A rolling horizon approach for material requirement planning under fuzzy lead times

Manuel Díaz-Madroñero[a], Josefa Mula[a], Mariano Jiménez[b] and David Peidro[a]

[a]*Research Centre on Production Management and Engineering (CIGIP), Universitat Politècnica de València, Alicante, Spain;* [b]*Department of Applied Economy, Universidad del País Vasco (UPV-EHU), San Sebastian, Spain*

This paper proposes a fuzzy multi-objective integer linear programming (FMOILP) approach to model a material requirement planning (MRP) problem with fuzzy lead times. The objective functions minimise the total costs, back-order quantities and idle times of productive resources. Capacity constraints are included by considering overtime resources. Into the crisp MRP multi-objective model, we incorporate the possibility of occurrence of each uncertain lead time using fuzzy numbers. Then FMOILP is transformed into an auxiliary crisp mixed-integer linear programming model by a fuzzy goal programming approach for each fuzzy lead time combination. In order to defuzzify the set of solutions associated with each fuzzy lead time combination, a solution method based on the centre of gravity concept is addressed. Model validation with a numerical example is carried out by a novel rolling horizon procedure where uncertain lead times are updated during each planning period according to the centre of gravity obtained. For illustration purposes, the proposed solution approach is satisfactorily compared to a rolling horizon approach in which lead times are allocated when the possibility of occurrence is established at one.

Keywords: MRP; uncertainty; fuzzy methods; multi-objective optimisation; rolling horizon

1. Introduction

Material requirement planning (MRP) remains the most widely used production planning system in the world, and is combined with other techniques, such as just-in-time, lean manufacturing, agile manufacturing or quick response manufacturing, among others. In manufacturing contexts, there are many forms of uncertainty that could affect MRP plans. Ho (1989) identifies two main sources of uncertainty: (i) environmental uncertainty, which includes uncertainty in demand and supply; (ii) system uncertainty, which is related to operation yield uncertainty, production lead time uncertainty, quality uncertainty, production system failure and changes in the product structure. Generally speaking, uncertainty in demand is more broadly addressed in the scientific literature (Grabot et al. 2005; Mula et al. 2006; Mula, Peidro, and Poler 2010; Campuzano, Mula, and Peidro 2010; Guillaume, Thierry, and Grabot 2011; among others). According to Dolgui and Prodhon (2007), MRP models under uncertainty are adapted to unexpected situations, mainly through setting safety stocks (Grubbström and Tang 1999; Mula et al. 2014), safety lead times (Wijngaard and Wortmann 1985) or lot-sizing rules. Other approaches are found in Mula et al. (2006). Regarding MRP models under uncertainty in lead times, it is necessary to highlight the pioneering works by (Yano 1987a, 1987b, 1987c) based on stochastic lead times, and also the works by Dolgui and Ould-Louly (2002) and Ould-Louly and Dolgui (2004). Other approaches are found in Dolgui et al. (2013), Aloulou, Dolgui, and Kovalyov (2013) and Guillaume, Thierry, and Grabot (2011).

The mathematical programming formulation of MRP problems emerge in the works of Karni (1981) and Billington, McClain, and Joseph Thomas (1983), conducted to optimise production planning in terms of the total costs subject to capacity constraints. Subsequently, different uncertain parameters were introduced into these mathematical programming formulations by stochastic programming (Escudero and Kamesam 1993) and fuzzy programming approaches (Mula, Poler, and Garcia 2006; Mula, Poler, and Garcia-Sabater 2007, 2008). In this paper, the consideration of fuzzy lead times in MRP models is addressed through fuzzy modelling. Quite frequently in manufacturing contexts, lead times are defined by decision-makers in terms of a range of values associated with degree of occurrence; e.g. Supplier A requires three, four or six days, with a possibility of occurrence of 80, 50 and 20%, respectively, in order to deliver its products. In order to incorporate fuzzy lead times into MRP mathematical programming models, we propose a novel approach based on the centre of gravity concept and a rolling horizon validation procedure. The main contributions of this paper are twofold: first to provide a tool to develop MRP calculations where lead times are considered fuzzy; second to present a novel validation procedure based on a rolling horizon framework using a numerical example, where lead times are periodically updated instead of traditional demand updating (Baker 1977).

The remainder of the paper is organised as follows. Section 2 reviews the scientific literature about uncertainty in lead times. Section 3 presents a multi-objective formulation of the MRP problem under fuzzy lead times. Section 4 proposes a solution approach for the addressed problem. Section 5 validates and evaluates the results of our proposal with a numerical example. Section 6 provides the main conclusions, managerial implications and further research of this paper.

2. Literature review

In real-world MRP systems, input data or parameters are imprecise or uncertain. Initially several authors, such as Whybark and Williams (1976) identified two sources of uncertainty in such production planning systems. The first corresponds to changes or demand variability, while the second is related to scheduled receptions or supply uncertainty. These variabilities can generate disruptions and unavailability of finished goods, components and raw materials. Several literature reviews were published to study the effects and modelling approaches of uncertainty in production supply chain planning models: e.g. Aloulou, Dolgui, and Kovalyov (2013), Ben Ammar et al. (2013), Dolgui and Prodhon (2007), Dolgui et al. (2013), Mula et al. (2006), Peidro et al. (2009).

