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# STUDY OF THE EFFECTS OF THE TEMPERATURE ON THE FUNCTIONING OF THE LINEAR MOTOR OF 3D PRINTER BY DESIGNING AND MODELLING ON OPENMODELICA.

THESIS OF INDUSTRIAL ENGINEERING

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## INTRODUCTION:

Nowadays 3D printers are gaining more importance on our society and on the future, we will need them to produce all the products. The investigations on all the fields related to them will be value to keep improving them. Some of them use an important type of motor as the linear motor is, but with the different type of products that we want to build, the conditions where the motors work are changing and then, it appears some difficulties on the motors.

The aim of this investigation is to model a realistic system which represent a linear motor on the program Open Modelica to simulate it at different temperatures and see which the parameters of the motor are affected and why the accuracy of the motor is lower the temperature of the functioning of the motor decrease.

# CHAPTER 1: 3D PRINTER

## 1.1: PREFACE

First of all, before begin working with the real problem of this investigation, we should understand in which machine and systems are we working with. 3D printing is any of various process when material is joined or solidified under computer design and control to create three-dimensional object.

During the last decades the importance of this type of machines and production is growing, due to the several benefits that we can obtain. For that reason, it is our duty to keep working and investigation on any field of this type of production like other different way to produce the products and how to improve the results.

We can divide the process in three general principles.

- Modelling

First, we need to produce a digital model of the product that we want to obtain. This model should be created with a computer aided design (CAD) software. That can be produce by a 3D scanner, a digital camera or create the model directly.

- Printing

Before printing we should check out the model trying to observe any mistake that can affect the final result of the product. Then, the model is processed by different software to transform the model into some instructions than the printer can understand. Depending on the resolution of the printer that we are working with, we will be able to produce a better product, any physical impact that can affect this resolution should be investigated, which it is the aim of this thesis.

Depending on the material and product that we want to obtain we will have to apply a different technique of production. As the historical material used in this type of machines is a polymer, injection moulding is the process more investigated and applied.

- Finishing

Finally, after achieving our product we should make a revision about little mistakes that can be produced through higher-resolution subtractive process. Then other process like printing are done to achieve the final form of our product.

## 1.2: COMPONENTS OF 3D PRINTER:

Depending on which printer do you have, and which technique are you use could be different, but the principal components are:



- Print bed:

It is the surface where the objects are printed on to. Typically, it will consist of sheet of glass, a heating element and some kind of surface on top to help the material stick

- Extruder:

The extruder is where the material gets drawn in, melted, and pushed out. Normally it consists of two parts, hot end and cold end.

- Thermistor:

Sensors for determining the temperature the process and the parts of the extruder. They will be located where the maximum and minimum temperatures are achieved.

- Motor:

It is the mechanism which will move the extruder into the different points of the resolution trying to form the product.

- Power supply:

- Motherboard:

It is the brain of the printer. It takes the commands given by the computer and transform them to an execution.

- End stops:

The end stops show the printer where it is and where it is the beginning. It will be one for any axis.

## 1.3 TECHNIQUES TO PRODUCE THE MODEL:

### 1.3.1: Fused deposition modelling:

As we said, typically the material used in this type of machines is thermoplastic. There are many techniques developed but the most common is fused deposition modelling.

It is the fastest, most affordable way to create customer goods. It uses a thermoplastic filament, which is heated to its melting point and then extruded to create a three-dimensional object. The most common printing material for FDM is acrylonitrile butadiene styrene (ABS), or thermoplastics, like polycarbonate (PC) or polyetherimide (PEI).

### 1.3.2: Cryogenic 3D printing:

Even FDM is the most common technique it is not the only one. With the development of new materials it has been necessary to find new forms to produce the products. With the increase of chronic diseases, injuries and traumas one of the “new” fields investigated is the design and production of supportive structures, generally defined as scaffolds. Here is where our motor is going to work.

It will be necessary a solution which contains chitosan (CH) a natural polysaccharide with good combination of biocompatibility and biodegradability non-toxic and not expensive. Physically this process will be different in two main changes. A new deposition system that uses a syringe pump and by replacing the hot plate with a cooling system. One of the meaning problems of this method is the low range of temperature where the motor must work which can produce a few disfunctions.

#### 1.4: MOTOR THORLABS (DDS 220/M).

Here, we will see the features of the real linear motor of the company. Our models will be based on these characteristics trying to replicate the motor and its functioning as much as we can.

##### Features

- High Speeds: Up To 300 mm/s.
- High Repeatability: 0.25  $\mu\text{m}$ .
- Positional Accuracy: <3.0  $\mu\text{m}$ .
- Low Profile: 44 mm (1.73").
- Integrated, Brushless DC Linear Servo Motor Actuators.
- Linear Optical Encoders.
- High-Quality, Precision-Engineered Linear Bearings.
- 5-40, 8-32, and 1/4"-20 or M3, M4, and M6 Tapped Holes for Mounting Optomechanics.

Key Specifications	
Travel Range	220 mm (8.6")
Velocity (Max)	300 mm/s
Min Achievable Incremental Movement	0.1 $\mu\text{m}$
Bidirectional Repeatability <sup>b</sup>	$\pm 0.25 \mu\text{m}$
Horizontal Load Capacity (Max)	3.0 kg (6.6 lbs)
Actuator Type	Brushless DC Servo Motor
Cable Length	2.7 m (8.9 ft)
Recommended Controller	<u>apt™ Servo Motor Controllers</u>

Table 1. Specifications of the motor.

DDS220 Stage		
<b>Travel Range</b>	220 mm (8.6")	
<b>Speed (Max)</b>	300 mm/s	
<b>Acceleration (Max)</b>	5000 mm/s <sup>2</sup>	
<b>Bidirectional Repeatability<sup>a</sup></b>	±0.25 µm	
<b>Backlash<sup>b</sup></b>	N/A	
<b>Load Capacity<sup>c</sup></b>	3.0 kg (6.6 lb)	
<b>Incremental Movement (Min)<sup>d</sup></b>	0.1 µm	
<b>Absolute On-Axis Accuracy</b>	±2.0 µm	
<b>Home Location Accuracy (Unidirectional)</b>	±0.25 µm	
<b>Straightness/Flatness</b>	± 5.0 µm	
<b>Pitch<sup>e</sup></b>	±175 µrad	
<b>Yaw<sup>e</sup></b>	±175 µrad	
<b>Continuous Motor Force</b>	7.0 N	
<b>Peak Motor Force (2 sec)</b>	15 N	
<b>Weight (Including Cables)</b>	2.4 kg (5.3 lbs)	
<b>Limit Switches</b>	Yes	
<b>Operating Temperature Range</b>	5 to 40 °C (41 to 104 °F)	
<b>Bearing Type</b>	Precision Linear Bearing	
<b>Motor Type</b>	Brushless DC Linear Motor	
<b>Dimensions</b>	370.0 mm x 90.0 mm x 44.0 mm (14.57" x 3.54" x 1.73")	
<b>Weight</b>	2.4 kg (5.3 lbs)	
<b>Recommended Controller</b>	BBD201 (1-Channel) BBD202 (2-Channel)	

*Table 2. Features of the motor.*

We can see that the operating temperature range is between 5 to 40 °C, and the Cryogenic 3D printing needs temperatures below 15 °C, but we need to know which are the effects that these different temperatures produce on our model.

## CHAPTER 2: ELECTROMECHANICAL SYSTEMS

### 2.1: Introduction to electro mechanics.

We speak about electromechanical machine where there is interaction of electrical and mechanical systems. It is usually understood to refer to devices which involve an electrical signal to create mechanical movement

We can classify the electrical machines between:

- Direct current (DC)
- Alternating current (AC)
  - Asynchronous motor.

It is a machine in which the electric current in the rotor needed to produce torque is obtained by electromagnetic induction from the magnetic field of the stator winding. The asynchronous nature of induction-motor operation comes from the slip between the rotational speed of the stator field and slower speed of the rotor.

- Synchronous motor.

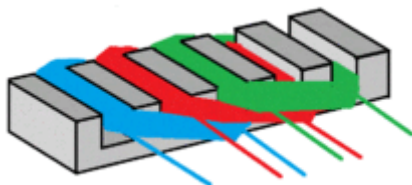
It is a motor in which, the rotation of the shaft is synchronized with the frequency of the supply current. The magnetic field rotates in time with the oscillations of the line current and the rotor turns in step with the stator field at the same rate.

### 2.2: Linear actuator

A linear actuator or linear motor is an electric motor that has had its stator and rotor “unrolled” so that instead of producing a torque (rotation) it produces a linear force along its length.

#### 2.2.1. Permanent magnet synchronous linear motor (PMSLM).

In this type of linear motor, the rate of movement of the magnetic field is controlled, usually electronically, to track the motion of the rotor. We will use permanent magnets on the stator.



*Figure 1. Three phase linear induction motor.*

The force is produced by a moving linear magnetic field acting on conductors in the field. The conductor, in this case, a loop, will have currents induced in it thus creating an

opposing magnetic field, in accordance with Lenz's law. The two opposite fields will repel each other, thus creating motion as the magnetic field sweeps through the metal.

## 2.3: Basic theory

### 2.3.1. Lorentz force.

The Lorentz force is the combination of electric and magnetic force on a point charge due to electromagnetic fields. A particle of charge  $q$  moving with velocity  $v$  in the presence of an electric field  $E$  and a magnetic field  $B$  experiences a force. A positively charged particle will be accelerated in the same linear orientation as the  $E$  field but will curve perpendicularly to the velocity  $v$  and the  $B$  field according to the right-hand rule. The term  $qE$  is called the electric force, while the term  $qv \times B$  is called the magnetic force.

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

*Figure 2. Lorentz force.*

### 2.3.2. Law of Ampere-Maxwell.

The law of Ampere-Maxwell relates the integrated magnetic field around a closed loop to the electric current passing through the loop. If we consider a wire carrying a steady current  $I$ . If we consider that the field will act on a circle of radius  $r$  centred on the wire. The magnetic field-strength is the same at all points on the loop and the value will be:

$$B = \frac{\mu_0 I}{2\pi r}.$$

*Figure 3. Law of Ampere Maxwell.*

### 2.3.3. Law of Faraday-Lenz.

Faraday's law of induction is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force. It is the fundamental operating principle of transformers, inductors and many types of electrical motors, generators and solenoids.

The induced electromotive force in any closed circuit is equal to the negative of the time rate of change of the magnetic flux enclosed by the circuit.

If we have a tightly wound coil of wire composed of  $N$  identical turns, each with the same  $\Phi B$ . Faraday's law of induction will be:

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Figure 4. Law of Faraday-Lenz.

Where N is the number of turns of wire and  $\Phi_B$ , is the magnetic flux through a single loop.

### 2.3.4. Biot-Savart Law.

The Biot-Savart Law is an equation describing the magnetic field generated by stationary electric current. It relates the magnetic field to the magnitude, direction, length and proximity of the electric current.

The law is evaluated over the path C in which the electric current flow.

$$B(r) = \frac{\mu_0}{4\pi} * \int_C \frac{I * dl * r'}{r'^3}$$

Equation 1. Biot-Savart Law.

Where dl is a vector along the path C, whose magnitude is the length of the differential element of the wire in the direction of the current.  $r' = r - l$  is the full displacement vector from the wire element (dl) to the point which the field is being computed (r), and  $\mu_0 = 4\pi * 10^{-7}$  Wb/Am is the magnetic constant which represents the permeability of the vacuum.

### 2.3.5. Force between two surfaces.

The mechanical force between two nearby magnetized surfaces can be calculated with the following equation.

$$F = \frac{\mu_0 * H^2 * A}{2} = \frac{B^2 * A}{2 * \mu_0}$$

Equation 2. Force between two surfaces.

Where:

A is the area of each surface, in m<sup>2</sup>.

H is their magnetizing field, in A/m.

$\mu_0$  is the permeability of space (T\*m/A).

B is the flux density, in T.

### 2.3.6. Friction on Linear Motor.

Another important part on the development of the Linear Motor is the friction. We are going to follow the descriptions made on [6] about a model which describes the equations to show the behaviour of the friction on the motor. We can see that the model consists of two components: The relationships between displacement and friction force under microscopic motion (non-linear spring characteristic) and a model for the relationship between velocity and friction force (Stribeck curve).

The Stribeck curve is a fundamental concept in the field of the tribology (science and engineering of interacting surfaces in relative motion, it includes the study and application of the principles of friction, lubrication and wear). It shows that friction in fluid-lubricated contacts is a non-linear function of the contact load, the lubricant viscosity and the lubricant entrainment speed.

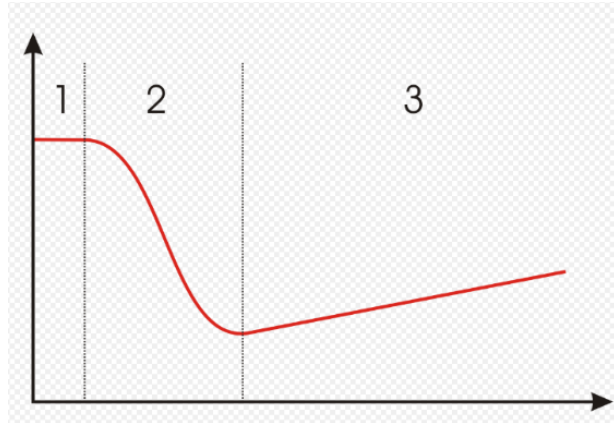


Figure 5. Stribeck curve.

We can see that differences in grease viscosity strongly influence the damping of vibrations for the step response. The friction force of linear guide behaves as a spring, and thus the relationship between displacement and friction force gives a hysteresis loop. This characteristic, referred to as a non-linear spring, influences the dynamic behaviour of the feed drive system. The friction characteristics depend on the lubricant viscosity, ball retainer, pre-load, and guide size.

### 2.3.7. Resistance of copper.

If we want to create a realistic model which describes the motor, we should to keep in mind the important paper that have the materials that we are using. In this case, we are talking about the copper and how its properties change when temperature does. More specifically we are going to take in mind how the resistance changes when the temperature of the copper change.

The temperature coefficient of copper (near room temperature) is +0.393 percent per degree C. This means if the temperature increases 1°C, the resistance will increase 0.393%.

The generic formula for temperature affects on resistance is as follows:

$$R = R_{ref}[1 + \alpha(T - T_{ref})]$$

Equation 3. Resistance of the copper with the temperature.

Where:

- $R$  = Conductor resistance at temperature “ $T$ ”.
- $R_{ref}$  = Conductor resistance at reference temperature  $T_{ref}$ , usually  $20^{\circ}\text{C}$ , but sometimes  $0^{\circ}\text{C}$ .
- $T$  = Conductor temperature in degrees Celsius.
- $T_{ref}$  = Reference temperature that  $\alpha$  is specified at the conductor material.



## CHAPTER 3: LINEAR ELECTROMAGNETIC MOTOR.

### 3.1: Composition.

#### 3.1.1. Permanent Magnets of stator.

Inside the large field of permanent magnets, we can find the Rare earth magnets which are the strongest magnets produced from alloys of rare earth elements. There are two principal types, samarium-cobalt or neodymium magnets. In our model we will be using the last ones.

Neodymium magnets (NdFeB, NIB or Neo) are the most widely used, its made from an alloy of neodymium, iron and boron. We can calculate that our model is composed by two lines of 15 blocks (15x8x20mm).

To calculate the value of the B field on the symmetry axis of an axially magnetised block magnet we can follow:

$$B = \frac{Br}{\pi} \left[ \arctan\left(\frac{LW}{2z * \sqrt{4z^2 + L^2 + W^2}}\right) - \arctan\left(\frac{LW}{2 * (D + z) * \sqrt{4(D + z)^2 + L^2 + W^2}}\right) \right]$$

*Equation 4. Value of B field on the symmetry axis.*

Where:

Br: Remanence field, independent of the magnet's geometry.

z: Distance from a pole face on the symmetry axis.

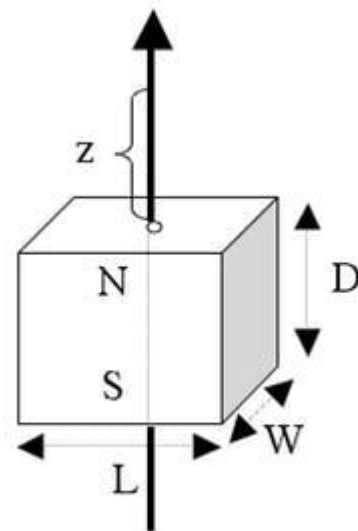
L: Length of the block.

W: Width of the block.

D: Thickness (or height) of the block.

So, if we are using Nd<sub>2</sub>Fe<sub>14</sub>B we will have: Br = 0.7, L = 0.020 m, W = 0.015 m, D = 0.008 m, z = 0.005 mm.

B = 6.51 T.



### 3.1.2. Rotor

The rotor of our motor will be composed by 3 solenoids. Each solenoid is independent from the others, they will be represented by the same circuit but with a different way of voltage source.

In order to calculate the force necessary to move the rotor, first, we need to calculate the magnetic field created by the solenoid. To make an approximation we will use the Biot-Savart law (Equation 1) which can be used to calculate the magnetic field along the axis of the solenoid. If we can assume that this value does not change if we move far to the axis and near to the coil, following the calculations made on [2].

$$B(r) = \frac{\mu_0 * i}{4\pi} \int_C \frac{dl * r'}{r'^3}$$

Equation 5.

Where  $dl$  and  $r'$  are the line segment and distance vector from the viewpoint to the source charge;  $\mu_0 = 4\pi * 10^{-7}$  Wb/Am represent the permeability of the vacuum. By the use of  $dl = R * d\psi \psi^\wedge$ ,  $r' = z * z^\wedge - r * r^\wedge$  and  $|r'| = \sqrt{z^2 + r^2}$ , the magnetic field at an arbitrary point P on the axis of the current loop becomes:

$$B_{sol,z} = \frac{\mu_0 * i * r^2}{4\pi} * \int_0^{2\pi} \frac{d\psi}{(r^2 + z^2)^{\frac{3}{2}}} z^\wedge = \frac{\mu_0 * i * r^2}{2 * (r^2 + z^2)^{\frac{3}{2}}} z^\wedge$$

Equation 6

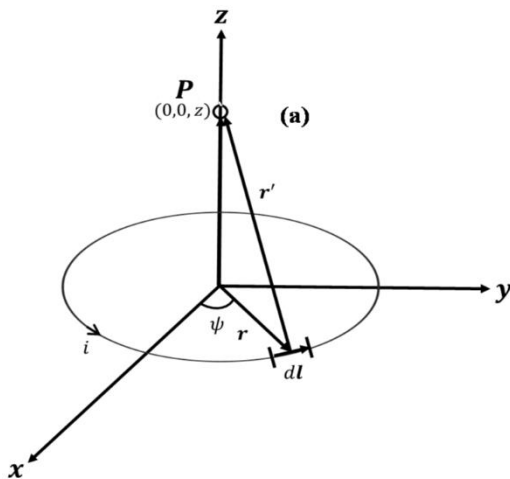


Figure 6. Magnetic field at an arbitrary point P.

If we replace:

-  $r = \text{width of pole}/2 = 14.5/2 \text{ mm} = 0.0072 \text{ m}$

-  $z = \text{Height of teeth} + \text{air gap length}/2 = 15\text{mm} + 1.1/2 = 15.55 \text{ mm} = 0.0155 \text{ m}$

$B_{\text{sol},z} = 5.1923 * \mu_0 * i.$

*Equation 7*

### 3.1.3. State of art.

If we investigate we can find many models to represent the linear motor. We are going to follow the representation showed on [3]. Here we can see that the best way to operate the motor is to use unipolar rectangular impulse direct (constant) voltage of power supply.

The change of the amplitude is evaluated by the voltage factor (coefficient):

$$y = \frac{U}{U_m}$$

*Equation 8. Voltage factor.*

The duration of the voltage impulse is characterized by the casual power supply voltage impulse duration factor (coefficient):

$$\beta = \frac{t_{imp}}{T} = t_{imp} * f_{imp}$$

*Equation 9. Casual power supply voltage impulse duration factor.*

The maximum whole number of communications could be made according to this expression:

$$n_{max} = \frac{L}{\frac{t}{3}}$$

*Equation 10. Maximum number of communications.*

Where:

-  $L = \text{the length of the rectilinear way of the motor.}$

-  $t = 2 * b = \text{the polar step of the secondary teathed element.}$

-  $b = \text{the width between the similar tooth and spans of the secondary element.}$

#### 3.1.3.1. The traction force of the motor:

The expression can be got using theory of the change of the electromagnetic energy. According to this theory, to get the extraction is necessary to know the law of the change of the induction of the x coordinate.

For engineering calculations, the experimental curve could be approximated by the formula:

$$L(x) = L_0 + L_m * \cos\left(\frac{2\pi}{t} x\right)$$

Equation 11. Experimental curve.

$$L_0 = \frac{L_1 + L_2}{2}$$

Equation 12. "L<sub>0</sub>" of experimental curve.

$$L_m = \frac{L_1 - L_2}{2}$$

Equation 13. "L<sub>m</sub>" of experimental curve.

Where:

- L<sub>0</sub>, L<sub>m</sub> - The direct (constant) and changeable impulse of excitation induction coil.
- L<sub>1</sub>, L<sub>2</sub> – The evaluation of the induction of coil, when the teeth and span poles are just at the middle position between the tooth.

The expression of the traction force of a linear motor with one induction coil:

$$f_x = -\frac{\pi}{t} * L_m * i^2 * \sin\left(\frac{2\pi}{t} x\right)$$

Equation 14. Traction force of a linear motor with one induction coil.

### 3.1.4. Force generated.

After defining the magnetic field created by the rotor and the stator we want to calculate which is the interaction between them.

Following the Equation 2 we have:

$$F = \frac{B^2 * A}{2 * \mu_0} = \frac{B_{sol} * B_{permagnet} * A}{2 * \mu_0} = \frac{6.51 * 5.1923 * \mu_0 * i}{2 * \mu_0} = 16.90 * i$$

Equation 15.

In our case, as we do not have two magnets, there is not really an area A to be represented as a geometric object. It becomes a coefficient. If we follow [3], we can see how is the form of the equation that represent each solenoid.

$$f - x = (\text{Force generated}) * \sin\left[\frac{2\pi}{t} \left(x - \frac{t}{3}\right)\right]$$

Equation 16. Force of -x solenoid.

$$f_{0x} = (\text{Force generated}) * \sin\left(\frac{2\pi}{t}x\right)$$

*Equation 17. Force of 0x solenoid.*

$$f_{+x} = (\text{Force generated}) * \sin\left[\frac{2\pi}{t}\left(x + \frac{t}{3}\right)\right]$$

*Equation 18. Force of +x solenoid.*

So, if we substitute the force generated for the value calculated on (Equation 15), and the names of each solenoid, we will have:

$$f_A = (16.90 * i) * \sin\left[\frac{2\pi}{t}\left(x - \frac{t}{3}\right)\right]$$

*Equation 19. Force of A solenoid.*

$$f_B = (16.90 * i) * \sin\left(\frac{2\pi}{t}x\right)$$

*Equation 20. Force of B solenoid.*

$$f_C = (16.90 * i) * \sin\left[\frac{2\pi}{t}\left(x + \frac{t}{3}\right)\right]$$

*Equation 21. Force of C solenoid.*

Where:

- $t = 2 * b$  (the polar step of the secondary teathed element)
- $b$  = the width between the similar tooth and spans of the secondary element.

We can see, that the force between the stator and the rotor depends on the intensity that flows through the solenoid and the position along the axis x.

