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DTU Electrical Engineering
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Cost-optimal ATCs in zonal electricity markets considering partial coordination between zones

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Abstract

Over the last years, the penetration of non-dispatchable variable renewable energy is constantly increasing, bringing new sources of uncertainty and challenging the traditional electricity market designs. To properly integrate these energy sources is currently a main research topic, specially in zonal electricity markets, in which market prices are constant across each zone without spatial differentiation.

This thesis addresses this problem exploring new methods to determine the Available Transfer Capacities (ATCs), which limit the trade of power between zones in the day ahead stage. The methodology used, extensively analyzed in the previous literature, defines ATCs aiming to minimize the operational costs and decoupling them from the physical grid, unlike the current methods in which ATCs are determined according to security and reliability constraints. To define this set of optimal ATCs, a external entity gathers all the necessary information of the different zones of the zonal network and solves a bilevel stochastic optimization problem.

The aim of this thesis is to use the previous models of the literature as a benchmark, analyze and extend them in order to align them with the current practice. Two main contributions are listed in this master thesis work: (i), previous work determine the cost-optimal ATCs minimizing the total expected operating cost, here the analysis is extended not only to the total costs, but also to the costs of each one of the zones of the zonal network. (ii), the model is reformulated and solved in a distributed fashion, using a distributed optimization technique, in order to avoid an excessive share of information and respect the privacy of the entities involved. The resulting algorithm is applied to different case studies, in order to compare and analyze the results of both approaches, i.e the centralized one, with full share of information to the central entity (named the full coordination model) and the distributed approach, named partial coordination model.

Preface

This thesis was prepared at the department of Electrical Engineering at the Technical University of Denmark. The thesis is set to be 30 ECTS and is submitted in fulfillment of the requirements for acquiring a Master of Science in Engineering (Sustainable Energy). The study was carried out from August 2017 to January 2018 under the supervision of Professor Jalal Kazempour, Professor Pierre Pinson and PhD. student Lejla Halilbasic from the Center for Electric Power and Energy at the Department of Electrical Engineering DTU.

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Almost seven years ago I started my engineering adventure at the Polytechnic University of Valencia. During this long and exciting journey I have had the opportunity to be affiliated to two extraordinary universities, to meet awesome people and to learn more than I could imagine. Definitely, it has been an incredible experience.

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I would also like to thank from the bottom of my heart to my friends, from both Copenhagen and Valencia, for their constant support. Last but not least, thanks to my family for being always with me, for your help, love and patience. *Gracias por todo, sin vosotros esto no hubiera sido posible.*

Nomenclature

ATC	Available Transfer Capacity.
MPEC	Mathematical problem with equilibrium constraints.
IEEE	Institute of Electrical and Electronics Engineers.
KKT	Karush–Kuhn–Tucker optimality conditions .
LMP	Locational marginal price.
MILP	Mixed-integer linear problem.
TSO	Transmission system operator.
VRE	Variable renewable energy.
DSO	Distribution system operator.
ADMM	Alternating direction method of multipliers.
TTC	Total transfer capacity.
NTC	Net transfer capacity.
TRM	Transmission reliability margin.
LL	Lower level.
UL	Upper level.

Indexes and sets

z	Index for zones.
n, m	Index for nodes.
g	Index for generators.
e	Index for links between zones.
l	Index for lines between nodes.
s	Index for renewable production scenarios.
k	Index for renewable generators.
d	Index for loads.
v	Index for iteration of ADMM.
Z	Set of zones.
N	Set of nodes.
G	Set of generators.
E	Set of links.
S	Set of renewable production scenarios.
K	Set of renewable generators.
D	Set of loads.
l^*	index for lines connecting nodes from different zones.

Parameters

Q_g^{max}	Installed capacity of generator g [MW].
C_g	Price offer of generator g [€/MWh].
C_{up}	Up-regulation extra cost [€/MWh].
C_{down}	Down-regulation extra cost [€/MWh].
L_d	Load level [MW].
$B_{n,m}$	Susceptance between nodes n and m [p.u].
$R_{k,s}^{RT}$	Realized renewable production of renewable generator k [MW].
$F_{n,m}^{max}$	Maximum capacity of the line connecting nodes n and m [MW].
$\overline{R_g}$	Upper bound difference between the production in day ahead and real time stages [MW].
$\underline{R_g}$	Lower bound difference between the production in day ahead and real time stages [MW].
R_k^{DA}	Installed capacity of renewable generator k [MW].
$VOLL$	Value of lost load [€/MWh].
π_s	Probability of scenario s [%].
ρ	Penalty parameter of the ADMM.

Variables

q_g^{DA}	Production of generator g in the day ahead stage [MW].
$q_{g,s}^{RT}$	Production of generator g in scenario s in the real time stage [MW].
$q_{g,s}^{up}$	Up-regulation of generator g [MWh].
$q_{g,s}^{down}$	Down-regulation of generator g [MWh].
r_k^{DA}	Production of renewable generator k in DA [MW].
$r_{k,s}^{DA}$	Production of renewable generator k in scenario s in RT [MW].
$f_{n,m,s}^{RT}$	Flow between nodes n,m in RT in scenario s [MW].
$FDA_{z,zo}$	Flow between zones z and zo in DA [MW].
$l_{d,s}^{shed,RT}$	Load shedding in real time [MW].
$ATC_{z,zo}$	Available transfer capacity between zones z and zo [MW].
$\theta_{n,s}$	Voltage angle in node n [p.u].
λ_z^{DA}	Market price in zone z in DA (€/MWh).
$\underline{\mu}_g^{Q,DA}, \overline{\mu}_g^{Q,DA}$	Dual variables of the conventional generator capacity constraint in day ahead per generator g.
$\underline{\mu}_k^{R,DA}, \overline{\mu}_k^{R,DA}$	Dual variables of the renewable generator capacity constraint in day ahead per renewable generator k.
$\underline{\gamma}_{z,zo}^{F,DA}, \overline{\gamma}_{z,zo}^{F,DA}$	Dual variables of the flow in day ahead limit constraint per pair of connected zones z, zo.
λ_e	Dual variable of the coupling constraint of flows in day ahead per link e.

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CHAPTER 1

Introduction

1.1 Motivation

Over the last few years, the penetration of non-dispatchable variable renewable energy generators (VREs) has been constantly increasing. In Denmark, the 43.4% of the electricity consumption was entirely supplied by land and sea turbines in 2017, setting a new historical maximum. Only 8 years ago, in 2009, this share was around a 20% [10]. On the 28th of October 2016, the 24.6% of the European electricity demand was covered with wind energy, being Denmark and Germany the main producers. These trends are expected to continue in the future, and policies such as the Energy Roadmap 2050 [7] or the E.U 2030 Energy strategy [6] support that statement. From a environmental point of view, the reduction of the fossil fuel dependence is highly positive. However, the increase of penetration of VREs challenges the current market designs, and how to properly integrate the renewable generation is currently a main research topic.

In the classical electricity market designs the generation was usually centralized and provided by large-scale conventional generators, while the loads were considered almost a passive element of the grid. In that case, to optimally match the supply and demand is pretty straightforward, as conventional generators usually have low levels of uncertainty, and demand levels usually follow a pattern. However, in the current practice, VREs represent an important share of the supply side, so the levels of uncertainty are increased. As a consequence, the power fluctuations that VREs induce entails an increase of balancing resources needed to compensate them.

The effective integration of uncertain VREs is even more challenging in zonal electricity markets, in which the market prices are constant across each zone. In the previous literature many different alternatives have been analyzed to address this issue. In [9] a new methodology to define the Available Transfer Capacities (ATCs) in zonal electricity markets is proposed. This method is implemented in a European scale test system, proving that the total expected operating system costs can be reduced compared to the current definition of ATCs. However, the model, as defined in [9] needs different modifications in order to adapt it to the current European practice. Mainly, a suitable model should avoid an excessive share of information between the different zones of the zonal system and should be implemented in a distributed way.

The aim of this thesis is to use the model of [9] as a benchmark, analyze it and extend it to align it with the current practice. First, this thesis gives an overview of how the ATCs are defined in the current practice, and why it could be beneficial to determine them in a data-driven manner. Then, the thesis is divided in two main contributions: on the one hand, the original model of [9] is reformulated and applied to two different case studies to analyze not only the effects of the ATCs on the total system cost, but also on the costs of each one of the zones of the zonal network. On the other hand, the model is solved in a distributed fashion, using a distributed optimization technique. The resulting algorithm is applied to different case studies, in order to compare and analyze the results of both approaches (i.e the centralized model of [9] and the distributed model of this thesis). Finally, different conclusions are derived from the results, and different ideas for future work are proposed.

1.2 Electricity markets

Electricity markets have been traditionally organized in a centralized way. Generation, transmission, distribution and retail were controlled by vertically integrated, state-owned utilities. This organization was unchanged until the 1980s, when the first steps to liberalize the electricity sector were proposed in Chile. Now, most of the liberalized electricity markets worldwide have separated generation, transmission, distribution and retail. Competition is encouraged in generation and retail, while transmission acts as a monopoly [19].

In the simplest approach, an electricity market consists of a day ahead market and a balancing market. In day ahead market production is scheduled the day prior to the energy delivery on an hourly basis. On the other hand, the balancing market closes several minutes prior to the power delivery. Besides the day ahead and the balancing markets, the futures market permits to trade energy from one week to several years in advance [8].

Relevant participants related to the electricity markets are the Transmission System Operator (TSO), Distribution System Operator (DSO), Regulator, Market Operator, generating companies, and retailers. The TSO operates the transmission assets. The DSO operates the distribution grid. The regulator designs the markets and sets the rules. The market operator organices and operates the market. Generators offer its production through the different markets, while the retailers buys electricity from them [24].

The day ahead market is cleared one day in advance by the market operator. The producers submit offers in the form of production blocks, with an associated price, and the consumers submit bids in the form of consumption blocks, with an associated price as well. Then, the market operator determines aggregate sale and purchase curves using the merit order principle, i.e offer bids are sorted by increased price, and the demand bids in the inverse order [19].

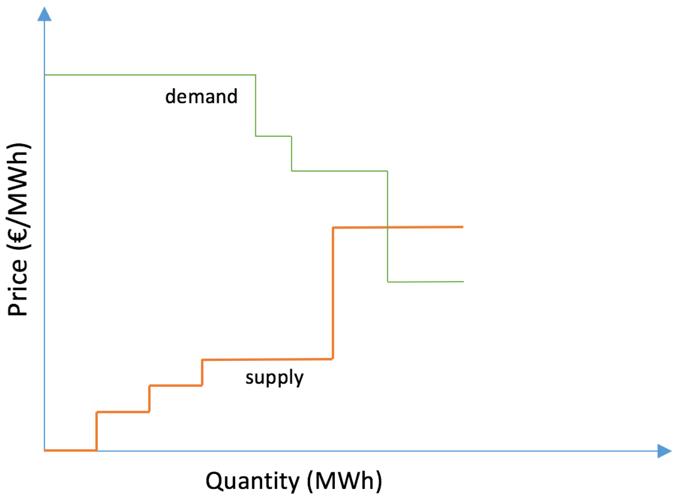

Figure 1.1: Energy supply and demand curve.

If during the market clearing procedure the transmission grid is not considered, the intersection between the demand and supply curves, as it can be seen in Figure 1.1, determines the equilibrium price, and this price applies for all market participants. On the other hand, if during the market clearing the transmission network considered a locational market price (LMP) is defined for each node of the grid, instead of a single market price [8]. During this thesis the models to be developed aim to find the cost-optimal ATCs in **zonal** electricity markets. Thus, the day ahead market clearing results in a single market price per zone (i.e, the grid is not considered in the day ahead stage).

Finally it is important to comment the impact of renewable energy sources on electricity markets. Marginal costs of sources such as wind or solar is close to zero, and they can even take negative values. Thus, in the merit order curve, renewable generators are scheduled before conventional generators. As a consequence, a high amount of scheduled renewable production shift the offer curve to the right, lowering the equilibrium price, while a low scheduled production shifts the curve to the left, increasing the equilibrium price. One direct consequence is that regions with high installed renewable capacity tend to have lower prices, but its volatility is higher [19].

1.3 ATCs in current practice

As mentioned in the previous section, in zonal networks the market clearing in day ahead results in one single market price per zone, i.e there is no spatial differentiation across a zone. So, when clearing interconnected markets in a zonal network, each one of the transmission systems is highly simplified, as seen in figure 1.2.

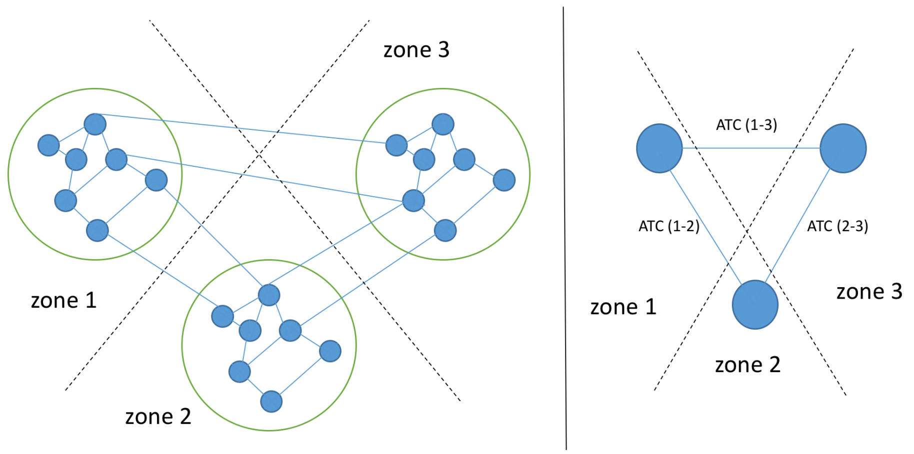

Figure 1.2: Simplification of the transmission network in the day ahead stage. Left: real system network; right: simplified network in day ahead .

The transmission network of a zone is formed by a certain number of nodes and lines, as represented on the left of figure 1.2. In the day ahead stage each zone represents the whole transmission network, and the interconnection between zones is reduced to a tie-line. The trade of power between zones is limited by the Available Transfer Capacity (ATC).

The values of these ATCs are, in practice, determined by the affected TSOs, based principally on reliability and security measures. The process to calculate them is shown in figure 1.3.

The total Transfer Capacity (TTC) is the maximum transmission of active power in accordance with the system security criteria which is permitted in transmission cross-sections between the areas [21]. In the literature different approaches are derived to calculate this value. For instance, in [4] a mathematical model is proposed, in which the TTT is calculated as the maximum cross-border flow between two zones, subject to different constraints related to flow balances and operational security standards.

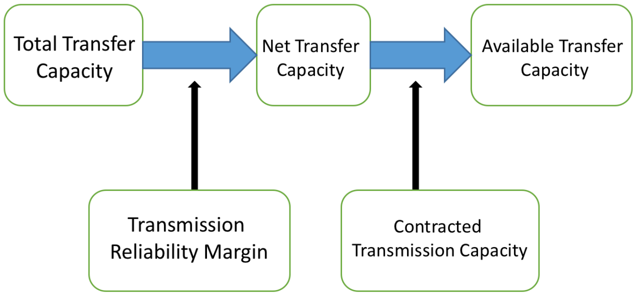

Figure 1.3: Flowchart of the process to determine ATCs in the current practice.

The transmission Reliability Margin (TRM) is a security margin that copes with uncertainties on the computed TTC values, including unintended deviations of physical flows during operations due to physical functioning of load-frequency regulation; emergency exchanges between TSOs and inaccuracies. The Net Transfer Capacity (NTC) results from subtracting the TRM to the TTC. NTC is, per definition, the maximum exchange program between two areas compatible with security standards applicable in both areas, and considering future network conditions and uncertainties [21]. Finally, to derive ATCs from NTCs it is necessary to consider the previously contracted transmission capacity [4].

Note that, in the aforementioned calculation process, ATCs are determined according to the physical characteristics of the grid, and they are limited by security and reliability constraints. Thus, in the current practice, ATCs are considered a purely physical parameter. However, these ATCs values highly influence the market clearing in the day ahead and, consequently, the final dispatch in the real time market.

In [9] it is suggested to define ATCs as financial parameters, and to decouple them from the current physical ATCs definition. The main idea of this model is that, if the determined ATCs allow trades in day ahead that result in efficient and reliable dispatches in real time, TSOs should not care about their specific values. In other words, as long as the resulting power flows in real time respect the security and reliability requirements of the grid, financial ATCs could have higher values than the current physical ATCs, and even higher than the capacity limits of the interconnection lines. Thus, [9] defines a new market entity, named ATC optimizer, that solves a bilevel stochastic optimization problem and finds the set of financial ATCs that minimizes the total system cost in a multi-zonal network.

The results obtained in [9] are promising, but the proposed model present different drawbacks. First, the system assumes a full coordination between all the TSOs of the zonal network, thus, it is assumed a high share of information between them. Moreover, the set of financial ATCs minimize the total system costs, but they do not necessarily minimize the system cost of each one of the system zones, so different participants may perceive that they are taking a losing position. Finally, the model is solved in a centralized fashion, and in order to implemented in practice, it should be implemented in a distributed and decentralized way.