Related to uncertainty in lead times from suppliers, Ben Ammar et al. (2013) and Dolgui et al. (2013) classify models according to the structure of the supply chain (serial or assembly/MRP type) by considering single-level or multi-level product structures, and single-period or multi-period models.

Dolgui, Portmann, and Proth (1995) and Dolgui (2001) propose a MRP model with constant demand and random lead times for a lot-for-lot order policy with several finished products, and different components and raw materials. The same problem is addressed by Proth et al. (1997), who present a heuristic procedure based on priorities to determine procurement and assembly production planning. Based on the same demand pattern and random lead times for components, Louly and Dolgui (2002) and Dolgui and Ould-Louly (2002) present generalised newsboy vendor models with integer decision variables to determine the optimal values for planned lead times with a Markov modelling approach. One particular case of this problem, which also considers set-up costs, is addressed by Ould-Louly and Dolgui (2004) who use the same analytical approach according to the assumption that supply lead times follow the same distribution probability, and that the holding costs per period for the different components are the same. Louly, Dolgui, and Hnaien (2008a) also develop an exact algorithm based on a branch-and-bound procedure to solve this problem, which was enriched by Louly, Dolgui, and Hnaien (2008b) by introducing a service-level constraint.

Louly and Dolgui (2011) present another generalisation of their previous newsboy model for the parameterisation of MRP systems with periodic order quantity policies, but without the restrictive identical probability distribution assumption for procurement lead times. This algorithm is also adapted to the consider service level constraints by Louly and Dolgui (2013). A recent approach for the optimisation of planned lead times was proposed by Ben Ammar, Dolgui and Marian (2014), which is based on genetic algorithms.

All the above contributions consider mono-objective approaches and the uncertainty of lead times using probability distributions or stochastic approaches. However, according to Díaz-Madroñero, Mula, and Jiménez (2014), an MRP model operates in an uncertain scenario in which statistical data might not be altogether reliable, or even available. In this sense, it is quite commonplace that inventory levels are updated in planning systems when products are supplied, but without calculating the time that has elapsed from order placement to their arrival, i.e. lead times are considered fixed and poorly updated. In this context, it can scarcely be admitted that the future values of certain parameters take a frequentistic nature and are, therefore, likely to be treated by a stochastic approach. Therefore, when statistical data are not that reliable or are unavailable, the determination-based models of these probability distributions may not be the best option. Hence, fuzzy mathematical programming can prove to be an alternative approach to model the different types of uncertainty in MRP systems (Mula, Poler, and Garcia 2006; Mula, Poler, and Garcia-Sabater 2007, 2008; Grabot et al. 2005; Li et al. 2009; Mula, Peidro, and Poler 2010; Díaz-Madroñero, Mula, and Jiménez 2014). Another fuzzy mathematical programming approach for production planning with fuzzy lead times is seen in Peidro et al. (2010), who apply the approach by Jiménez et al. (2007) based on fuzzy ranking numbers in order to manage fuzzy lead times by obtaining a fuzzy solution based on alpha cuts.

After a review process, we highlight the following issues related to MRP models under lead times uncertainty: (1) shortage of models with a multi-objective approach that allow the simultaneous optimisation of several conflicting objectives; (2) shortage of models that consider the uncertainty related to supply lead times with fuzzy approaches; (3) the scarce or null validation of models which periodically update lead times in a rolling horizon context.

3. Multi-objective model formulation for MRP

3.1 *Assumptions*

The following assumptions were considered based on an automotive industry problem that used a simplified data-set from a real-world company, with some extended variations compared to previous studies in terms of fuzzy lead times (Mula, Poler, and Garcia 2006; Mula, Poler, and Garcia-Sabater 2007, 2008; Díaz-Madroñero, Mula, and Jiménez 2014).

- A multi-product manufacturing environment. With the term product, we refer to finished goods, components, raw materials and subassemblies structured in a bill of materials.
- A multi-level production system where subsets of components are assembled independently.
- A multi-period planning horizon that comprises a set of consecutive and integer time periods of the same length.
- The inventory of each product (finished goods, raw materials and components) is the volume available at the end of a given period.
- The backlog of the demand of a product at the end of a period is defined as the non-negative difference between the cumulated demand and the volume of available product.
- The master production schedule (MPS), which specifies the quantity to produce of each finished good during each planning horizon period, and the MRP, which provides the net requirements of raw materials and components for each planning period, are jointly solved.
- Scheduled receptions are considered.
- Production capacity constraints.
- The lead time of a product is the number of consecutive and integer periods that are required for their finalisation.
- Fuzzy lead time for finished goods, components and raw materials.
- Fuzzy lead times are represented using different discrete values associated with different degrees of possibility for each one.
- By combining the different possible discrete values of lead times, several instances are obtained, each one with a possibility of occurrence equal to the minimum between the possibilities of occurrence of its components.

3.2 *Models formulation*

Three objective functions are considered to minimise: (1) the total costs over the time periods that are computed consider variable production costs, inventory holding costs and overtime costs; (2) the back order quantities over the whole planning horizon; and (3) the idle time of productive resources. The main reasons for selecting these three objectives to be minimised are to evaluate the performance of the production planning system in terms of optimising production capacities by maximising the utilisation of productive resources through minimising idle times and penalising overtime without increasing inventory levels, but by maximising the service level measured by the number of back orders. Some relevant works have previously used these criteria in a single-objective cost function (Peidro et al. 2009, 2010). However, it is sometimes very hard to quantitatively compare or estimate in terms of cost values as idle times or back orders, which are so subjective to measure, and multi-objective formulations are an adequate approach to address them.