### 3.1.5. Controller.

Other important part of the motor will be the controller. It will be the part where we will introduce our objective of position of the motor, and the signal of the real position. It will produce the necessary signal that will supply the energy to our motor to achieve this position. We are going to use the model explained on [6].

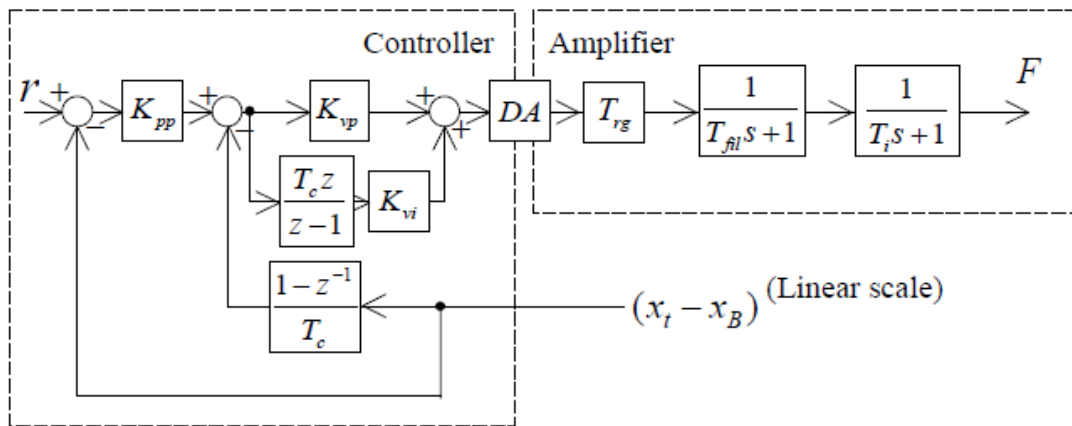


Figure 7. Controller and Amplifier.

We will use the controller part, then on our model we will have a gain that will be used as amplifier.

Where:

- r: Reference position that we want to achieve.
- $(x_t - x_b)$ : Real position from sensor.
- $K_{pp}$ : Proportional gain position loop [1/s].
- $K_{vp}$ : Proportional Gain for velocity loop. [s/m].
- $K_{vi}$ : Integral Gain of the velocity loop [1/m].
- $T_c$ : Control frequency [s].

### 3.1.6. Friction.

We will have a model which consists on a table (part of the motor which is moved), a rod (part through the table is moved) and a base plate.

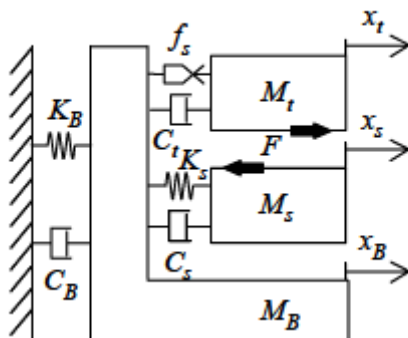


Figure 8. Friction system.

Where  $f_s$  (friction force due to the rods),  $K$  (stiffness),  $C$  (viscosity),  $M$  (mass),  $t$  (table),  $s$  (rod),  $b$  (base plate). The equations which describes the dynamic model are what follows:

$$M_t * \ddot{x}_t + C_t * (\dot{x}_t - \dot{x}_B) + f_s = F$$

Equation 22. Friction of table.

$$M_s * \ddot{x}_s + C_s * (\dot{x}_s - \dot{x}_B) + K_s * (x_s - x_b) = -F$$

Equation 23. Friction of rod.

$$M_B * \ddot{x}_B + C_B * \dot{x}_B + K_b * x_B = C_t * (\dot{x}_t - \dot{x}_B) + f_s + C_s * (\dot{x}_s - \dot{x}_B) + K_s * (x_s - x_b)$$

Equation 24. Friction of base plate.

### 3.1.6.1. Relationship between velocity and friction force:

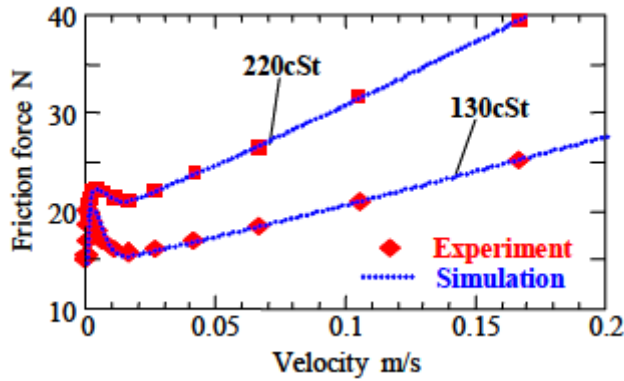


Figure 9. Relationship between velocity and friction force.

If we follow the simulation made on [6], the friction force linearly increases with the increasing velocity up a certain point. This occurs irrespective of the grease viscosity employed. The relationship between velocity and friction force may be approximated to:

$$f_v(v) = \left\{ f_s(x) + C_{t1} * v^\alpha * \exp\left(-\left(\frac{v}{v_a}\right)^\beta\right) \right\} \text{sgn}(v) + C_{t2} * v^\gamma$$

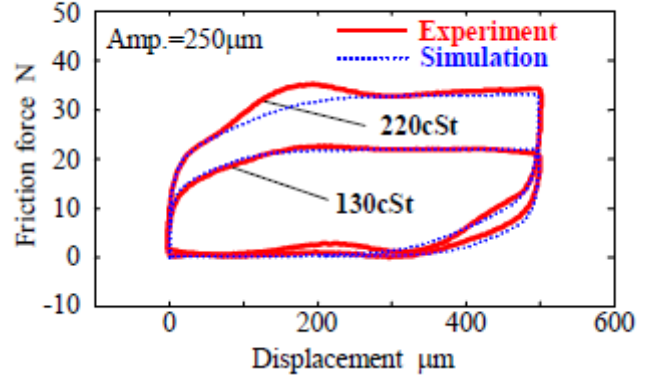
Equation 25. Approximation of relationship between velocity and friction force.

Where  $f_s(x)$  is friction force for the non-linear spring characteristic which is a function only of displacement. Because movement is in the order of millimetres,  $f_s(x)$  is equal to the constant friction force (Coloumb friction force).

### 3.1.6.2. Non-Linear spring characteristics:

Figure 10. Hysteresis loops produced by the displacement and friction force.

We can see in the figure the hysteresis loops produced by the displacement and friction force. The friction force non-linearly increases with increasing displacement. Above this displacement the friction force becomes constant. This characteristic is modelled as follows:



$$f_s(x) = K(x) * x$$

Equation 26. Friction force of displacement.

Where  $K(x)$  is a function of spring constant which expresses the non-linear behaviour and  $x$  is displacement. Therefore  $K(x)$  can be expressed as follows:

$$K(x) = \frac{K_1}{\left|\frac{x}{x_1}\right|^a + 1} + \frac{K_2}{\left|\frac{x}{x_2}\right|^b + 1} + \frac{K_3}{\left|\frac{x}{x_3}\right|^c + 1}$$

Equation 27.  $K(x)$  of friction force displacement.

This shows that the hysteresis loops can be accurately represented by adjusting the parameters. So, the equations can be rewritten as follows:

$$M_t * \ddot{x}_t + \frac{df_v(v)}{dv} * (\dot{x}_t - \dot{x}_B) + f_s(x) = F$$

Equation 28. Friction of the table adjusted.

$$M_s * \ddot{x}_s + C_s * (\dot{x}_s - \dot{x}_B) + K_s * (x_s - x_b) = -F$$

Equation 29. Friction of the rod adjusted.

$$\begin{aligned} & MB * \ddot{x}_B + CB * \dot{x}_B + Kb * x_B \\ &= \frac{df_v(v)}{dv} * (\dot{x}_t - \dot{x}_B) + f_s(x) + C_s * (\dot{x}_s - \dot{x}_B) + K_s * (x_s - x_b) \end{aligned}$$

Equation 30. Friction of the base plate adjusted.

We need to calculate the derivate of  $f_v(v)$  if we want to use the equation 25:

$$f_v(v) = \left\{ f_s(x) + C_{t1} * v^\alpha * \exp\left(-\left(\frac{v}{v_a}\right)^\beta\right) \right\} \text{sgn}(v) + C_{t2} * v^\gamma$$



We have two summands, the second is easy to calculate but the first is more difficult. First, we have to know that the derivative of the function sgn is the double of delta of Dirac. If we apply the calculation of derivatives we can achieve the final equation of:

$$df_v(v) = e^{\left(-\left(\frac{v}{v_a}\right)^\beta\right)} * (Ct1 * a * v^{(a-1)} + Ct1 * V^a) + 2g(v) * \left(f_s(x) + Ct1 * V^a * e^{\left(-\left(\frac{v}{v_a}\right)^\beta\right)}\right) + Ct2 * \gamma * v^{\gamma-1}$$

*Equation 31. Derivative of friction depending on the velocity.*

### 3.1.7. Resistance f copper.

If we use the Equation 3 explained on point 2.3.7, to calculate the maximum and minimum resistance that the copper achieves, knowing that our motor works between -10°C and -15°C, we will have:

$$R = R_{ref} [1 + \alpha(T - T_{ref})]$$

Substituting:

- $\alpha = 0.004041$ .
- $T_{ref} = 20^\circ\text{C} + 273 = 293^\circ\text{K}$ .
- $T = -10^\circ\text{C} + 273 = 263^\circ\text{K}$ .
- $R_{ref}(20^\circ\text{C}) = 1.68 * 10^{-8}\text{ohm}$ .

$$R(-10^\circ\text{C}) = 1.476 * 10^{-8} \text{ ohm (12.15\% less)}$$

$$R(-15^\circ\text{C}) = 1.4423 * 10^{-8} \text{ ohm (14.14\% less)}$$

As we see, the temperature affects a lot to our system, because due to the different resistance of the copper, the intensity will change and with that, all our system.

We are going to do the calculations with the resistance of our motor. If they are 32.7 ohms of each solenoid. If  $R_{ref}(20^\circ\text{C}) = 32.7 \text{ ohm}$ .

$$R(-10^\circ\text{C}) = 28.735 \text{ ohm}$$

$$R(-15^\circ\text{C}) = 28.075 \text{ ohm}$$

## 3.2: Design of Modelica Model.

### 3.2.1. Power supply.

There are many ways to supply the necessary power to our motor, we can supply voltage or current. We will do it both ways, if we want to control it better, it will be more useful if we use supply current, but if we want to learn how the motor works when the temperatures changes, the current will be different, and if we use current pulse will be the same. So, in order to calculate it better we will use supply voltage.

but as the generated force is proportional to the current, to control it better, the current will be more useful than the voltage. In [5] we can see one solution, it is called six-step commutation. It is the powering of the three-phases of the motor stator with three different waveforms, each 120 degrees out of phase with the others. We will have three different states, “fully positive”, “fully negative” and zero.

Phase A goes positive, zero, negative, negative, zero then positive.

Phase B goes negative, negative, zero, positive, positive, zero.

Phase C goes zero, positive, positive, zero, negative, negative.

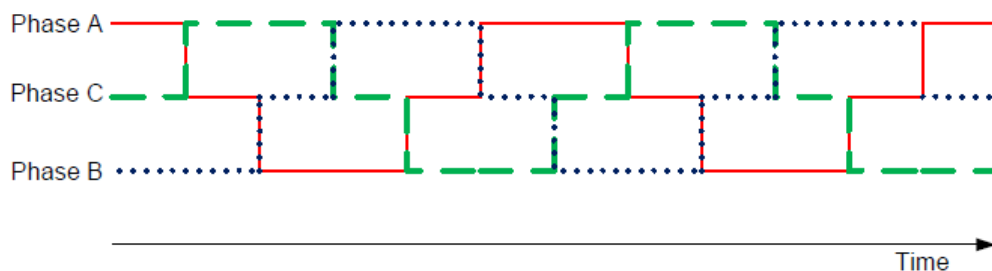


Figure 11. Wave of three phases.

If we want to represent it, we will need two models of pulse current or pulse voltage for each solenoid. One of them will represent the positive part of the wave and the other the negative part. It will have the same characteristics, but each wave of the solenoid will start at a different time, it will be 120 degrees out of phase with each other.

Phase A:

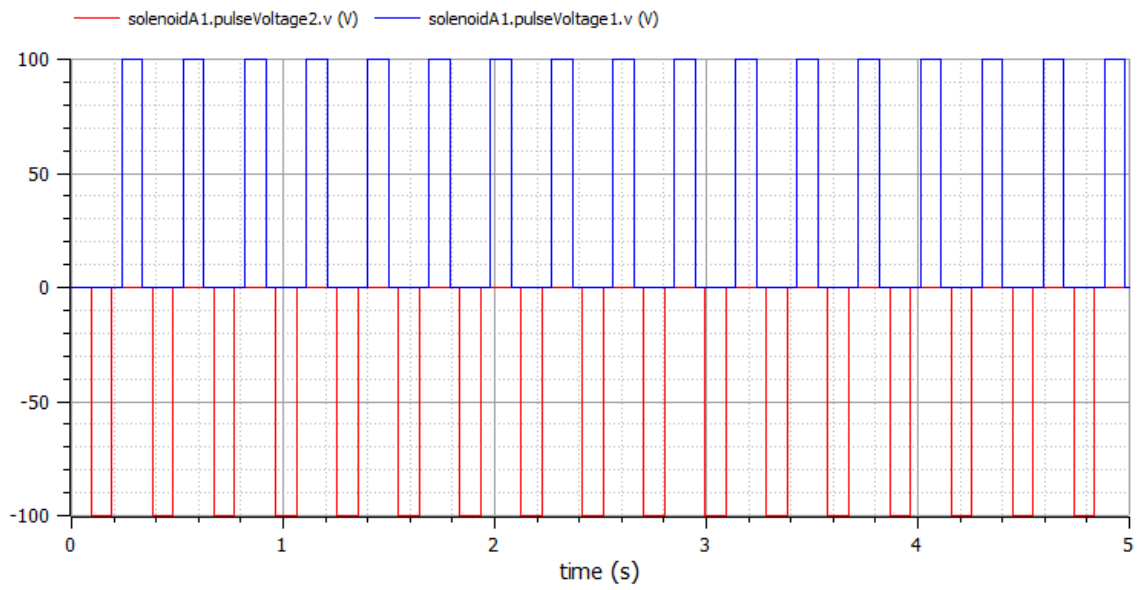


Figure 12. Wave of phase A with supply voltage.

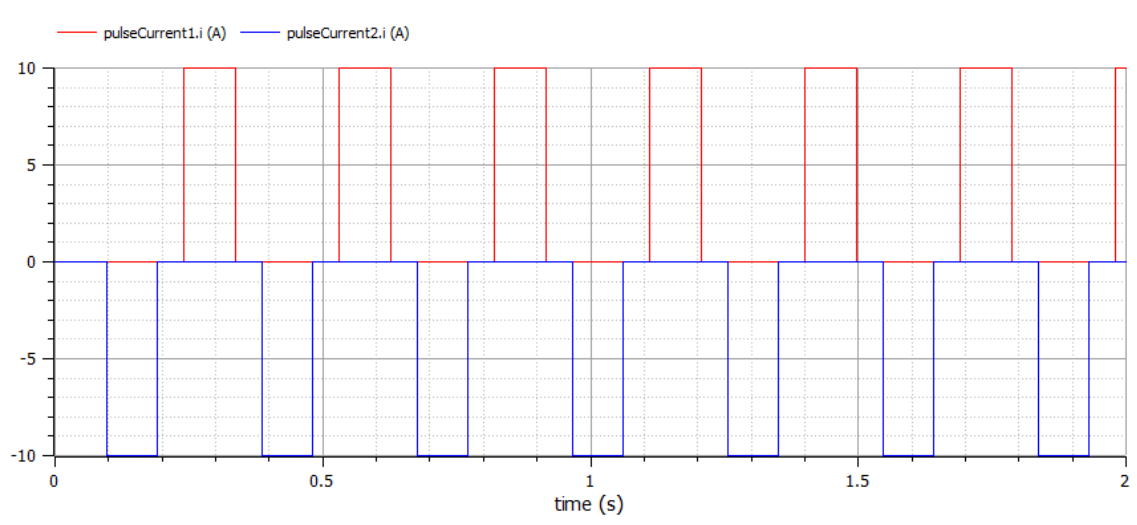


Figure 13. Wave of phase A with supply current.

Phase B:

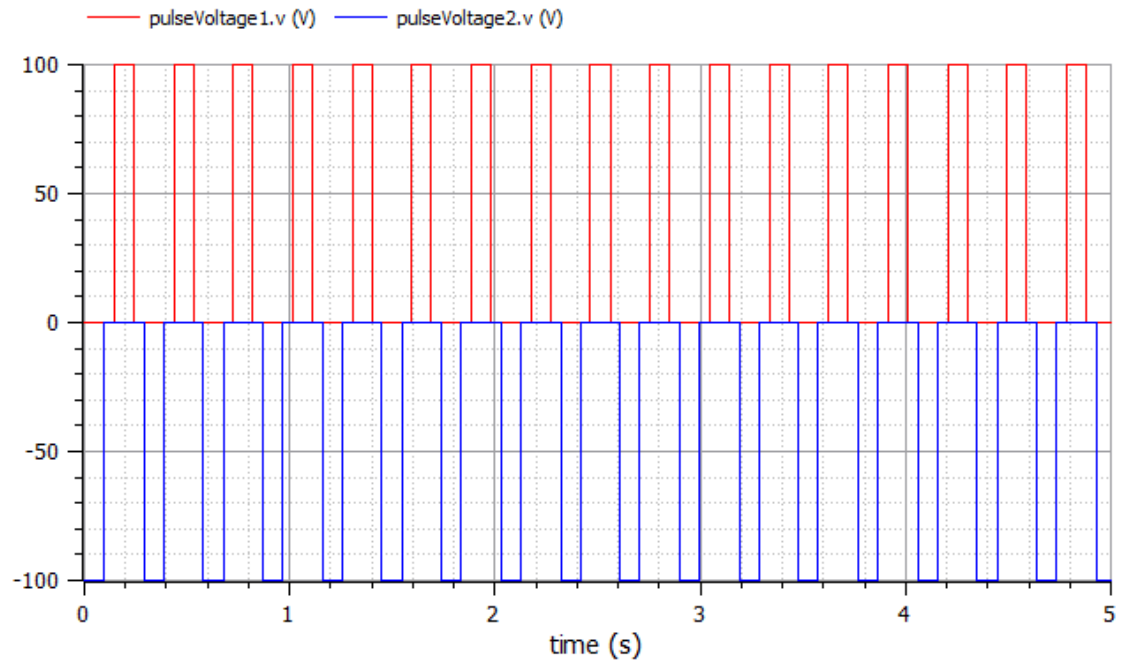


Figure 14. Wave of phase B with supply voltage.

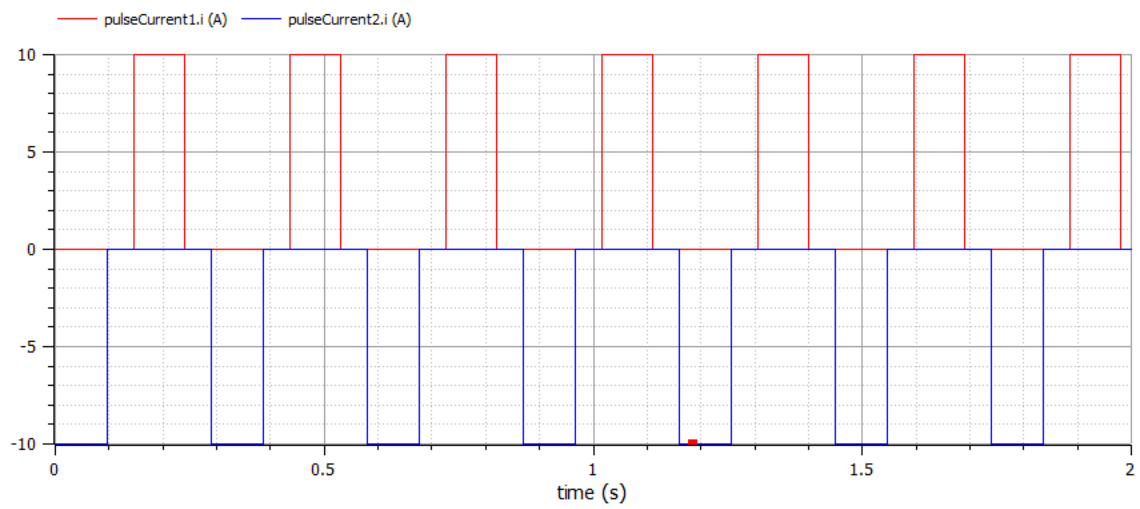


Figure 15. Wave of phase A with supply current.

Phase C:

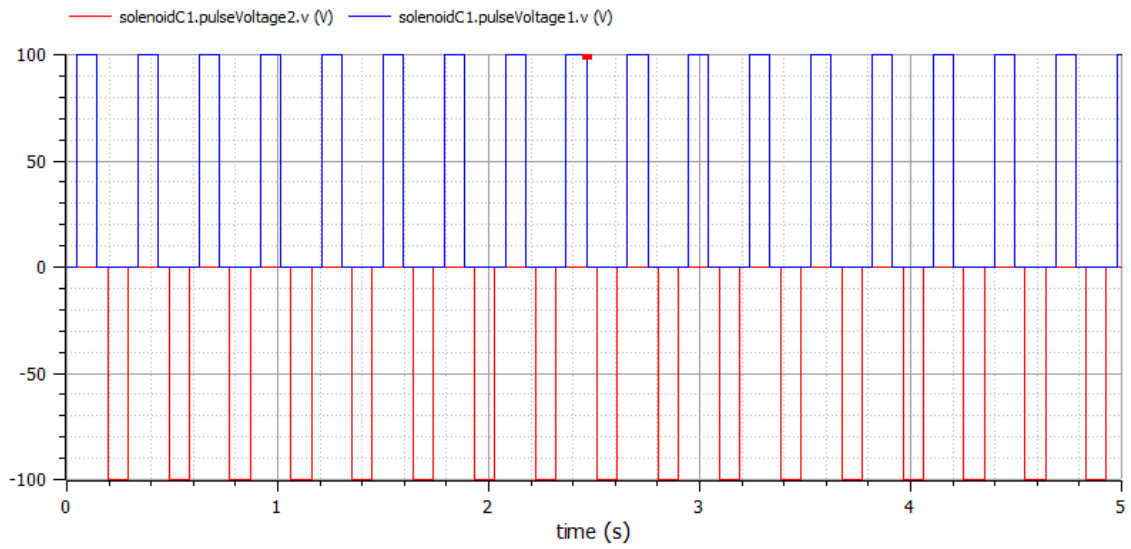


Figure 16. Wave of phase C with supply voltage.