1.4 Thesis objectives

All things considered, the objectives that this thesis aims to fulfill are listed as follows:

- Formulate the mathematical model of [9], and suggest alternatives to simplify it.
- Implement the model of [9] in different case studies and analyze the impact of the resulting ATCs not only in the overall costs of the system, but also in the particular cost of each one of the participating zones
- Find a suitable distributed optimization technique to solve the model of [9] in a distributed (i.e partially coordinated) fashion
- Formulate and implement the model of [9] using the selected distributed optimization algorithm
- Analyze and compare the ATCs derived from the centralized case (full coordination) and the distributed one(partial coordination)

1.5 Thesis organization

The rest of the thesis is structured as follows. In chapter 2 the cost-optimal ATCs with full coordination between zones model of [9] is widely described and formulated. Finally, it is applied to two different case studies, to find which are the sets of cost-optimal ATCs minimizing not only the total system cost, but also the individual cost of each zone. In chapter 3 a distributed algorithm is chosen and the mathematical model is reformulated to fit the algorithm requirements. Finally, the partial coordination model is applied to a set of case studies, in order to compare the set of ATCs determined in both the centralized and distributed approaches. Chapter 4 provides the final remarks, conclusion and future research directions.

CHAPTER 2

Cost-optimal ATCs with full coordination between zones

2.1 Chapter scope

The objective of this chapter is to analyze and expand the Cost-optimal ATCs in zonal electricity markets model proposed by Tue Vissing et al. in [9]. The idea behind this model is to find the set of optimal ATCs that minimizes the total expected operational costs of the system. In order to find this set, it is assumed a full coordination between all the zones.

The main goal of this thesis is to find the set of cost-optimal ATCs, but considering a non-cooperative fashion, so each zone seeks to optimize its own costs and revenues. Thus, the objective of this chapter is to use the full coordination model to analyze not only how ATCs affect the total system costs, but also to determine how they affect each one of the regions of the system. The expected output is to prove that the set of ATCs that minimize the total system costs may be sub-optimal for different individual zones.

Throughout this chapter two mathematical models will be formulated and applied to a case studio in order to analyze the full-coordination output. In the first one, described in section 2.2, the ATCs are determined considering sequential Day Ahead (DA) and Real Time (RT) market operation. Thus, as the DA market is cleared prior to the real time one, the merit order is respected. From now on, this model will be named the ”sequential market model”. In the second one, described in section 2.4, the ATCs are determined considering a stochastic DA and RT market clearing. In this case, the merit order can be violated, and the mathematical model is purely convex. This model will be referred to as the ”stochastic market model”.

This chapter is organized as follows. Section 2.2 describes both Cost-optimal ATCs models: one with sequential market clearing, and another with stochastic market clearing. Sections 2.3 and 2.4 define the mathematical formulation of both models and the different reformulations to make them computationally implementable. Section 2.5 gives an overview of the congestion rent and its relevance in the proposed models. Finally, in section 2.6 the models are applied to different systems.

2.2 Model description and overview

2.2.1 Cost-optimal ATCs determination with sequential market clearing

The cost-optimal ATCs determination was introduced by Tue Vissing et al. in [9]. In the current practice, ATCs are tipically set attending only to physical and security restrictions. However, ATCs values influence the market clearing. For instance, too restrictive ATCs may increase the system cost, specially in day ahead, because fewer trades are permitted. On the other hand, lax ATCs may cause excessive costs in the real time stage due to the necessity of massive balancing actions to ensure the security and reliability of the system. Moreover, current trends show that renewable penetration is increasing more and more each year [16]. As a consequence, the uncertainty cost associated to these rise of VRES production increases its relevance. It is necessary to develop new market mechanisms to handle these issues.

In this sense, Tue Vissing et al. proposed a model in which ATCs are considered financial parameters. This model finds the set of ATCs that minimize the total system cost, assuming full coordination between zones. In order to find this set of ATCs, a external entity, named ATC optimizer, is defined. This model can be seen as a Stackelberg game, in which the ATC optimizer is the leader and the day ahead and real time markets are the followers.

In this chapter, a simplified version of the model defined by Vissing et al. is used. A sketch of this model can be seen in figure 2.1. The Stackelberg game is formulated as a bilevel model, in which the upper level (ATC optimizer) seeks to minimize the total cost of the system, consisting of the day ahead cost and the expected real time cost; and the lower level problem, for a given value of ATCs, clears the day ahead market seeking to minimize the day ahead cost.

It is important to note that, in this case, the real time market clearing is not considered a lower level problem. This is because the upper level problem does not provide direct information to the RT market, as ATCs do only directly affect to the day ahead one. Considering that the upper level objective function seeks to minimize the total cost, including the real time stage, real time market constraints can be placed in the upper level and the model is equivalent.

It is also important to consider that there is a set of real time constraints per scenario, in which each scenario represents a different renewable production.

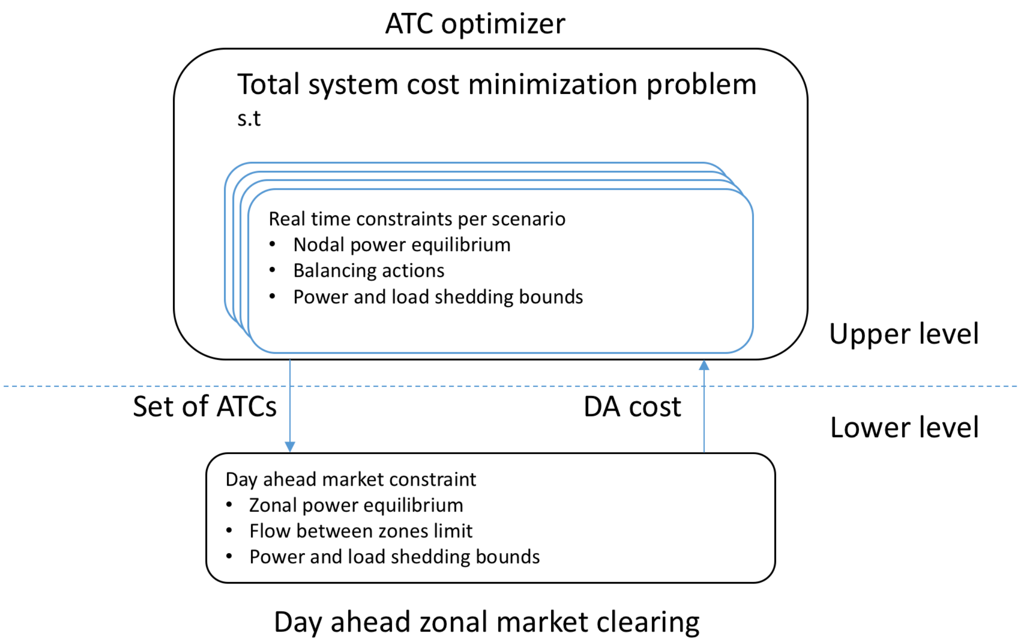

Figure 2.1: Bilevel model sketch.

2.2.2 Cost-optimal ATCs determination with stochastic market clearing

In the sequential model the merit order is respected because of its bilevel structure: the day ahead market is cleared aiming to optimize exclusively the cost in day ahead, so that the demand is served using the cheapest available generators. This sequential model is in general non-convex as shown in [9]. Further comments on this will be discussed in chapter 3, but overall, distributed optimization techniques applied to non-convex problems are not in general easy-to-solve.

The structure of ATC optimizer considering a stochastic market clearing is summarized in figure 2.2. In this case, the DA and RT markets are cleared simultaneously, aiming to optimize the objective function of both stages. Thus, merit order is not necessarily respected, as the day ahead market is not cleared considering only its own costs.

This model is, in fact, a simplification of the sequential one: the constraints associated to the DA and RT markets are the same, but in this case the model is a single-level optimization problem. Therefore, the resulting cost of the stochastic model will be lower or equal than the sequential one, so it can be used as a lower-bound. Moreover, one interesting feature of the stochastic model is that is mathematical formulation is purely convex, easing the application of a distributed optimization technique (further discussion in chapter 3). The main drawback of the stochastic model is that, as mentioned before, merit order may be violated in the DA stage.

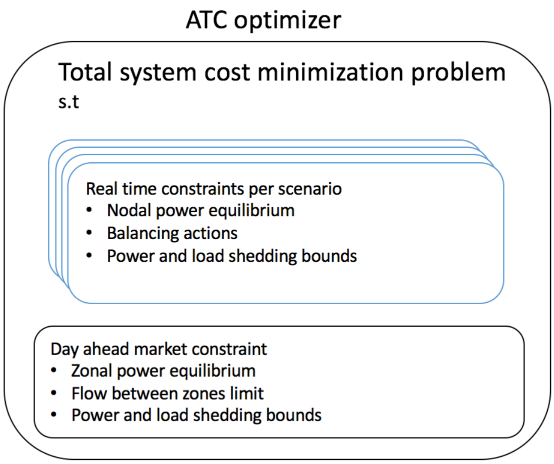

Figure 2.2: Single-level model sketch.

Note that, with this single-level structure, the variable ATC does not need to be included in the model. With the previous bilevel structure, ATCs are defined in the upper level in order to constraint the flow exchanged in day ahead in the lower level problem. Here, as both markets are cleared in a stochastic fashion, there is no need to constraint the flow traded in the DA stage. In order words, as it can be seen in Figure 2.1, ATCs are used to connect and, somehow, exchange information between both levels. Thus, when the system collapses to one level, ATCs variables are not needed anymore.

As a consequence, this model provides the optimal power flow to be traded in day ahead, and the ATCs have to be defined a posteriori according to this flow. Naturally, these ATCs must be higher or equal to the optimal power flow in order to respect the optimal dispatch. Thus, during this thesis, we assume that the optimal ATCs in this model are equal to the optimal flow in day ahead.

2.3 Sequential market model: Mathematical formulation

2.3.1 Bilevel model

The set of cost-optimal ATCs is determined by a bilevel optimization problem. The upper level objective function seeks to minimize the total cost of the system, including day ahead production cost and real time expected cost.

$$\begin{aligned} \underset{\Xi^{UL}}{\text{minimize}} \quad & \sum_g C_g \cdot q_g^{DA} + \sum_s \pi_s \Bigg[\sum_g \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) \\ & + \sum_d VOLL_d \cdot l_{d,s}^{shedRT} \Bigg] \end{aligned} \tag{2.1}$$

The objective function can be divided into two main terms:

1. The term $\sum_g C_g \cdot q_g^{DA}$ represents the total production cost in the day ahead stage, and it is calculated as the product of the production costs and energy production scheduled in day ahead of all generators.

2. The term $\sum_s \pi_s \left[\dots \right]$ represents the total expected cost in the real time operation of the system. It includes the production cost of the difference between the scheduled production in day ahead and the actual one in real time, the up and down regulation costs and the cost of load shedding.

It is important to note that as the model's aim is to minimize the total overall costs of the system and full coordination is assumed, revenues due to the exchange of power between zones are not considered. However, when analyzing each zone separately, and to determine optimal situations per zone, revenues should be included.

The upper-level constraints are listed in the following equations:

$$ATC_e \geq 0 \qquad \forall e \tag{2.2a}$$

$$\sum_{g \in \psi_n} q^{RT}_{g,s} + \sum_{k \in \psi_n} r^{RT}_{k,s} + \sum_{d \in \psi_n} (l^{RT}_{d,s} - L_d) = \sum_{m \in \delta_n} f^{RT}_{n,m,s} \qquad \forall n, \forall s \tag{2.2b}$$

$$q^{RT}_{g,s} = q^{DA}_g + q^{up}_{g,s} - q^{down}_{g,s} \qquad \forall g, \forall s \tag{2.2c}$$

$$f^{RT}_{n,m,s} = B_{n,m}(\theta_{n,s} - \theta_{m,s}) \qquad \forall n, \forall m \in L \tag{2.2d}$$

$$0 \leq q^{RT}_{g,s} \leq Q^{max}_g \qquad \forall g, \forall s \tag{2.2e}$$

$$0 \leq q^{up}_{g,s}, 0 \leq q^{down}_{g,s} \qquad \forall g, \forall s \tag{2.2f}$$

$$0 \leq r^{RT}_{k,s} \leq R^{RT}_{k,s} \qquad \forall k, \forall s \tag{2.2g}$$

$$0 \leq l^{shed,RT}_{d,s} \leq L_d \qquad \forall d, \forall s \tag{2.2h}$$

$$-F^{max}_{n,m} \leq f^{RT}_{n,m,s} \leq F^{max}_{n,m} \qquad \forall n, \forall m \in L, \forall s \tag{2.2i}$$

$$\theta_{n=1,s} = 0 \qquad \forall s \tag{2.2j}$$

$$-\underline{R_g} \leq (q^{RT}_{g,s} - q^{DA}_g) \leq \overline{R_g} \qquad \forall g, \forall s \tag{2.2k}$$

1. The equation 2.2a defines the non-negativity condition for all the ATCs linking different zones.

2. The set of equations 2.2b-2.2k represent the constraints of the real time stage of the market operation. In Eq 2.2b the power equilibrium in all the nodes of the system is defined. Eq 2.2c connects the day ahead and real time production through the balancing actions. Eq 2.2d defines the flow between nodes. Eq 2.2e-2.2i define the bounds of the different real time variables. Eq 2.2j fixes to zero the voltage angle of the reference node. Finally, Eq 2.2k defines a limit between the difference of the scheduled production in day ahead and the production in real time.

3. The set of upper-level variables $\Xi^{UL} = (ATC_e, q^{RT}_{g,s}, q^{up}_{g,s}, q^{down}_{g,s}, r^{RT}_{k,s}, l^{shed,RT}_{d,s}, f^{RT}_{n,m,s}, \theta_{n,s})$ comprises the real time production of the generators, the up and down regulation per generator, the flow per line, the voltage angles and the ATCs between zones.

The lower level problem of the bilevel model corresponds to the day ahead market clearing. The lower level objective function is shown in the following equation.

$$\underset{\Xi^{LL}}{\text{minimize}} \quad \sum_g C_g \cdot q^{DA}_g \tag{2.3}$$

The objective function seeks to minimize the total production cost in the day ahead

stage and it is calculated as the product of the production costs and energy production scheduled in day ahead of all generators.
The lower-level constraints are listed in the following equations:

$$\sum_{g\in\psi_z} q_g^{DA} + \sum_{k\in\psi_z} r_k^{DA} - \sum_{d\in\psi_z} L_d = \sum_{zo\in\delta_z} FDA_{z,zo} \qquad : \lambda_z^{DA} \qquad \forall z \tag{2.4a}$$

$$FDA_{z,zo} + FDA_{zo,z} = 0 \qquad : \lambda_e \qquad \forall e \tag{2.4b}$$

$$0 \leq q_g^{DA} \leq Q_g^{max} \qquad : \underline{\mu}_g^{Q,DA}, \overline{\mu}_g^{Q,DA} \qquad \forall g \tag{2.4c}$$

$$0 \leq r_k^{DA} \leq R_k^{ins} \qquad : \underline{\mu}_k^{R,DA}, \overline{\mu}_k^{R,DA} \qquad \forall g \tag{2.4d}$$

$$-ATC_e \leq FDA_{z,zo} \leq ATC_e \qquad : \underline{\gamma}_{z,zo}^{F,DA}, \overline{\gamma}_{z,zo}^{F,DA} \qquad \forall z, zo = e \in E \tag{2.4e}$$

1. Equation 2.4a defines the power equilibrium in all the zones of the system. In equation 2.4b the flow between two zones is defined as positive for one of them and negative for the other. Equations 2.4c-2.4e set bounds for the day ahead variables.

2. The set of lower-level variables $\Xi^{LL} = (q_g^{DA}, r_k^{DA}, FDA_{z,zo})$ comprises the scheduled day ahead production of the generators, the forecast renewable production and the flow exchanged between zones. The dual variables associated to the lower-level problem are listed on the right side of the Equations 2.4a-2.4e. $\Xi^{LL,dual} = (\lambda_z^{DA}, \lambda_e, \underline{\mu}_g^{Q,DA}, \overline{\mu}_g^{Q,DA}, \underline{\mu}_k^{R,DA}, \overline{\mu}_k^{R,DA}, \underline{\gamma}_{z,zo}^{F,DA}, \overline{\gamma}_{z,zo}^{F,DA})$

2.3.2 MPEC model

The bilevel structure of the model proposed in section 2.2.1 is not implementable in most of the current optimization software, so it is necessary to reformulate the mathematical model. As the lower level problem (equations 2.3,2.4a-2.4e) is linear and thus convex, it can be replaced by its Karush-Kuhn-Tucker conditions, so that the bilevel model is transformed into a Mathematical Problem with Equilibrium Constraints (MPEC).

It is important to note that this MPEC is not appropriate for large-scale systems, as it would probably require a very large computational time. However, as the scope of this chapter is to analyze the effects of the model in small size setups this approach is sufficient.

