cr} , ϵ_1 and ϵ_2 are expressed as per thousand [‰]; E_{c0} is the concrete elasticity
 8 modulus; $k_1 = 1$ for bending; and l_{cs} is the critical length in metres ($l_{cs} = \min(s_m, h -$
 9 $x)$), where s_m is the average separation between cracks, h is the height of the T-beams
 10 and x is the depth of the neutral axis. According to EHE-08 [1] for SFRC with softening
 11 behaviour and with $f_{R3} < 3$ MPa (Table 3), the separation s_m can be calculated without
 12 considering the steel fibre contribution (as plain concrete). The separation s_m was less
 13 than $(h - x)$ in all specimens, therefore the critical length l_{cs} was equal to s_m . Critical
 14 length l_{cs} was 141 mm for beams V1-20, V1-30, V2-30, V3-30, V2-40, V3-40 and was
 15 115 mm for beams V2-20, V3-20, V1-40.

16 The constitutive law for reinforcements is elastoplastic. Elasticity modulus E_s and yield
 17 stress f_y are the same as those shown Table 4.

1 4.2.3. Determination of the compressive resultant load in flange N_f .

2 In the general case, the specimens' cross section can be divided into three rectangles,
3 as indicated in Figure 17.a. However, there were three specimens whose failure
4 occurred by longitudinal shear according to a horizontal plane formed over web stirrups:
5 V2-0, V3-0 and V2-20. In this case, the cross section was simplified into four rectangles,
6 as shown in Figure 17.b. For these three specimens, the resultant compressive load on
7 flange N_{f2} was obtained from the two rectangles of width b_{eff} that constituted the
8 flange on each side of the core, plus the central rectangle with a height that equalled
9 geometric concrete cover r . The fourth rectangle formed the rest of the web: 200 mm
10 wide and $400-r$ height.

11 The process to determine the resultant compressive load on the flange according to a
12 vertical failure plane (N_f , see Figure 17.a), and the resultant compressive load on the
13 flange according to a horizontal failure plane (N_{f2} , see Figure 17.b), consisted of
14 dividing the section into the elements indicated in Figure 17 and following these steps:

15 (1) An initial value was established for effective width: $b_{eff} = 1200$ mm if the procedure
16 began. Otherwise the effective width took the value of the previous iteration.

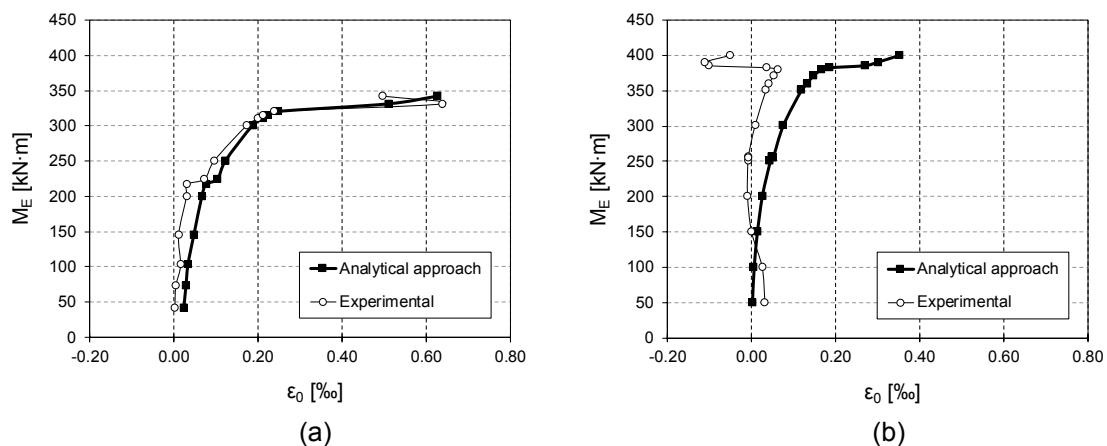
17 (2) From the axial force equilibrium (Equation (12)), the value of the strain at the
18 reference centre of sectional forces ε_0 was obtained as the experimental curvature for
19 each load level was known.

20 (3) After obtaining ε_0 , the bending moment equilibrium was stated (Equation (13)) to
21 determine effective width b_{eff} . For this purpose, bending moment M_t was compared
22 with experimental moment M_E . If $|M_{tot} - M_E| \leq 0.001$ kNm, then a satisfactory solution
23 was found and step (4) was followed. If $M_t - M_E > +0.001$ kNm, a lower value was
24 adopted for b_{eff} and step (2) was repeated. If $M_t - M_E < -0.001$ kNm, a higher value
25 was adopted for b_{eff} and step (2) was repeated.