As it is said before, by combining the different possible discrete values of lead times, several instances are obtained, each with its corresponding possibility of occurrence. For each scenario, the following multi-objective mathematical programming model (with the nomenclature indicated in Table 1 based on Díaz-Madroñero, Mula, and Jiménez 2014) is obtained:

$$\text{Min } z_1 \cong \sum_{i=1}^{I}\sum_{t=1}^{T}(cp_i P_{it} + ci_i INVT_{it}) + \sum_{r=1}^{R}\sum_{t=1}^{T}(ctov_{rt} Tov_{rt}) \tag{1}$$

$$\text{Min } z_2 \cong \sum_{i=1}^{I}\sum_{t=1}^{T} B_{it} \tag{2}$$

$$\text{Min } z_3 \cong \sum_{r=1}^{R}\sum_{t=1}^{T} Tun_{rt} \tag{3}$$

Table 1. Nomenclature.

Symbol	Description
Sets of indices	
T	Number of periods in the planning horizon (t = 1 … T)
I	Number of products (i = 1 … I)
J	Number of parent products in the bill of materials (j = 1 … J)
R	Number of resources (r = 1 … R)
Decision variables	
P_{it}	Quantity of product i to be produced during period t
$INVT_{it}$	Inventory of product i at the end of period t
B_{it}	Backlog of product i at the end of period t
Tun_{rt}	Undertime hours of resource r during period t
Tov_{rt}	Overtime hours of resource r during period t
Data	
d_{it}	Demand of product i during period t
α_{ij}	Required quantity of i to produce one unit of product j
$T\tilde{S}_i$	Lead time of product i
SR_{it}	Scheduled receipts of product i during period t
$INVT_{i0}$	Inventory of product i during period 0
B_{i0}	Backlog of product i during period 0
Objective function cost coefficients	
cp_{it}	Variable cost of the normal production of a finished good unit or the purchase of a unit of raw material or component i
ci_{it}	Inventory cost of a unit of product i
$ctov_{rt}$	Overtime hour cost of resource r during period t
Technological coefficients	
AR_{ir}	Required time of resource r for one unit of production of product i
CAP_{rt}	Available capacity of resource r during period t

3.3 *Constraints*

The following constraints were included.

$$INVT_{i,t-1} + P_{i,t-T\tilde{S}_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I} a_{ij}\left(P_{jt} + SR_{jt}\right) + B_{it} = d_{it} \quad \forall i \forall t \tag{4}$$

$$\sum_{i=1}^{I} P_{it}A_{ir} + Tun_{rt} - Tov_{rt} = CAP_{rt} \quad \forall r \forall t \tag{5}$$

$$B_{iT} = 0 \quad \forall i \tag{6}$$

$$P_{it}, INVT_{it}, B_{it}, Tun_{rt}, Tov_{rt} \geq 0 \quad \forall i \forall r \forall t \tag{7}$$

$$P_{it}, INVT_{it}, B_{it} \in Z \quad \forall i \forall t \tag{8}$$

Constraint (4) is the inventory balance equation for all the products. Constraint (5) establishes the available capacity for normal and overtime production. Constraint (6) finishes with the delays in the last planning horizon period (T). Constraint (7) contemplates the non-negativity for the decision variables and Constraint (8) establishes the integrity conditions for some decision variables. The solution for each one of these models is associated with a lead times instance and, therefore, has the same possibility of occurrence, i.e. the minimum between the possibilities of occurrence of lead times that make up the corresponding instance.

4. Solution methodology

In order to solve all the multi-objective models (1)–(8) obtained for each lead time scenario, we propose the goal programming (GP) approach. However, determining a precise aspiration level for each objective in the complex context of our problem could prove a difficult task. Given this circumstance, decision-makers should find it more convenient if they can express their preferences through a linguistic expression, like 'cost should be essentially less than …'.

4.1. *A FGP approach*

The fuzzy set theory introduced by Zadeh (1965) and Bellman and Zadeh (1970) provides an adequate tool for modelling problems that contain linguistic terms to describe imprecise targets. Goals with imprecise aspiration levels are modelled by fuzzy sets, thus the GP model becomes a fuzzy goal programming (FGP) model. Each fuzzy goal is described by a membership function that reflects the decision-maker's degree of satisfaction about achieving the target.

For the sake of simplicity, we use linear membership functions to obtain a linear programming problem. In any case, and as Verdegay (2015) states: 'It was shown that possible further changes of those membership functions do not affect the former optimal solution,…. This sensitivity analysis … shows the convenience of using linear functions instead of other more complicated ones'.

As the three goals are of the minimising type in our case, the three membership functions are non-increasing:

$$\mu_k = \begin{cases} 1 & z_k < z_k^l \\ \frac{z_k^u - z_k}{z_k^u - z_k^l} & z_k^l < z_k < z_k^u \\ 0 & z_k > z_k^u \end{cases} \tag{9}$$

where μ_k is the membership function of z_k, while z_k^l and z_k^u are, respectively, the lower and upper bounds of the fuzzy aspiration level of z_k.