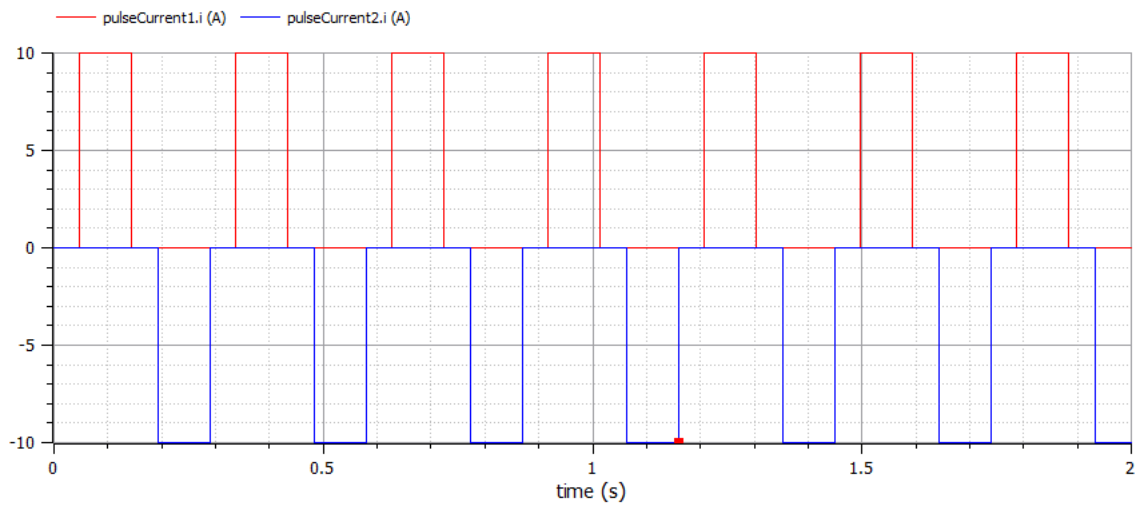


Figure 17. Wave of phase A with supply current.

### 3.2.2. Motor

#### 3.2.2.1. Example of a simply linear motor.

First, we are going to build an example of a linear motor, but this model will be simpler. We only want to build some model, without taking in mind the values of the magnitudes, which produce electromagnetic force and moves a mass.

This class will be designed by one constant voltage of 100V, one resistor of  $32.7\Omega$ , one mass of 2.4 kg, one inductor which will be the representation of the solenoid of 30H and one translational EMF with transformation constant of 5 N/A. We took the values of the resistance, voltage and inductor of [3].

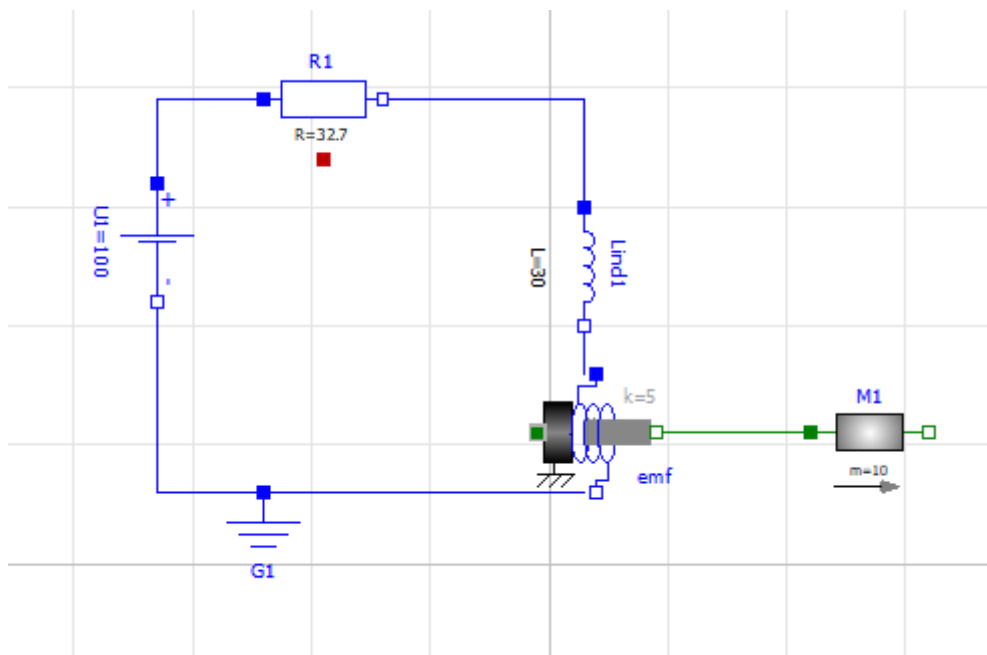


Figure 18. Example of simply linear motor.

If we simulate our model during a few seconds we are able to see, how the force is produced. The force (green line) is not constant due to the intensity of the circuit neither is, so this force will produce an acceleration during these seconds, which will be translated to the mass to achieve a constant velocity. Here, we are not taking in consideration any external force that is opposed to ours, so the mass will be moving indefinitely.

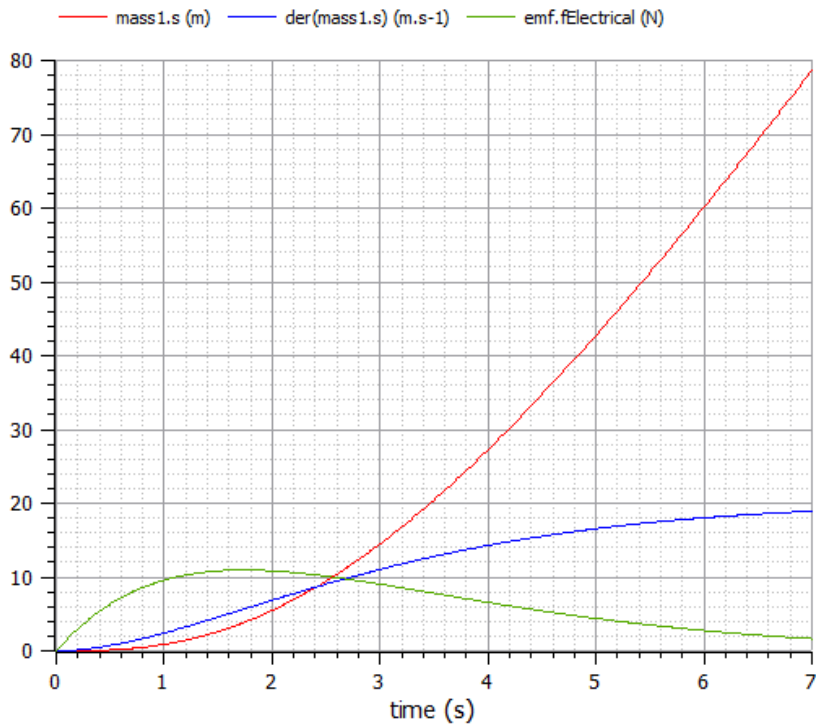


Figure 19. Simulation of example of simply linear motor.

If we try to build a more realistic linear motor, we can introduce the model of pulse voltage in exchange of the constant voltage with the characteristics explained on 3.2.1. Also, we can introduce the equation of the inductor, and the force generated along the axis x.

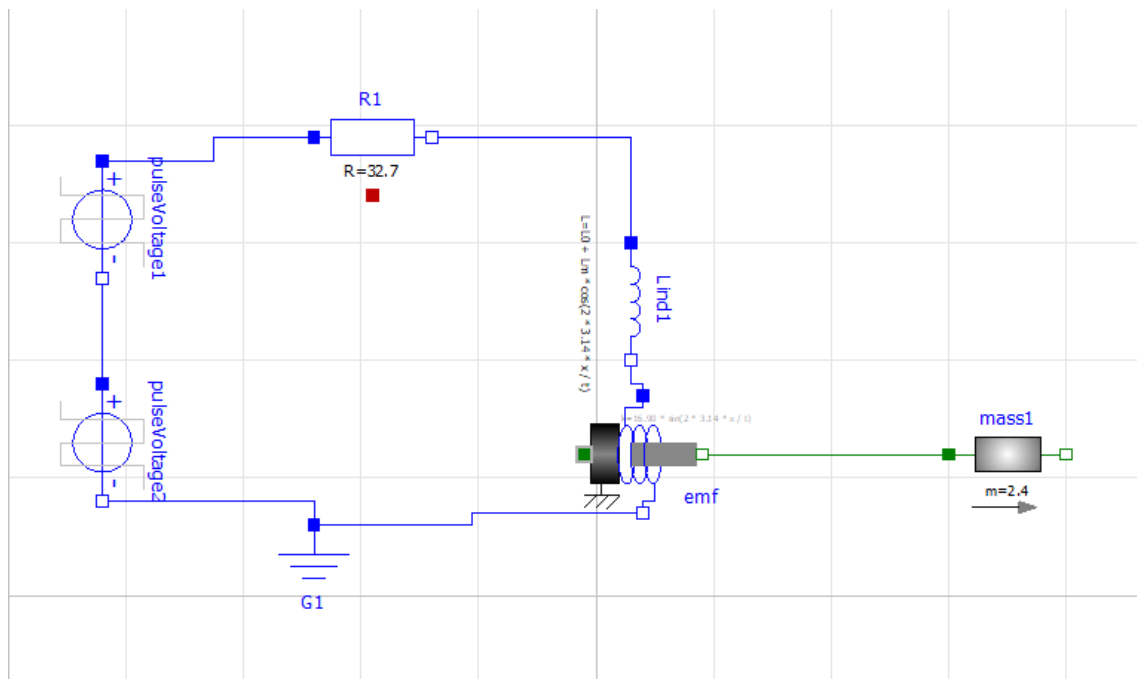


Figure 20. Example of simply linear motor with supply of pulse voltage.

### 3.2.2.2. Stator

The stator of the motor is formed by permanent magnets alternated between each tooth, that we have calculated in the previous points. So, in OpenModelica, we will have the value of the magnetic field created by the magnets along the axis x, positive or negative, depending on the teeth. This value appears on the constant of the model of the translational emf.

If we want to introduce the positive value of the permanent magnet and the negative value, we can duplicate our circuit. We will have the positive value of the magnet on the constant of the electromechanical force of the positive pulse current and the negative value on the other side. Then, when each pulse work, will produce an acceleration on the same direction directly related to the value of the pulse, so it will be easier to control it.

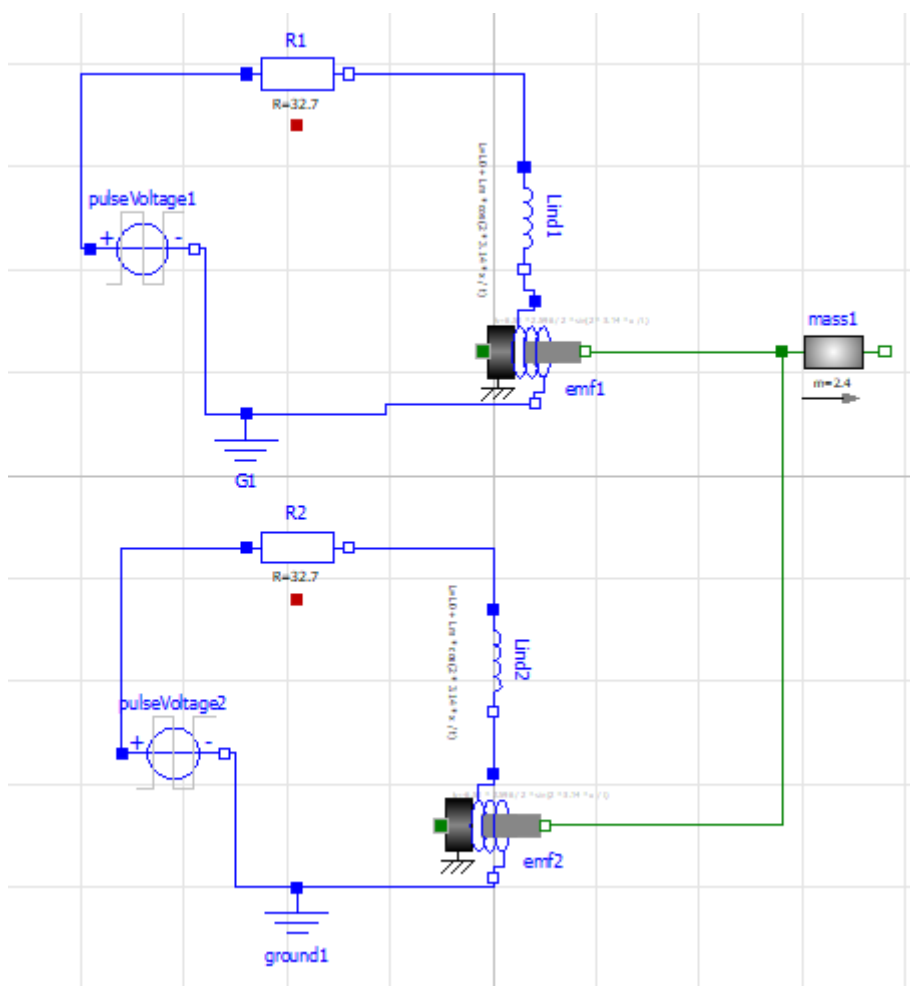


Figure 21. Stator with pulse divided.

If we simulate our model we will see how the pulse generate the force on the mass, and how the acceleration and velocity change.



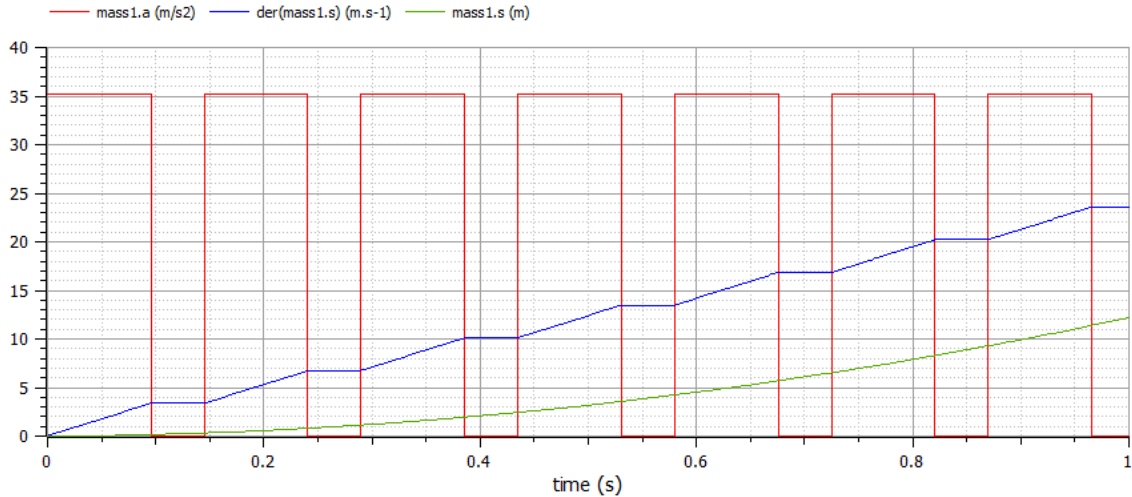


Figure 22. Simulation of the model with the stator divided.

### 3.2.2.3. Rotor.

The representation of the rotor will be combined the 3 solenoids. There will be 3 equal circuits isolated representing each coil. As we know the solenoid can be represented by a R-L circuit. So, we will have the signal voltage which transforms the input signal into a voltage, a resistor which represent the resistance of the circuit, one inductor which represent the coil of the solenoid and one translational emf, which will act as the transformation of the electrical current into a translational movement which will be applied to a mass, acting as the mass of the motor which must be moved.

Each solenoid will produce a force, these three forces will be joined, and the resultant strength will be applied to the mass. If we create the three models of each solenoid with the different pulses and we join the forces produced, we will have:

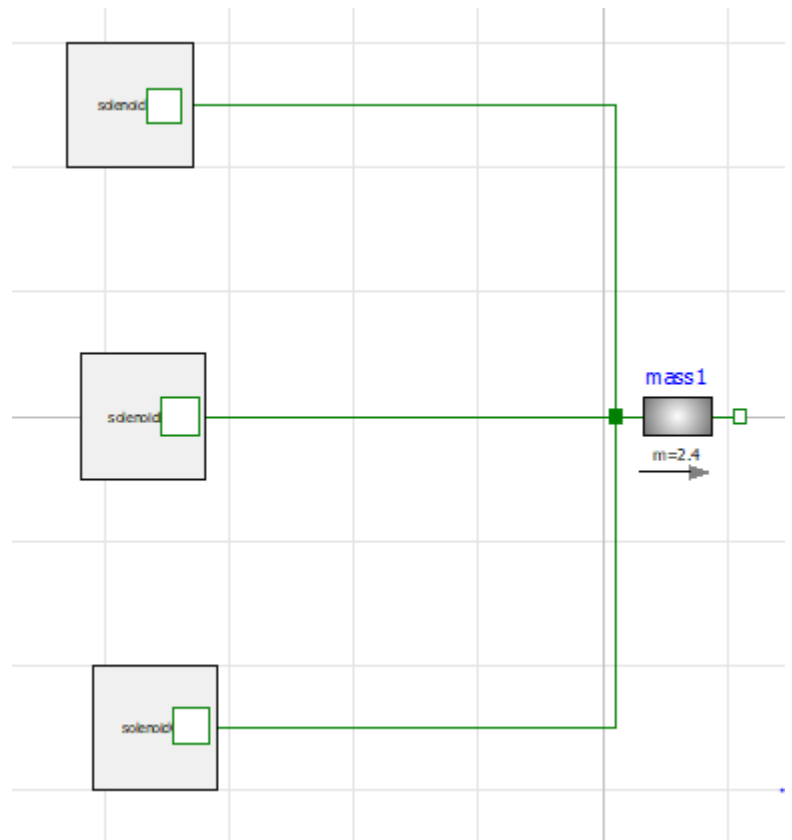


Figure 23. Construction of rotor.

The value of the inductor will come from the state of art, Equation 11.

$$L(x) = L_0 + L_m * \cos \frac{2\pi}{t} x$$

On the other part, we know that the inductor is not constant, it depends on a cosinus that depends of the displacement of the motor. So, with the objective to improve the accuracy of our model we are going to introduce a variable inductor which depends on the entrance signal. We will derivate another output of the displacement which comes to the solenoids. This signal will be derivate to a model which transforms and calculate the equation of the inductor.

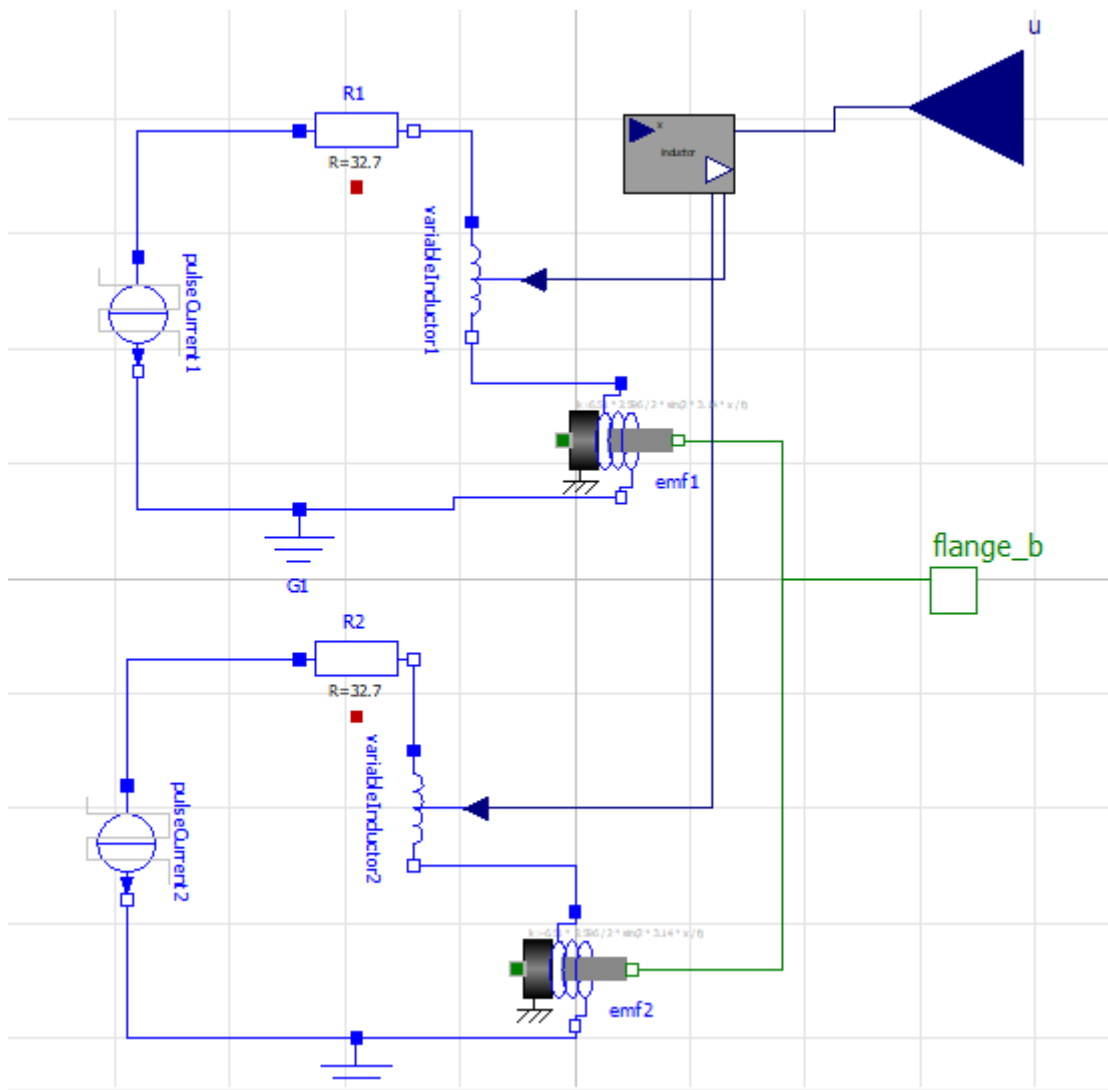


Figure 24. Solenoid with variable inductor.

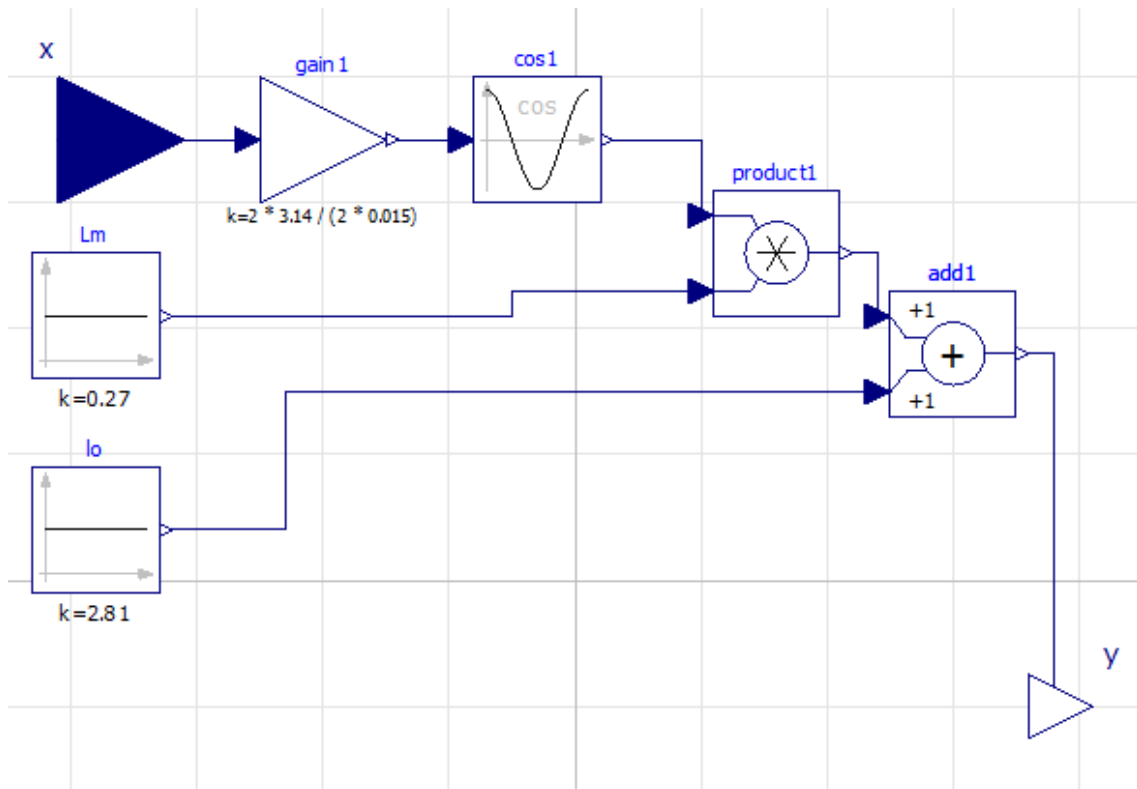


Figure 25. Model of the equation of variable inductor.