1 (4) Finally, when ε_0 and b_{eff} were obtained, the axial compressive force of flange N_f
2 was determined. The mean longitudinal shear stress at the web-flange junction τ_{xy} was
3 obtained as the quotient between the force N_f and the failure surface along the
4 considered beam region.

5 (5) Go back to step (1) until the maximum moment in the section under the loading
6 point.

7 The analytical model was validated based on experimental results. Figure 22 compares
8 the experimental bending moment M_E with both then strain at the reference centre of
9 sectional forces ε_0 obtained experimentally and ε_0 obtained analytically. The
10 experimentally obtained strain ε_0 was computed from strain gauges located in both the
11 longitudinal reinforcements (ε_{s2} and ε_{s3}) (Figure 8) and the most compressed concrete
12 side (ε_{c6}) at the midspan section (Figure 8). The analytical values of ε_0 are computed
13 according to equations (15) and (16). The experimental moment (M_E) – strain (ε_0)
14 diagram is represented in this figure. A sufficient approximation between the
15 experimental results and the analytical model is observed in this figure.



16
17 **Figure 19: Experimental verification of the analytical model: (a) Specimen V1-20 (Web-flange**
18 **shear V failure); (b) Specimen V3-0 (Web-flange shear H failure)**

19

1 **4.2.4.Results.**

2 **Table 9** shows the results for each specimen. The most significant situations were
 3 selected: the maximum resultant axial force situation in flange $N_{f,max}$ and the maximum
 4 moment situation supported by beam $M_{E,max}$. Both the results of the resultant
 5 compressive force in flange N_f and the compressive forces of the two flanges and
 6 central section N_{f2} are provided in the three specimens whose failure occurred by
 7 longitudinal shear according to a horizontal surface (V2-0, V3-0 and V2-20). In addition,
 8 the moment – curvature diagrams ($M_E-\chi_E$) and the moment – compressive resultant
 9 force on the flange (M_E-N_f) of two specimens, whose web-flange longitudinal shear
 10 failure was along a vertical plane, are displayed (**Figure 20** and **Figure 21**).

Beam	Situation	M_E (mkN)	b_{ef} (mm)	N_f (kN)	N_{f2} (kN)
V1-0	$N_{f,max}$	240.3	334.2	300.3	-
	$M_{E,max}$	281.2	189.4	254.2	-
V2-0	$N_{f,max}$	374.0	264.7	362.9	837.9
	$M_{E,max}$	379.7	293.9	356.7	809.5
V3-0	$N_{f,max}$	380.3	420.7	515.2	1127.5
	$M_{E,max}$	399.7	352.9	493.3	1089.4
V4-0	$N_{f,max}$	384.9	392.3	504.2	-
	$M_{E,max}$	402.2	359.8	496.4	-
V1-20	$N_{f,max}$	315.0	394.4	429.0	-
	$M_{E,max}$	341.9	284.8	374.2	-
V2-20	$N_{f,max}$	-	-	-	-
	$M_{E,max}$	441.8	421	574.6	1244.2
V3-20	$N_{f,max}$	-	-	-	-
	$M_{E,max}$	518.4	478.1	682.0	-
V1-30	$N_{f,max}$	380.2	338.8	487.2	-
	$M_{E,max}$	410.2	365.4	453.4	-
V2-30	$N_{f,max}$	-	-	-	-
	$M_{E,max}$	475.9	459.1	645.4	-
V3-30	$N_{f,max}$	-	-	-	-
	$M_{E,max}$	497.6	443.7	680.0	-
V1-40	$N_{f,max}$	359.9	482.6	501.9	-
	$M_{E,max}$	371.1	429.1	496.0	-
V2-40	$N_{f,max}$	-	-	-	-
	$M_{E,max}$	398.2	392.9	538.1	-
V3-40	$N_{f,max}$	370.2	472.8	524.9	-
	$M_{E,max}$	372.2	438.1	511.2	-

