SETTLEMENT MONITORING MATHEMATICAL DATA TREATMENT

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SUMMARY

Formulae are presented to separate building movement into rigid and deformation movements. Since any solid movement can be superimposed to a deformation movement, the solid movement is obtained by minimisation. This way allows to describe what is happening in terms of few simple variables. An example is shown where the obtained description gives an important clue to the repair solution.

RESUMEN

Se presentan fórmulas para separar los movimientos medidos en movimientos como sólido rígido y movimientos que producen deformación. Como cualquier movimiento como sólido rígido se puede superimponer al movimiento deformacional, aquel se obtiene mediante minimización. Esto permite describir lo que pasa mediante pocas variables sencillas. Se presenta un ejemplo en que la descripción obtenida da una importante clave para la solución de reparación.

KEYWORDS: Foundation movement, settlement measurements

INTRODUCTION

Burland and Wroth (I) and many other researchers have mentioned that, when investigating the effects of differential settlement on buildings, the movement as a rigid body has to be taken off the overall movement. This paper shows a way to do this.

The movement of a deformable body can be decomposed in a variable deformation and a translation plus rotation, as a rigid body.

If $\mathbf{u}$ is the displacement vector, the deformation tensor is

$$\mathbf{e}_{ij} = \frac{\partial \mathbf{u}_i}{\partial \mathbf{x}_j}$$

The unit strain tensor is the symmetrical part of the strain tensor

$$\mathbf{e}_{ij} = \frac{1}{2}\left[\frac{\partial \mathbf{u_i}}{\partial \mathbf{x_j}} + \frac{\partial \mathbf{u_j}}{\partial \mathbf{x_i}}\right]$$

and the rotation is the antisymmetrical part

$$\mathbf{e}_{ij} = \frac{1}{2}\left[\frac{\partial \mathbf{u_i}}{\partial \mathbf{x_j}} - \frac{\partial \mathbf{u_j}}{\partial \mathbf{x_i}}\right]$$

It is easy to show that the displacements of a rigid body form an antisymmetric and constant tensor, so that the addition of a rigid body movement does not alter the unit strain tensor.

This paper shows a way of separating the rigid body movement from the strain movement by minimizing the norms of the strain movement vectors.

THE STRAIN VECTORS

If in a body displacements are measured $\mathbf{u}_i$ at $\mathbf{N}$ points placed at positions represented by their situation vectors, $\mathbf{r}_i$, a decomposition of movements is possible in the form

$$\mathbf{u}_i = \mathbf{u}_{di} + \mathbf{u}_{ri}$$

where the index $\mathbf{d}$ refers to the strain and the index $\mathbf{r}$ refers to the rigid movements. If the rigid movement is written as a moment field

$$\mathbf{u}_i = \mathbf{u}_{di} + \mathbf{u}_o + (\mathbf{r}_o - \mathbf{r}_i)\,\Delta w$$

where $\mathbf{r}_o$ is an arbitrary point and w is a rotation.

The norm of the strain vector, $\| \mathbf{u}_{di} \|$, is the square of its modulus, and the function to be minimized is

$$\mathbf{F} = \Sigma \| \mathbf{u}_{di} \|$$

MINIMIZATION

In the minimization problem, the unknowns are the three components of the uo vector and the three components of the rotation vector, w. If the ro arbitrary point is any one, a set of six equations with six unknowns is obtained. Direct observation of these equations shows that, if the arbitrary point is the center of gravity of the measured points, the system is reduced to two systems of three equations with three unknows each, where the center displacement is separated from the tilt. The system is undetermined, since a straight line and not a point is enough to situate the rotation.

The center displacement is

$$\mathbf{u}_G = (\Sigma \mathbf{u}) / \mathbf{N}$$

that is, the average of the maesured displacements.

If the position origin is the center of gravity of the measured points, the tilt equations can be reduced to the vectorial equation

$$W \cdot \Sigma \mathbf{r}^2 - \Sigma \mathbf{r}\,(\mathbf{r} \cdot w) - \Sigma r\,\Delta\,\mathbf{u} = \mathbf{0}$$

Using the foregoing equations, a rigid body displacements field is obtained which has the property that the sum of deformation displacements vanishes, because

$$\Sigma \mathbf{u}_d = \Sigma \mathbf{u} - \mathbf{N} \cdot \mathbf{u}_G + (\Sigma \mathbf{r})\,\Delta w$$

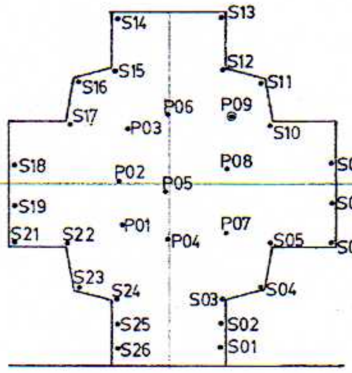

Figura 1.- Puntos de control.