The KKT conditions of the lower level problem are listed as follows:

$$\frac{\partial \mathcal{L}}{\partial q_g^{DA}} = C_g + \lambda_{z:g\in\psi_z}^{DA} - \underline{\mu}_g^{Q,DA} + \overline{\mu}_g^{Q,DA} = 0 \qquad \forall g \tag{2.5a}$$

$$\frac{\partial \mathcal{L}}{\partial r_k^{DA}} = \lambda_{z:k\in\psi_z}^{DA} - \underline{\mu}_k^{R,DA} + \overline{\mu}_k^{R,DA} = 0 \qquad \forall k \tag{2.5b}$$

$$\frac{\partial \mathcal{L}}{\partial FDA_{z,zo}} = \lambda_z^{DA} + \lambda_e - \underline{\gamma}_{z,zo}^{F,DA} + \overline{\gamma}_{z,zo}^{F,DA} = 0 \qquad \forall z, zo \in E \tag{2.5c}$$

$$0 \leq \underline{\mu}_g^{Q,DA} \perp q_g^{DA} \geq 0 \qquad \forall g \tag{2.5d}$$

$$0 \leq \overline{\mu}_g^{Q,DA} \perp Q_g^{max} - q_g^{DA} \geq 0 \qquad \forall g \tag{2.5e}$$

$$0 \leq \underline{\mu}_k^{R,DA} \perp r_k^{DA} \geq 0 \qquad \forall k \tag{2.5f}$$

$$0 \leq \overline{\mu}_k^{R,DA} \perp R_k^{ins} - r_k^{DA} \geq 0 \qquad \forall k \tag{2.5g}$$

$$0 \leq \underline{\gamma}_{z,zo}^{F,DA} \perp FDA_{z,zo} + ATC_e \geq 0 \qquad \forall z, zo \in E \tag{2.5h}$$

$$0 \leq \overline{\gamma}_{z,zo}^{F,DA} \perp ATC_e - FDA_{z,zo} \geq 0 \qquad \forall z, zo \in E \tag{2.5i}$$

Finally, the resulting MPEC can be written as follows:

$$\begin{aligned} &\underset{\Xi^{UL}\cup\Xi^{LL}\cup\Xi^{LL,dual}}{\text{minimize}} && (2.1) \\ &\text{Subject to} && (2.2a)-(2.2k), (2.4a)-(2.4b), (2.5a)-(2.5i) \end{aligned}$$

2.3.3 Linearization of the MPEC

Now, the bilevel original model has been transformed into a single-level MPEC, which in principle is easier to implement in the current optimization programs. However, the complementarity conditions 2.5d-2.5i introduce non-linear terms to the system, due to the product of variables. There are different ways to linearize complementarity conditions. In this case, the Fortuny-Amat McCarl linearization is applied. In this method, also known as Big M method, each complementarity constraint is substituted by four inequalities. For instance, the complementarity constraint 2.5d is linearized as follows.

$$\underline{\mu}_g^{Q,DA} \geq 0 \tag{2.6a}$$

$$q_g^{DA} \geq 0 \tag{2.6b}$$

$$\overline{\mu}_g^{Q,DA} \leq M \cdot u \tag{2.6c}$$

$$q_g^{DA} \leq M \cdot (1-u) \tag{2.6d}$$

where u is a binary variable and M is a large enough scalar. The selection of the M values is critical in this method. If the values of M are wrongly selected, there might be situations in which the KKT conditions are not satisfied.

The complementarity constraints 2.5e-2.5i can be linearized using the Big M method in the same way as in Equation 2.6. The following equations show the constraints related to these linearizations.

$$Q_g^{max} - q_g^{DA}, r_k^{DA}, R_k^{ins} - r_k^{DA}, FDA_{z,zo} + ATC_e, ATC_e - FDA_{z,zo} \geq 0 \tag{2.7a}$$

$$\overline{\mu}_g^{Q,DA}, \underline{\mu}_k^{R,DA}, \overline{\mu}_k^{R,DA}, \underline{\gamma}_{z,zo}^{F,DA}, \overline{\gamma}_{z,zo}^{F,DA} \geq 0 \tag{2.7b}$$

$$\overline{\mu}_g^{P,DA} \leq M \cdot u \tag{2.7c}$$

$$Q_g^{max} - q_g^{DA} \leq M \cdot (1 - u) \tag{2.7d}$$

$$\underline{\mu}_k^{R,DA} \leq M \cdot u \tag{2.7e}$$

$$r_k^{DA} \leq M \cdot (1 - u) \tag{2.7f}$$

$$\overline{\mu}_k^{R,DA} \leq M \cdot u \tag{2.7g}$$

$$R_k^{ins} - r_k^{DA} \leq M \cdot (1 - u) \tag{2.7h}$$

$$\underline{\gamma}_{z,zo}^{F,DA} \leq M \cdot u \tag{2.7i}$$

$$FDA_{z,zo} + ATC_e \leq M \cdot (1 - u) \tag{2.7j}$$

$$\overline{\gamma}_{z,zo}^{F,DA} \leq M \cdot u \tag{2.7k}$$

$$ATC_e - FDA_{z,zo} \leq M \cdot (1 - u) \tag{2.7l}$$

Note that, for the sake of simplicity, all binary variables have been named ”u”. In reality, each complementarity constraint has associated one different binary variable, so each pair of constraints (i.e 2.7c-2.7d) has its own one associated.

Finally, if all the complementarity conditions are substituted, the MPEC is tranformed into a Mixed Integer Linear Problem (MILP), which can be easily implemented and solved. The resulting MILP can be written as follows:

$$\begin{aligned} \underset{\Xi^{UL} \cup \Xi^{LL} \cup \Xi^{LL,dual}}{\text{minimize}} \quad & (2.1) \\ \text{Subject to} \quad & (2.2a) - (2.2k), (2.4a) - (2.4b), (2.5a) - (2.5c), (2.6), (2.7) \end{aligned}$$

2.4 Stochastic market model: Mathematical formulation

The mathematical formulation in this case is determined by a single-level optimization problem. The objective function seeks to minimize the total system cost, including day ahead and expected cost in real time, exactly as in the upper level of the sequential market model (equation 2.1).
The constraints of the problem are, again, the real time market constraints per scenario (2.2b-2.2k) and finally the day ahead market constraints (2.4b-2.4d).
The resulting single-sevel optimization problem can be written as follows:

$$\begin{aligned} \underset{\Xi^{SL}}{\text{minimize}} \quad & (2.1) \\ \text{Subject to} \quad & (2.2b) - (2.2k), (2.4b) - (2.4d) \end{aligned}$$

Note that in this model, the set of variables Ξ^{SL} is equivalent to the union of the sets Ξ^{LL} and Ξ^{UL}, excluding the variable ATC_e.
This model, convex and with a single-level structure, is perfectly implementable in the current optimization software, so it is not necessary to reformulate it.

2.5 Congestion rent relevance

The congestion rent is created when there is a power flow between two zones with different area prices. In other words, if the power flow from a zone A to a zone B is equal to the total capacity between both zones (congested line), zone B will be paying a price higher than the revenue that zone A is receiving, because the market price of B will be higher than the market price of A. This difference between the payment and the revenue is called congestion rent.

During this chapter the aim is to evaluate if the optimal ATCs obtained in a full-coordinated manner are also optimal for each zone individually. In order to do so, it is not enough to analyze only the generation costs of each of the zones, the revenues and costs of the power trade between zones should also be considered.

As a consequence, congestion rent plays a crucial role when analyzing the revenues and costs of each of the zones. If congestion rent is not considered, there might be situations in which increasing the flow from a zone with a lower market price to a zone with a higher one will not have an impact in the revenue and cost functions. For instance, if the flow from a zone A to a zone B is increased in 20 MW and in both cases there is congestion and the market prices do not vary, zone A will produce 20 extra MW at its own market price and will receive as a revenue 20 MW at its own market price, so there will not be any difference in their overall revenue. However, considering congestion rent, the second scenario is in principle more profitable, as there is a long-term extra revenue to be considered.

All things considered, it is important to define how this congestion rent is divided among the implied zones. In Nordpool, for instance, the congestion rent is shared according to different agreements between the different transmission system operators [20]. These agreements are regularly updated. For instance, in Nordpool in 2006 the share of the congestion rent was defined as 31.91% to Energinet, 12.77% to Fingrid, 17.45% to Statnett and 37.87% to Svenska Kraftnät. In 2011, the congestion rent was agreed to be divided in two equal shares between the two affected TSOs. This half and half method is also applied, for instance, between Western Denmark and Germany, due to an agreement between Energinet and E.ON Netz.

For the case studies to be analyzed in the following section, and considering the current agreements in Nordpool, it is assumed that the congestion rent is divided equally among the affected TSOs.

2.6 Case studies

In this section, the two proposed models will be implemented in two small-size setups: a system composed of 2 different zones with 3 nodes per zone, and a extension with 3 zones and 9 nodes. Both systems have different generators, wind farms and loads attached.

2.6.1 Setup description

2.6.1.1 2 zones model

The setup considered in this case is shown in figure 2.3. There are 2 zones, each of them with 3 different nodes. The capacity of all the lines of the system is set as 300 MW. Relevant data (extracted from [25]) of installed capacity, production costs and loads is shown in Table 2.1.

Table 2.1: Relevant parameters of case study 2 zones [25].

	Conventional generator capacity (MW)	Renewable installed capacity (MW)	Load level (MW)	Marginal price generator (€/MW)
n1	300	-	-	10
n2	150	100	-	10
n3	300	-	243	20
n4	-	-	271	-
n5	400	-	243	12
n6	200	100	243	12

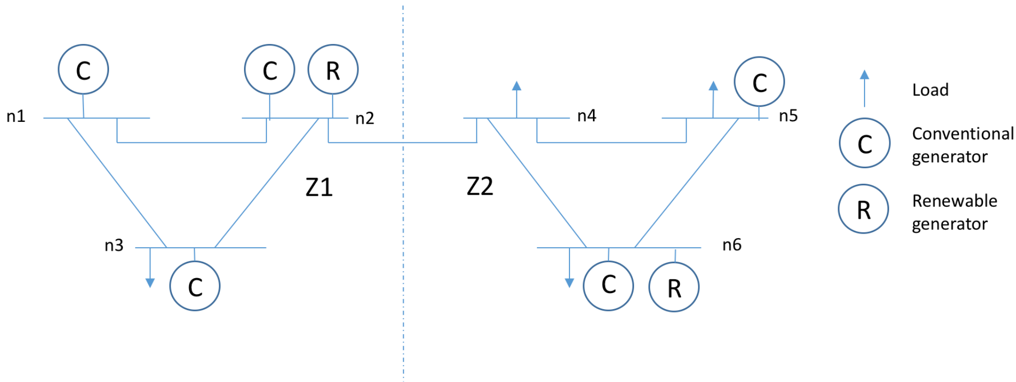

Figure 2.3: Sketch of the 2 zones model.

The only source of uncertainty in the system is the production of the renewable generators in the real time stage. To model this uncertainty, three different scenarios are considered: a real time production of 50%, 75% and 100% of the installed capacity with a probability of 0.3, 0.3 and 0.4 respectively.

2.6.1.2 3 zones model

In this case, the setup is extended to 3 zones. The sketch is shown in figure 2.4. The capacity of all lines is again set to 300 MW. Zones 1 and 2 maintain the same structure, generators and loads than in the previous case. The relevant data regarding zone 3 is shown in table 2.2.

Regarding the uncertainty characterization, here there are also 3 different renewable production scenarios with the same probabilities than in the 2 zones case.

Table 2.2: Relevant parameters of the 3 zones system.

	Conventional generator capacity (MW)	Renewable installed capacity (MW)	Load level (MW)	Marginal price generator (€/MW)
n7	200	-	-	12
n8	100	-	271	15
n9	200	100	243	13

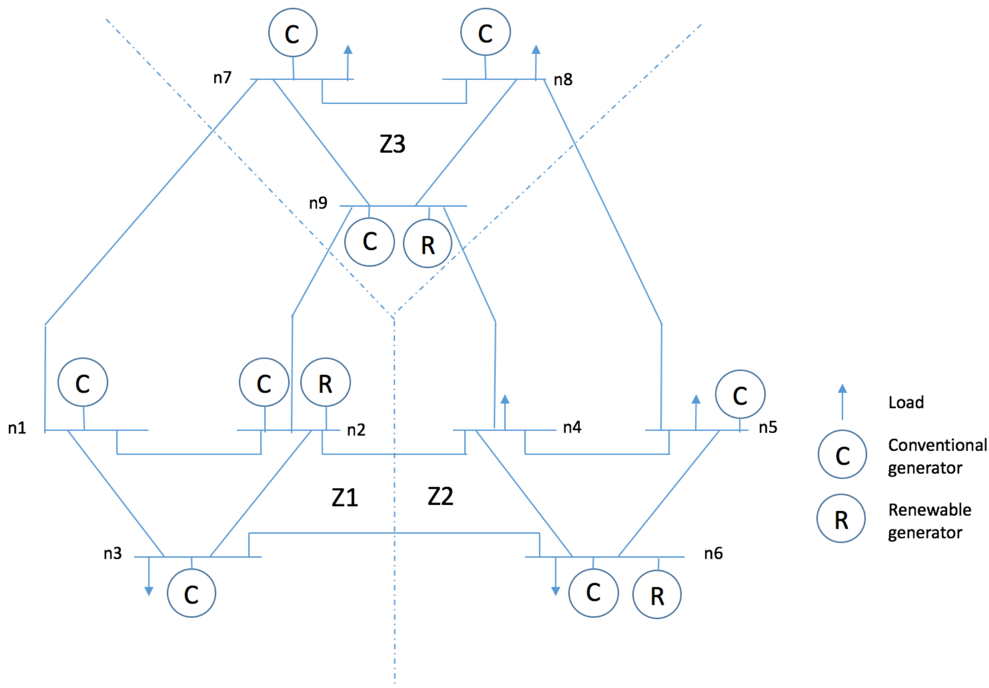

Figure 2.4: Sketch of the 3 zones model.

2.6.2 Cost and Revenue calculation

During this case study, we aim to calculate the Cost-optimal ATCs minimizing the total system cost, but we also want to determine how ATCs affect to the costs and revenues of each individual zone, to find the optimal points of each one and compare them with the centralized solution. In this section different equations to calculate these costs and revenues are suggested.

The production costs per zone are calculated as total scheduled cost in day ahead plus the expected cost during the real time operation.

$$Production \;\; costs = \sum_{g \in z} C_g \cdot q_g^{DA} + \sum_{s} \pi_s \left[\sum_{g \in z} \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) + \sum_{d \in z} VOLL_d \cdot l_{d,s}^{shedRT} \right] \tag{2.8}$$

The revenues (or costs) in day ahead are calculated as the scheduled flow between zones in day ahead times the market price.

$$Revenues_{DA} = FDA_{z,zo} \cdot \lambda_z^{DA} \tag{2.9}$$

The revenues (or costs) in the real time stage are calculated as the product of the difference between the scheduled flow in day ahead and the actual flow between zones in real time and the nodal marginal price of the border nodes.

$$Revenues_{RT} = \sum_{s} \pi_s \left[\left(\sum_{n,m \in \delta_b} f_{n,m,s}^{RT} - FDA_{z,zo} \right) \cdot \frac{\lambda_n^{RT}}{\pi_s} \right] \tag{2.10}$$

Finally, the congestion rent is calculated as the difference between the market price of zones multiplied by the flow between them. As assumed before, it is considered that this rent is equally divided among zones.

$$Congestion \;\; rent = FDA_{z,zo} \cdot abs[(\lambda_z - \lambda_{zo})] \tag{2.11}$$

So, finally, the total cost per zone is calculated as follows:

$$cost - revenues = (2.7) - (2.8) - (2.9) - \frac{(2.10)}{2} \tag{2.12}$$

2.6.3 Results

Sequential model, 2 zones-6 nodes system
The mathematical model has been implemented using GAMS. In this system the cost-optimal ATC has been determined to be 300 MW. With this ATC the total cost of the system is 9662.8€. In order to analyze what is the impact of the ATCs not only

in the total system costs but also in the costs and revenues of each zone, a sensitivity analysis has been performed.

The relevant results of the sensitivity analysis are plotted in Figure 2.5. The blue line represents the total cost of the system, which is the objective function to minimize in the proposed MILP. The green and red lines represent the cost minus revenues of each of the zones (equation 2.11). The first important comment is that each zone has a different optimal ATC. The ATC that minimizes the total cost of the system is 300 MW. This value also minimizes the cost minus revenues of zone 2. However, the optimal value for zone 1 is slightly lower, 257 MW. So the first insight that these results show is that the cost optimal ATCs with full coordination are not necessarily the best solution for each one of the zones.

It is also interesting to comment the particular shape of the curves associated to each one of the zones. From ATC=0 MW to 257 MW both of them decrease with a constant slope. The reason is that while the ATC is increasing, the flow from zone 1 to zone 2 in day ahead is also increasing. However, the market prices of both zones do not vary while increasing the ATC. As a consequence, while the flow from zone 1 to zone 2 is increasing, congestion rent is also increasing, so the cost minus revenues decreases at a constant pace. Above 257 MW of ATC there is one generator of zone 2 pushed out of the market, so the marginal price of zone 2 decreases. Therefore, higher values of ATC are more favourable in zone 2 rather than in zone 1. Above 300 MW of ATC the system is not congested, so both market prices are the same and the system reaches an equilibrium. A summary of the results is shown in Table 2.3.

Another relevant point is to evaluate how ATCs affect to the costs in the DA and RT stages. This is shown in figure 2.6. Results are pretty logical and in line with the previous comments: with lower values of ATC the trades in day ahead are restricted, so the cost in DA is increased. If ATCs are increased, the trades between zones in day ahead are higher, and the cost in DA is reduced. However, there is an increase in the balancing resources needed in real time, so the expected cost in RT is increased.

Sequential model, 2 zones-6 nodes system with additional generation
In the previous case, according to the plot in figure 2.5, there are two abrupt variations in the slope of the curves (in 257 MW and 300 MW), which coincide with the optimal values of the zones. As mentioned before, these variations are caused because, at

Table 2.3: Summary of results.