Following the extension principle of Bellman and Zadeh, Zimmermann (1978) and most of the former researchers used the MAXIMIN convolution to solve an FGP model. Later, the weighted additive approach was incorporated into FGP problems resolution (Tiwari, Dharmar, and Rao 1987; Yaghoobi and Tamiz 2006).

By adapting the ideas of Romero (2001) in relation to ordinary GP, we state what follows about the FGP case: The MAXIMIN approach seeks the maximisation of minimum satisfaction; that is to say, it provides the most balanced solution between satisfactions of different goals (maximum equity), whereas the additive approach provides the maximum aggregated satisfaction of goals (maximum efficiency). Therefore, it can be stated that they represent opposite poles between efficiency and equity. For the above reasons, a convex combination between the MAXIMIN approach and the weighted additive approach could provide a good compromise between the two opposite views of optimising: efficiency and equity. This hybrid approach was applied by Torabi and Hassini (2008) in a supply chain application. The FGP approach by Torabi and Hassini (2008), the convex combination of the lower bound for the degree of satisfaction of objectives plus the weighted sum of these degrees of achievement is adopted as the basis of this solution methodology. Therefore, FGP models are transformed into a single crisp objective model as follows:

$$\text{Max } \lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_k \theta_k \mu_k(x)$$

subject to

$$\theta_k \lambda_0 \le \mu_k \quad \forall k$$

$$x \in F(x) \tag{10}$$

where μ_k represents the degree of satisfaction of the k-th objective function, θ_k is the relative importance of the kth objective, and parameter $\gamma \in [0,1]$ is a coefficient of compensation between efficiency and equity. $F(x)$ is the set of feasible solutions. We should point out that on the left-hand sides of the constraint of the model (10), λ_0 is multiplied by θ_k to seek an equity solution, in which the ratio of the achieved levels comes as close to the ratio of the weights as possible (Lin 2004).

Then the equivalent auxiliary crisp mathematical programming model is formulated as follows:

$$\text{Max } \lambda(x) = \gamma\lambda_0 + (1-\gamma)(\theta_1 \cdot \mu_1 + \theta_2 \cdot \mu_2 + \theta_3 \cdot \mu_3) \tag{11}$$

subject to

$$\mu_1 \le \frac{z_1^u - z_1}{z_1^u - z_1^l} \tag{12}$$

$$\mu_2 \le \frac{z_2^u - z_2}{z_2^u - z_2^l} \tag{13}$$

$$\mu_3 \leq \frac{z_3^u - z_3}{z_3^u - z_3^l} \tag{14}$$

$$\theta_1 \lambda_0 \leq \mu_1 \tag{15}$$

$$\theta_2 \lambda_0 \leq \mu_2 \tag{16}$$

$$\theta_3 \lambda_0 \leq \mu_3 \tag{17}$$

$$0 \leq \lambda_0 \leq 1 \tag{18}$$

$$0 \leq \mu_1 \leq 1 \tag{19}$$

$$0 \leq \mu_2 \leq 1 \tag{20}$$

$$0 \leq \mu_3 \leq 1 \tag{21}$$

and Constraints (4)–(8).

where z_1, z_2 and z_3 correspond to Constraints (1), (2) and (3), respectively. z_1^u, z_2^u, z_3^u and z_1^l, z_2^l, z_3^l are their corresponding upper and lower bounds.

4.2 *Converting solutions into a decision. A centre of gravity defuzzification method*

By solving models (11)–(21) for each lead time instance, a fuzzy set of solutions S is generated. As we mentioned before, the possibility of occurrence of each of these solutions equals the minimum between the possibilities of occurrence of lead times that make up the corresponding instance. In order to make a decision at this point, we have to look for the most representative solution. That is to say, the obtained fuzzy set of solutions S should be defuzzified. Given its efficiency, the centre of gravity defuzzification method is the most widely used in practical applications (Van Broekhoven and De Baets 2006). In order to obtain the centre of gravity of our solutions set S, we propose a procedure inspired in the way in which the centre of gravity of a particle system is calculated in physical sciences. As it is well known, if we have n coordinate particles (x_i, y_i, z_i) and a mass m_i, the coordinates of their centre of gravity are:

$$x^G = \frac{\sum_1^n m_i x_i}{\sum_1^n m_i}, \quad y^G = \frac{\sum_1^n m_i y_i}{\sum_1^n m_i}, \quad z^G = \frac{\sum_1^n m_i z_i}{\sum_1^n m_i} \tag{22}$$

This procedure is adapted to our problem by considering the solution of models (11)–(21) for each instance Ii to be a particle in space with coordinates as its optimal objective values $\left(z_{1i}^*, z_{2i}^*, z_{3i}^*\right)$ and an associated mass that corresponds to the value of achievement function $\lambda_i{}^*(x)$, multiplied by its possibility degree π_i. Then Expression (22) is written as follows:

$$z_1^G = \frac{\sum_{i=1}^n \lambda_i^* \cdot \pi_i \cdot z_{1i}^*}{\sum_{i=1}^n \lambda_i^* \cdot \pi_i}, \quad z_2^G = \frac{\sum_{i=1}^n \lambda_i^* \cdot \pi_i \cdot z_{2i}^*}{\sum_{i=1}^n \lambda_i^* \cdot \pi_i}, \quad z_3^G = \frac{\sum_{i=1}^n \lambda_i^* \cdot \pi_i \cdot z_{3i}^*}{\sum_{i=1}^n \lambda_i^* \cdot \pi_i} \tag{23}$$

Our decision has to correspond to one of the considering lead times instances. However, the centre of gravity obtained in (23) did not correspond to any of them. In this case, we choose for our action the solution $\left(z_{1i}^*, z_{2i}^*, z_{3i}^*\right)$that has the minimum Euclidean distance to the centre of gravity $\left(z_1^G, z_2^G, z_3^G\right)$.