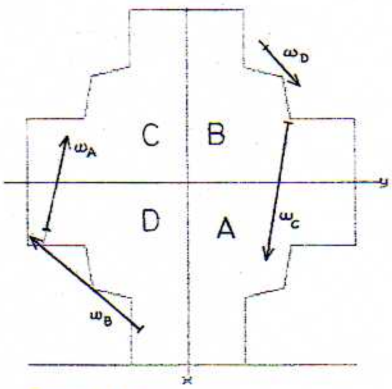

Figura 2.- Giros.

and $\sum \mathbf{r} = \mathbf{0}$ because $\mathbf{G}$ is the center of gravity.

AN EXAMPLE

A study was made in the Communion Chapel of St Mary's Church, Elche, Alicante (Spain) to determine the cause of fissures and apparent movements, 34 points were leveled in the 14X14 m plant. Levels were taken at the floor and the assumption that these were the structure movements was made.

Figure 1 shows the situation of the measured points and the subdivision of the chapel, made after the main fissure lines. Table 1 gives the levels referred to an arbitrary datum (point S14)

Table 2 gives the translation and rotation of each of the parts. The rotations are shown in their situation and magnitude in figure 2. It can be seen that each portion tends to rotate in a form similar to the opening of an orange in four parts. The values of rotations give separations in the upper part which are in agreement with other calculations based in structural reasons, as shown by Abdilla and others (2)

REFERENCES

(I) Burland J. B. and Wroth C. P.; "Settlement of buildings and associated damage". Conference on settlement of structures. Pentech, London, 1975.

(2) Abdilla E., Monfort J. and Rechea M. ; "Estudio de las causas de los daños y análisis de posibles soluciones en la Capilla de la Comunión de la Iglesia de Sta María de Elche.'" This volume.

Table 1. - Control points coordinates.

SECTOR	POINT	X COORD (m)	Y COORD (m)	Z COORD (m)
A	S01	6.600	2.600	-0.007
	S02	5.600	2.600	-0.009
	S03	4.600	2.600	-0.011
	S04	4.300	4.300	-0.011
	S05	2.600	4.600	-0.005
	S06	2.600	7.500	-0.014
	S07	1.000	7.500	-0.021
B	S08	-1.000	7.500	-0.012
	S10	-2.600	4.600	-0.022
	S11	-4.300	4.300	-0.027
	S12	-4.700	2.600	-0.029
	S13	-7.000	2.600	-0.010
C	S14	-7.000	-2.600	0.000
	S15	-4.700	-2.600	-0.018
	S16	-4.300	-4.300	-0.027
	S17	-2.600	-4.600	-0.020
	S19	-1.000	-7.200	-0.016
D	S20	1.000	-7.200	-0.010
	S21	2.600	-7.200	-0.004
	S22	2.600	-4.600	-0.013
	S23	4.300	-4.300	-0.010
	S24	4.700	-2.600	-0.005
	S25	5.700	-2.600	-0.008
	S26	6.600	-2.600	-0.006
FLOOR	P01	1.792	-2.163	-0.012
	P02	-1.280	-2.173	-0.018
	P03	-2.463	-1.593	-0.002
	P04	2.397	-0.008	-0.016
	P05	0.305	-0.120	-0.020
	P06	-3.004	-0.050	-0.029
	P07	1.793	2.668	-0.012
	P08	-0.651	2.596	-0.023
	P09	-2.911	2.743	-0.030

Table 2. - Calculated Rotations situation and values

SECTOR	AXIS OF ROTATION SITUATION			ROTATION		
	x (m)	y (m)	z (m)	ω_X (rad)	ω_Y (rad)	ω_Z (rad)
GLOBAL	21.1598	0.0000	-0.0004	-0.000443	-0.000694	0.000000
A	-27.2420	0.0000	0.0162	-0.002097	0.000409	0.000001
B	8.0190	0.0000	-0.0012	-0.002177	-0.002494	0.000001
C	29.9410	0.0000	0.0296	0.003174	-0.000541	0.000003
D	-11.0090	0.0000	0.0010	0.001099	0.000727	-0.000000