	Optimizing total cost	Optimizing zone 1	Optimizing zone 2
ATC (MW)	300	257	300
Total cost (€)	9662.8	9680	9662.8

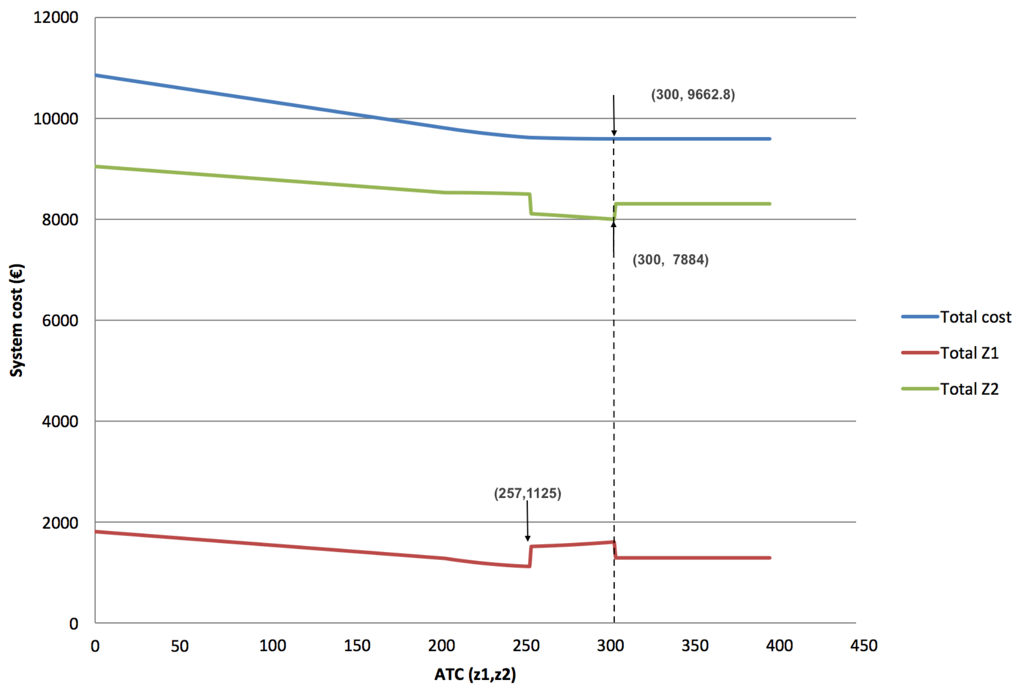

Figure 2.5: zonal cost minus revenues versus ATC: base case.

some point, the variation of the ATCs modifies the generation mix of a certain zone (i.e, generators are pushed out of the market, or new generators start producing). Thus, market prices may vary when ATC is increased/decreased.

In the considered 2-zones 6 nodes setup there are only 7 generators (5 conventional plus 2 renewable). During the previous simulation, only 2 of these generators change their status. This, in fact, is a problem when trying to analyze the results and extract solid conclusions, since this sudden abrupt variations can, somehow, buckle the results. In order to try to minimize this impact one solution is to add new generators. To keep the balance between load level and generation, each one of the generators of table 2.1 is divided into 6 smaller generators, so as the installed capacity of these 6 generators is equal to the installed capacity of the original one. The marginal price of the new generators is kept between a $+-10\%$ interval, and different from generator to generator. In this new setup there are 30 conventional generators. Again, a sensitivity analysis is performed to show how different values of ATC affect to the system. Results are plotted in figure 2.7.

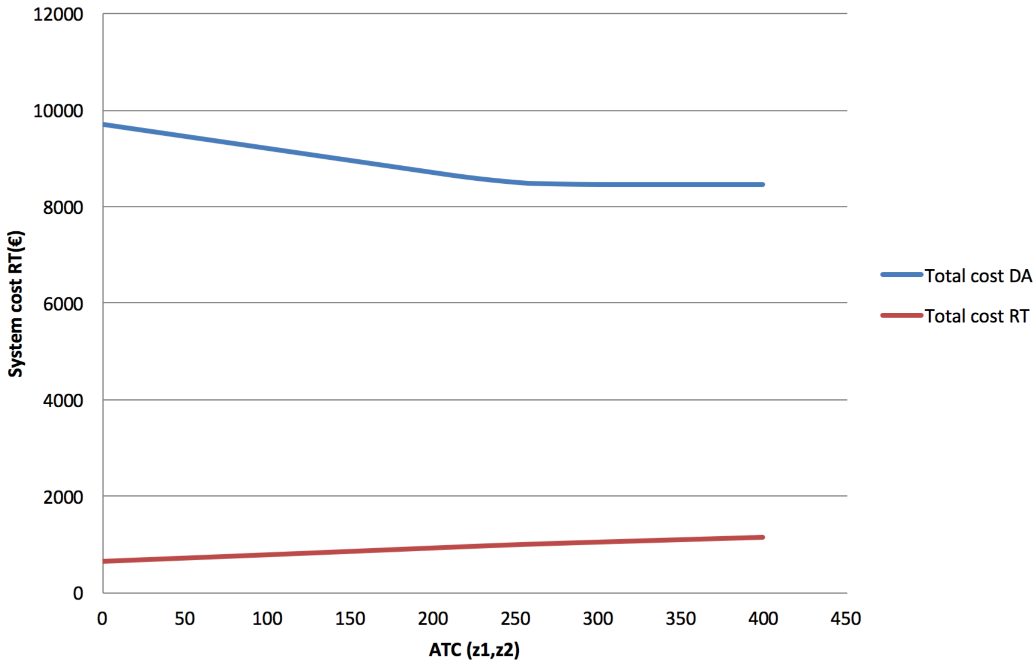

Figure 2.6: Comparison of costs in DA and RT stages.

As it can be seen, in this case, the variations because of the change in the generation mix are more distributed and less abrupt. In this case, the optimal ATC is 300 MW, leading to a total system cost of 9519€. This value also optimizes the cost minus revenues of zone 2. Zone 1 minimizes its function with an ATC of 277 MW. Note that the total system costs are slightly different compared to the previous case, because of small differences in the generation mix. Again, both zones have different ATCs minimizing its cost minus revenues function.

Sequential model, 3 zones-9 nodes system

The model has been implemented in GAMS, and the minimum system cost is 15150€ with a $ATC_{z1,z2} = 243$ MW, a $ATC_{z1,z3} = 64$ MW and a $ATC_{z2,z3} = 0$ MW. In this point, the flow in day ahead is congested in all lines, that means that the flow between zones 1 and 2 is 243 MW, the flow between zones 1 and 3 is 64 MW, and there is no flow between zones 2 and 3.

Evaluating the optimal ATCs for each one of the zones is not as straightforward with this system, because there are three degrees of freedom, as there are three differ-

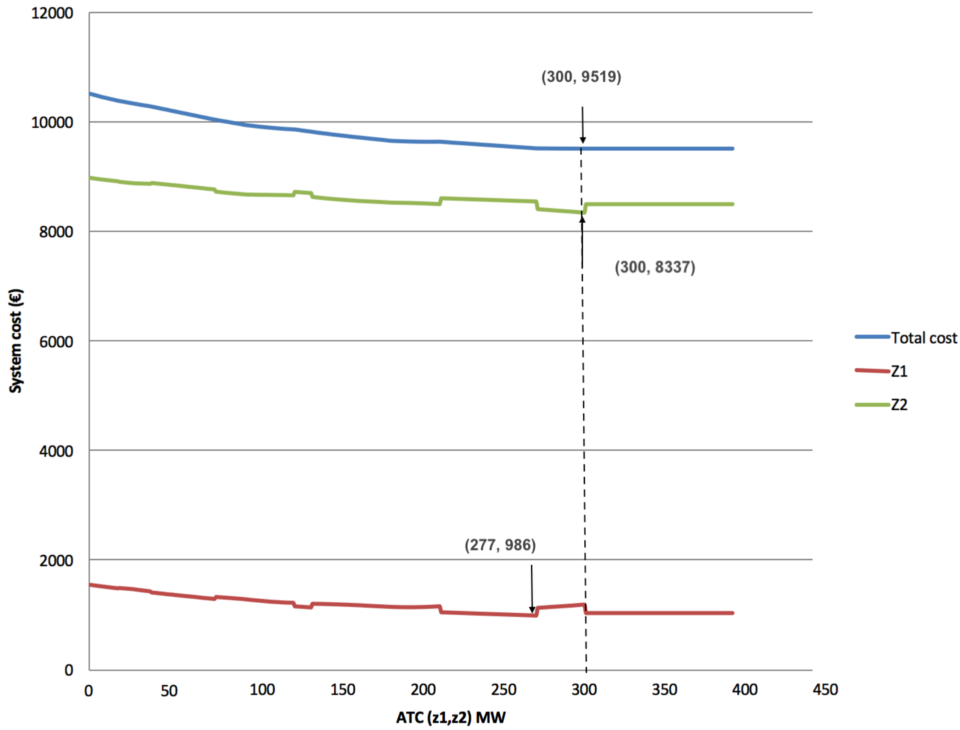

Figure 2.7: zonal cost minus revenues versus ATC: extended generation.

ent ATCs. To try to get insights of how the variation of the ATCs affects the costs of the different zones, first a sensitivity analysis is performed varying only the $ATC_{z1,z3}$ and fixing the other two ATCs to 0 MW. The results are plotted in figure 2.3.

The blue line represents the total system costs. The yellow, orange and grey curves represent the zonal costs minus the revenues, calculated the same way as in the 2 zones setup. These results allow to reach similar conclusions to the ones obtained with the 2 zones system. In this case, and considering only the ATC between zones 1 and 3, each of the zones have a different optimal point. It is important to note that, although the ATC considered does not include zone 2, there is a big impact in its costs and revenues. This shows that ATCs, indeed, have a big impact in the overall system.

Finally, another sensitivity analysis is performed, considering the impact of varying two ATCs in the costs and revenues of one zone. The selected ATCs are the two that

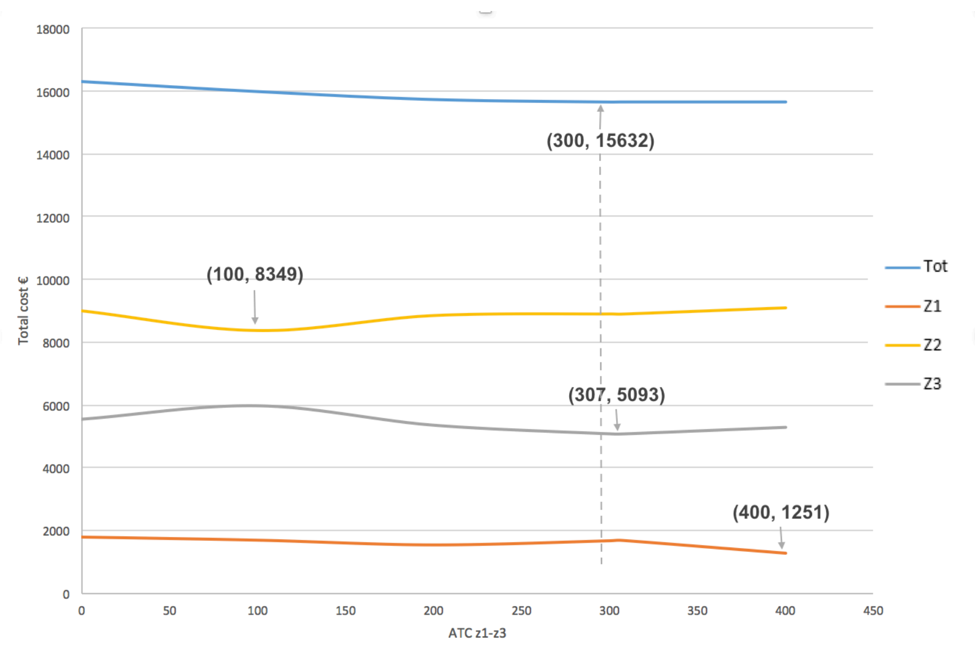

Figure 2.8: zonal cost minus revenues versus ATC: 3 zones case.

have a non-zero value in the optimal point, $ATC_{z1,z2}$ and $ATC_{z1,z3}$. The selected zone to evaluate its costs and revenues is zone 1. The obtained results are plotted in figure 2.4.

The optimal point is marked in the figure 2.9 with a grey circle, and it corresponds to an $ATC_{z1,z2} = 150$ MW and an $ATC_{z1,z3} = 200$ MW, with a cost minus revenues equal to 810€. Note that the ATC values are quite different to the optimal ones considering full coordination, which were 243 and 64 MW, respectively, and are marked with a black circle. Again, it looks clear that the optimal ATCs for each zone do not match with the optimal ATCs for the overall system.

Stochastic market model, 2 zones-6 nodes system

The main difference between the sequential market model and the named stochastic market model is that the former always respects the merit order in DA and the latter can eventually violate it.

In the cases in which the stochastic market model results in a dispatch which respects the merit order in day ahead, the results of both models are in practice pretty similar.

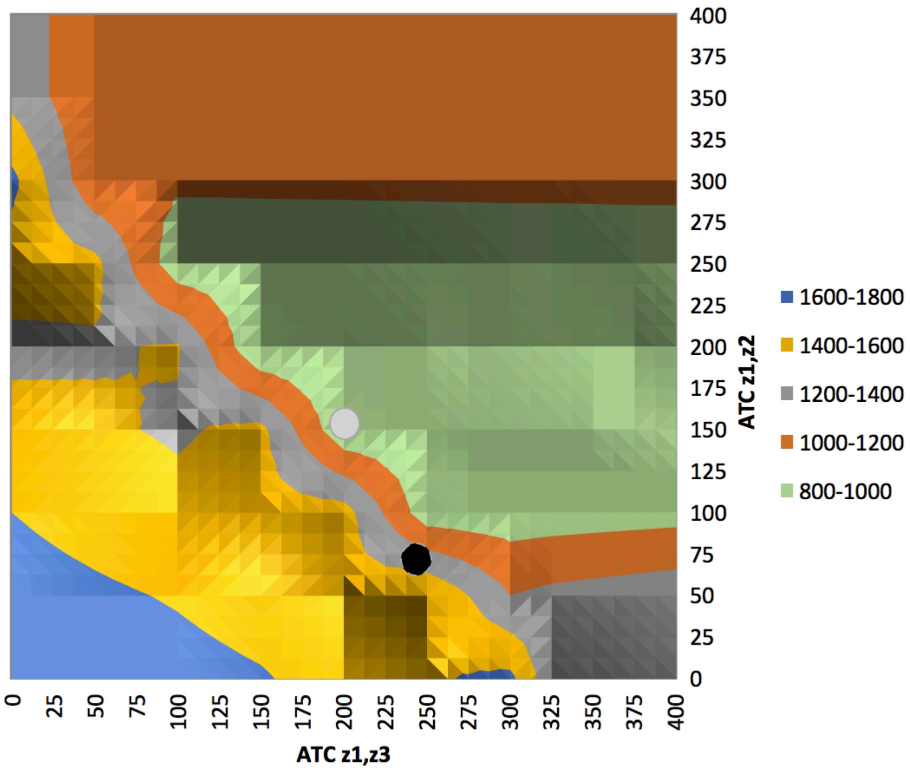

Figure 2.9: zone 1 cost minus revenues versus $ATC_{z1,z3}$.

In large-scale setups this coincidence may be remote, but in smalls systems such as the 2 zones-6 nodes this is more likely to happen.

In order to find situations in which both models result in different solutions, different simulation cases are proposed.

- Case 1: original 3 scenario model.
- Case 2: original 3 scenario model reducing the installed wind capacity to 50%.
- Case 3: original 3 scenario model increasing the installed wind capacity in a 50%.
- Case 4: original model extended to 10 scenarios.

Results of the simulations are shown in table 2.4.

Table 2.4: Comparison between stochastic and sequential model.

	Case 1	Case 2	Case 3	Case 4
Optimal ATC sequential model (MW)	300	257	300	300
Total system cost sequential model (€)	9662.8	10740	8477	9715
Optimal ATC stochastic model (MW)	307	272	307	307
Total system cost stochastic model (€)	9612.8	10630	8354	9680

As it can be seen, in that small-size setup results of both models are quite similar, but in the stochastic model the total system cost is always slightly lower, and the defined ATCs higher. This results are pretty logical, as the stochastic model is a less constrained optimization problem, thus, it provides a lower bound in the solution.

Note that, in all cases, the stochastic model violates the merit order while the sequential one is enforced to respect it.

2.7 Conclusions

During this chapter the cost-optimal ATCs determination models considering full coordination are applied to two different systems, aiming to analyze the impact not only in the total system costs, but also in the costs and revenues of each one of the zones inside the system. The obtained results prove that the set of ATCs that minimize the total system cost does not necessarily coincide with the optimal points of each region.

However, these optimal points of each region are highly dependent on the market prices of the neighbouring zones. In this chapter we have found these optimal points gathering all the necessary information from all different zones and analyzing it. But, from a non-coordinated perspective, none of the zones can find them using only its own information.

Congestion rent has proven to be relevant to analyze the optimal points of each zone. If congestion rent is not considered, the different regions do not seem to have incentives to trade power. The assumption of dividing it equally among the implied TSOs is consistent according to the current regulatory framework in the Nordic countries. However, the optimal ATCs per zone are really sensitive to variations in this percentage, so it may be a interesting idea to explore how the congestion rent division may affect the different optimal points.