4.3 *Summarised solution procedure*

In accordance with the foregoing, we can summarise our procedure as follows:

Step 1: Formulate the MRP problem (1)–(8).
Step 2: Estimate the fuzzy lead times.
Step 3: Specify the corresponding linear membership functions for all the fuzzy objectives (upper and lower limits).
Step 4: Determine the corresponding relative importance of the objective functions (θ_k) and the coefficient of compensation (γ).
Step 5: Transform the original MRP problem into an equivalent single-objective problem using models (11)–(21).

Step 6: Generate problem instances that are related to all the possible combinations of product lead time values.
Step 7: Solve the proposed auxiliary crisp single-objective model for each problem instance and obtain a fuzzy set of solutions.
Step 8: Defuzzify the obtained solution by applying the centre of gravity method.
Step 9: Determine the Euclidean distance of each solution to the centre of gravity obtained in step 8.
Step 10: Select the solution with a minimum distance to the defuzzified crisp solution.

5. Computational experiments. A numerical example

The proposed model was implemented and solved with Gurobi 6.5. The input data and the model solution values were processed with the PostgreSQL database. A numerical example (25 instances) to validate and evaluate the results of our proposal is presented in order to gain a better understanding of the proposed procedure in Section 4.3, although it could be applied to a more complex production problem.

5.1 *Assumptions*

The study considers a finished good (final product) with a product structure composed of two components (Figure 1). In order to validate our approach, we have used a representative part, which allows us to generalise its behaviour to any part in the system, also to understand the behaviour of this single part can be the basis for additional research of the proposed models (Mula, Poler, and Garcia 2006).

Decision variables, P_{it}, $INVT_{it}$ and B_{it} are considered to be integer. A planning horizon of 25 periods is considered. Only the finished good has external demand. Firm orders cannot be rejected, although the backlog for the finished good is considered. A single productive resource restricts production: the assembly line. Fuzzy lead times are represented using three different values associated with the different possibility degrees of each one: Product 1 (lead time/possibility degree): {1/1, 1/0.5, 3/0.2}; Product 2 (lead time/possibility degree): {1/1, 5/0.7, 7/0.3}; Product 3 (lead time/possibility degree): {3/1, 7/0.8, 8/0.4}. The fuzzy lead times of components are always longer than or equal to the finished good lead times. Thus, by combining the discrete values of lead times the following instances were generated (Table 2).

5.2 *Solution procedure*

The model is executed for a rolling horizon over 25 daily time periods. Figure 2 depicts the execution of the models based on the rolling horizon technique. Each model calculation in the different planning horizon periods (after applying the defuzzification method to the 19 instances) updates the data for the time period being considered, and the results of the decision variables for the remaining periods are ruled out.

Some of the stored decision variables are used as input data to solve the model in the following time periods. These data include: demand backlog, programmed receptions and inventory. This process is repeated for all the time periods of the rolling horizon planning. The results of the model are evaluated from the data of the decision variables stored during each model execution. Experiments are run in an Intel I5 PC, at 3.3 GHz and with 8 GB of RAM memory.

5.3 *Evaluation of the results*

In order to validate our proposal, the model was executed according to the solution procedure described in the previous Section and was compared to additional executions by always considering a lead time combination of (1,1,3); i.e. with a possibility of occurrence equal to one. Tables 3 and 4 show the detailed results provided by the two different

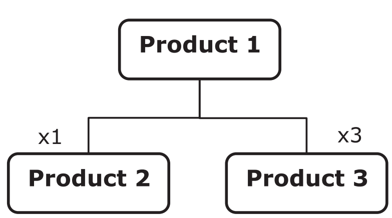

Figure 1. Product structure.

Table 2. Instances generated (1).

Instance	Lead times	Possibility (minimum of the possibility degrees)
I1	1,1,3	1
I2	1,1,7	0.8
I3	1,1,8	0.4
I4	1,5,3	0.7
I5	1,5,7	0.7
I6	1,5,8	0.4
I7	1,7,3	0.3
I8	1,7,7	0.3
I9	1,7,8	0.3
I10	3,5,3	0.5
I11	3,5,7	0.5
I12	3,5,8	0.4
I13	3,7,3	0.3
I14	3,7,7	0.3
I15	3,7,8	0.3
I16	5,5,7	0.2
I17	5,5,8	0.2
I18	5,7,7	0.2
I19	5,7,8	0.2