### 3.2.2.4. Friction.

#### 3.2.2.4.1. $F_s(x)$

In order to try to build the models which represents the interaction of the friction forces on the motor we will have to build the equations. First of all, we have the equation of  $f_v(v)$ , inside of it, we have  $f_s(x)$ .  $F_s(x)$ , it is a force that depends of the displacement  $x$ . So, we will have to build a sensor which take the displacement, conduct it to a model which transforms this displacement on the force produced. The model will be:

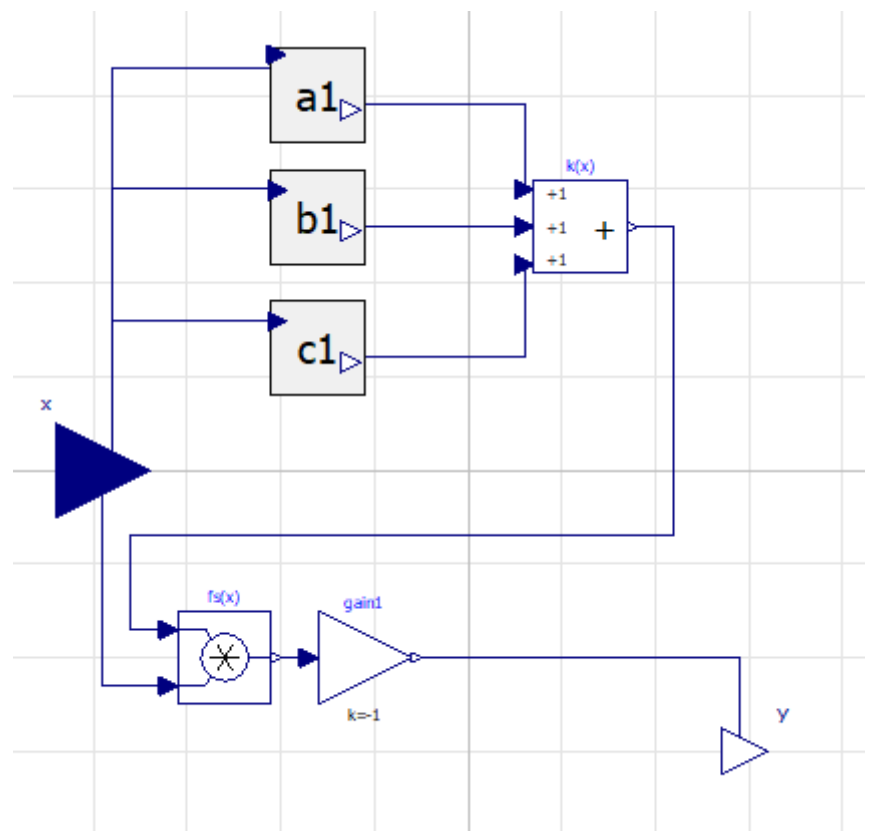


Figure 26. Model of  $f_s(x)$ .

On the equation we can see two parts, the constant  $K(x)$  that we must multiply for  $x$  to obtain  $f_s(x)$ , and the signal  $x$ . But  $K(x)$  also depends of the displacement  $x$ , so we will have the signal first conducted to calculate  $K(x)$  and then conducted to obtain the product. Finally, we have introduced a negative gain to change the direction of the force in order to be contrary to the force produced by the solenoids.

Inside of  $K(x)$  we have also three summands, each of them depending of  $x$ , with the same body but with different parameters.

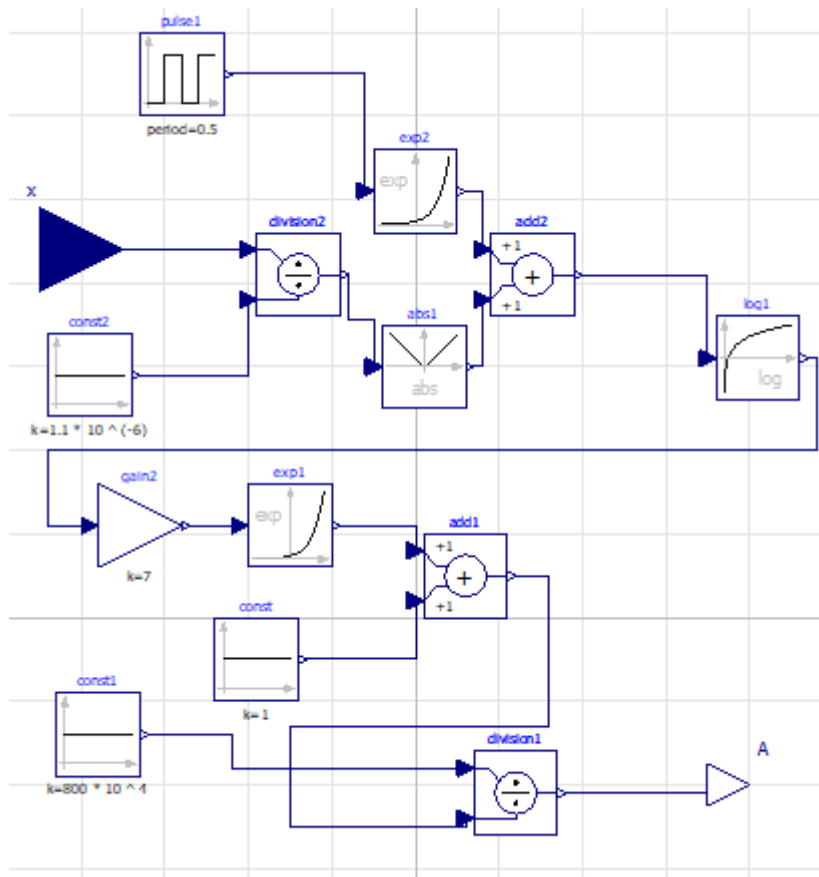


Figure 27. Model of part A of  $f_s(x)$ .

This is the representation of the first part of the equation of  $K(x)$ . More clearly:

$$\frac{K_1}{\left| \frac{x}{x_1} \right|^a + 1}$$

Equation 32. First part of equation of  $f_s(x)$ .

Where “ $k_1$ ” is divided by the sum of “1” plus the positive value of “ $x$ ” divided by “ $x_1$ ” elevated to “ $a$ ”. The values of  $K_1$ ,  $K_2$ ,  $K_3$ ,  $x_1$ ,  $x_2$ ,  $x_3$ ,  $a$ ,  $b$ ,  $c$  are obtained from 220 cSt of the paper [6]. To build the number elevated to  $a$ , we have had to use the logarithms. When we model it, we will find a problem at the initialization due to the class “log 1”, because as we know, the log (0) does not exist so we need to introduce a

pulse of 1 which works a few thousandths after the initialization. So, first the pulse will be 0, and the  $\exp(0) = 1$ , and then will be 0.

Parameter	Unit	Value	
		130 cSt	220 cSt
$x_1$	m	$0.75 \times 10^{-6}$	$1.1 \times 10^{-6}$
$x_2$	m	$8.0 \times 10^{-6}$	$8.0 \times 10^{-6}$
$x_3$	m	$120 \times 10^{-6}$	$180 \times 10^{-6}$
$K_1$	N/m	$680 \times 10^4$	$800 \times 10^4$
$K_2$	N/m	$70 \times 10^4$	$100 \times 10^4$
$K_3$	N/m	$8.0 \times 10^4$	$6.0 \times 10^4$
$a$	—	2.5	7
$b$	—	2	2
$c$	—	4	6
$C_{t1}$	N/m	5.2	9
$C_{t2}$	N/m	63	145
$v_a$	m/s	$1.3 \times 10^{-2}$	$8.0 \times 10^{-3}$
$\alpha$	—	$3.0 \times 10^{-2}$	$2.0 \times 10^{-1}$
$\beta$	—	2	2
$\gamma$	—	1.1	1.1

Figure 28. Parameters of friction of paper [6].

#### 3.2.2.4.1. Fv (v).

Then, we want to build the equation that shows Fv (v), which it will have fs (x) inside.

$$f_v(v) = \left\{ f_s(x) + C_{t1} * v^\alpha * \exp\left(-\left(\frac{v}{v_a}\right)^\beta\right) \right\} sgn(v) + C_{t2} * v^\gamma$$

We are going to follow the same system that we use to build fs(x). We need as a input signal the velocity and the displacement. Then we will be transforming the signal of the velocity as the equation does. Finally, we will have the force produced depending on the velocity.

As we did with fx (x), we will have as output signal the signal of fv (v) in both ways, as a real signal and as a force, in case we need it.



- Model A =  $Ct1 * V^a$

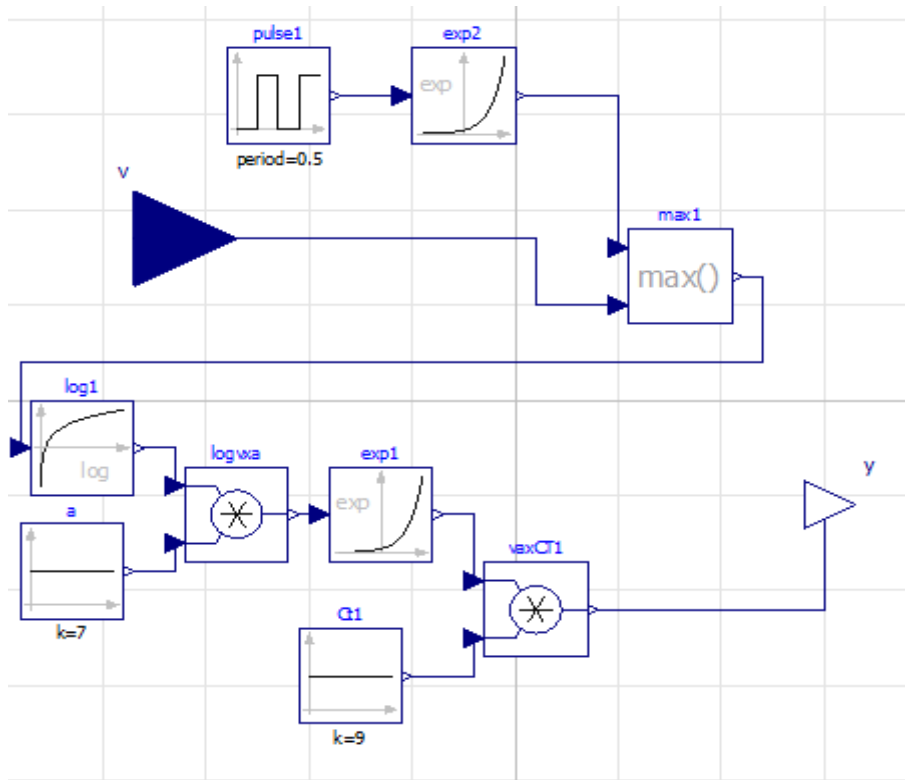


Figure 30. Model of  $Ct1 * V^a$ .

- Model B =  $e^{-(\frac{v}{v_a})^\beta}$

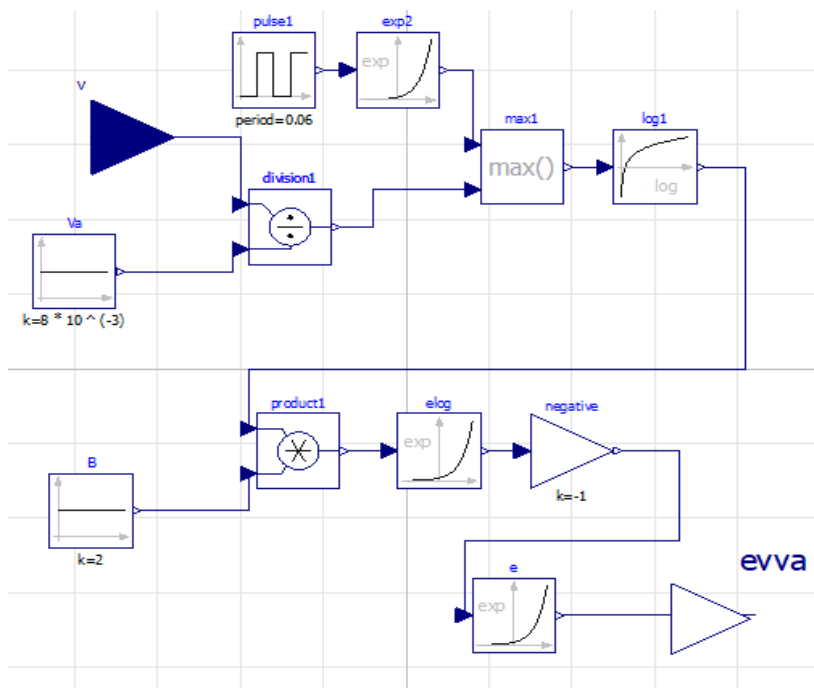


Figure 31. Model of  $e^{-(v/v_a)^\beta}$

- Model  $C = C_{t2} * \gamma * v^{\gamma-1}$

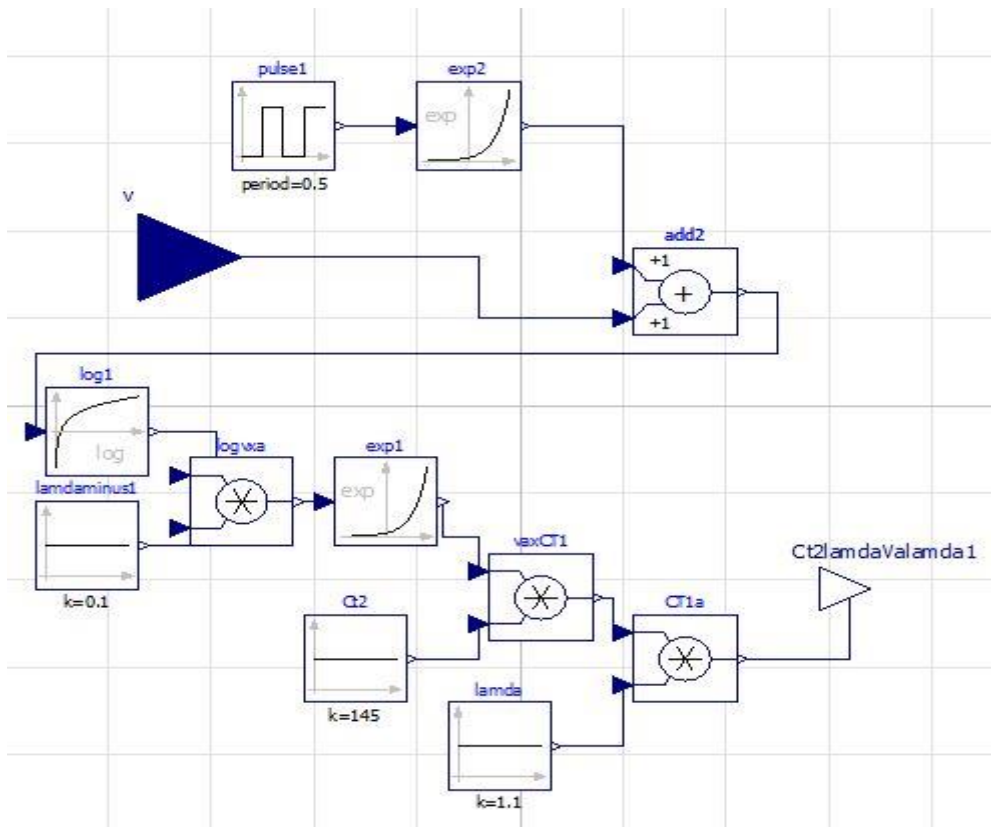


Figure 32. Model of  $C_{t2} * \gamma * v^{\gamma-1}$ .

- Model  $dA = Ct1 * a * v^{(a-1)}$

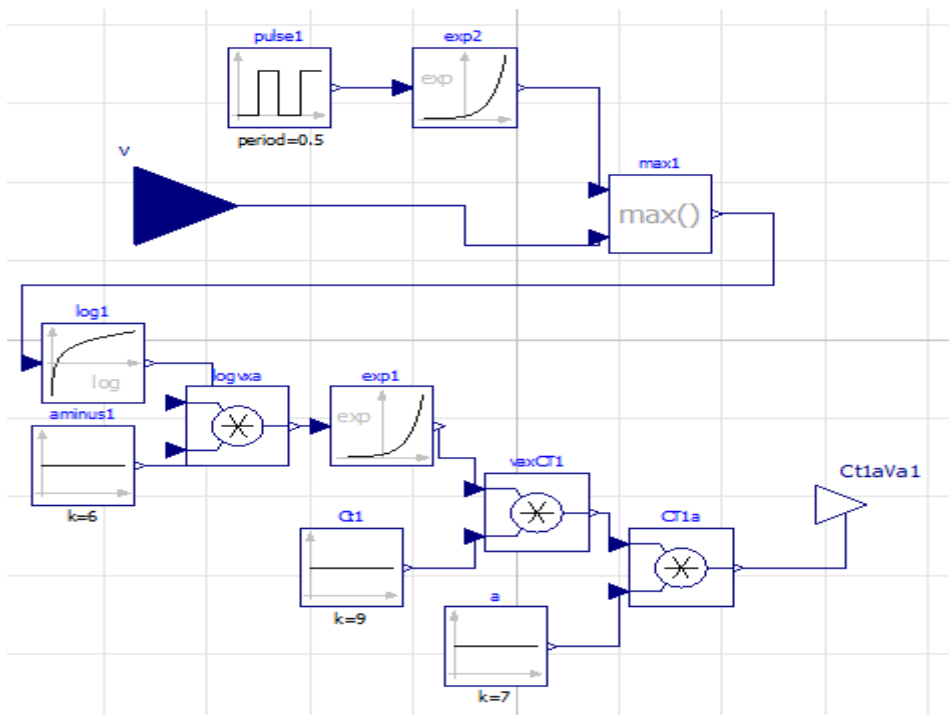


Figure 33.  $Ct1 * a * v^{(a-1)}$



- Model  $gv = 2g(v)$

Here we have, the derivative of the función  $\text{sgn}(x)$ . We know that the signum function is differentiable with derivative 0 everywhere except at 0 where it is not differentiable. So, it will be huge when the velocity is 0 and 0 when it is not.

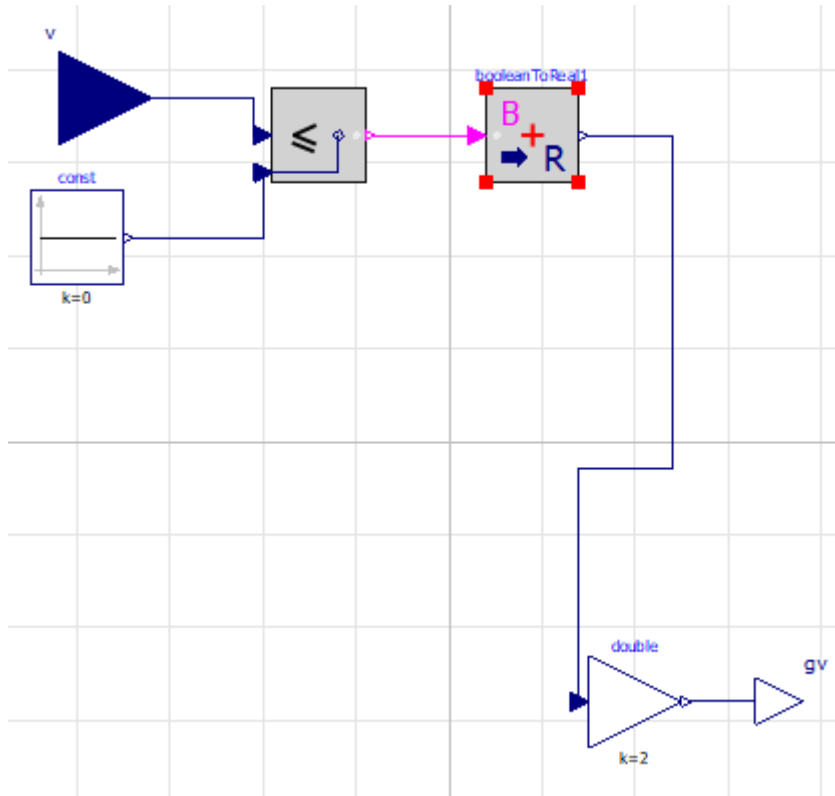


Figure 34. Model of  $2g(v)$ .

Here we have a Boolean model which will have a true signal if the velocity is equal or less to 0. Then, we will have this signal transformed to a real signal multiply to 2. So, when the velocity it's different from 0, we will have a 0 on the output, and a 5 when its 0.

In some models we have introduced a pulse and an exponential, followed as a model of max, due to the first moments of the simulation, with velocity near to 0 the logarithm cannot be calculated. So, the first seconds the pulse will produce a little signal and then, the real signal.

Finally, if we want to mix everything we will have to build this model:

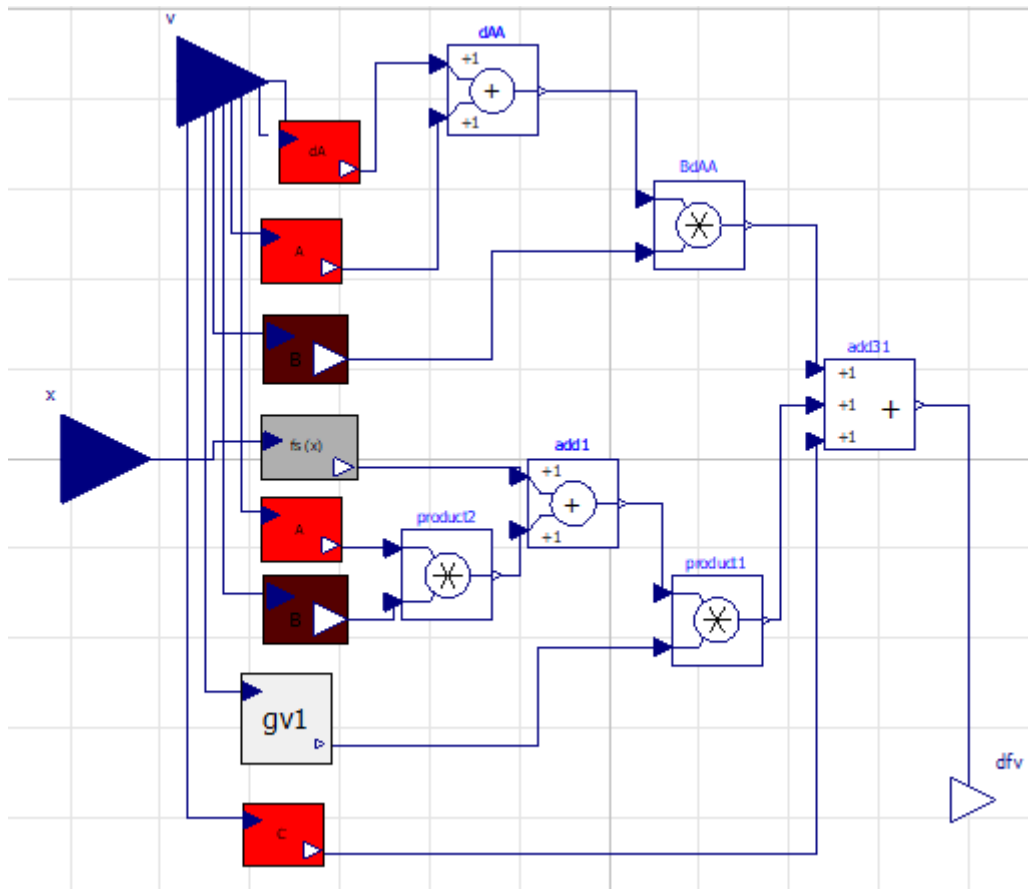


Figure 35. Model of  $dfv(v)$ .