CHAPTER 3

Cost-optimal ATCs with partial coordination between zones

3.1 Chapter scope

The models proposed in Chapter 2 reduce significantly the expected operation cost compared to other models with static ATCs [22]. However, these models are not aligned with the current European practice, and they present different drawbacks. The proposed entity, "ATC optimizer", has to collect information from all the different zones. In a real-case scenario, it is difficult to believe that all Transmission System Operators (TSO) are in favour of sharing its detailed information to third parties. So, in a realistic approach, it should be necessary to determine the Cost-optimal ATCs keeping the privacy of the involved TSOs. Thus, applying distributed optimization methods to solve the Cost-optimal ATCs problems arise as a solution.

The objective of this chapter is to analyze and discuss a distributed approach to solve the ATCs optimization problem. The expected output is to apply this approach to different case studies and compare the resulting ATCs obtained in the centralized-full coordinated method and in the distributed-partially coordinated one.

This chapter is organized as follows. Section 3.2 presents a brief literature review of different distributed optimization methods available in the literature, trying to find and justify the proper choice. In section 3.3 the Alternating Direction Method of Multipliers (ADMM) will be described and analyzed. In section 3.4 the ADMM method will be implemented in our two optimization problems. In section 3.5 the resulting algorithm will be applied to different simple case studies. Section 3.6 discusses different alternatives to improve the resulting models. Finally, in section 3.7 a conclusion is derived.

3.2 Literature review

Distributed optimization methods have been widely reviewed and applied in the previous literature, as they have been proven as a really powerful tool to handle large-scale complex optimization problems. In the power systems area, these algorithms have been extensively implemented in Optimal Power Flow (OPF) problems. The Cost-Optimal ATCs determination problem is obviously not an OPF problem, but both of them have enough similarities to consider OPF a proper source of inspiration.

It is important to recall that in Chapter 2 we introduced two models, one of them with a bilevel structure, and in general non-convex, and the other one with a single-level structure, and purely convex. It is necessary to consider this feature when exploring different distributed optimization methods, as usually non-convex problems are not easy-to-solve in a distributed way.

Many different analysis and overviews of various distributed optimization techniques are present in the literature. For instance, in [2], six decomposition coordination algorithms are studied (analytical target cascading, optimality condition decomposition, alternating direction method of multipliers, auxiliary problem principle, consensus+innovations and proximal message passing). These algorithms are applied to a DC-OPF problem, which is convex, and the computational time and amount of data exchanged are analyzed. In [18] a similar analysis of different methods is performed, but with a deeper analysis in non-convex OPF problems. Both reviews conclude that analytical target cascading, alternating direction method of multipliers and auxiliary problem principle need a lower amount of data exchanged, but they require, in general, a higher computational effort. On the other hand, optimality condition decomposition and proximal message passing exchange more data per iteration, with a lower required computational effort.

In [14] a non-convex OPF problem is solved applying ADMM. The main contribution of this paper is that the system considered is a large-scale one (Polish 2383-bus system). In such a real-scale system, ADMM converges to a solution close to the centralized one, proving its robustness. ADMM is also explored in [13], in which the parameter selection and penalty parameter tuning is analyzed. In [17] a non-convex OPF problem is solved applying the Consensus ADMM, a specific application of the generic ADMM.

Besides ADMM, other methods have been succesfully used in the literature to solve non-convex problems, for instance, in [18] analytical target cascading is used. However, ADMM seems to be the most widespread method to solve this kind of problems. All things considered, in this chapter ADMM will be used to solve the Cost-Optimal ATCs problem determination in a distributed way.

3.3 Background of the alternating direction method of multipliers (ADMM)

3.3.1 General ADMM structure

The alternating direction method of multipliers was firstly proposed in the 1970s by Glowinski, Marrocco, Gabay and Mercier [12]. Afterwards, it has been extensively analyzed and expanded, for instance in [5].

ADMM is a really powerful optimization algorithm which permits to decompose a large problem into smaller subproblems, which are solved independently. The solutions to these smaller subproblems are then coordinated in an iterative fashion, in order to find a solution to the original problem. ADMM intends to combine the benefits of two previous algorithms: dual decomposition and augmented Lagrangian.

Formally, ADMM solve problems in the form:

$$\begin{aligned} \text{minimize} \quad & f(x) + g(y) \\ \text{Subject to} \quad & Ax + By = c \quad : \mu \end{aligned} \tag{3.1}$$

With $x \in \mathbf{R}^n$, $y \in \mathbf{R}^m$, A $\in \mathbf{R}^{qxn}$, B $\in \mathbf{R}^{qxm}$ and c $\in \mathbf{R}^q$

The next step is to define the augmented Lagrangian, listed as follows.

$$L_\rho(x, y, \mu) = f(x) + g(y) + \mu^T(Ax + By - c) + \frac{\rho}{2} \parallel Ax + By - c \parallel_2^2 \tag{3.2}$$

In which $\rho \geq 0$ is the augmented Lagrangian parameter or penalty parameter.

Then, the ADMM algorithm consist of the following iteration steps:

$$x^{\upsilon+1} := argminL_\rho(x, y^\upsilon, \mu^\upsilon) \tag{3.3}$$

$$y^{\upsilon+1} := argminL_\rho(x^{\upsilon+1}, y, \mu^\upsilon) \tag{3.4}$$

$$\mu^{\upsilon+1} := \mu^\upsilon + \rho(Ax^{\upsilon+1} + By^{\upsilon+1} - c) \tag{3.5}$$

We have three iteration steps: two variable updates (x and y) and the dual variable update (μ). Note that the two variable steps are completely independent between them, as the values of external variables are fixed to the value of the previous iteration (for instance, in the $x^{\upsilon+1}$ step, y^{υ} and μ^{υ} are fixed, i.e they are taken as parameters). Thus, applying these steps to the optimization problem 3.1 means to solve it in a distributed fashion: in the x-step the objective function is only function of x; and in the y-step the objective function is only function of y.

This distributed algorithm would allow to keep the privacy in our Cost-Optimal ATCs case. Imagine a simple case in which $f(x)$ represents the cost function of a zone A, and $g(y)$ represents the cost of a zone B. The coupling constraint $Ax + By = c$, in this case, could be for instance the flow in day ahead exchanged between zones. Note that, solving this simple example with the three steps (3.3)-(3.5) a consensus solution would be found in which the total cost is minimized. Moreover, during each step each zone would minimize only its own cost function, taking into account only its own variables. Thus, a common solution would be achieved, and only information related to the coupling constraint would be exchanged.

As stated in [5] the ADMM algorithm can be reformulated in the so called "Scaled Form", which is more convenient in certain cases. With this form, the three steps are expressed as follows:

$$x^{\upsilon+1} := argmin(f(x) + \frac{\rho}{2} \parallel Ax + By^{\upsilon} - c + u^{\upsilon} \parallel_2^2) \tag{3.6}$$

$$y^{\upsilon+1} := argmin(g(y) + \frac{\rho}{2} \parallel Ax^{\upsilon+1} + By - c + u^{\upsilon} \parallel_2^2) \tag{3.7}$$

$$u^{\upsilon+1} := u^{\upsilon} + Ax^{\upsilon+1} + By^{\upsilon+1} - c \tag{3.8}$$

In which u is the scaled dual variable, calculated as follows.

$$u = \frac{1}{\rho} \cdot \mu \tag{3.9}$$

3.3.2 Convergence and stopping criteria

ADMM convergence has been widely analyzed in the previous literature. In [5] it is proved that, if the following assumptions are satisfied:

- Function $f(x)$ and $g(y)$ are proper, closed and convex.
- The augmented Lagrangian L_p has a saddle point.

Then, the ADMM algorithm will converge to the following results:

$$
\begin{gathered}
\lim_{\upsilon \to \infty} (Ax^{\upsilon} + By^{\upsilon} - c) = 0 \\
\lim_{\upsilon \to \infty} (f(x^{\upsilon}) + g(y^{\upsilon})) = Q^{*} \\
\lim_{\upsilon \to \infty} (\mu^{\upsilon}) = \mu^{*}
\end{gathered} \tag{3.10}
$$

Being Q^{*} the optimal value of the objective function, and μ^{*} the optimal value of the dual variables. Note that this convergence criteria is only applicable to convex problems, while one of the models proposed in Chapter 2 is, in general, non-convex.

Regarding the stopping criteria, again, many different analysis have been proposed in the previous literature. In [5] the suggested one is related to the primal and dual residuals, listed as follows:

$$
r_{p}^{\upsilon} = Ax^{\upsilon} + By^{\upsilon} - c \tag{3.11}
$$

$$
r_{d}^{\upsilon} = \rho A^{T} B(y^{\upsilon} - y^{\upsilon-1}) \tag{3.12}
$$

Where r_{p}^{υ} is the primal residual in the υ iteration, and r_{d}^{υ} is the dual residual in the υ iteration. When these residuals are smaller than certain limit, the iterative process can be stopped.

3.3.3 ADMM in the non-convex case

In the previous section, the convergence of the ADMM has been analyzed considering a purely convex optimization problem. However, this convergence criteria is not applicable to non-convex problems, such as the Cost-Optimal ATCs determination with sequential market clearing proposed in chapter 2.

As mentioned before, despite ADMM is a method originally conceived to solve convex optimization problems, it has been succesfully applied to many different non-convex cases. This has been extensively explored in [5], showing two relevant features:

- If ADMM is applied to a non-convex problem, convergence is not guaranteed, and even if there is convergence, it is not guaranteed to converge to an optimal point.

- ADMM can achieve convergence to different points in the same non-convex optimization problem, depending on the initial values x^0, μ^0 and the penalty parameter ρ. Thus, unlike in the convex case, ADMM do not necessarily converge to a single objective function value.

3.3.4 Choice of rho and initialization parameters

Attending to the discussion in the previous sections, the choice of the initialization parameters and the penalty parameter is highly relevant in order to correctly implement the ADMM algorithm. This choice is not trivial, and has been extensively analyzed in the previous literature.

In the convex problem case, the choice of the penalty and initialization parameters does not affect to the optimal solution, as proved before, but it can highly affect to the computational time (i.e number of iterations). Also, a wrong choice of these parameters can cause a failure in the algorithm (i.e, no convergence achieved). There is not a generic way to optimally tune the parameters, it is highly dependent on the problem to solve. In the literature, different tuning strategies and discussion have been presented, for instance in [13] and [11]. In [15] a varying penalty parameter is proposed, which has been extensively used during the last years. This extension is shown as follows:

$$
\begin{aligned}
\rho^{\upsilon} &:= a \cdot \rho^{\upsilon} \quad if \parallel r_p^{\upsilon} \parallel_2 > \ \sigma \parallel r_d^{\upsilon} \parallel_2 \\
\rho^{\upsilon} &:= \rho^{\upsilon}/b \quad if \parallel r_d^{\upsilon} \parallel_2 > \ \sigma \parallel r_p^{\upsilon} \parallel_2 \\
\rho^{\upsilon+1} &:= \rho^{\upsilon} \quad otherwise
\end{aligned}
\tag{3.13}
$$

In which σ, a and b are parameters with value higher than 1.

In the nonconvex case the parameter selection is even more complicated, as different values can highly affect the final solution. In [13] some insights regarding this tuning are presented. However, in recent applications of ADMM to power system problems, such as a nonconvex OPF solved in [14], the tuning is based on a try-and-error basis. Another usual practice is to the starting points according to the so called warm start, in which the initial parameters are set to a feasible operating point. However, there is not a standardized way to set these initial values, as in general, in the power system applications, they are highly dependent on the system considered.

3.3.5 Scalability of ADMM

The algorith proposed in (3.3)-(3.5) to solve problems in the form (3.1) is the most generic ADMM. However, in certain cases, this approach is not the most appropriated [17]. For instance, considering a generic optimization problem, in which the objective function can be partitioned in different blocks, listed as follows:

$$\begin{aligned} \underset{x_i}{\text{minimize}} \quad & \sum_{i=1}^{N} f(x_i) \\ \text{Subject to} \quad & \sum_{i=1}^{N} A_i x_i = c \quad : \mu \end{aligned} \tag{3.14}$$

With $x_i \in \mathbf{R}^{n_i}$, A $\in \mathbf{R}^{mxn}$ and c $\in \mathbf{R}^m$

Apparently, considering the proposed ADMM algorithm in Equations (3.3)-(3.5), this problem could be solved in a distributed fashion, with i different x_i steps carried out sequentially, and a dual variable update afterwards. However, as proved in [17], this approach may fail to converge when the number of blocks $N > 3$. Thus, in principle this approach could be interesting to solve small systems in which the number of different blocks is reduced, but in order to solve large-scale systems other approaches need to be explored.

One solution proposed in the literature to handle this issue is the so-called Consensus ADMM [17],[5]. This specific ADMM-based methods are specially interesting considering the models that we aim to solve in this thesis. In the Cost-Optimal ATCs determination with partial coordination each one of the zones seeks to minimize its own cost, while the cross-border power transfer and coupling constraints are agreed in a consensus manner. One specific case of the Consensus ADMM, introduced in [5] which is interesting to this thesis case is the Optimal Exchange ADMM. Consider a problem in the form:

$$\begin{aligned} \underset{x_i}{\text{minimize}} \quad & \sum_{i=1}^{N} f_i(x_i) \\ \text{Subject to} \quad & \sum_{i=1}^{N} x_i = 0 \quad : \mu \end{aligned} \tag{3.15}$$

With $x_i \in \mathbf{R}^{n_i}$.
This is a exchange problem. In an electricity market perspective, it could be seen as a problem in which each zone aims to minimize its cost function $f(x_i)$ while all zones agree in a consensus manner in their coupling constraints $\sum_{i=1}^{N} x_i = 0$ (for instance,

the sum of the power flow sent from one zone and the power received from the other must be zero).

In [5] an ADMM algorithm to solve this problem, which is scalable to a large number of blocks (N) is proposed. This algorithm is listed as follows.

$$x_i^{\upsilon+1} := argmin_{x_i}(f_i(x_i) + \mu^{\upsilon T} x_i + \frac{\rho}{2} \parallel x_i - (x_i^{\upsilon} - \overline{x}^{\upsilon}) \parallel_2^2) \tag{3.16}$$

$$\mu^{\upsilon+1} := \mu^{\upsilon} + \rho \overline{x}^{\upsilon+1} \tag{3.17}$$

Where $\overline{x}^{\upsilon+1}$ is the average of all the x_i in the previous iteration.

Thus, applying (3.16)-(3.17) to a consensus problem in the form of (3.15) would permit to extend a distributed algorithm to large-scale setups.

3.4 ADMM implementation

Now, in this section, the objective is to apply the aforementioned ADMM method to solve the two Cost-Optimal ATCs problems proposed in chapter 2. Recall that one of the models was purely convex, and the other one was, in general, non-convex. Thus, each one of the models has a different ADMM implementation.

3.4.1 Cost-Optimal ATCs determination with stochastic DA and RT market clearing case

The first model to consider is the Cost-Optimal ATCs determination with stochastic market clearing, in which the merit order in day ahead is not respected. An illustrative sketch of the differences between the centralized solution approach (i.e the scope of chapter 2) and the distributed application of ADMM is shown in Figure 3.1.

As it can be seen, in the centralized approach there is only one single level problem, which minimizes the total system cost, as described in chapter 2. The ATC optimizer is an external entity which collects all the system information and determines the set of optimal ATCs. On the other hand, as it can be seen in Figure 3.1 right, in the distributed approach each zone solve its own single-level cost minimization problem. Then, after each ADMM iteration, each zone shares with a central coordinator only the minimum required information (i.e, the information related to the coupling constraints). Thus, the privacy of each TSO is kept, and when the algorithm converges, a consensus solution between all different zones is reached.

As the single level model of the Cost-Optimal ATCs determination in the stochastic

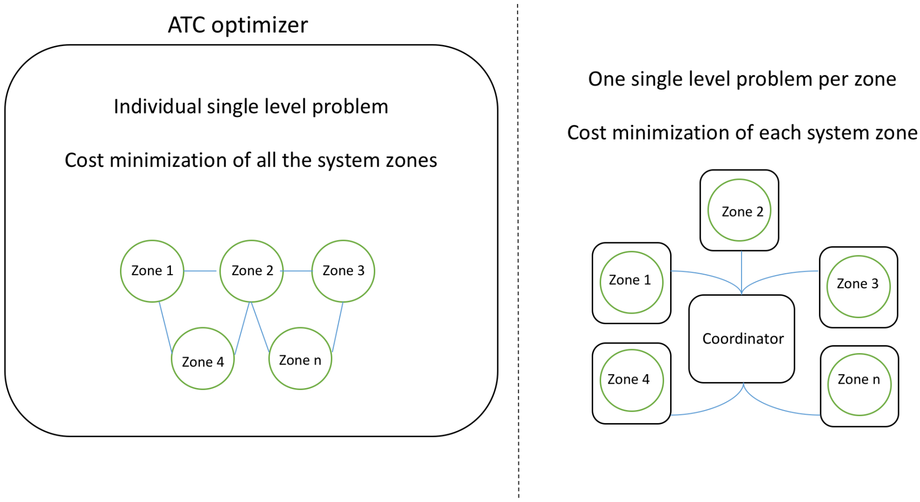

Figure 3.1: Left, centralized approach. Right, distributed ADMM approach.

DA+RT market clearing case is convex, the solution of the centralized approach and the distributed ADMM algorithm must be the same. Thus, in this specific case, even without full coordination between the different TSOs, the final solution will minimize the total system cost while minimizing the share of information.

In order to properly apply the ADMM algorithm to this particular case, the mathematical model has to fulfill two conditions:

- The objective function has to be separable in different blocks, one per zone.
- Each single level problem must contain only variables belonging to its own zone, i.e, in the same single level problem there can not be variables related to different zones.