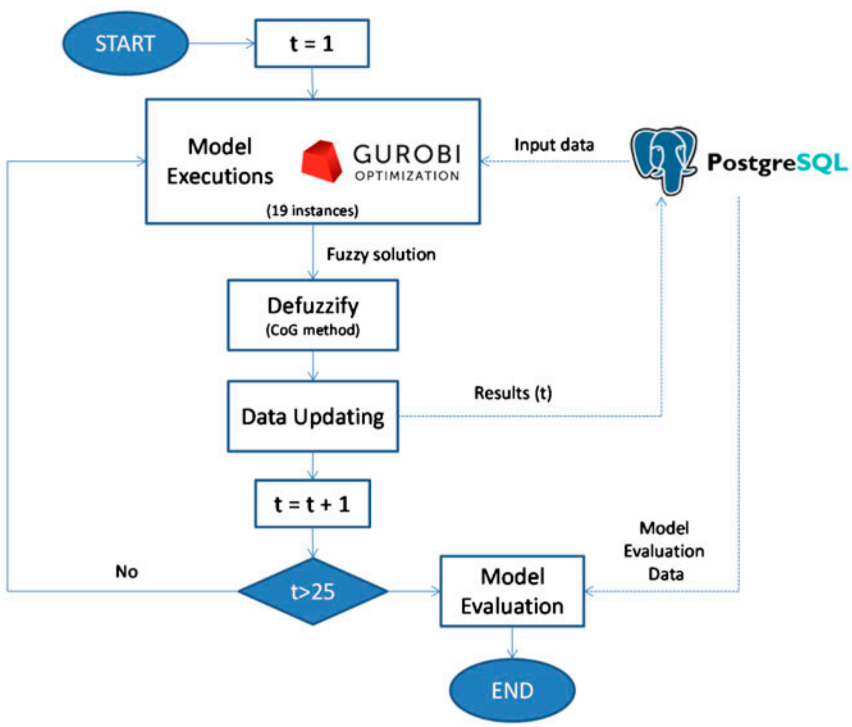

Figure 2. Computational experiment diagram.

executions. First, Table 3 presents the results when the combination of lead times considered is always (1,1,3). Then Table 4 provides the results when lead times are obtained from the centre of gravity method. Furthermore, orders are simulated to arrive later owing to back orders caused by supplier or transport problems (scenario row), which might already be the case in practice. Table 5 summarises the results by comparing both the experimentation approaches;

Table 3. Results by assigning crisp lead times, only those with possibility = 1 (1,1,3), through the rolling horizon.

	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$	$t=11$	$t=12$	$t=13$	$t=14$	$t=15$	$t=16$	$t=17$	$t=18$	$t=19$	$t=20$	$t=21$	$t=22$	$t=23$	$t=24$	$t=25$	TOTAL
Demand	612	684	471	647	531	778	369	554	685	535	761	401	435	654	508	472	625	758	762	370	445	573	762	488	433	14,313
$INVT_{i=1,t}$	0	697	616	191	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	87	1607
$INVT_{i=2,t}$	612	222	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	834
$INVT_{i=3,}t$	12,195	7677	11,529	10,683	9942	8430	8430	8430	3351	1512	1512	3261	3261	303	303	2553	1041	1041	3327	3327	1512	1671	159	0	1512	106,962
$P_{i=1,t}$	279	390	222	282	247	504	0	0	1693	792	0	1008	0	1490	504	0	504	0	504	0	605	2654	504	504	0	12,686
$P_{i=2,t}$	0	282	247	504	454	1008	1693	0	792	504	0	986	0	504	504	504	347	0	605	0	2307	504	504	3464	0	15,713
$P_{i=3,t}$	0	0	0	228	0	0	537	1725	2820	1512	0	1512	2250	1512	3798	0	2445	4164	0	0	1353	1512	0	0	0	25,368
$B_{i=1,t}$	0	0	0	0	340	589	958	1512	1693	535	504	905	1340	986	4	476	597	851	1613	1983	1924	1892	0	0	0	18,702
Tun	11,250	5700	14,100	11,100	12,850	0	25,200	25,200	0	0	25,200	0	25,200	0	0	25,200	0	25,200	0	25,200	0	0	0	0	25,200	256,600
Tov	0	0	0	0	0	0	0	0	59,450	14,400	0	25,200	0	49,300	0	0	0	0	0	0	5050	107,500	0	0	0	260,900
Scenario	1,1,6	1,2,3	1,2,7	2,2,8	1,1,4	3,6,6	2,2,3	4,6,4	1,1,3	1,4,4	2,2,3	2,2,3	1,1,3	1,1,8	2,4,4	1,1,4	1,5,5	4,4,4	2,2,4	2,4,6	1,1,3	1,1,3	1,1,8	1,2,8	1,4,3	
Lead times with possibility = 1	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	1,1,3	

Table 4. Results by assigning fuzzy lead times through the rolling horizon.