So, we will have one model with two inputs, the distance and the velocity, by the equations we will have the result of the derivative of the force  $fv(v)$ .

Now we will see the models of friction introduced to the rotor:

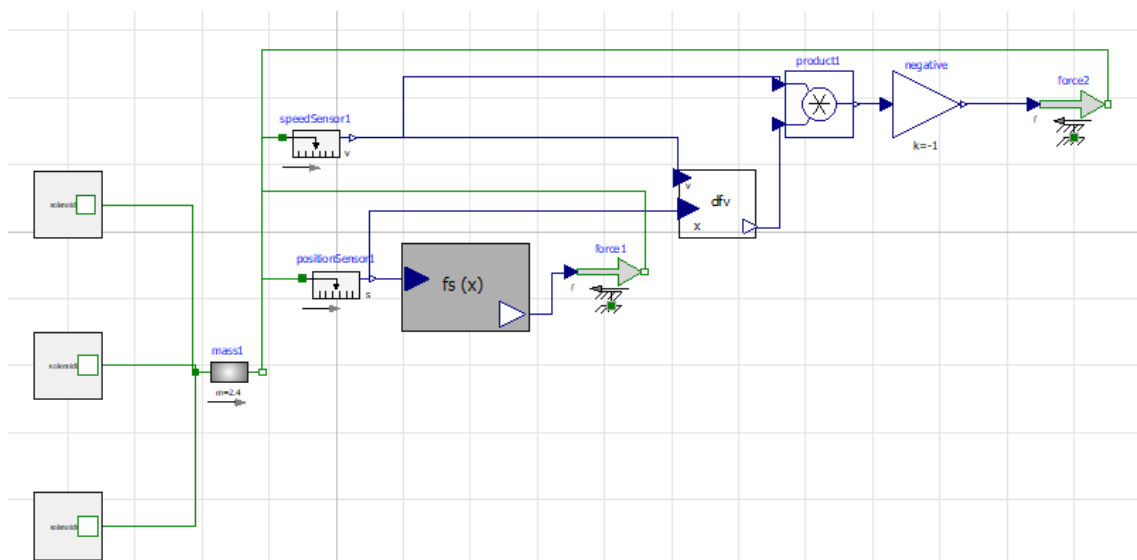


Figure 36. Model of rotor with equations of friction.

### 3.2.1.5 Controller.

If we want to transform the blocks of the controller on a model on Open Modelica we will have to proceed as follows:

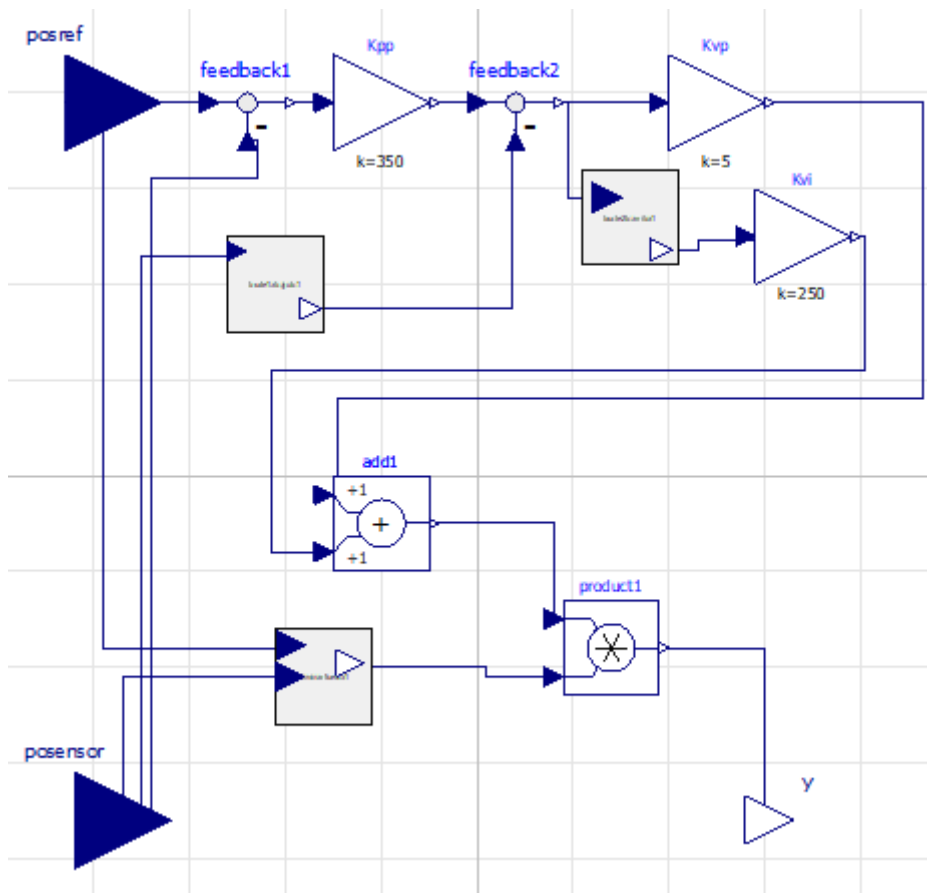


Figure 37. Model of controller.

We will have two inputs, one designated to the reference position of the motor and the other the signal coming of the position sensor, and one output, the signal which will be command the supply energy to our motor.

First, we have a feedback between the position reference and the position sensor. This signal with the proportional gain of the position loop is directed to our second feedback, which is coming from the position sensor through a model named Loop 1. Here we have, the equation and the model.

$$\frac{1-z^{-1}}{T_c}$$

Figure 38. Equation of first block of controller.

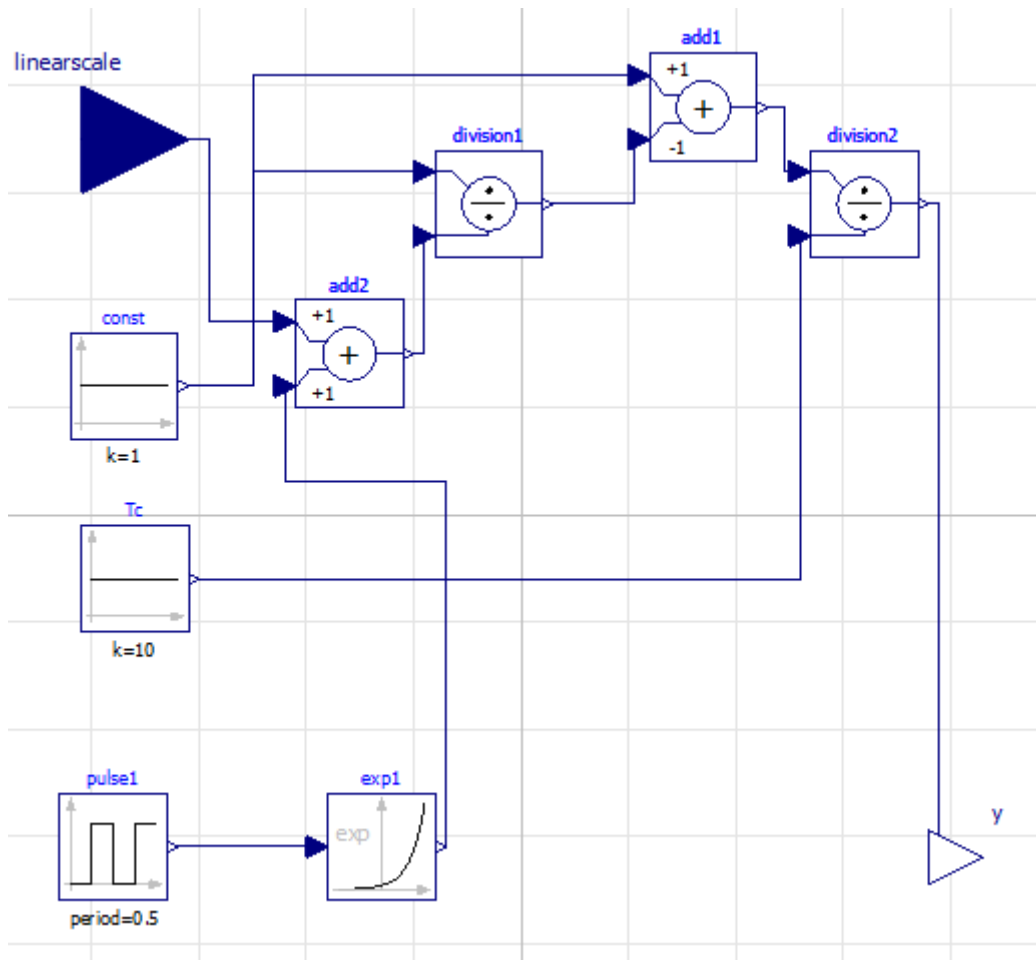


Figure 39. Model of first block of controller.

We have had to introduce the pulse, the exponential and the add2 due to a simulation mistake when the signal of the position sensor is 0 and is the divisor of the division 1. So, the pulse1 will produce a little signal a few seconds, while the signal of the sensor grows a little.

If we continue with our model we can see that from the feedback2 we have two ways, that conclude with the final add1. One is through the proportional gain of velocity loop, and the other is directed by the model called Loop 2 and the Integral gain of the velocity loop. As we did before, we will see the equation and the model of Loop 2.

$$\frac{T_c z}{z-1}$$

Figure 40. Equation of second block of controller.

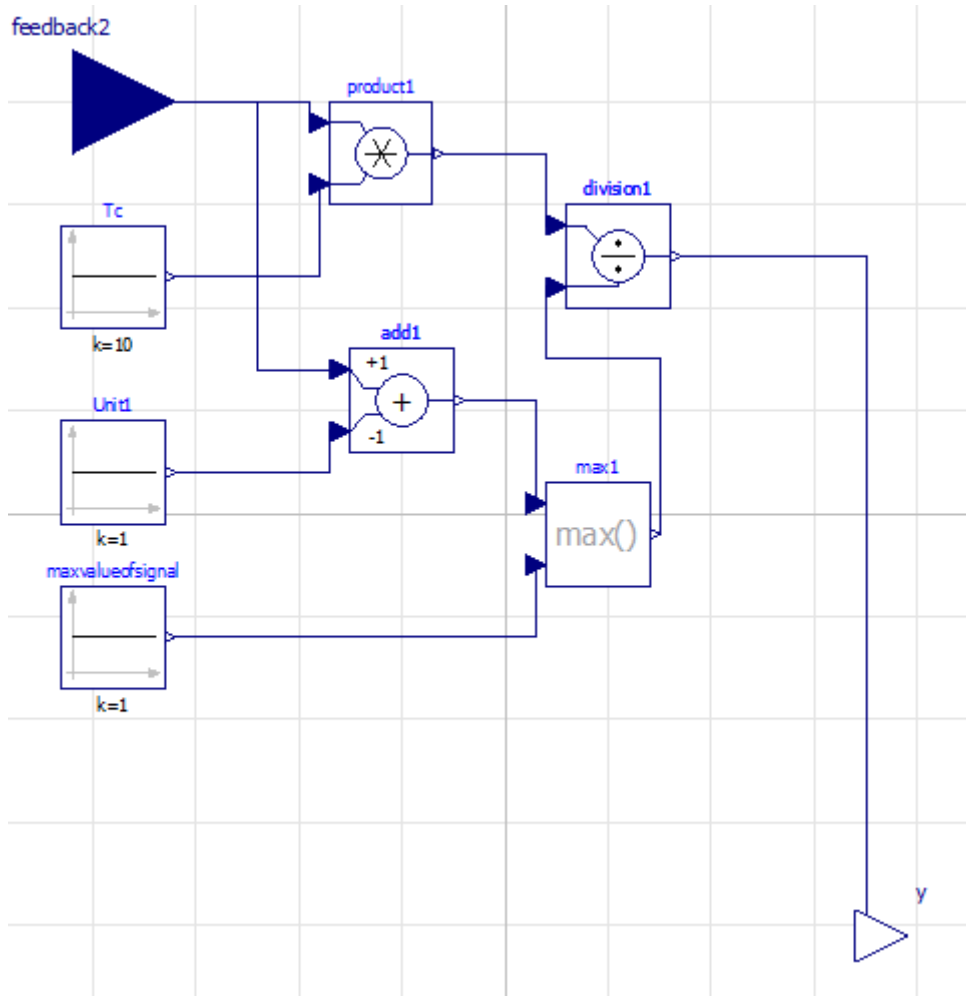


Figure 41. Model of second block of controller.

Here, it can be observed the equation, but due to some mistakes during the simulation we have added a minimum value on the divisor of the division1. When the reference position is less than 0.5 m, when the mass arrive to its destiny, appears a mistake due to the signal of the feedback is 0 so the division cannot be calculated. So, the model of max value, will have as an output signal, the normal signal, but when this signal is lower than 1, the output and the divisor will be 1, and the simulation will work correctly.

We have had to introduce another model because during the simulation appears a mistake when the position sensor is near to the reference position. We need to stop the functioning of the motor before it arrives to the destiny, but when we simulate our model we will observe that although we stop the current the inductor will need a few quantities of time to stop, so the current that it provides will create another force. This current will depend on the reference position that we want to achieve, so we must

create a model that depending on the reference position produce a signal that will represent the distance to the reference position where our motor must stop.

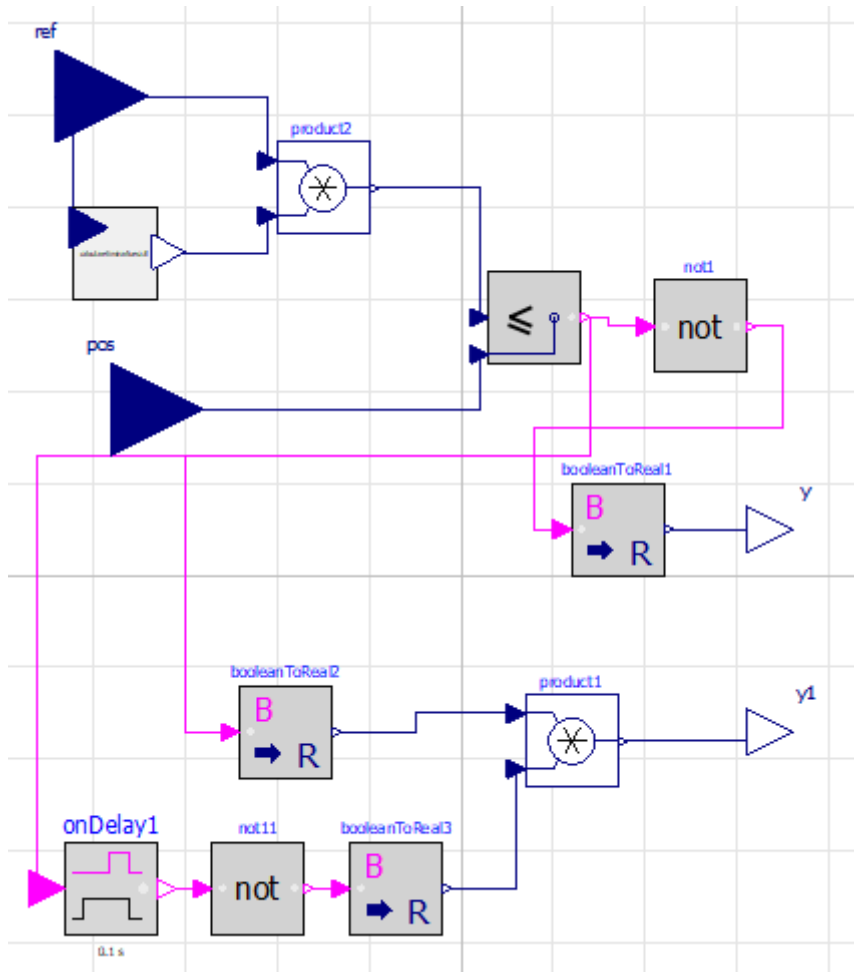


Figure 42. Model of eliminate force.

First, we have the two inputs, the reference position and the position from the sensor. We will use the reference position redirected to a model which calculate the proportional value of reference position where the motor must stop.

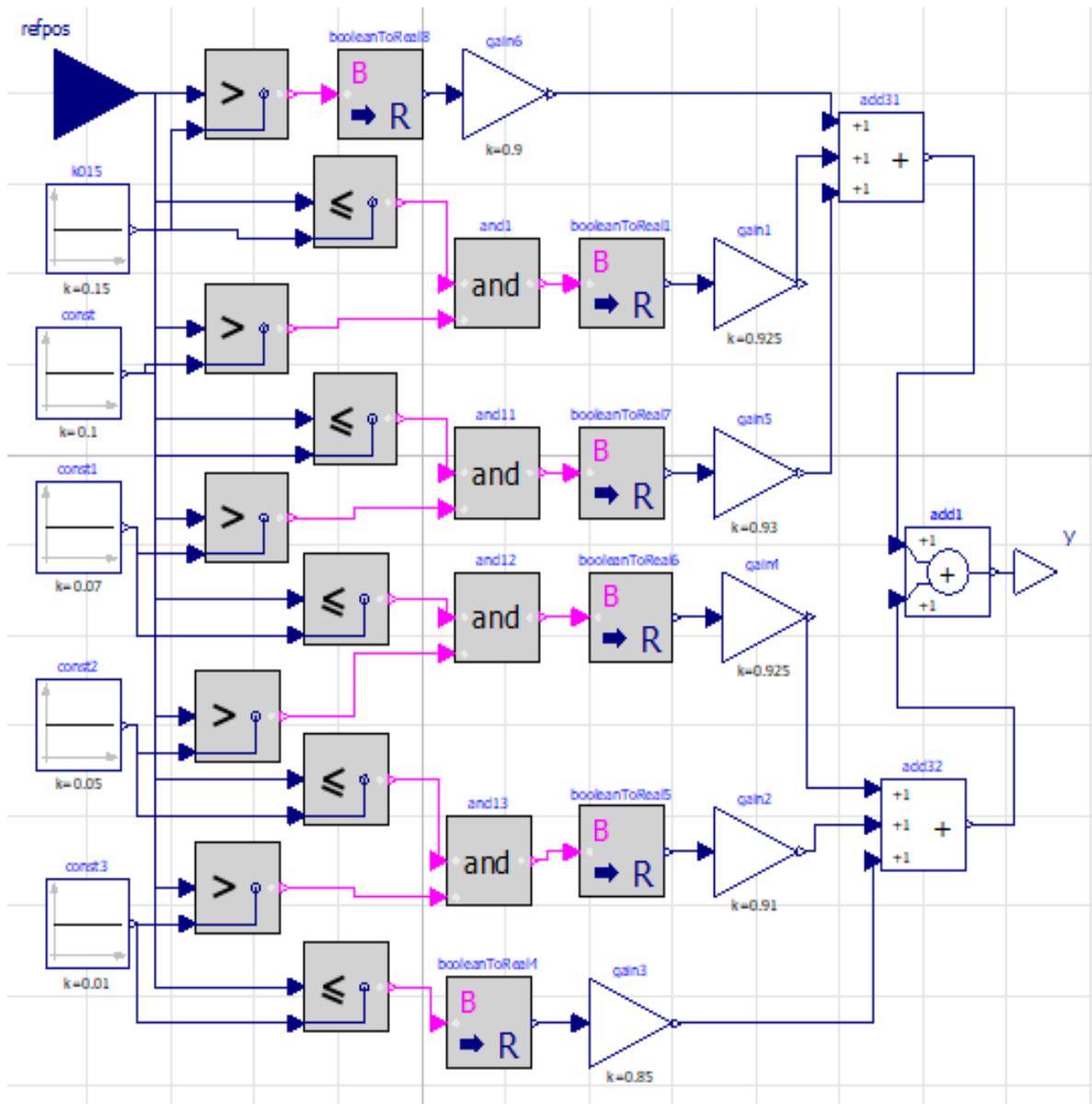


Figure 43. Model to calculate gain of reference position.

All the range of the reference position is divided into 6 steps with the models of more than, and less and equal than. Each Boolean signal will be transformed to a real signal that will be 1 if we are in the range or 0 if we are not. We will multiply each way to the value that we want on each part and all will be summed.

- Less than 0.01: 0.85.
- 0.01-0.05: 0.91.
- 0.05-0.07: 0.925.
- 0.07-0.1: 0.93.
- 0.1-0.15: 0.925.
- More than 0.15: 0.9.

Then a model which has a true output if the input 1 is less than input 2. We have negated the output and then redirected to the main controller.

Secondly, we have introduced another part to our model, besides cancel the force that moves the motor, we need another force that slow down the velocity and acceleration that we have achieved. So, we have created another way from the first Boolean signal, this signal will be positive when the real position has arrived at the reference, so we want that signal transformed to a Boolean and be summed up to the same signal delayed but negated. We have done this because, we want that the force that slow down our motor, only work a few thousandths, so the signal that will be 1 when we want that force is redirected to a model which will delay the rise of the signal, and before being delayed.

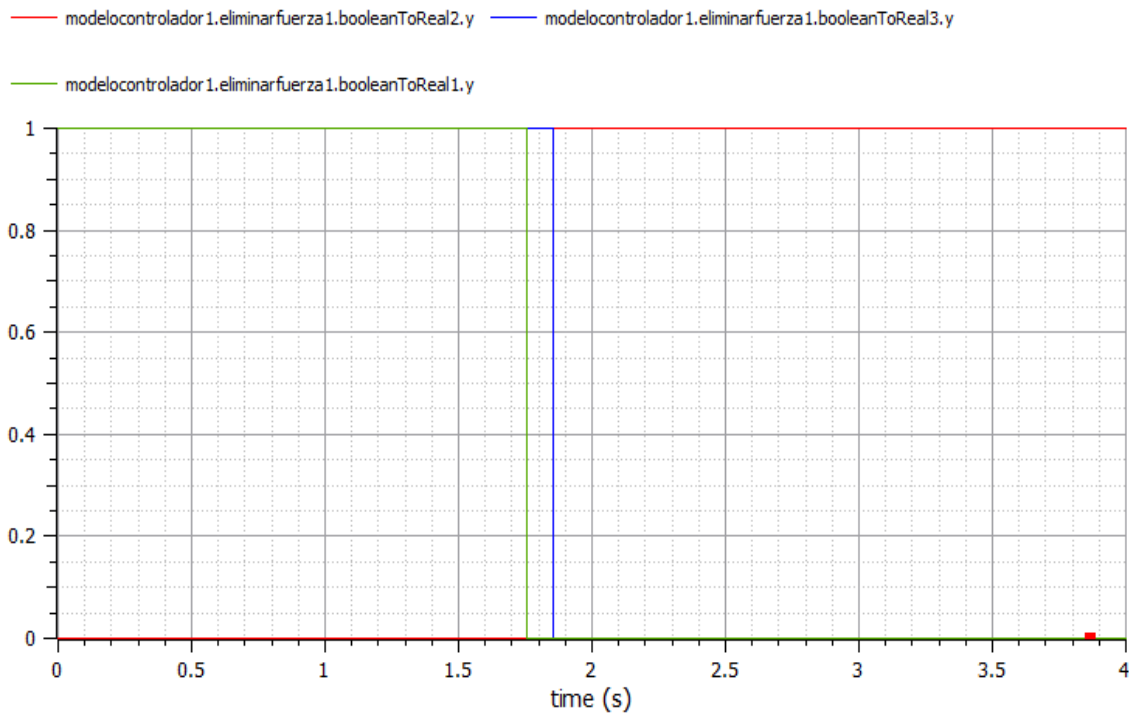


Figure 44. Simulation of the signals and the delay.