It is necessary to adapt the mathematical model in order to accomplish this requirements. First, recall the Objective function of the model (derived in Chapter 2).

$$\underset{\Xi^{SL}}{\text{minimize}} \quad \sum_{g} C_g \cdot q_g^{DA} + \sum_{s} \pi_s \Bigg[\sum_{g} \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) + \sum_{d} VOLL_d \cdot l_{d,s}^{shedRT} \Bigg] \tag{3.18}$$

In this objective function the total production cost is minimized, including the total production cost in the day ahead stage, and the total expected cost in the real time operation.
The cost in day ahead is calculated as the product of the production costs and energy production scheduled in day ahead of all generators. If we consider that each conventional generator is situated only in one zone, the total cost in day ahead can be decomposed in blocks, one per zone.

$$\sum_{g} C_g \cdot q_g^{DA} = \sum_{g \in z1} C_g \cdot q_g^{DA} + \sum_{g \in z2} C_g \cdot q_g^{DA} + ... + \sum_{g \in z_i} C_g \cdot q_g^{DA} \tag{3.19}$$

The cost in real time includes the production cost of the difference between the scheduled production in day ahead and the actual one in real time, the up and down regulation costs, and the cost of load shedding. If we assume, again, that each generator pertains only to one zone, and that each load is situated inside a single node, the real time expected cost can be also decomposed in blocks, one per zone.

$$\sum_{s} \pi_s \left[\sum_{g} ... + \sum_{d} ... \right] = \sum_{s} \pi_s \left[\sum_{g \in z1} ... + \sum_{d \in z1} ... \right] + ... + \sum_{s} \pi_s \left[\sum_{g \in z_i} ... + \sum_{d \in z_i} ... \right] \tag{3.20}$$

So the objective function of the mathematical problem can be expressed as the sum of the objective functions of the different zones.

Then, it is necessary to identify the complicating constraints and to ensure that each single level problem contains only variables belonging to one zone. Recall the constraints of the Cost-Optimal ATCs problem with stochastic day ahead and real time market clearing, listed as follows.

$$\sum_{g \in \psi_n} q_{g,s}^{RT} + \sum_{k \in \psi_n} r_{k,s}^{RT} + \sum_{d \in \psi_n} (l_{d,s}^{RT} - L_d) = \sum_{m \in \delta_n} f_{n,m,s}^{RT} \qquad \forall n, \forall s \tag{3.21a}$$

$$q_{g,s}^{RT} = q_g^{DA} + q_{g,s}^{up} - q_{g,s}^{down} \qquad \forall g, \forall s \tag{3.21b}$$

$$f^{RT}_{n,m,s} = B_{n,m}(\theta_{n,s} - \theta_{m,s}) \qquad \forall n, \forall m \in L \tag{3.21c}$$

$$0 \leq q^{RT}_{g,s} \leq Q^{max}_{g} \qquad \forall g, \forall s \tag{3.21d}$$

$$0 \leq q^{up}_{g,s}, 0 \leq q^{down}_{g,s} \qquad \forall g, \forall s \tag{3.21e}$$

$$0 \leq r^{RT}_{k,s} \leq R^{RT}_{k,s} \qquad \forall k, \forall s \tag{3.21f}$$

$$0 \leq l^{shed,RT}_{d,s} \leq L_d \qquad \forall d, \forall s \tag{3.21g}$$

$$-F^{max}_{n,m} \leq f^{RT}_{n,m,s} \leq F^{max}_{n,m} \qquad \forall n, \forall m \in L, \forall s \tag{3.21h}$$

$$\theta_{n=1,s} = 0 \qquad \forall s \tag{3.21i}$$

$$-\underline{R_g} \leq (q^{RT}_{g,s} - q^{DA}_{g}) \leq \overline{R_g} \qquad \forall g, \forall s \tag{3.21j}$$

$$\sum_{g \in \psi_z} q^{DA}_{g} + \sum_{k \in \psi_z} r^{DA}_{k} - \sum_{d \in \psi_z} L_d = \sum_{zo \in \delta_z} FDA_{z,zo} \qquad \forall z \tag{3.21k}$$

$$FDA_{z,zo} + FDA_{zo,z} = 0 \qquad \forall e \tag{3.21l}$$

$$0 \leq q^{DA}_{g} \leq Q^{max}_{g} \qquad \forall g \tag{3.21m}$$

$$0 \leq r^{DA}_{k} \leq R^{ins}_{k} \qquad \forall g \tag{3.21n}$$

The constraints related to conventional generators, renewable generators and loads do not need any modification. If we again assume that each generator and load pertains only to one single zone, these constraints never include variables related to different zones. So constraints (3.21b),(3.21d)-(3.21g),(3.21j)-(3.21n) are unchanged.

Constraints related to the power flow in real time ($f^{RT}_{n,m,s}$) do need a modification. Note that this flow between two nodes is defined in constraint (3.21c) as a function of the voltage angles of them. Thus, in the tie-lines connecting two different zones, the flow is function of voltage angles of two zones. One alternative to solve this issue, and decouple different zones, is to create fictitious nodes, duplicating existing ones [1]. An illustrative example is shown in Figure 3.2.

In this example, without the fictitious nodes, the flow in real time between nodes n and m is calculated as follows:

$$f^{RT}_{n,m,s} = B_{n,m}(\theta_{n,s} - \theta_{m,s}) \tag{3.22}$$

Expression including variables of two different zones. If the fictitious nodes are taken into account, the flow can be calculated from the two different sides.

$$f^{RT}_{n,nx,s} = B_{n,nx}(\theta_{n,s} - \theta_{nx,s}) \tag{3.23}$$

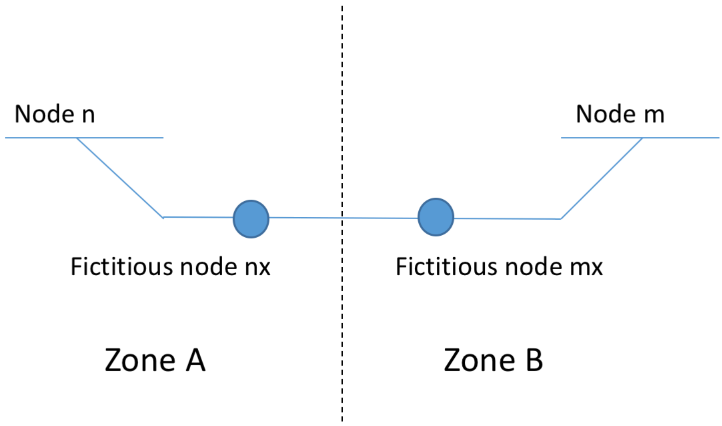

Figure 3.2: Illustrative example of node duplication.

$$f^{RT}_{m,mx,s} = B_{m,mx}(\theta_{m,s} - \theta_{mx,s}) \tag{3.24}$$

In these two new expressions, to calculate the flow requires only variables of one zone. So, applying this node duplication, all the constraints related to the flow in real time contain only variables of one zone. To ensure that the flows across these ficticious nodes coincide, it is necessary to add a coupling constraint, which is common to all zones. In this illustrative example, it is written as follows:

$$f^{RT}_{m,mx,s} + f^{RT}_{n,nx,s} = 0 \qquad : \mu^{RT}_{l,s} \qquad \forall n, m = l \in l^* \tag{3.25}$$

Constraints related to the flow in day ahead $FDA_{z,zo}$ do not need modifications: (3.21k) is already decoupled per zone, and (3.21l), which includes variables of different zones, is a coupling constraint.

All things considered, the Cost-Optimal ATCs model with stochastic DA and RT market clearing can take the generic form:

$$\underset{x_i}{\text{minimize}} \quad \sum_{i=1}^{N} f(x_i) \tag{3.26}$$

$$\text{Subject to} \qquad x_i \in \chi_i \quad \forall i \tag{3.27}$$

$$\sum_{i=1}^{N} A_i x_i = 0 \quad : \mu \tag{3.28}$$

In which (3.26) is the objective function (3.18), decomposed per zone; constraint (3.27) enforces feasibility constraints of each zone, that is, constraints (3.21a)-(3.21k), (3.21m)-(3.21n) with the stated modifications to decouple them per zone; and finally constraint (3.28) represent the coupling constraints, defined previously in (3.21l) and (3.25).

Note that, with the mathematical formulation (3.26)-(3.28), as stated in the previous sections, it is possible to apply ADMM.

3.4.1.1 General ADMM algorithm

Here the objective is to develop the exact algorithm to apply the general ADMM algorithm, stated in equations (3.1)-(3.4), applied to the Cost-Optimal ATCs problem with stochastic DA and RT market clearing. First, it is necessary to calculate the augmented Lagrangian. The Lagrangian of the recasted problem (3.26)-(3.28) is listed as follows.

$$L_p(x_1, x_2, ..., x_n, \mu) = \sum_{i}^{N} f(x_i) + \mu^T(\sum_{i=1}^{N} A_i x_i) + \frac{\rho}{2} \parallel \sum_{i=1}^{N} A_i x_i \parallel_2^2 \tag{3.29}$$

For the sake of simplicity, the different terms of the Lagrangian are listed separately.

$$\sum_{i}^{N} f(x_i) = \sum_{g} C_g \cdot q_g^{DA} + \sum_{s} \pi_s \left[\sum_{g} \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) + \sum_{d} VOLL_d \cdot l_{d,s}^{shedRT} \right] \tag{3.30}$$

The first term corresponds to the cost function of the system, which is separable per zone, as discussed before.

$$\mu^T(\sum_{i=1}^{N} A_i x_i) = \sum_{s} \pi_s \sum_{n,m=l \in l^*} \mu_{l,s}^{RT} (f_{m,mx,s}^{RT} + f_{n,nx,s}^{RT}) + \sum_{z,zo=e} \mu_e^{FDA} (FDA_{z,zo} + FDA_{zo,z}) \tag{3.31}$$

The second term corresponds to the augmented coupling constraints. Note that $\mu_{l,s}^{RT}$ and μ_e^{FDA} are the dual variables of the two aforementioned coupling constraints.

$$\frac{\rho}{2} \parallel \sum_{i=1}^{N} A_i x_i \parallel_2^2 = \sum_s \pi_s \sum_{n,m=l \in l^*} \frac{\rho}{2} \parallel f_{m,mx,s}^{RT} + f_{n,nx,s}^{RT} \parallel_2^2 + \sum_{z,zo=e} \frac{\rho}{2} \parallel FDA_{z,zo} + FDA_{zo,z} \parallel_2^2 \tag{3.32}$$

Finally, the third term includes all the penalty parameters.

To have the ADMM algorithm, it is necessary to apply the iterative steps of (3.3)-(3.5) to the Lagrangian. This results in the following equations.

$$x_1^{\upsilon+1} := argmin_{x1} L_p(x_1, x_2^{\upsilon}, ..., x_n^{\upsilon}, \mu^{\upsilon}) \tag{3.33}$$

$$x_2^{\upsilon+1} := argmin_{x2} L_p(x_1^{\upsilon+1}, x_2, ..., x_n^{\upsilon}, \mu^{\upsilon}) \tag{3.34}$$

$$x_n^{\upsilon+1} := argmin_{xn} L_p(x_1^{\upsilon+1}, x_2^{\upsilon+1}, ..., x_n, \mu^{\upsilon}) \tag{3.35}$$

$$\mu_e^{FDA,\upsilon+1} = \mu_e^{FDA,\upsilon} + \rho(FDA_{z,zo}^{\upsilon+1} + FDA_{zo,z}^{\upsilon+1}) \qquad \forall z, zo = e \tag{3.36}$$

$$\mu_{l,s}^{RT\upsilon+1} = \mu_{l,s}^{RT,\upsilon} + \rho(f_{m,mx,s}^{RT,\upsilon+1} + f_{n,nx,s}^{RT,\upsilon+1}) \qquad \forall n, m = l \in l^*, \forall s \tag{3.37}$$

3.4.1.2 Optimal Exchange-Consensus sharing ADMM algorithm

Here the aim is to develop the algorithm to solve the problem but applying the Optimal Exchange ADMM, discussed in section 3.3.5. The main benefit of this algorithm is its scalability to large systems, and that it can be carried in parallel, unlike the general ADMM developed in the previous section.

Recall the Optimal exchange algorithm.

$$x_i^{\upsilon+1} := argmin_{x_i}(f(x_i) + \mu^{\upsilon T} x_i + \frac{\rho}{2} \parallel x_i - (x_i^{\upsilon} - \overline{x}^{\upsilon}) \parallel_2^2) \tag{3.38}$$

For the sake of simplicity, the different terms are analyzed separately.

$$f(x_i) = \sum_{g \in z_i} C_g \cdot q_g^{DA} + \sum_s \pi_s \left[\sum_{g \in z_i} \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) + \sum_{d \in z_i} VOLL_d \cdot l_{d,s}^{shedRT} \right] \tag{3.39}$$

The first term corresponds to the cost function of each zone i.

$$\mu^{\upsilon T} x_i = \sum_s \pi_s \sum_{n,m=l \in l^* \in z_i} \mu_{l,s}^{RT,\upsilon} (f_{n,nx,s}^{RT}) + \sum_{zi,z=e} \mu_e^{FDA,\upsilon} (FDA_{zi,z}) \tag{3.40}$$

$$\frac{\rho}{2} \parallel x_i - (x_i^\upsilon - \overline{x}^\upsilon) \parallel_2^2 = \sum_s \pi_s \sum_{n,m=l \in l^* \in z_i} \frac{\rho}{2} \parallel f_{n,nx,s}^{RT} - (f_{n,nx,s}^{RT,\upsilon} - \overline{f}^{RT,\upsilon}) \parallel_2^2 + \sum_{zi,z=e} \frac{\rho}{2} \parallel FDA_{zi,z} - (FDA_{zi,z}^\upsilon - \overline{FDA}^\upsilon) \parallel_2^2 \tag{3.41}$$

Second and third terms are shown in equations (3.42) and (3.43).
Finally, the dual variables are updated as follows.

$$\mu_e^{FDA,\upsilon+1} = \mu_e^{FDA,\upsilon} - \rho(\overline{FDA}^{\upsilon+1}) \qquad \forall z, zo = e \tag{3.42}$$

$$\mu_{l,s}^{RT\upsilon+1} = \mu_{l,s}^{RT,\upsilon} - \rho(\overline{f}^{RT,\upsilon+1}) \qquad \forall n, m = l \in l^*, \forall s \tag{3.43}$$

Some comments are required at this point:

- Note that, for the sake of simplicity, the averages in the previous notation are not indexed (for instance, $\overline{FDA}$). These averages are calculated per coupling constraint. For instance, in the update of the dual variable μ_e^{FDA}, $\overline{FDA}$ is the average of flows through the link e, not the average of all the flows of the system.
- In the dual update step stated in equation (3.17) the term $\rho \overline{x_i}^{\upsilon+1}$ is added up, while in (3.44)-(3.46) it is subtracted. This modification is necessary, because in the original algorithm a different sign criteria is considered (in [5] a negative value of x_i means contribution to the system, in our model it is the opposite).

3.4.2 Cost-Optimal ATCs determination with sequential DA and RT market clearing case

Now, the model to be considered is the Cost-Optimal ATCs determination with sequential DA and RT market clearing, in which the merit order in day ahead is respected. Figure 3.3 shows a brief sketch summarizing the main differences between the centralized approach, solved in chapter 2, and the distributed approach, which is the aim of this section.

The differences here are quite similar than in the previous case. In this case, in the

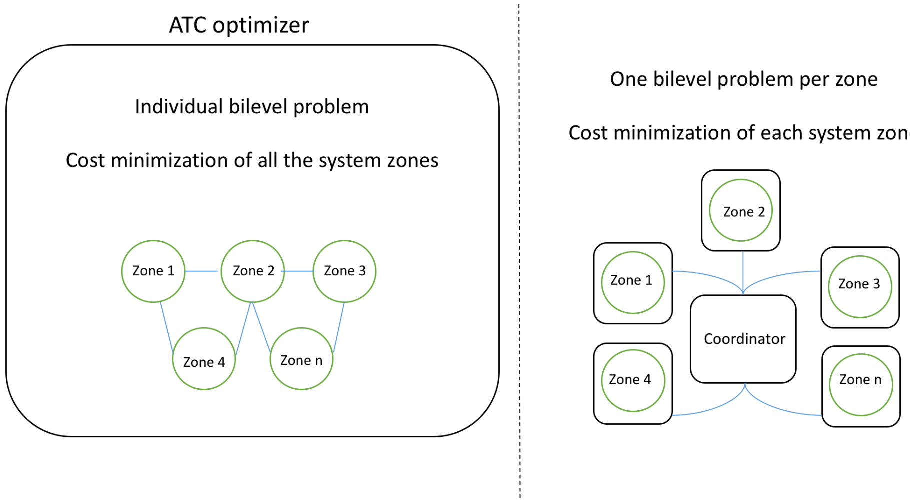

Figure 3.3: Left, centralized approach. Right, distributed ADMM approach.

centralized approach one bilevel model is solved to optimize the total system cost, while in the distributed approach each zone solve its own bilevel model, optimizing its own system cost, and coordinating between themselves sharing information related to the coupling constraints.

As the mathematical model derived for the Cost-Optimal ATCs determination with a sequential DA and RT market is, in general, non convex, the solution of the centralized approach and the distributed ADMM algorith do not necessarily have to be the same [5]. Thus, it is interesting to analyze and compare the solutions provided by both approaches.