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	$t = 10$	$t = 11$	$t = 12$	$t = 13$	$t = 14$	$t = 15$	$t = 16$	$t = 17$	$t = 18$	$t = 19$	$t = 20$	$t = 21$	$t = 22$	$t = 23$	$t = 24$	$t = 25$	TOTAL
Demand	612	684	471	647	531	778	369	554	685	535	761	401	435	654	508	472	625	758	762	370	445	573	762	488	433	14,313
$INVT_{i=1,t}$	0	922	845	879	348	1042	673	119	185	257	401	504	573	927	923	451	834	760	0	0	0	446	188	204	0	11,481
$INVT_{i=2,t}$	387	2719	2038	1374	1048	3352	3105	2601	2417	1512	1008	504	0	3056	3024	2520	1836	1332	828	324	0	1237	733	504	0	37,459
$INVT_{i=3,}t$	11,520	10,338	8295	6303	3879	2367	3024	1512	1203	6291	4779	12,642	11,130	9618	8106	6594	4542	3030	5229	4542	3570	3711	2199	1512	0	135,936
$P_{i=1,t}$	504	394	681	664	808	504	247	504	607	905	504	504	504	504	504	504	684	504	504	504	324	504	504	229	504	13,103
$P_{i=2,t}$	2726	0	482	711	2097	0	423	174	0	2394	0	456	536	472	0	0	1237	504	0	0	0	0	0	0	0	12,212
$P_{i=3,t}$	1398	0	7803	1791	0	4158	0	3426	0	0	0	0	0	966	3711	825	0	687	0	0	0	0	0	0	0	24,765
$B_{i=1,t}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	372	313	0	0	0	0	687
Tun	0	5500	0	0	0	0	12,850	0	0	0	0	0	0	0	0	0	0	0	0	0	9000	0	0	13,750	0	41,100
Tov	0	0	8850	8000	15,200	0	0	0	5150	20,050	0	0	0	0	0	0	9000	0	0	0	0	0	0	0	0	66,250
Scenario	1,1,6	1,2,3	1,2,7	2,2,8	1,1,4	3,6,6	2,2,3	4,6,4	1,1,3	1,4,4	2,2,3	2,2,3	1,1,3	1,1,8	2,4,4	1,1,4	1,5,5	4,4,4	2,2,4	2,4,6	1,1,3	1,1,3	1,1,8	1,2,8	1,4,3	
Lead times by centre of gravity	3,5,7	1,5,3	3,5,8	3,5,8	5,5,7	1,5,8	1,1,3	3,5,3	1,1,8	5,5,7	1,1,7	1,7,7	1,5,3	1,7,7	3,5,7	1,1,8	3,5,3	1,7,7	1,1,7	1,5,7	1,1,7	1,5,7	1,1,3	1,5,3	1,1,3	

Table 5. Summary of results.

Total costs	Back orders	Idle time	Overtime
+8.44%	−96.33%	−83.98%	−74.61%

specifically, the results from our proposal, based on the centre of gravity defuzzification method, are related to those based on the assignation of lead times with a greater possibility of occurrence. We have shown, despite total costs being slightly higher with our proposal (8.44%), as the model produces earlier to face uncertain lead times and higher inventory costs are generated, satisfactory improvements are obtained in terms of minor back orders, idle time and overtime production.

Figure 3 shows how the production of the different solution approaches adapts to demand. We can see how running our proposal provides production levels that adapt considerably better to the demand levels.

5.4 *Comparative with an alternative approach*

Additionally, in order to strengthen our proposal, we have compared it with an alternative approach. Thus, according to the previous literature review, we can conclude that the approach by Jiménez et al. (2007) later applied by (Peidro et al. 2010) in a supply chain planning problem considering fuzzy lead times is the closer one to address a similar problem. Nevertheless, the main difference between the two approaches, apart from the solution methodology is that while the approach by Jiménez et al. (2007) provides a fuzzy solution based on alpha-cuts that the DM must select based on his perception our new proposal is able to offer a defuzzified solution. Therefore, by applying the approach by Jiménez et al. (2007) Constraint (4) in the previous original model is transformed into the following two equivalent crisp constraints:

$$INVT_{i,t-1} + P_{i,t-TS_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I} a_{ij}\left(P_{jt} + SR_{jt}\right) + B_{it} = d_{it} \quad \forall i \forall t \tag{24}$$

$$FTS_i \leq \frac{\alpha}{2}\frac{FTS_{i1} + FTS_{i2}}{2} + \left(1 - \frac{\alpha}{2}\right)\frac{FTS_{i3} + FTS_{i4}}{2} \quad \forall i \tag{25}$$

$$FTS_i \geq \frac{\alpha}{2}\frac{FTS_{i3} + FTS_{i4}}{2} + \left(1 - \frac{\alpha}{2}\right)\frac{FTS_{i1} + FTS_{i2}}{2} \quad \forall i \tag{26}$$

where α represents the degree that, at least, all the constraints are fulfilled; that is, α is the feasibility degree of a decision x; and FTS_{i1}, FTS_{i2}, FTS_{i3}, FTS_{i4} are the extreme values which represent the trapezoidal fuzzy number associated with the corresponding fuzzy lead time ($\tilde{TS_i}$). Table 6 provides the results generated by applying the Jiménez et al. (2007) approach to our problem by selecting arbitrarily an alpha cut of 0.8. It is seen that, apart from the advantage of getting a defuzzified solution, our approach has provided better results, mainly, in terms of minor inventory levels and overtime and, consequently, total costs.

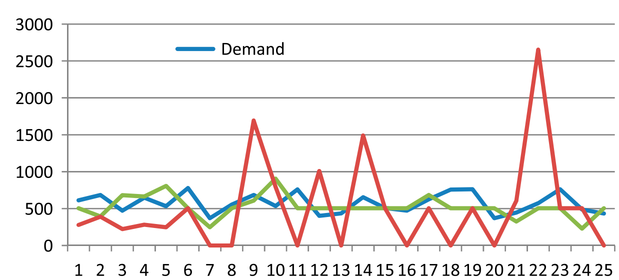

Figure 3. Demand and production levels.