Here we have the demonstration of the model, green line represents the force that moves our motor, red line the signal that is going to slow down our motor, it will rise when the other comes down. Finally, blue line will eliminate the force of slowing down our motor.

### 3.2.1.5.1. Example of functioning.

We are going to build a simpler model with the intention to calculate if the controller is working as we need. We will have a signal constant, that represents the reference position that we want to achieve, the controller, directed to a model which transforms the input signal into a force. We have introduced a gain, that will reduce the value of the controller achieving the necessary force, it will be calculated by experimental way. This signal will be conducted to a mass1 representing the motor and a sensor which calculate the position of the mass and redirect the position of the controller closing the loop.



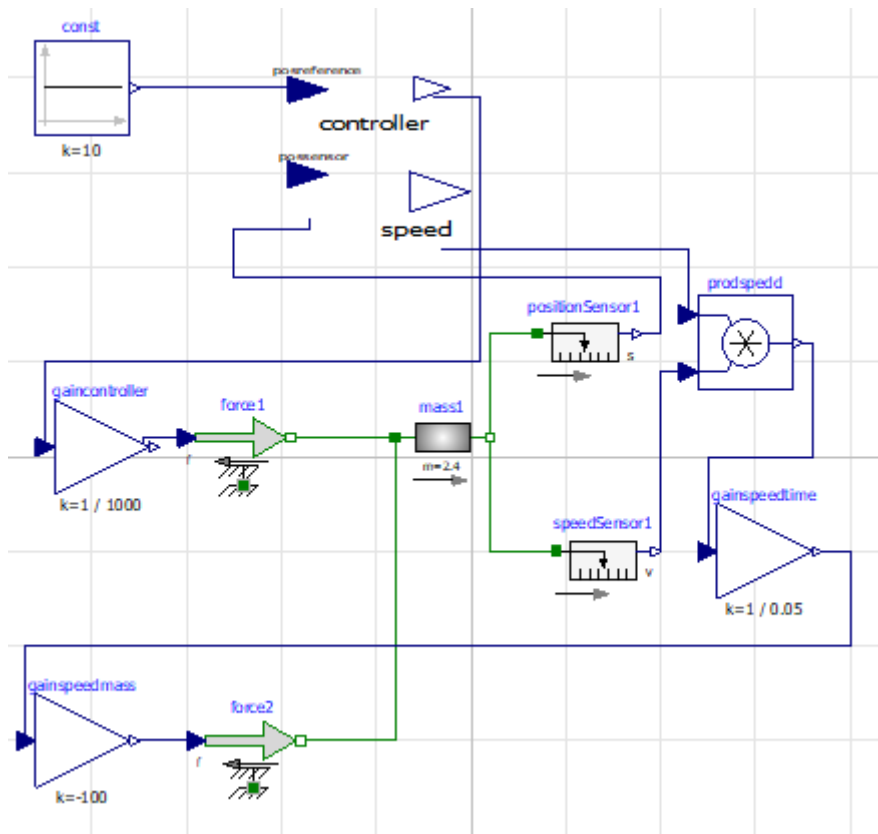


Figure 45. Example of functioning of the controller.

After the simulation we have had to introduce and extend our model, because we need to cancel the energy that the mass have achieved when it arrives to its position. So we will take the velocity of the mass, and with the circuit explained on the model of delating force, will be multiply for the boolean signal to be activated or deactivated. Then, we will be multily by  $1/0.05$  because we want to reduce the velocity on 0.05 seconds and then multiply for the necessary gain to achieve the position and not be exceeded.

Now we will be the results of the simulation.

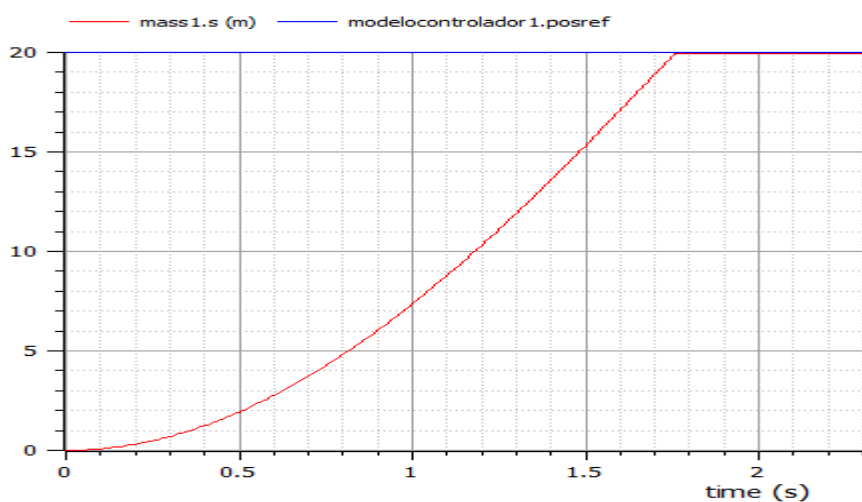


Figure 46. Simulation of the example of distance and reference position.

Here we see the difference between the reference position (blue line) and the real position of the motor (red line). As we see, it moves until the position wanted and then it is slowed down.

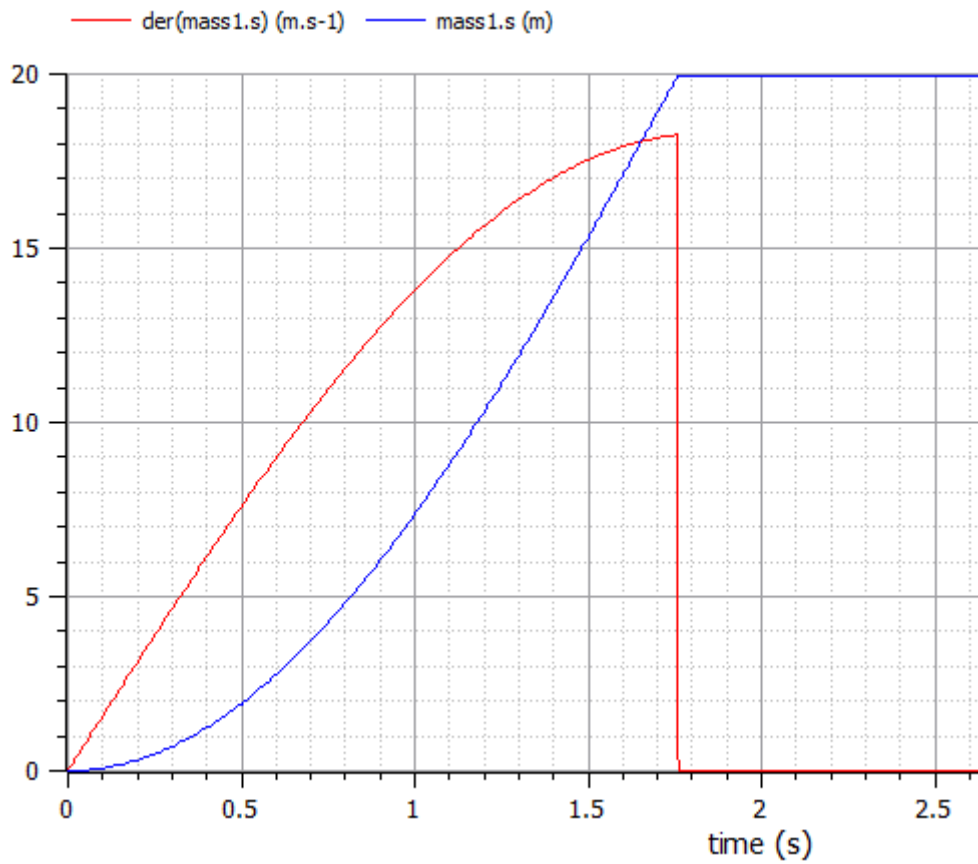


Figure 47. Simulation of example of distance and velocity.

Consequently, we see the position and the velocity. We can observe how the velocity decrease with high accuracy when we achieve our position.

### 3.2.1.5.2. Real application.

Now, we are going to apply the controller to the real model including the solenoids, and the forces produced by the friction.

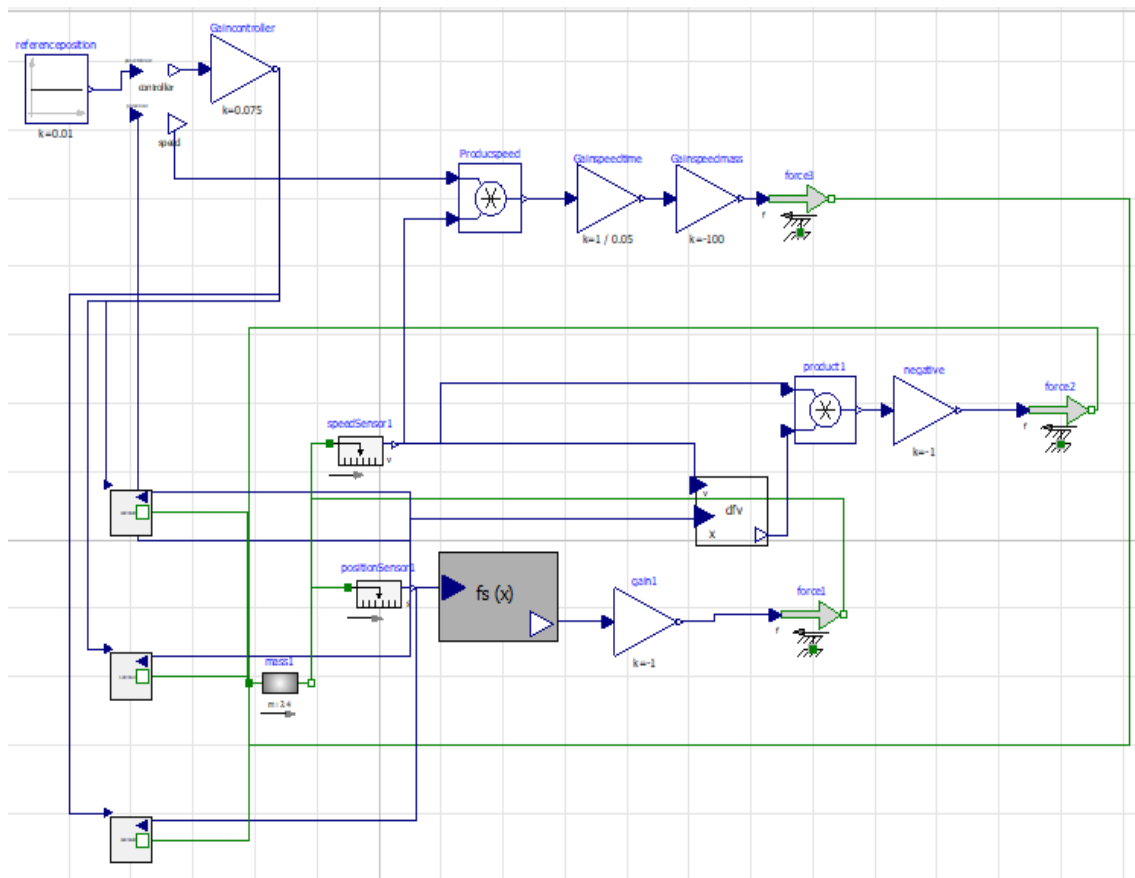


Figure 48. Model of the complete system of the motor.

We have a constant signal representing the reference position that we want to achieve with the mass. This signal will be redirected to the controller with the signal from the position sensor. As we explained before, the controller will produce the signal that is going to control the pulses that produce the current signal of the solenoids through the necessary gain controller. This gain has been calculated to arrive to all the different position with great accuracy as 0.075.

As we have a continuous signal from the controller we have design a model inside each solenoid, that will multiply the signal by two pulses, each one to each current signal. Each pulse source will produce a unitary pulse with the frequency necessary of each solenoid, so with the same frequency that is needed, it will multiply the necessary value calculated by the controller. These signals will conduct the signal current, and the signal current will produce the necessary force through the electromagnetic force.

We will have a position sensor and a speed sensor from the mass, to calculate through the models explained before the friction forces produced inside the motor. We will use the signal from the speed sensor to also calculate the necessary force to slow down the motor when it arrives at its position, by the gains of the speed, we will have the necessary force redirected to the mass.

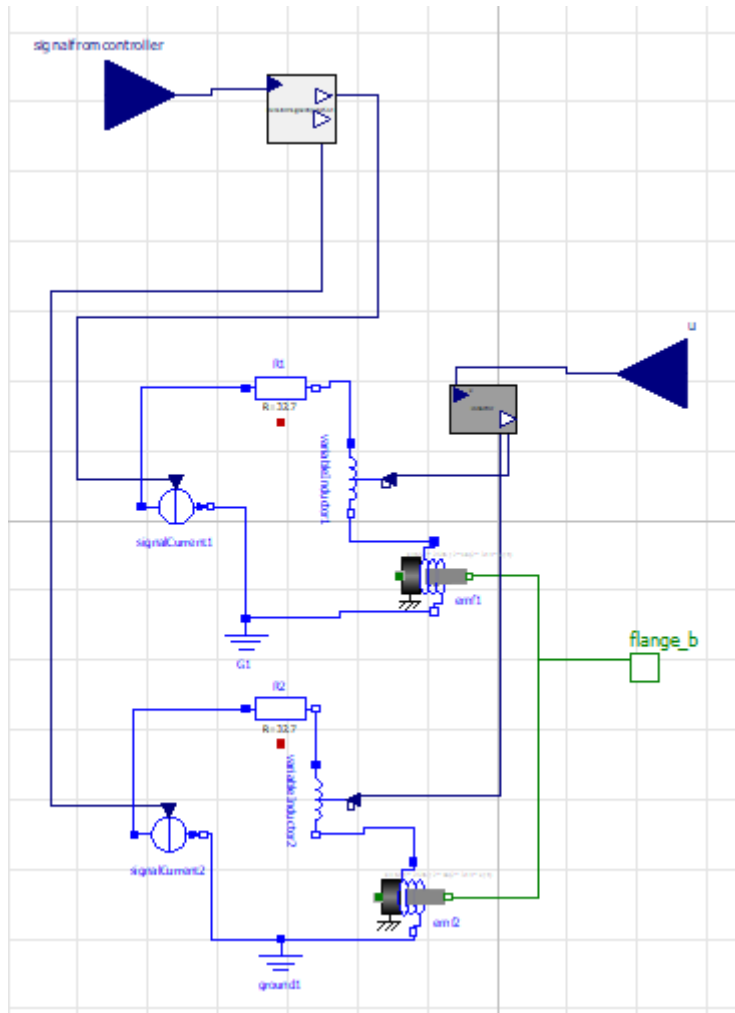


Figure 49. Model of the complete solenoid.

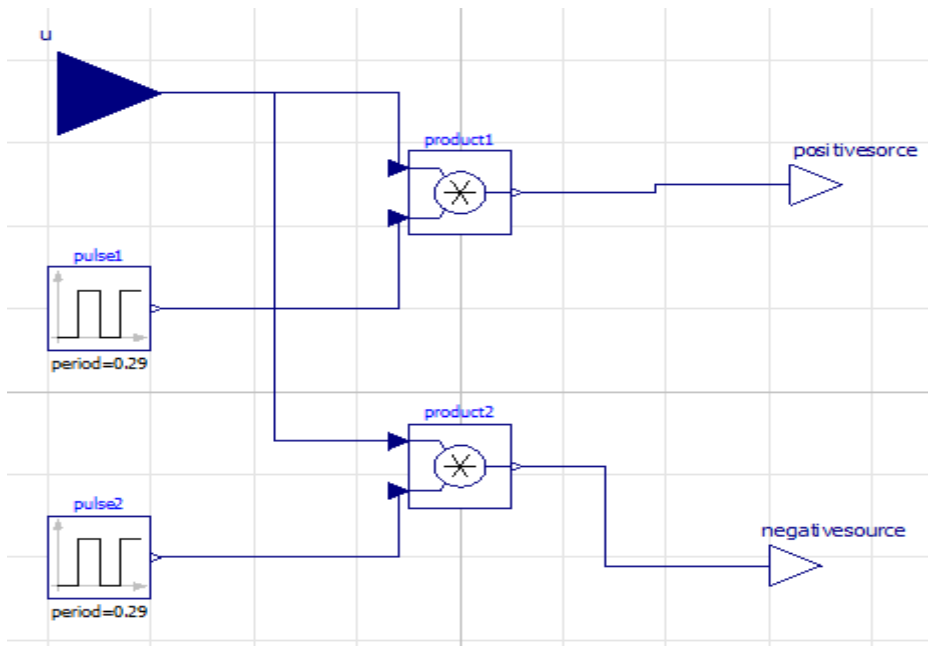


Figure 50. Model to transform signal to pulse.

The simulation will be produced to ensure the accuracy of our simulation, so we will introduce a reference position of 0.2m (the maximum length that our motor is able to achieve) and a reference position of 0.001m. All the distances between them will be validated.

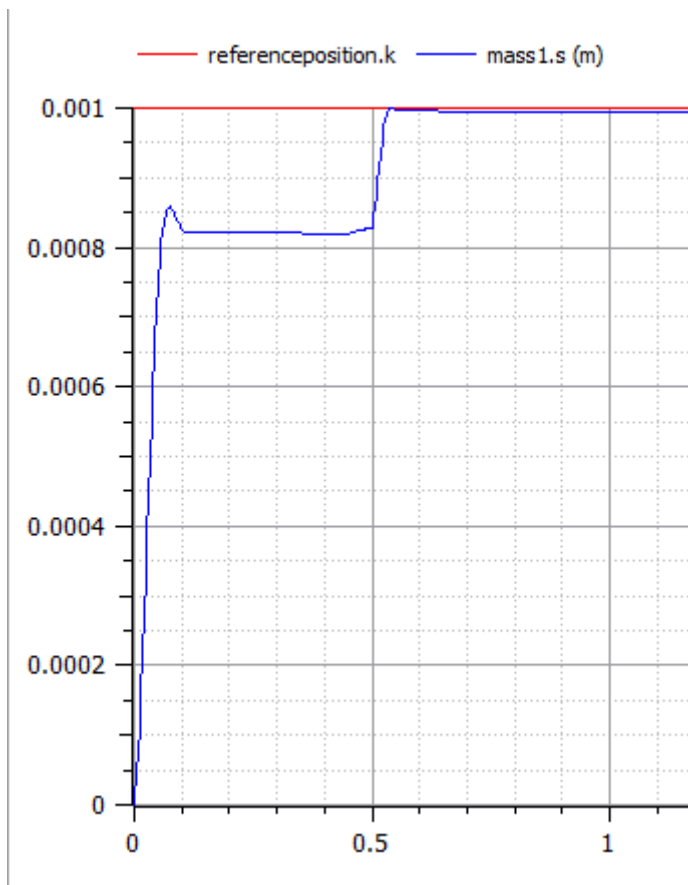


Figure 51. Distance and reference position of complete model.

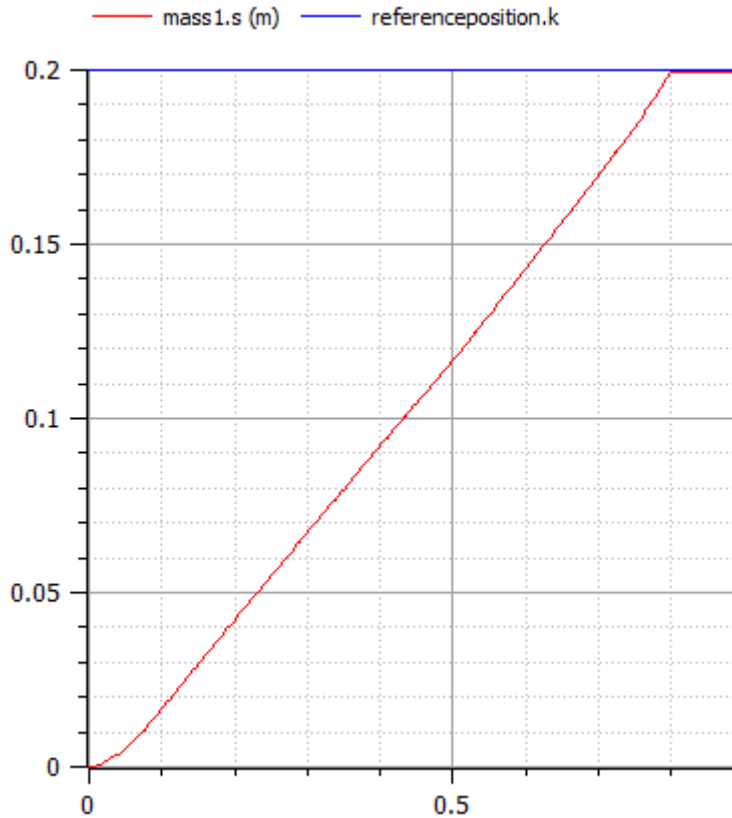
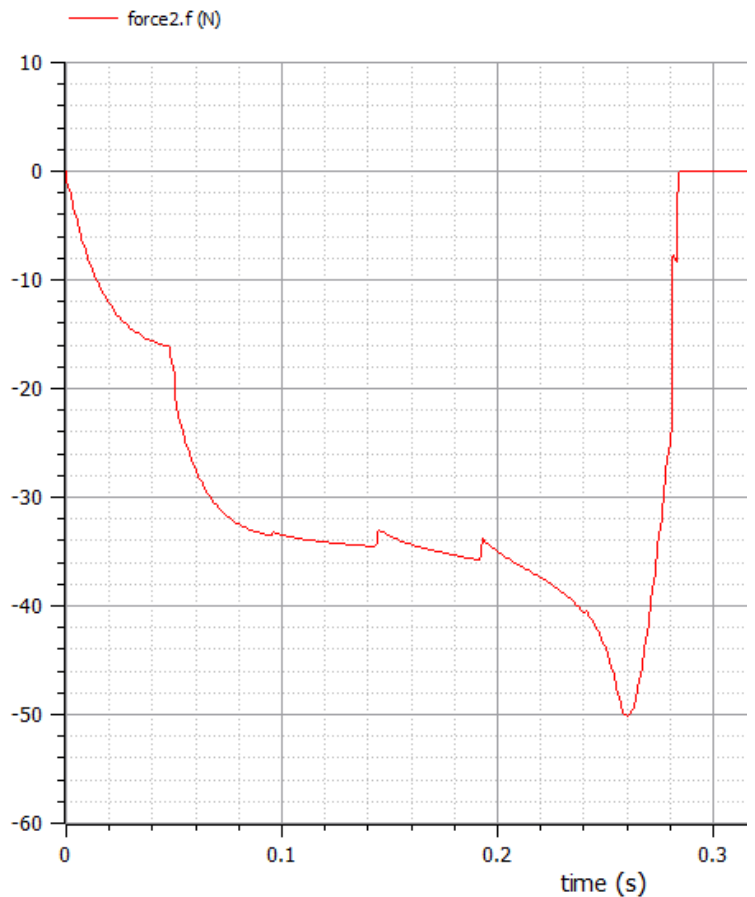
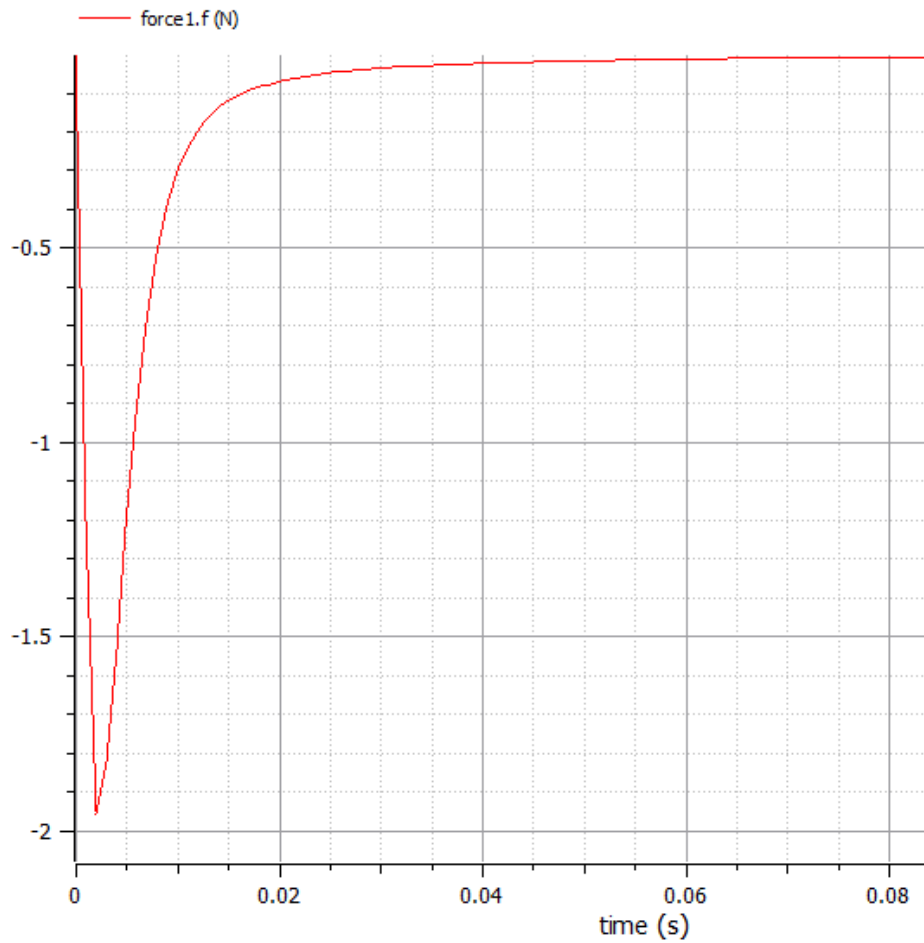


Figure 52. Distance and reference position of complete model 2.