Again, to succesfully apply ADMM two conditions must be respected:

- The objective function has to be separable in different blocks, one per zone.
- Each bilevel problem must contain only variables belonging to its own zone, i.e, in the same single level problem there can not be variables related to different zones.

The first condition is fulfilled, as it was proved in equations (3.18)-(3.20) (note that the objective function of both models is the same: the total system cost minimization).

Then, it is necessary to identify the complicating constraints, and to reformulate the model to ensure that the rest of the constraints can be decomposed per zone. Recall the constraints of the Cost-Optimal ATCs problem with sequential DA and RT market clearing. Note that these are the constraints after applying KKT conditions, so the resulting model is no longer bilevel.

$$ATC_e \geq 0 \qquad \forall e \tag{3.44a}$$

$$\sum_{g\in\psi_n} q^{RT}_{g,s} + \sum_{k\in\psi_n} r^{RT}_{k,s} + \sum_{d\in\psi_n} (l^{RT}_{d,s} - L_d) = \sum_{m\in\delta_n} f^{RT}_{n,m,s} \qquad \forall n, \forall s \tag{3.44b}$$

$$q^{RT}_{g,s} = q^{DA}_{g} + q^{up}_{g,s} - q^{down}_{g,s} \qquad \forall g, \forall s \tag{3.44c}$$

$$f^{RT}_{n,m,s} = B_{n,m}(\theta_{n,s} - \theta_{m,s}) \qquad \forall n, \forall m \in L \tag{3.44d}$$

$$0 \leq q^{RT}_{g,s} \leq Q^{max}_{g} \qquad \forall g, \forall s \tag{3.44e}$$

$$0 \leq q^{up}_{g,s}, 0 \leq q^{down}_{g,s} \qquad \forall g, \forall s \tag{3.44f}$$

$$0 \leq r^{RT}_{k,s} \leq R^{RT}_{k,s} \qquad \forall k, \forall s \tag{3.44g}$$

$$0 \leq l^{shed,RT}_{d,s} \leq L_d \qquad \forall d, \forall s \tag{3.44h}$$

$$- F^{max}_{n,m} \leq f^{RT}_{n,m,s} \leq F^{max}_{n,m} \qquad \forall n, \forall m \in L, \forall s \tag{3.44i}$$

$$\theta_{n=1,s} = 0 \qquad \forall s \tag{3.44j}$$

$$- \underline{R_g} \leq (q^{RT}_{g,s} - q^{DA}_{g}) \leq \overline{R_g} \qquad \forall g, \forall s \tag{3.44k}$$

$$\sum_{g\in\psi_z} q^{DA}_{g} + \sum_{k\in\psi_z} r^{DA}_{k} - \sum_{d\in\psi_z} L_d = \sum_{zo\in\delta_z} FDA_{z,zo} \qquad \forall z \tag{3.44l}$$

$$FDA_{z,zo} + FDA_{zo,z} = 0 \qquad \forall e \tag{3.44m}$$

$$C_g + \lambda^{DA}_{z:g\in\psi_z} - \underline{\mu}^{Q,DA}_{g} + \overline{\mu}^{Q,DA}_{g} = 0 \qquad \forall g \tag{3.44n}$$

$$\lambda^{DA}_{z:k\in\psi_z} - \underline{\mu}^{R,DA}_{k} + \overline{\mu}^{R,DA}_{k} = 0 \qquad \forall k \tag{3.44o}$$

$$\lambda^{DA}_{z} + \lambda_e - \underline{\gamma}^{F,DA}_{z,zo} + \overline{\gamma}^{F,DA}_{z,zo} = 0 \qquad \forall z, zo \in E \tag{3.44p}$$

$$(2.6) - (2.7) \tag{3.44q}$$

For the sake of simplicity, linearized complementarity constraints (equation 3.44q) are not listed, as in principle they are trivial to decompose per zone.

Constraints (3.44b)-(3.44m) were analyzed in the previous section, as these constraints are common to both models, so to decouple them per zone we can repeat the previous approach, i.e:

- Duplicate nodes in the lines connecting zones, as in Equation (3.23)-(3.24), and add the coupling constraint (3.25).

After that, constraints (3.44b)-(3.44m) are decoupled per zone. Constraints (3.44n)-(3.44o) are decoupled per se, as they include only variables related to one single conventional generator and one single renewable generator respectively.

Constraints related to ATC_e require modifications. In the centralized model, we defined ATCs per link e, so as the ATC optimizer assigns a ATC to each couple of zones. In the distributed model, each zone defines a ATC with their neighbouring zones, so the variable ATC_e must be duplicated. For instance, if a link e connects two zones z and zo, ATC_e can be duplicated into $ATC_{z,zo}$ and $ATC_{zo,z}$.

Moreover, it is necessary to add a new coupling constraint, to ensure that ATCs defined by different neighbouring zones coincide. This coupling constraint is listed as follows.

$$ATC_{z,zo} = ATC_{zo,z} \qquad : \mu_e^{ATC} \qquad \forall z, zo = e \in E \tag{3.45}$$

Constraint (3.44p) needs to be modified. Note that the variable λ_e is defined per link between two zones. In a distributed fashion, each zone will have its own variable, so it is necessary to duplicate it into $\lambda_{z,zo}$ and $\lambda_{zo,z}$ (same approach as with ATC_e). Moreover, it is necessary to add a coupling constraint to ensure that these two variables are equal. This constraint is written as follows.

$$\lambda_{z,zo} = \lambda_{zo,z} \qquad : \xi_e \quad \forall z, zo = e \in E \tag{3.46}$$

All things considered, with this modifications, the Cost-Optimal ATCs determination with sequential DA-RT market model takes the form stated in (3.26)-(3.28), i.e, an objective function separable per zones, a set of constraints per zone, and a set of coupling constraints. Thus, ADMM can be properly applied.

3.4.2.1 General ADMM algorithm

The procedure to derive the ADMM algorithm is analogous to section 3.4.1.1. First, it is necessary to derive the Lagrangian, as in Equation (3.29). The different terms are listed separately as follows.

$$\sum_{i}^{N} f(x_i) = \sum_{g} C_g \cdot q_g^{DA} + \sum_{s} \pi_s \left[\sum_{g} \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) \right.$$
$$\left. + \sum_{d} VOLL_d \cdot l_{d,s}^{shedRT} \right] \tag{3.47}$$

The first term corresponds to the cost function of the system, which is separable per zone, as discussed before.

$$\mu^T (\sum_{i=1}^{N} A_i x_i) = \sum_{z,zo=e} \mu_e^{ATC} (ATC_{z,zo} - ATC_{zo,z}) + \sum_{s} \pi_s \sum_{n,m=l \in l^*} \mu_{l,s}^{RT} (f_{m,mx,s}^{RT} + f_{n,nx,s}^{RT})$$
$$+ \sum_{z,zo=e} \mu_e^{FDA} (FDA_{z,zo} + FDA_{zo,z}) + \sum_{z,zo=e} \xi_e (\lambda_{z,zo} - \lambda_{zo,z}) \tag{3.48}$$

The second term corresponds to the augmented coupling constraints. Note that μ_e^{ATC}, $\mu_{l,s}^{RT}$, μ_e^{FDA} and ξ_e are the dual variables of the three aforementioned coupling constraints.

$$\frac{\rho}{2} \parallel \sum_{i=1}^{N} A_i x_i \parallel_2^2 = \sum_{z,zo=e} \frac{\rho}{2} \parallel ATC_{z,zo} - ATC_{zo,z} \parallel_2^2 + \sum_{s} \pi_s \sum_{n,m=l \in l^*} \frac{\rho}{2} \parallel f_{m,mx,s}^{RT} + f_{n,nx,s}^{RT} \parallel_2^2$$
$$+ \sum_{z,zo=e} \frac{\rho}{2} \parallel FDA_{z,zo} + FDA_{zo,z} \parallel_2^2 + \sum_{z,zo=e} \frac{\rho}{2} \parallel \lambda_{z,zo} - \lambda_{zo,z} \parallel_2^2 \tag{3.49}$$

Finally, the third term includes all the penalty parameters.

To have the ADMM algorithm, it is necessary to apply the iterative steps of (3.3)-(3.5) to the Lagrangian. The x steps, in this case, are the same as the ones in the previous section, i.e Equations (3.33)-(3.35). The same applies for the update of the dual variables μ_e^{FDA} and $\mu_{l,s}^{RT}$, listed in (3.36)-(3-38). The update of the dual variables ξ_e and μ_e^{DA} are listed as follows.

$$\xi_e^{v+1} = \xi_e^{v} + \rho(\lambda_{z,zo}^{v+1} - \lambda_{zo,z}^{v+1}) \quad \forall z, zo = e \tag{3.50}$$

$$\mu_e^{ATC,v+1} = \mu_e^{ATC,v} + \rho(ATC_{z,zo}^{v+1} - ATC_{zo,z}^{v+1}) \qquad \forall z, zo = e \tag{3.51}$$

3.4.2.2 Optimal Exchange-Consensus sharing ADMM algorithm

The procedure to derive the ADMM algorithm is analogous to section 3.4.1.2. The different terms of the x steps of the algorithm (Equation (3.38)) are listed as follows:

$$f(x_i) = \sum_{g \in z_i} C_g \cdot q_g^{DA} + \sum_s \pi_s \Bigg[\sum_{g \in z_i} \left(C_g \cdot (q_{g,s}^{RT} - q_{g,s}^{DA}) + C_{up} \cdot q_{g,s}^{up} + C_{down} \cdot q_{g,s}^{down} \right) + \sum_{d \in z_i} VOLL_d \cdot l_{d,s}^{shedRT} \Bigg] \tag{3.52}$$

The first term corresponds to the cost function of each zone i.

$$\mu^{vT} x_i = \sum_{zi,z=e} \mu_e^{ATC,v}(ATC_{zi,z}) + \sum_s \pi_s \sum_{n,m=l \in l^* \in z_i} \mu_{l,s}^{RT,v}(f_{n,nx,s}^{RT}) + \sum_{zi,z=e} \mu_e^{FDA,v}(FDA_{zi,z}) + \sum_{zi,z=e} \xi_e^v(\lambda_{zi,z}) \tag{3.53}$$

$$\frac{\rho}{2} \parallel x_i - (x_i^v - \overline{x}^v) \parallel_2^2 = \sum_{zi,z=e} \frac{\rho}{2} \parallel ATC_{zi,z} - (ATC_{zi,z}^v - \overline{ATC}^v) \parallel_2^2 + \sum_s \pi_s \sum_{n,m=l \in l^* \in z_i} \frac{\rho}{2} \parallel f_{n,nx,s}^{RT} - (f_{n,nx,s}^{RT,v} - \overline{f}^{RT,v}) \parallel_2^2 + \sum_{zi,z=e} \frac{\rho}{2} \parallel FDA_{zi,z} - (FDA_{zi,z}^v - \overline{FDA}^v) \parallel_2^2 + \sum_{zi,z=e} \frac{\rho}{2} \parallel \lambda_{zi,z} - (\lambda_{zi,z}^v - \overline{\lambda}^v) \tag{3.54}$$

Second and third terms are shown in equations (3.54) and (3.55).

The dual variable steps are carried out as in equations (3.44)-(3.46) for μ_e^{FDA} and $\mu_{l,s}^{RT}$. For ξ_e and μ_e^{ATC}, the dual variable updates are listed as follows.

$$\xi_e^{v+1} = \xi_e^v - \rho(\overline{\lambda}^{v+1}) \tag{3.55}$$

$$\mu_e^{ATC,v+1} = \mu_e^{ATC,v} - \rho(\overline{ATC}^{v+1}) \tag{3.56}$$

Two comments are required at this point:

- The proposed Optimal Exchange algorithm is based in a model in which the coupling constraints are in the form of $\sum x_i = 0$, so that in the optimal point, the average $\overline{x}$ should be equal to 0. Note that, in our particular case, the coupling constraint related to the ATCs is not defined in that way, due to the fact that ATCs are always considered positive. The simplest way to solve that issue is to calculate the average of ATCs ($\overline{ATC}$) assigning arbitrarily a negative sign to one of each pair of ATCs. Other way is to modify the model so that each pair connected zones defines the ATCs with a different sign.

- Similar to the ATC_e case, the coupling constraint related to ξ_e is not defined in the form $\sum x_i = 0$, so the average $\overline{\lambda}$ is not going to be zero in an equilibrium point. Thus, the solutions proposed to the ATC_e constraint are also applicable in this case.

3.5 Illustrative examples

In this section, both distributed algorithms are applied to the 2-zones 6-nodes system used in chapter 2. The goal is to compare the results obtained with the ADMM algorithms with the results of Chapter 2, with a centralized optimization. Thus, the system parameters (i.e generation capacity, load level, marginal prices, etc) are kept as in the previous simulations (see Table 2.1). All the models have been implemented using GAMS. Due to the small-scale of the system, and the fact that there are only 2 zones, the AMDD algorithm used is the general one (Section 3.4.1.1).

3.5.1 ADMM applied to the stochastic market model

Recall that the Cost-Optimal ATCs determination with stochastic DA and RT market clearing is a convex problem, thus if the ADMM algorithm converges, it must converge to the optimal solution, i.e the centralized solution. So here the aim is to compare the solutions of both centralized and distributed methods.

The initialization parameters and Lagrange multipliers are set to zero in the first iteration. The convergence criteria chosen is the primal feasibility. The algorithm is stopped when the primal residuals are reduced within a ϵ=10^{-3}.

As the algorithm is highly dependent on the assigned value of ρ, different initial values have been used. Note that, in this case ρ is kept constant during all the iterations. Relevant results are shown in Table 3.1.

As it can be seen, with values of $\rho > 1$ the model fails to converge. Moreover, with values of $\rho < 0.001$ the number of iterations required to achieve convergence shoots up. Around $\rho = 0.01$ the system converges in a reasonable number of iterations.

Table 3.1: Summary of results of the ADMM algorithm applied to the stochastic market model.

	ATC	Total system cost	Number of iterations
Centralized solution	307	9612.8	-
Distributed solution ρ=0.001	307	9612.8	91
Distributed solution ρ=0.005	307	9612.8	17
Distributed solution ρ=0.01	307	9612.8	9
Distributed solution ρ=0.05	307	9612.8	18
Distributed solution ρ=0.1	306.9	9613	33
Distributed solution ρ=1	-	-	-

In the cases in which the algorithm converges, the ATC and system cost coincides with the centralized solution (i.e the optimal solution). Note that small differences between both solutions are related to the ϵ defined in the convergence criteria. The lower the ϵ, the lower this error will be.

It is also interesting to analyze how the algorithm works if the initial values of some parameters are modified. Recall that in the previous case all the initialization parameters were set to zero in the first iteration. In Table 3.2 the results of the algorithm are shown for different initialization values of μ^{DA}, keeping $\rho = 0.01$.

Note that the model still converges to the optimal solution. In this case, the algorithm is sensitive to the initial value of μ^{DA}, and different values involves different number of iterations to converge, i.e, different computational time.

3.5.2 ADMM applied to the sequential market model

Recall that the Cost-Optimal ATCs determination with sequential DA and RT market clearing is, in general, a nonconvex problem, thus, the ADMM algorithm is not guaranteed to converge to a single local optimal solution. Depending on the initialization values it may converge to different nonoptimal points. So here the aim is to compare the solutions of both centralized and distributed methods.

For the first simulation, the initialization parameters and Lagrange multipliers are set to zero in the first iteration. The convergence criteria chosen is the primal feasibil-

Table 3.2: Relevant performance of the algorithm as a function of initialization parameters.

	ATC	Total system cost	Number of iterations
Centralized solution	307	9612.8	-
Distributed solution $\mu^{DA,0}$=0	307	9612.8	9
Distributed solution $\mu^{DA,0}$=2	307	9612.8	7
Distributed solution $\mu^{DA,0}$=5	307	9612.8	4
Distributed solution $\mu^{DA,0}$=10	307	9612.8	5
Distributed solution $\mu^{DA,0}$=20	307	9612.8	9
Distributed solution $\mu^{DA,0}$=100	307	9612.8	25

ity. The algorithm is stopped when the primal residuals are reduced within a ϵ=10^{-3}.

The algorithm here is, again, highly dependent on the assigned value of ρ. In this case, as the model is nonconvex, this value affects the computational time, and it might also affect the final solution found. Thus, the algorithm is implemented in the illustrative example with different initial values of ρ. Note that the value of ρ is unchanged during the iterations. Relevant results are shown in Table 3.3.

Note that, for all the chosen values of ρ, when the algorithm converges the solution found is the same local optimal point than the centralized model. For values of ρ<0.005 and ρ>0.1 the computational time required to converge grows up exponentially. With ρ>10 the system fails to converge.

In order to evaluate how the algorithm works with different initialization values, a sensitivity analysis is performed. ρ is set to a value of 0.01, and the initialization of μ^{DA} is modified. The rest of parameters are kept to 0. Results are shown in Table 3.4.

These results are in line with the previous simulations. The algorithm, again, converges to the same local optimal solution than the centralized model, and the variation of the initialization parameter affects only to the computational time.

Table 3.3: Summary of results of the ADMM algorithm applied to the sequential market model.

	ATC	Total system cost (€)	Number of iterations
Centralized solution	300	9662.8	-
Distributed solution ρ=0.001	300	9662.8	143
Distributed solution ρ=0.005	300	9662.8	24
Distributed solution ρ=0.01	300	9662.8	15
Distributed solution ρ=0.05	300	9662.8	23
Distributed solution ρ=0.1	300	9662.8	54
Distributed solution ρ=1	300	9662.8	423
Distributed solution ρ=10	-	-	-

Table 3.4: Relevant performance of the algorithm as a function of initialization parameters.