Table 6. Results by applying Jiménez et al. (2007) approach with $\alpha = 0.8$.

	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$	$t=11$	$t=12$	$t=13$	$t=14$	$t=15$	$t=16$	$t=17$	$t=18$	$t=19$	$t=20$	$t=21$	$t=22$	$t=23$	$t=24$	$t=25$	TOTAL
Demand	612	684	471	647	531	778	369	554	685	535	761	401	435	654	508	472	625	758	762	370	445	573	762	488	433	14,313
$INVT_{i=1,t}$	0	471	647	670	139	419	50	0	323	292	1480	1583	1652	2736	2732	2260	2493	2239	1477	1107	662	1056	798	814	885	26,985
$INVT_{i=2,t}$	838	1174	504	454	504	1008	504	0	870	504	0	469	295	1866	1362	1008	956	452	504	82	0	504	504	504	0	14,866
$INVT_{i=3,}t$	12,873	10,932	8922	7260	5748	4236	2724	1212	2520	2241	729	6120	2418	5598	5754	4692	3180	1668	11,283	10,017	9771	16,380	16,380	14,868	14,868	182,394
$P_{i=1,t}$	53	647	670	554	504	504	504	504	504	1949	504	504	1234	504	504	354	504	504	0	422	82	504	504	504	504	13,525
$P_{i=2,t}$	983	504	554	685	323	973	1374	1825	1583	250	1060	0	0	0	52	452	504	504	0	504	0	504	0	0	0	12,634
$P_{i=3,t}$	0	0	1650	1707	2820	1560	3918	1266	2370	3024	1668	1668	0	8121	9615	0	0	0	1512	0	0	0	0	0	0	40,899
$B_{i=1,t}$	0	0	0	0	0	0	0	504	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	504
Tun	22,550	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7500	0	0	25,200	4100	21,100	0	0	0	0	80,450
Tov	0	7150	8300	2500	0	0	0	0	0	72,249	0	0	36,500	0	0	0	0	0	0	0	0	0	0	0	0	126,699

6. Conclusions

This paper addresses the MRP problem under uncertainty associated with lead times by means of a fuzzy multi-objective decision model. Multi-objective models are necessary because of the difficulty that companies have defining production parameters as backlog costs or idle time costs, which tend to appear in single-objective traditional MRP models.

For the purpose of solving the multi-objective model, we propose a solution methodology based on FGP which considers lack of knowledge associated with lead times, and a defuzzification method based on the centre of gravity calculation. This proposal is applied to a numerical example with 25 different instances. A rolling horizon experimentation approach was designed to test this proposal. The results provide major improvements in terms of reducing back orders, idle time, production overtime and better adjustment from production to demand levels. Additionally, the approach was compared with the previous one by Jiménez et al. (2007). In this sense, the proposal was consistent with their results by decreasing total costs derived by minor inventory levels and overtime costs.

The advantages of this proposal are related to: (i) modelling and establishing priorities for production objectives that are traditionally measured through estimated costs, with the consequent difficulty for companies; considering different values for the product lead times associated with distinct possibility degrees, which provide the decision-maker with a broad decision spectrum that has different risks levels; (ii) validating the centre of gravity concept as a defuzzification method for production problems under fuzzy lead times; (iii) using rolling horizon experiments for validating problems under uncertainty in lead times.

Managerial implications are oriented, mainly, to the availability of models that can formalise the uncertainty in lead times additionally to model multiple objectives in different measure units. In this sense, decision-makers could integrate these types of models in their current information systems based on MRP for supporting the decision-making processes under uncertainty in external or internal supply, where suppliers or manufacturing processes are not very reliable. Therefore, decision-makers can account with an alternative approach to safety stocks, safety times and lot-sizing rules in order to face the epistemic uncertainty in lead times.

With respect to the limitations of our proposal, we have validated our approach using a numerical example with an only finished good and a unique BOM level. Nevertheless, in order to apply it to a real-life situation the same steps in Section 4.3 should be followed by replying it for all the desired finished goods and raw materials and components. Of course, at the same time that the input data in terms of finished goods and raw materials are growing the combinatorial explosion of lead times instances (from step 6 to step 9 in Section 4.3) also increases. Therefore, it would be desirable the development of a tailored software application to manage it for each specific real-life problem according to the summarised solution procedure in Section 4.3. Thus, further research proposals include: (i) developing a decision support system to systematise model configuration and running; (ii) examining the effect of more complex product structures and validating the proposed solution methodology in real-world MRP problems; (iii) proposing alternative solution methodologies for the addressed fuzzy problem and comparing them with the current proposal; (iv) comparing alternative approaches based on parameterisation methodologies and robust optimisation.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the Spanish Ministry of Education Projects: Design and Management of Global Supply Chains (GLOBOP) [Ref. DPI2012-38061-C02-01] and [ECO2011-26499].

ORCID

Manuel Díaz-Madroñero http://orcid.org/0000-0003-1693-2876
Josefa Mula http://orcid.org/0000-0002-8447-3387
Mariano Jiménez http://orcid.org/0000-0003-1417-7985
David Peidro http://orcid.org/0000-0001-8678-6881

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