Here, we can see how the motor is able to move the mass until the position wanted and then slow down the velocity and be there. So, the accuracy of our motor is validated. We are going to add also the friction forces produced on the simulation of 0.2m.

Figure 53. Friction force depending on velocity.





*Figure 54. Friction force depending on distance.*

We can obviously observe how the friction depending on the velocity increase with it and its 0 when the mass achieve its position and how the friction depending on the position is huge when the mass start to move but It reduces its value when the position increase.

## - CHAPTER 4: STUDY OF THE SYSTEM CHANGING TEMPERATURE.

### 4.1. Study of the functioning when the resistance change.

Finally, we are going to make the simulation of our model changing the temperature to see which the differences on the functioning of our motor are. First, we are going to simulate our model, and take images about the important factors. Then, we will introduce the difference values of the resistances (calculated on point 3.1.7) on the model and simulate again, seeing which the changes are. We will do the simulation on the worst case, with -15 grades, to see the differences clearer. We will show both pictures of each parameter together to see it better.

#### 4.1.1. Output gain controller.

Figure 55. Salida gain controller at 20 °C.

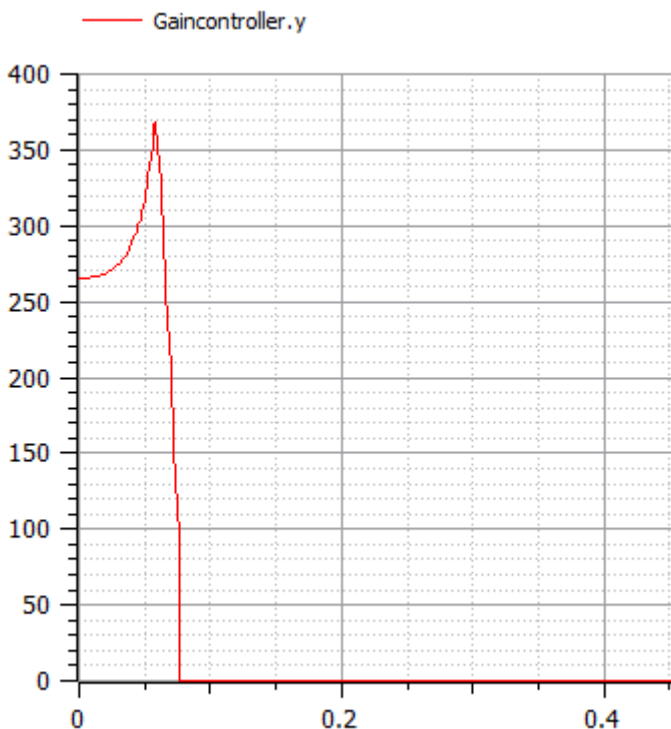
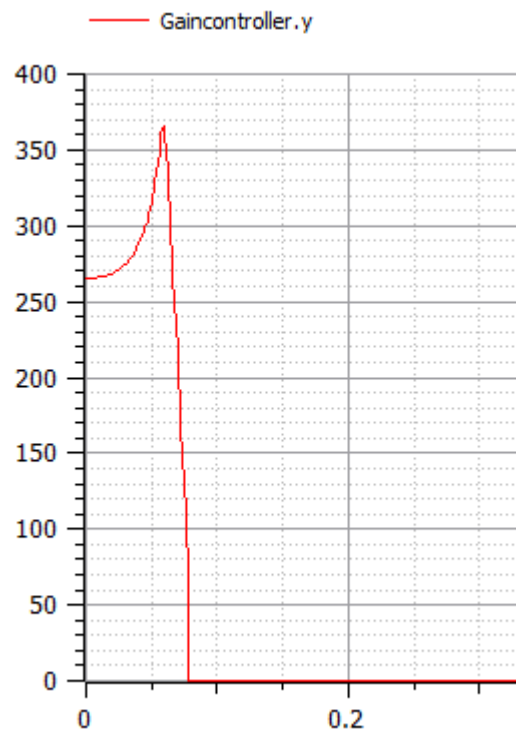


Figure 56. Salida gain controller at -15°C.





First, we are going to see the output of the gain controller. It is the output of the controller but multiplied for the necessary gain to have a magnitude that we can use on the supply voltage. As we can see there is not any difference between them. So, the temperature will not produce any change on the controller. If the temperature changes other magnitude, the controller will not adequate their functioning to achieve its destiny, it will work the same way.

#### 4.1.2. Intensity on the solenoid.

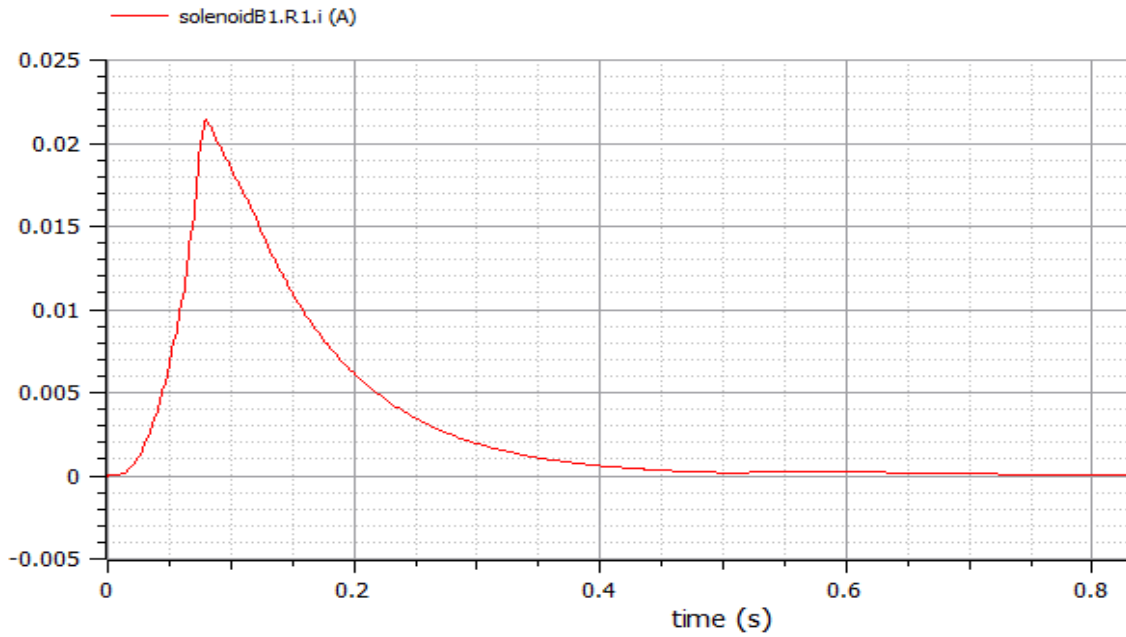


Figure 57. Intensity on the solenoid at 20 °C.

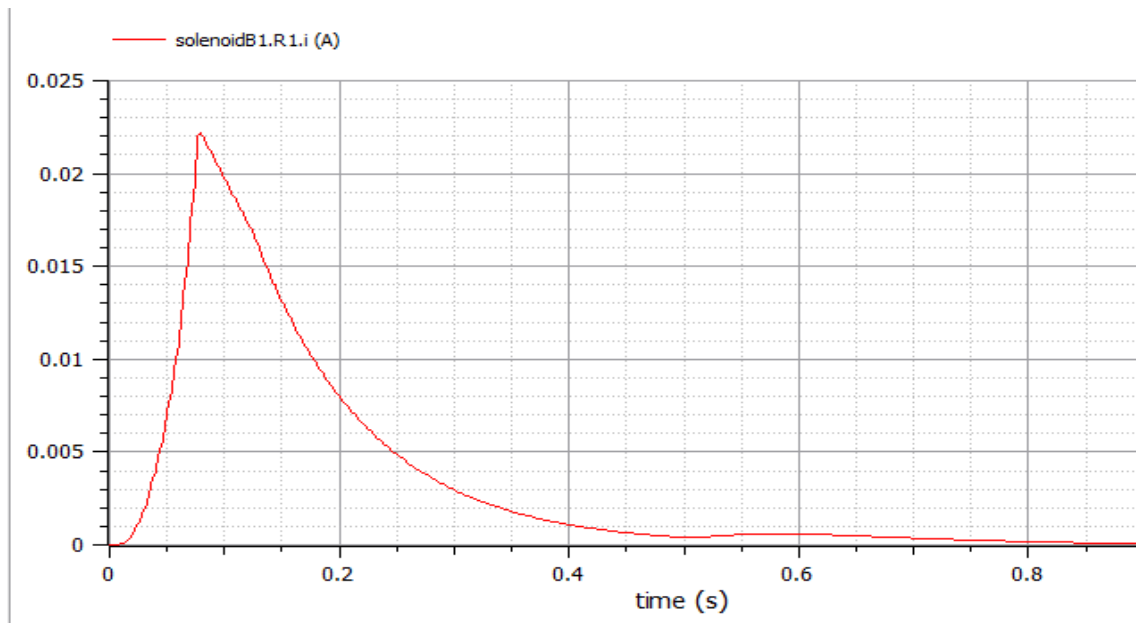


Figure 58. Intensity on the solenoid at -15 °C.

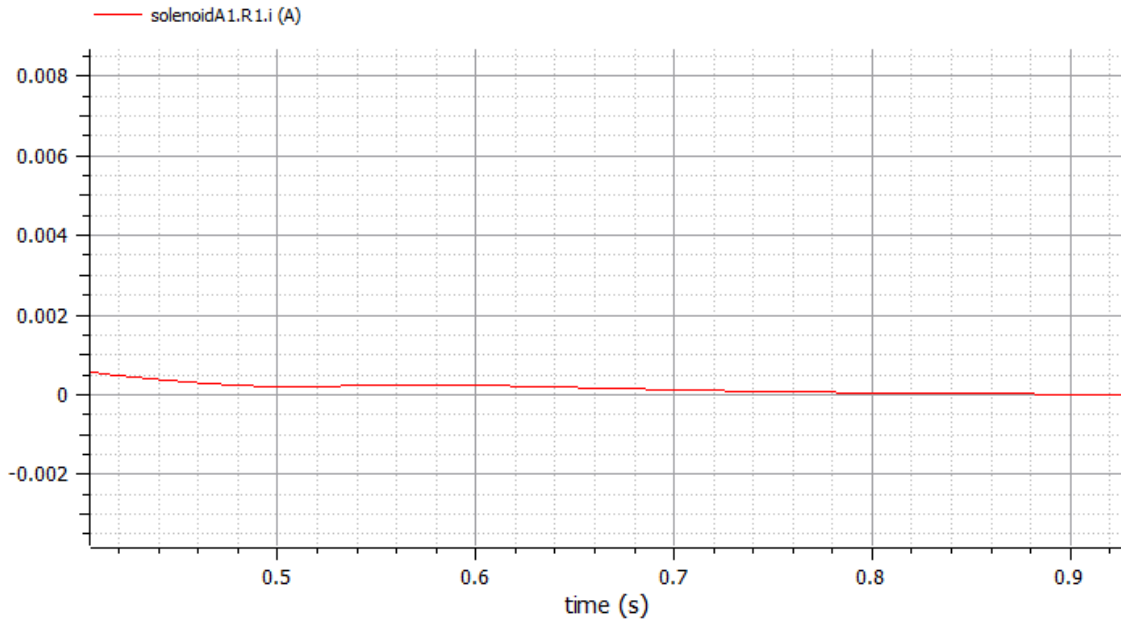


Figure 59. Zoom on the intensity of the solenoid at 20°C

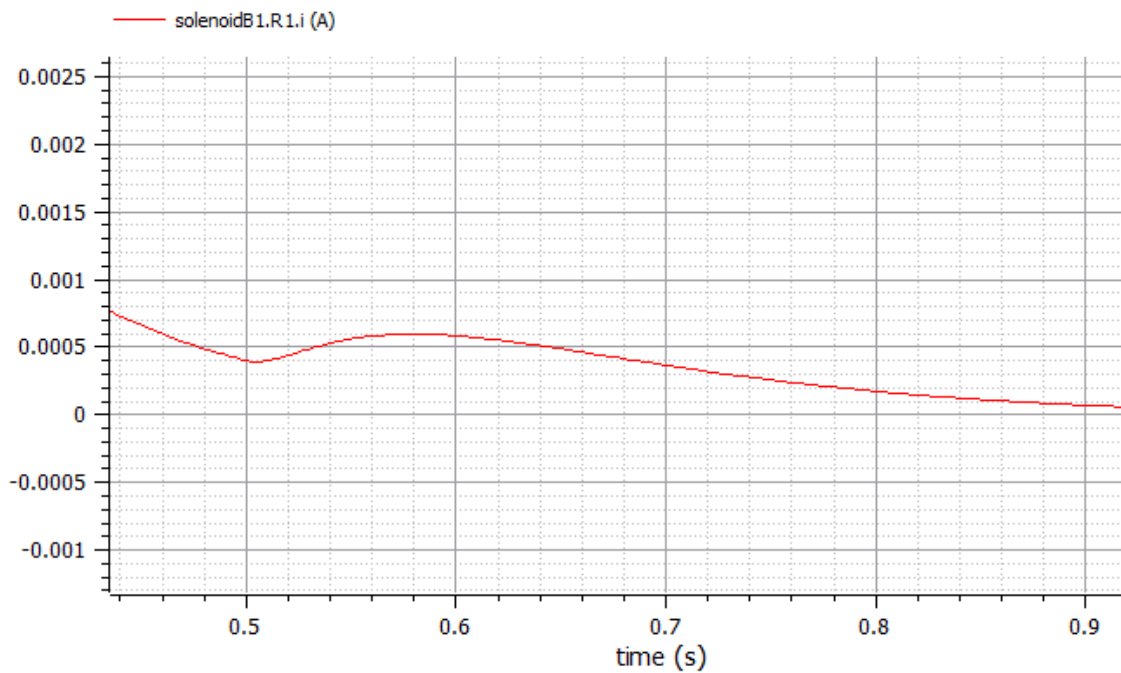


Figure 60. Zoom on the intensity of the solenoid at -15 °C.

We can see how the wave of the intensity through the solenoid that have not been activated by the controller are: the first part is similar when the temperature change. It is 0.021 A on the first photo and 0.022A on the second. But we can observe how appear another part on both waves. It should not appear more current on the circuit due to the voltage source is not working in that moment. That current will be provided by the inductor. The functioning on the motor is calculated to stop before and with that current achieve the necessary position. When the temperature change, that last impulse is higher (0.000596A at -15°C, and 0.000248°C at 20°C). But, if we look where the impulse

comes from we will see that on percentage is lower (34.054% at 20 °C and 15.32% at -15 °C) because the second step start at a higher point. (0.000185A to 0.000248A at 20°C and 0.000389A to 0.000596°C at -15 °C).

### 4.1.3. Voltage and forces.

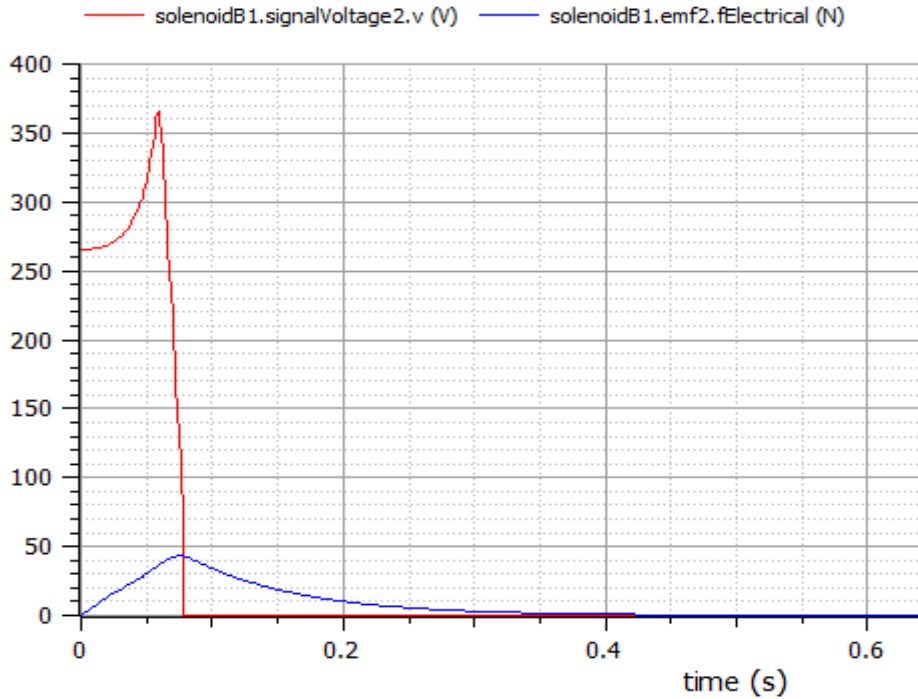


Figure 61. Voltage and force with voltage on solenoid at 20 °C.

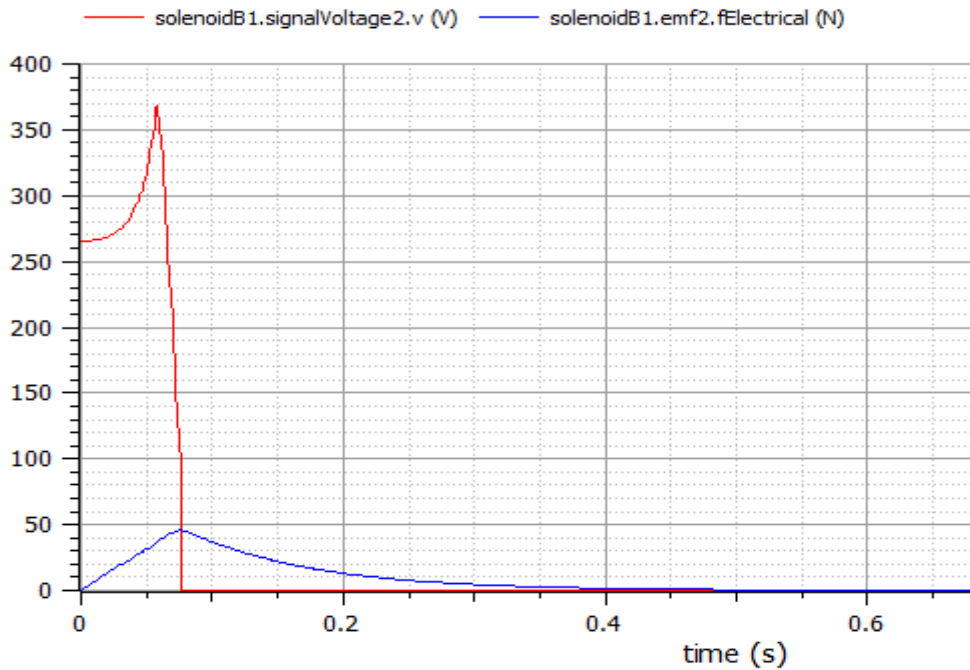


Figure 62. Voltage and force with voltage on solenoid at -15 °C.

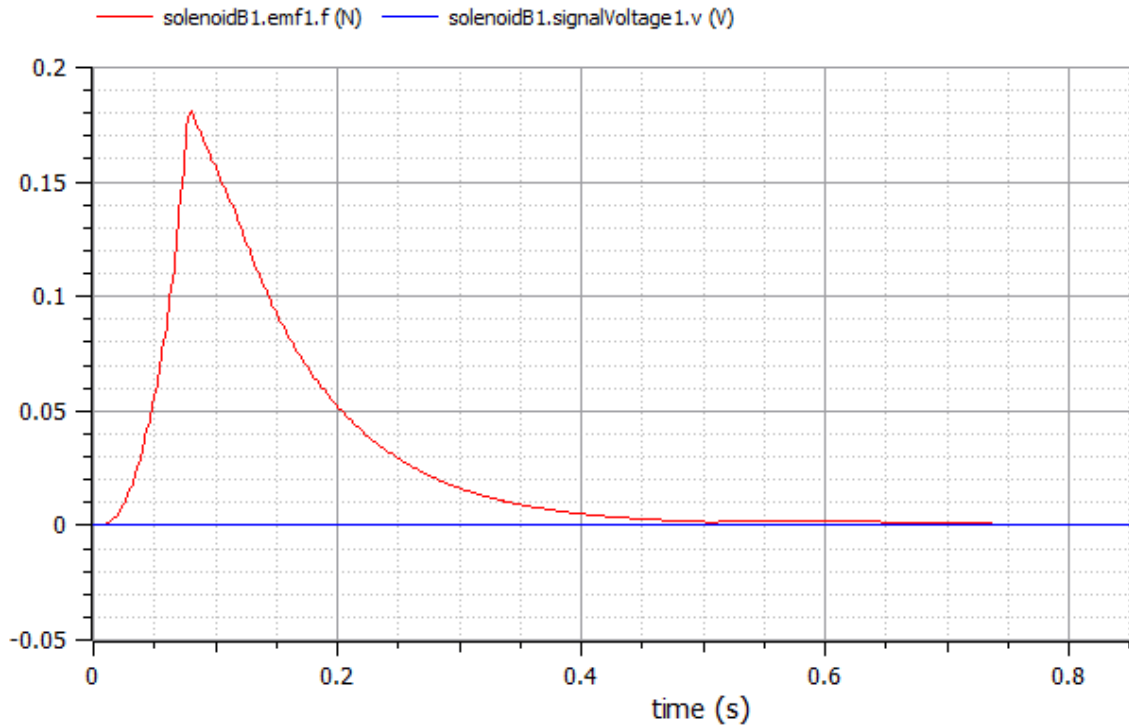


Figure 63. Voltage and force without voltage on solenoid at 20 °C.