11 **Table 9: The web-flange longitudinal shear assessment results.**

12

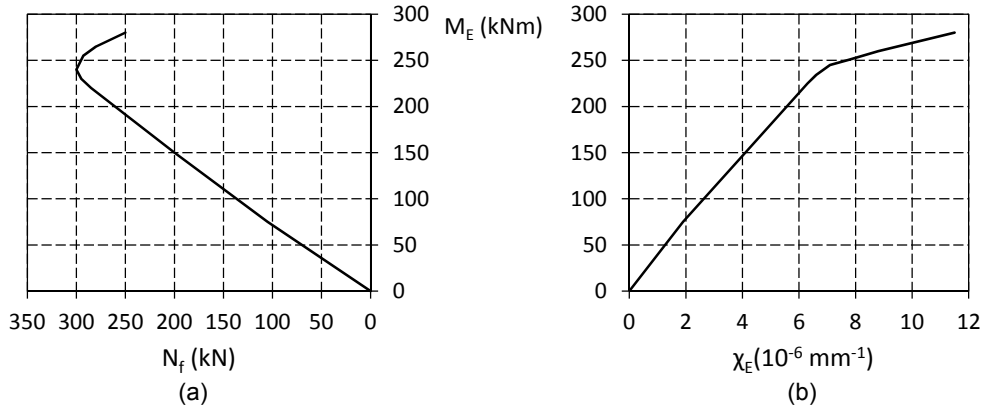


Figure 20: V1-0 specimen: (a) M_E-N_f diagram, (b) $M_E-\chi_E$ diagram.

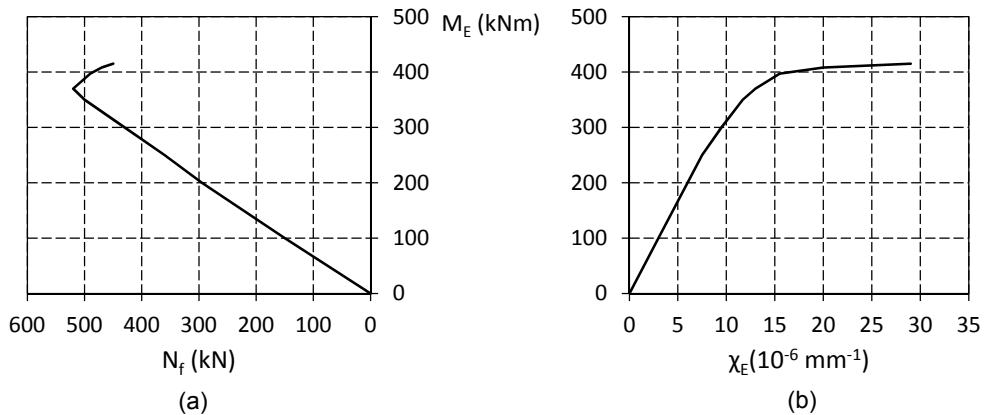


Figure 21: Specimen V1-30: (a) M_E-N_f diagram, (b) $M_E-\chi_E$ diagram.

The maximum compressive axial force value in flange N_f was reached before obtaining the maximum load level supported by beam $M_{E,max}$. Specimens V1-0, V1-20 and V1-30 reached the maximum N_f value at a vertical load of 14.5%, 7.9% and 7.3% lower than the maximum vertical load, respectively. This occurred because the web was able to increase the specimen's maximum bearing moment, even though flanges did not transmit more compressive axial force (Figure 20 and Figure 21). Effective width grew when the flange transverse reinforcement ratio was higher as flanges could transmit more compressive axial force before failure occurred.

Figure 22 displays the web-flange normalised shear stress t (Equation (16)) in relation to the flange transverse reinforcement mechanical ratio ω (Equation (18)).

$$t = \frac{\tau_{xy}}{f_c} \quad (16)$$

where:

1 τ_{xy} : Mean longitudinal shear stress at the web-flange junction.

$$\tau_{xy} = \frac{N_f}{a \cdot h_f} \quad (17)$$

2 a : Length between the support and loading point (Figure 15.a).

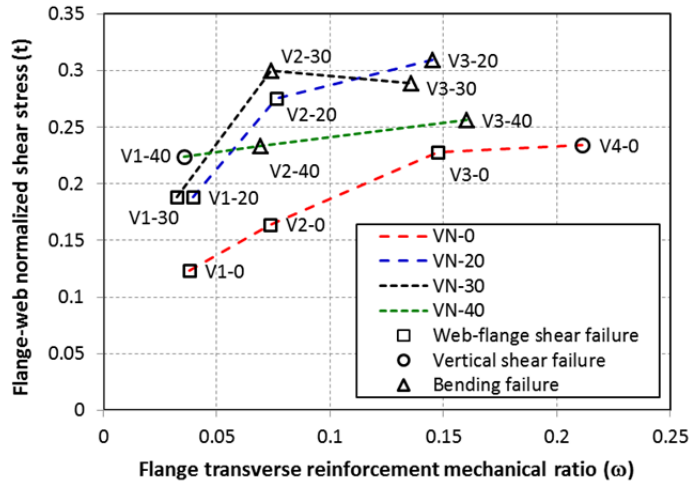
3 h_f : Flange thickness (Figure 15).