	ATC	Total system cost (€)	Number of iterations
Centralized solution	300	9662.8	-
Distributed solution $\mu^{DA,0}$=0	300	9662.8	15
Distributed solution $\mu^{DA,0}$=2	300	9662.8	13
Distributed solution $\mu^{DA,0}$=5	300	9662.8	6
Distributed solution $\mu^{DA,0}$=10	300	9662.8	4
Distributed solution $\mu^{DA,0}$=20	300	9662.8	7
Distributed solution $\mu^{DA,0}$=100	300	9662.8	38

3.5.3 Concluding remarks to the Illustrative example

This illustrative example shows that both ADMM algorithms are correctly implemented. The obtained results are satisfactory and in line with the initial insights.

Both algorithms converge to the same solution than the centralized models. In the case of the convex model, this is not a relevant output, because per definition a convex model has the same optimal value of the objective function in the centralized and distributed ADMM approach.

However, in the nonconvex model this results are highly relevant, because in that case the convergence to the same objective function value is not guaranteed. Previous simulations show that, in this particular 2-zones 6-nodes model the distributed and centralized approach end up in the same ATC and final expected system cost. Thus, the distributed solution allows (in this specific case) to achieve the same consensus solution respecting the privacy of each zone, and minimizing the share of information.

3.6 Case study

The results shown in the previous section are promising, but the illustrative example needs to be extended to find remarkable conclusions. Therefore, in this section the ADMM algorithms is applied to a extended system: the modified IEEE 24-Bus RTS [23], depicted in Figure 3.4. The system has been divided in two areas. Relevant data related to the generation capacity, load levels and transmission lines capacities considered in this setup are shown in Tables 3.5, 3.6 and 3.7.

Table 3.5: Capacity and cost data of the generators.

Unit	Zone	Installed capacity (MW)	Up-reserve capacity (MW)	Down-reserve capacity (MW)	Day-ahead offer price (€/MWh)
1	1	152	40	40	13.32
2	1	152	40	40	13.32
3	1	350	70	70	20.7
4	1	591	180	180	20.93
5	2	60	60	60	26.11
6	2	155	30	30	10.52
7	2	155	30	30	10.52
8	2	400	0	0	6.02
9	2	400	0	0	5.47
10	2	300	0	0	0
11	2	310	60	60	10.52
12	2	350	40	40	10.89

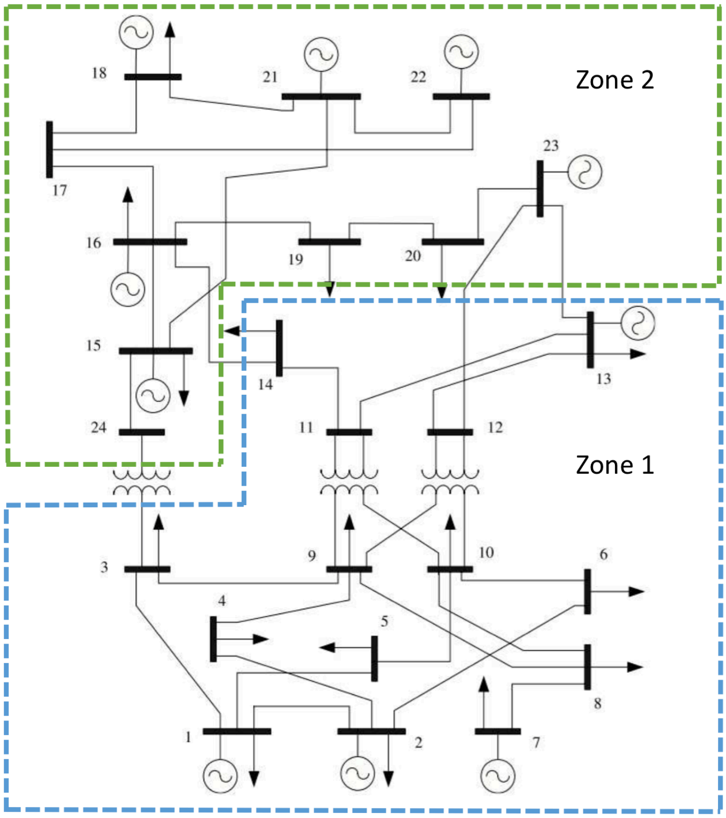

Figure 3.4: Two-area version of the IEEE 24-Bus RTS.

Table 3.6: Load levels in the system (corresponding to the hour 8 in [23]).

Load	Node	Level (MW)	Load	Node	Level (MW)
1	1	86	10	10	154
2	2	77	11	13	210
3	3	142	12	14	154
4	4	59	13	15	251
5	5	56	14	16	79
6	6	109	15	18	265
7	7	99	16	19	145
8	8	135	17	20	102
9	9	138			

Table 3.7: Capacity of the lines of the system.

From	To	Capacity (MW)	From	To	Capacity (MW)
1	2	175	11	13	500
1	3	175	11	14	500
1	5	350	12	13	500
2	4	175	12	23	500
2	6	175	13	23	250
3	9	175	14	16	250
3	24	400	15	16	500
4	9	175	15	21	400
5	10	350	15	24	500
6	10	175	16	17	500
7	8	350	16	19	500
8	9	175	17	18	500
8	10	175	17	22	500
9	11	400	18	21	1000
9	12	400	19	20	1000
10	11	400	20	23	1000
10	12	400	21	22	500

Following the recommendations of [23], six wind farms are included in the system, attached to the nodes 3, 5, 7, 16, 21 and 23, with a installed capacity of 200 MW each one. 10 wind scenarios are considered, and each wind generator production data per scenario is extracted from [3].

All things considered, the two ADMM algorithms are applied to the IEEE 24-Bus RTS system. All the models are implemented using GAMS. Due to the fact that the system is composed of 2 zones, the ADMM algorithm used in both cases is the general one (Sections 3.4.1.1 and 3.4.2.1).

3.6.1 Stochastic market model results

All initialization parameters and Lagrange multipliers are set to zero in the first iteration. The convergence criteria chosen is the primal feasibility with a $\epsilon = 10^{-3}$. Different values of ρ are used, keeping it constant during the iterative process.

The centralized model of Chapter 2 is also applied to the system, in order to compare the results of both approaches. Relevant results are shown in Table 3.8 and Figure 3.5.

Table 3.8: Summary of results of the ADMM algorithm in the 24-bus system with the stochastic market model.

	ATC (MW)	System cost (€)	Number of iterations
Centralized solution	1119	12295.81	-
Distributed solution ρ=0.001	1119	12295.81	117
Distributed solution ρ=0.005	1119	12295.81	55
Distributed solution ρ=0.01	1119	12295.81	9
Distributed solution ρ=0.05	1119	12295.81	20
Distributed solution ρ=0.1	1119	12295.81	41
Distributed solution ρ=1	1119	12295.81	223

The results show that the algorithm works properly in the 24-bus system. Convergence to the optimal solution (i.e, the same as the centralized) is achieved within a broad range of penalty parameters. Note that, for values of $\rho > 1$ and $\rho < 0.005$ the number of iterations required to reach convergence starts growing exponentially. Figure 3.5 shows how the total system cost (i.e the sum of the costs of each zone) evolves while the algorithm is iterating. The dashed line represents the centralized solution.

3.6.2 Sequential market model results

Again, all initialization parameters are set to zero in the first iteration. The convergence criteria chosen is the primal feasibility with a $\epsilon = 10^{-3}$. Different values of ρ are used, keeping it constant during the iterations. The centralized sequential model of Chapter 2 is also implemented in order to compare both results.

Table 3.9 summarize the results achieved. As it can be seen, the centralized approach solution coincides with the distributed one. The best choices of ρ, according to the number of iterations, are between 0.01 and 0.1. Figure 3.6 shows the evolution of the total system cost through the iterations of the ADMM are performed, until the distributed solution converges to the centralized one (dashed line).

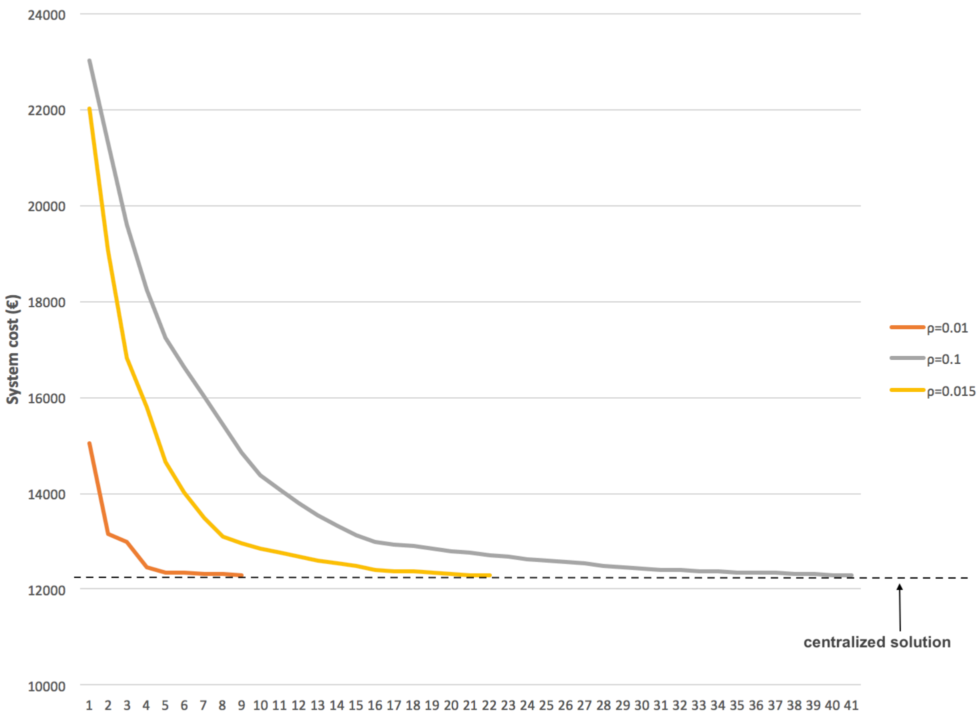

Figure 3.5: Evolution of the system cost along the iterations of the algorithm.

Table 3.9: Summary of results of the ADMM algorithm in the 24-bus system with the sequential market model.

	ATC (MW)	System cost (€)	Number of iterations
Centralized solution	1058	12801.93	-
Distributed solution ρ=0.001	1058	12801.93	254
Distributed solution ρ=0.005	1058	12801.93	71
Distributed solution ρ=0.01	1058	12801.93	22
Distributed solution ρ=0.05	1058	12801.93	17
Distributed solution ρ=0.1	1058	12801.93	28
Distributed solution ρ=1	1058	12801.93	358

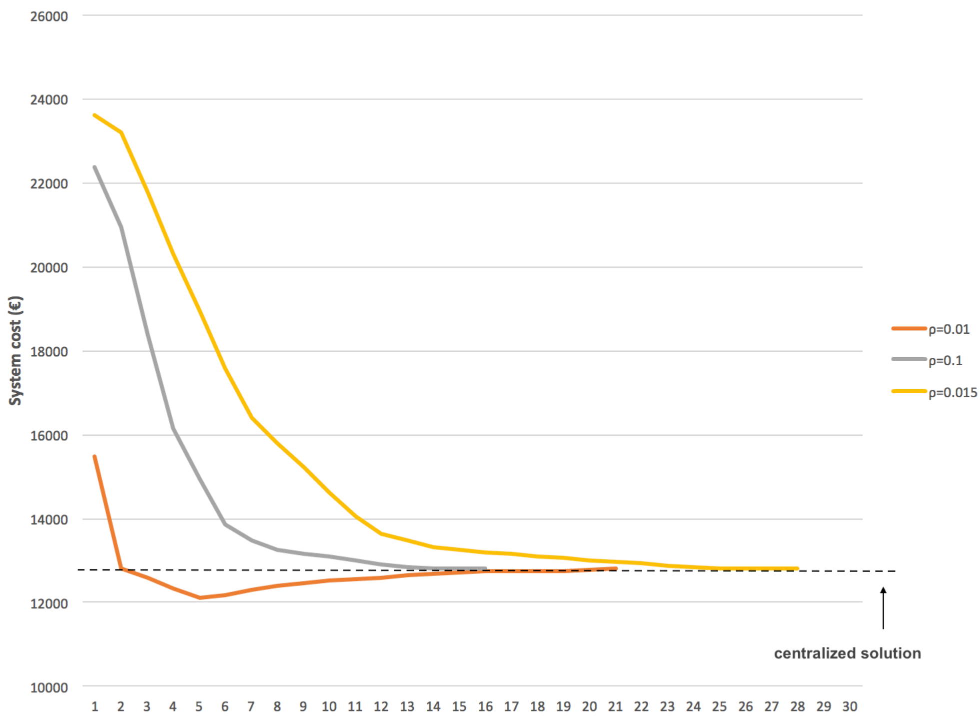

Figure 3.6: Evolution of the system cost along the iterations of the algorithm.

3.7 Summary and discussion of the results

In general, the results presented in this chapter are quite promising. The main conclusion that can be inferred from the case studies is that the centralized and distributed approaches in both models end up in the same values of cost-optimal ATCs. That is, the set of ATCs determined by the models of Chapter 2, in which all zones are perfectly coordinated and share all its network information, is the same that the set determined with the distributed approach (i.e Chapter 3) in which each zone aims to minimize only its own cost, and the share of information is minimum.

In the stochastic market model (i.e, the convex case) this coincidence was expected, due to the fact that the ADMM algorithm applied to a convex model converges to the optimal solution per definition. This model is useful to define a lower bound to the named sequential model, as its a less constrained version. Note that in the 24-bus case study the stochastic market model ends up with a total cost of 12295.81€, while the sequential model ends up with 12801.93€. In this case, respecting the merit order in day ahead implies a 4% higher cost.

Regarding the sequential model, due to its nonconvexity, this coincidence was not assured. Results in this chapter permit to infer that the model proposed in [9] and developed in chapter 2 can be solved in a distributed manner leading to the same set of ATCs. This model, as seen during this chapter, results in a higher system cost than the stochastic model, and requires a higher number of iterations to converge.

It has been also proved that the performance of the distributed algorithm is highly dependant on the values of the penalty parameter and the initialization parameters. For both distributed models, values of ρ in between 0.01 and 0.05 have been proven to reduce the number of iterations. In the nonconvex model, it is interesting to note that, in the systems considered, variations in the penalty parameters and initialization parameters do not modify the convergence solution (recall that, according to [5], in nonconvex models ADMM can give different solutions depending on these parameters). Thus, in the analyzed systems (6 buses and 24 buses) both algorithms are highly robust.

Finally, it is important to write some comments regarding the values of ATCs compared to the actual grid limits. One of the main motivations of the cost-optimal ATCs determination models is to decouple the financial ATC values from the physical grid, so as in day ahead they can have values higher than the real capacity of the interconnections between zones. However, in the case studies analyzed in this thesis (namely the 6 bus and the 24 bus system) the cost-optimal ATCs are lower than the grid capacity. Note that, for instance, in the 24 bus system, the capacity of all the combined tie-lines is 1850 MW, while the cost-optimal ATCs of the ADMM algorithms are 1119 MW and 1058 MW. In [9] it was proved that, in a large-scale European test system, cost-optimal ATCs may reach values higher than the cross border capacity (even more than double). However, in small-size setups such as the 6 bus or the 24 bus tested in this thesis, finding this specific cases can be more challenging.

In the 24 bus system, if a modification of the tie-line capacities is performed, reducing the capacity of the line 24-3 to 200 MW, line 14-16 to 250 MW, line 23-13 to 125 MW and line 23-12 to 165 MW, the total cross border capacity is reduced to 740 MW. In this setup, implementing the centralized and distributed models result in a ATC of 765 MW in the sequential model, and 790 in the stochastic model. So, even in this small-size setup, it is possible to find operating conditions in which the ATCs are higher than the cross-border capacity.

CHAPTER 4

Conclusion

4.1 Concluding remarks and future work

The main objective of these thesis was to solve the cost-optimal ATCs determination model of [9] considering a partial coordination between zones, that is, in a distributed way, and minimizing the share of information between zones. This objective has been achieved by applying the alternating direction method of multipliers.

Results of chapter 2 permit to demonstrate that the set of cost-optimal ATCs determined with full coordination between zones minimize the total system cost, but do not necessarily minimize the cost of each one of the zones of the system. In the case studies analyzed it has been found that the ATCs that are optimal for each single zone do not necessarily coincide. In chapter 3 the results show that, in the cases considered, the cost-optimal ATCs found with full and partial coordination are the same. Thus, the distributed approach ends up miniming the total system cost, while keeping the privacy of the participants preserved.

A simplification to the bilevel model of [9] has also been proposed, considering a stochastic market clearing in day ahead, leading to a single-level convex model. This model has ben tested in the same case studies, leading to a lower level system cost and a faster convergence ratio in the distributed approach.

To improve the method the tuning of the initialization parameters could be explored. In this thesis, the ADMM parameters were set in the first iteration according to a flat start. Different simulations have shown that the number of iterations is highly dependent on the change of this parameters, so a proper choice and tuning could improve the computational time. Finally, in the distributed approach it is considered that each TSO haves full information of its own zone. This approach is unrealistic, and the model could be improved adding uncertainty the cost and capacity of market participants.

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