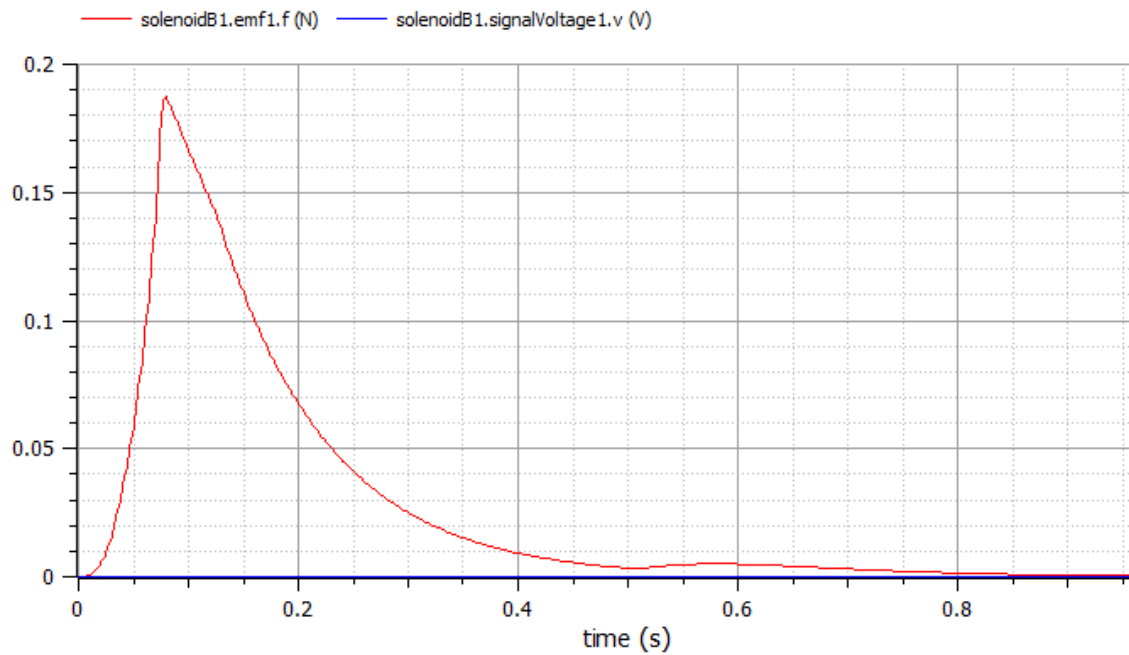


Figure 64. Voltage and force without voltage on solenoid at -15°C.

The first two photos show the solenoids that have been activated by the. We can see how the voltage increase and how that produce a proportional force on the mass. The wave of the voltage and the force is the same when we change the temperature. On the second two photos we can see the voltage and the force on the solenoids that have not

been activated by the controller. They have been felt a pulse which is lower than the other and have a similar wave than the intensity explained on point 4.1.3.

#### 4.1.4. Position.

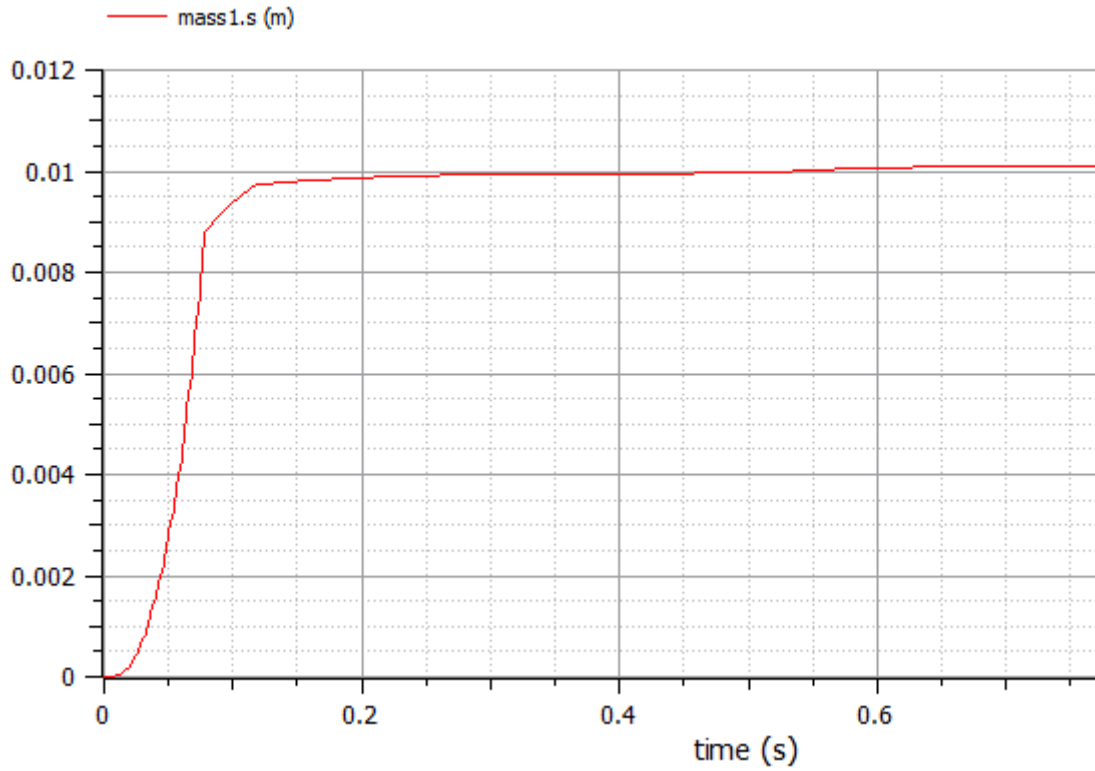


Figure 65. Position at 20 °C.

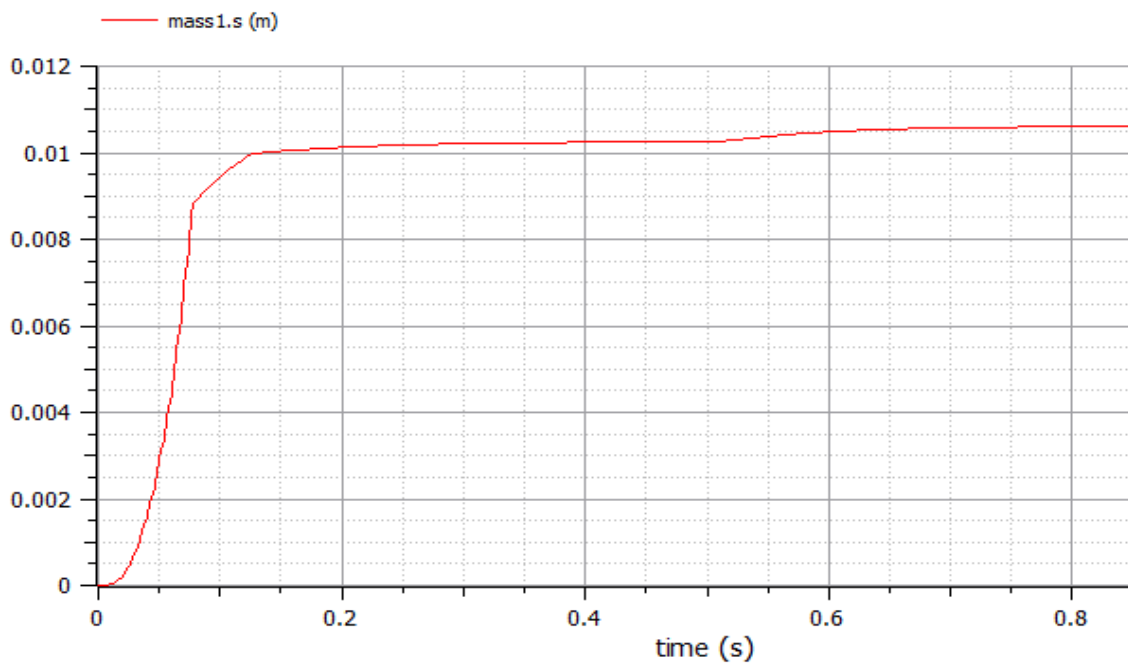


Figure 66. Position at -15 °C.

We can observe how the huge precision of the motor is lower when the temperature is lower. On the first case we achieve 0.0101m and on the second case 0.0106. It is a 5% more of the reference position wanted. We also can declare that the motor slow down its movement when it has achieved more distance.

#### 4.1.5. Velocity.

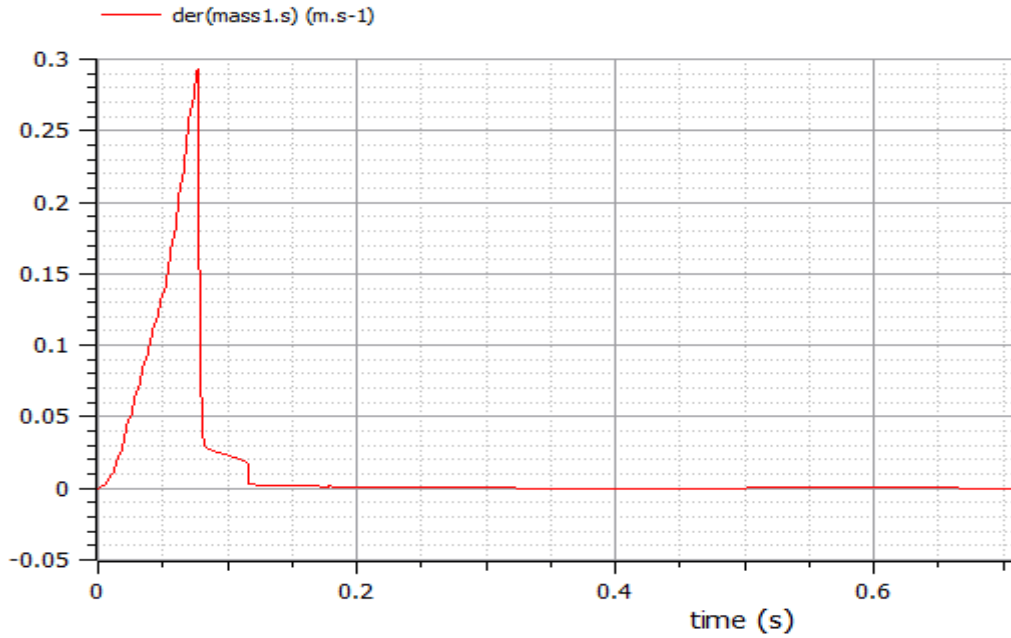


Figure 67. Velocity at 20°C.

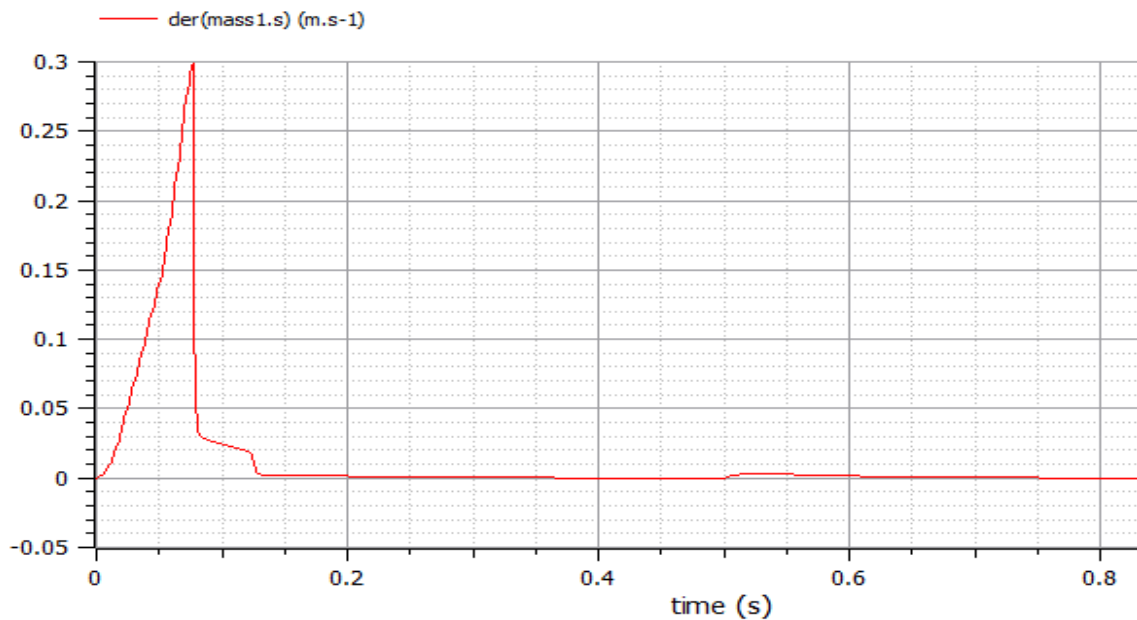


Figure 68. Velocity at -15°C.

In this case we can see that the velocity is greater when the temperature decrease, and so it is the velocity produced by the pulse for the current when the motor stop.

#### 4.1.6. Friction force of position.

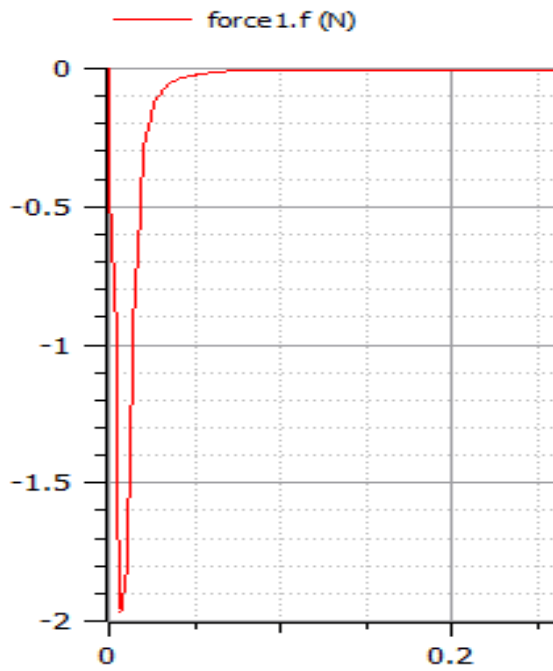


Figure 69. Friction force depending on the position at 20°C.

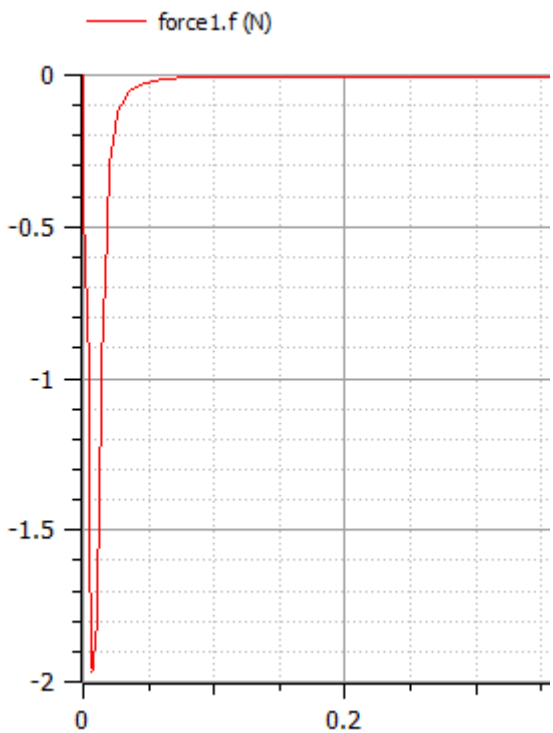


Figure 70. Friction force depending on the position at -15°C.

We can see how the friction force depending on the position is the same when we change the temperature. It has the same wave and the same magnitude.

#### 4.1.7. Friction force of velocity.

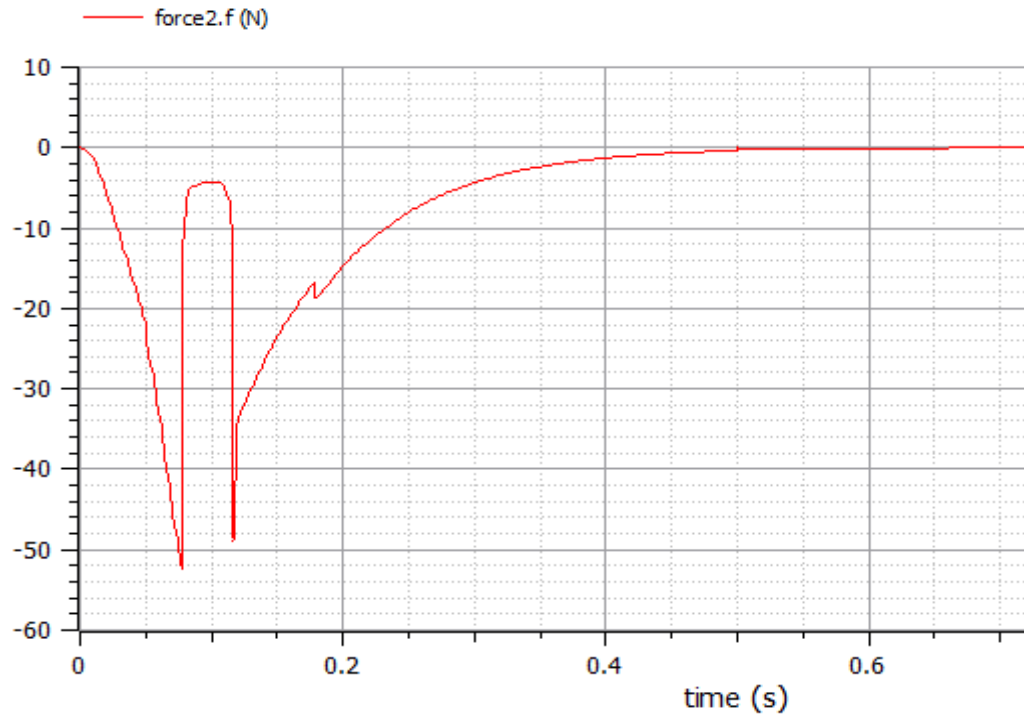


Figure 71. Friction force depending on the velocity at 20°C.

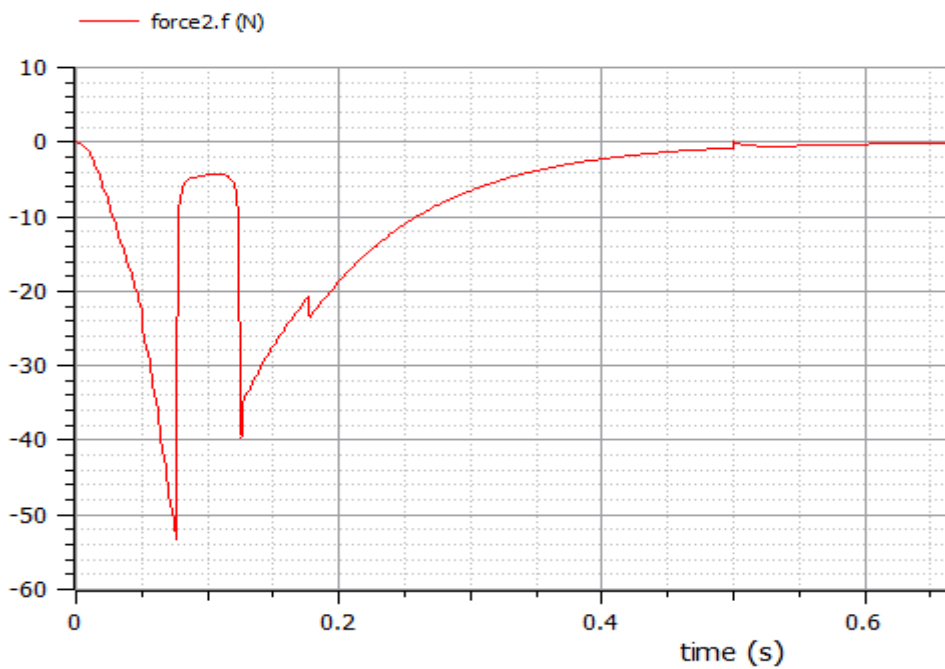


Figure 72. Friction force depending on the velocity at -15°C.

The analysis of the friction force that depends on the velocity shows that the wave is the same when we change the temperature. The value of the force is a little higher because the velocity has parts when is higher, but it is not enough big to be relevant on our study.



## - CHAPTER 5: CONCLUSION.

We have created a model of the motor that represents with great accuracy the functioning of a linear motor. Secondly, we have simulated that model changing the temperature where the motor is working. We have selected the most important parameters on the motor and they have been plotted at both temperatures. First at 20°C and then at -15°C, we have selected the worst case where the motor could work to see the differences better.

Referring to the study of the motor, we begin with the output of the controller. We can see how the controller is not affected by the temperature and it will provide the same output. It is a problem because they are parameters that change so with the same output we will have a different functioning that we want. Then, we have studied the intensity on the solenoids, the voltage, the forces produced, the position and the velocity. We can see how the voltage and force produced on the solenoids activated by the controller have a logical form, like the wave of the controller. On the other part if we see the other solenoids we can see how the wave of the intensity is divided on two peaks, the same way as the force, the first coincide with the output of the controller but the second not, the motor is modified to use this second peak to achieve the final destination, but with the lower temperature we will have a bigger peak. If we look on the plots of position and velocity we can see how this augment of intensity will provide a loss of precision on the motor. In our case, we will have a 5% less of accuracy. We should control this peak, to modify the functioning of the motor and use it to slow down the motor before and then achieve the position demanded depending on the temperature.

Finally, we have simulated also the friction forces that appear on the system, we have demonstrated that they are not affected in a relevant way. The friction depending on the position has the same value when we change the temperature. On the other part, the friction depending on the velocity change the value a bit but not the wave, and it is not enough change to be relevant on our study and to change the accuracy of the motor.

## BIBLIOGRAPHY:

[1]: Lisa Elviri, Ruben Foresti, Carlo Bergonzi, Francesca Zimetti, Cinzia Marchi, Annalisa Bianchera, Franco Bernini, Marco Silvestri, Ruggero Bettini “Highly defined 3D printed chitosan scaffolds featuring improved cell growth” – Parma: IOP Publishing, 2017.

[2]: Chang-Woo Song and Seung-Yop Lee “Design of a Solenoid Actuator with a Magnetic Plunger for Miniaturized segment Robots” – September 2015.

[3]: E. Matkevicius, L.Radzevicius “Mathematical Model of the Linear Motor” – Vilnius, Lithuania, 2007.

[4]: M.Molis, E. Matkevicius, L.Radzevicius “Calculation of the Mathematical Model of the Linear Motor” – Vilnius, Lithuania, 2009.

[5]: Qiulin Xie “Modeling and Control of Linear Motor feed drives for grinding machines” – Georgia Institute of Technology, May, 2008.

[6]: Satoshi KANEKO, Ryuta SATO, Masaomi TSUTSUMI “Mathematical Model of Linear Motor Stage with Non-Linear Friction Characteristics” – Tokyo, Japan, 2008.