4 ω : Flange transverse reinforcement mechanical ratio.

$$\omega = \frac{A_s \cdot f_y}{a \cdot h_f \cdot f_c} \quad (18)$$

5 A_s : Flange transverse reinforcement area.

6 f_y : Steel yield stress.



7
8 *Figure 22: Representation of the experimental results.*

9 **Figure 22** confirms the beneficial effect of the presence of steel fibres depending on
10 fibre content. This can be seen with the VN-20 family in relation to the VN-0 fibreless
11 family. The effect diminished in the VN-30 family, which also showed an anomalous
12 result with V3-30. However, it must be pointed out that the representation of failures by
13 another cause other than web-flange shear took a lower web-flange shear strength
14 value. The VN-40 family exhibited irregular behaviour. Beam V1-40 showed poor
15 concrete compaction and hollows. As a result, the failure was concrete crushing for the
16 compression of the strut of the web due to vertical shear. Notwithstanding, the
17 normalised shear stress in the ultimate state of V1-40 beam exceeded the shear
18 resisted by the three beams with an equal flange transverse reinforcement ratio ω and

1 lower fibre content (V1-0, V1-20 and V1-30), which all failed by web-flange shear. On
2 the contrary, the normalised shear stress of beams V2-40 and V3-40 was lower than
3 that of the beams with less fibre content (but exceeded that of the concrete beams
4 without fibres: V2-0 and V3-0). This is due to the concrete of beams V2-40 and V3-40
5 showed marked porosity on the lateral sides of the web, but did not show visible
6 hollows as in V1-40.

7 **5. Summary and Conclusions.**

8 The conclusions about this experimental campaign of 13 T-beams, made with concrete
9 with and without fibres, are the following:

- 10 - Steel fibres improved the longitudinal shear strength in T-beams with a lower
11 transverse reinforcement ratio than that required. Fibre content and the flange
12 transverse reinforcement ratio can determine the failure mode.
- 13 - The beam effective width depends on the applied load. A reduction in the effective
14 width was noted as the applied load level increased in the beams with a lower
15 flange transversal reinforcement ratio and with low steel fibre content. This result
16 could be due to the fact that the arranged transverse reinforcement was lower than
17 the minimum required according to Hendy and Smith [69].
- 18 - The tested beams went through the following stages in the ultimate state due to
19 web-flange shear: pronounced longitudinal cracking at the web-flange junctions,
20 yielding of flange transverse reinforcements, an increase in the longitudinal strains
21 in the web and a reduced flange effective width.
- 22 - A relatively small concrete cover thickness of compressed reinforcements plus a
23 high amount of these stirrups can cause web-flange shear failure along a
24 horizontal plane instead of along a vertical plane at the web-flange junction.
- 25 - According to the experimental results, longitudinal shear cracking started on the
26 lower side of the web-flange junction. This behaviour was also observed by Tizatto
27 and Shehata [32].

1 - Although crack formation followed a double symmetry pattern (transversal and
2 longitudinal symmetry), failure was not symmetrical.

3 - Effective width practically coincided with the real one for the first load levels, and
4 before web-flange cracking in both the tested and Tizatto beams [20].

5 The conclusions about cracking web-flange **longitudinal** shear determination are the
6 following:

7 - A procedure, based on the model of **Regan and Placas [18] y Tizatto [20]**, has
8 been developed to calculate the load that causes web-flange **longitudinal** cracking
9 and to determine if a minimum flange transverse reinforcement ratio to resist web-
10 flange **longitudinal** shear is necessary. A minimum flange transverse reinforcement
11 ratio is needed when the load that causes web-flange **longitudinal shear** cracking is
12 lower than the theoretical load that would produce beam failure due to either
13 bending or vertical shear. Otherwise, the beam could work perfectly without
14 transverse reinforcement, as long as the high lateral bending in flanges is avoided.
15 The theoretical **longitudinal shear cracking load** obtained for the tested beams
16 adequately came close to the experimental values.

17 - It has been analytically justified that longitudinal cracking in the flange starts on the
18 bottom side.

19 The **mean web-flange longitudinal shear** determination conclusions are the following:

20 - The maximum **web-flange longitudinal shear** situation does not have to coincide
21 with the maximum bending moment situation at the midspan section. The
22 maximum **longitudinal** shear situation may be reached earlier. In this case, web
23 strength capacity is able to increase the cross section's bearing moment, even
24 though effective width is reduced with increased load.

25 - The beneficial effect of including fibres in the concrete mass is confirmed. This
26 effect was stronger when including 20 kg/m^3 in relation to fibreless concrete.

1 Consequently, it is possible to replace transverse reinforcement with steel fibres to
2 resist web-flange shear.

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