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Unveiling rival´s offering prices in Electricity Markets

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Summary

In this Master Thesis, several alternatives are proposed in order to reveal offering prices in a spot market. This proposal to obtain hidden information within a system is considered as a very powerful mean to acquire a competitive advantage when proposing a market strategy.

Due to the opacity found in terms of methods to perform this exercise, the application proposed in this project is considered novel. Many proposals related to carrying out a strategic offering have been found although they result in nothing similar to what is proposed in this project. Taking as reference some aspects of these works already done, it is suggested as an improvement the use of a recursive filter.

For the purpose of obtaining of results, the modeling of a pair of representative electrical markets has been developed. Through them, outcomes will be generated in order to check if the suggested methods work properly. As a first step, an Inverse Optimization Problem will be applied to an electric market with three nodes and two interconnections. The outputs obtained through this methodology will be taken as a reference in order to improve them along an algorithm with simpler formulation.

Further on, an Ensemble Kalman Filter will be applied with the objective of reaching more feasible results than those obtained in the previous methodology. To do this, the same market will be used but reduced to a single node, although with more participants. In order to verify its robustness, three different situations will be proposed. In the first scenario, the *EnKF* model will remain static, which will lead to a better understanding of how the algorithm behaves. In the second scenario, the model will change to dynamic, and its parameters will be updated depending on the needs of the market. Finally, a third scenario will be proposed taking into account the formulation of the second scenario, although with different market conditions. In order to study the changes in demand, such as a failure in an interconnection or even a transition between year's seasons, extreme changes will be made between periods of simulation.

Resumen

En este proyecto final de máster, se proponen varias alternativas con el fin de revelar precios de oferta en un mercado del tipo spot. Esta propuesta de obtener información escondida dentro de un sistema, se considera como un medio muy potente para poder adquirir una ventaja competitiva al proponer una estrategia de mercado.

Debido a la opacidad encontrada en cuanto a la metodología se refiere para realizar dicho ejercicio, la aplicación propuesta en este trabajo es considerada del tipo novel. Muchas publicaciones relacionadas con optimizar una oferta estratégica se han encontrado aunque resulten en nada similar a lo propuesto en este proyecto. Tomando como referencia algunos aspectos de estos trabajos ya realizados, se propone como mejora el uso de un filtro recursivo.

Para la obtención de resultados, se ha realizado los modelos de un par de mercados eléctricos representativos. Mediante ellos, se generarán resultados con el fin de poder comprobar si los métodos sugeridos funcionan adecuadamente. En un primer lugar, se aplicará un Problema de Optimización Inverso a un mercado eléctrico con tres nodos y tres interconexiones. Los resultados obtenidos mediante esta metodología, se tendrán como referencia con el fin de mejorarlos con una formulación mas sencilla y potente.

Finalmente, se aplicará un Ensemble Kalman Filter con el fin de encontrar resultados más factibles que en los obtenidos en la metodología anterior. Para ello, se utilizará el mismo mercado pero reducido a un nodo aunque con un mayor número de participantes. Para poder comprobar su robustez, se propondrán tres escenarios distintos. En el primer escenario el modelo del *EnKF* será estático, lo que conllevará a entender mejor como funciona el algoritmo. En el segundo escenario, el modelo cambiará a dinámico, y sus parámetros se actualizaran dependiendo de las necesidades del mercado. Finalmente, un tercer escenario será propuesto teniendo en cuenta la formulación del escenario dos, pero con unas condiciones de mercado diferentes. Con el fin de estudiar los cambios de demanda, como puede ser un fallo en una interconexión o incluso un cambio de estación del año, se le realizaran cambios bruscos entre periodos de simulación.

Preface

This Master thesis was prepared at the department of Electrical Engineering at the Technical University of Denmark in fulfillment of the requirements for acquiring a Master of Science in Electrical Engineering.

This work will be presented both at the Technical University of Denmark and at the Polytechnic University of Valencia, as provided for in the agreement for the Double Degree program T.I.M.E..

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Symbols

PARAMETERS FOR THE INVERSE OPTIMIZATION PROBLEM

Symbol	Unit	Definition
λ^{G}_{dtnb}	€/MWh	Price offer for power block of the strategic unit at node n in time period t on day d.
λ^{O}_{dtnb}	€/MWh	Price offer for power block of the rival unit at node n in time period t on day d.
λ^{Otrue}_{nb}	€/MWh	Marginal cost of power block of the rival unit at node n.
λ^{Oini}_{dtnb}	€/MWh	Initial estimation of the marginal cost of power block of the rival unit at node n in time period t on day d.
λ^{D}_{dtnk}	€/MWh	Price bid of demand block at node n in time period t on day d.
P^{Gmax}_{nb}	MW	Upper bound of power block of the strategic unit at node n.
P^{Omax}_{nb}	MW	Upper bound of power block of the rival unit at node n.
P^{Dmax}_{dtnk}	MW	Upper bound of demand block at node n in time period t on day d.
P^{Gini}_{dn}	MW	Total power produced by the strategic unit at node n in the time period *(t=0)* prior to day d.
P^{Oini}_{dn}	MW	Total power produced by the rival unit at node n in the time period *(t=0)* prior to day d.
P^{max}_{nm}	MW	Transmission capacity of line n-m.

Symbol	Unit	Definition
B_{nm}	S	Susceptance of line n-m.
R_n^{Gdwn}	MW/h	Ramp-down limit of the strategic unit at node n.
R_n^{Gup}	MW/h	Ramp-up limit of the strategic unit at node n.
R_n^{Odwn}	MW/h	Ramp-down limit of the rival unit at node n.
R_n^{Oup}	MW/h	Ramp-up limit of the rival unit at node n.
λ^{MC}	€/MWh	Marginal cost of each company used in the model.

VARIABLES FOR THE MARKET CLEARING PROBLEM

Symbol	Unit	Definition
P_{dtnb}^{G}	MW	Power produced by block b of the strategic unit at node n in time period t on day d.
P_{dtnb}^{O}	MW	Power produced by block b of the rival unit at node n in time period t on day d.
P_{dtnk}^{D}	MW	Power consumed by block k of the demand at node n in time period t on day d.
δ_{dtn}	Radians	Voltage angle of node in time period t on day d.
λ_{dtn}	€/MWh	LMP at node n in time period t on day d (dual variable).

VARIABLE FOR THE INVERSE OPTIMIZATION PROBLEM

Symbol	Unit	Definition
λ_{dtnb}^{O}	€/MWh	Price offer of power block b of the rival unit at node n in time period t on day d.

PARAMETERS FOR THE SIMPLE KALMAN FILTER

Symbol	Unit	Definition
M_t	-	Constant which permits to propagate the *initial state estimate* variable into the *a priori state estimate* variable in each time period t.
w_t	-	White noise associated to the error committed when propagating the *initial state estimate* variable in each time period t.
Q_t	-	Noise covariance of parameter w_t.
H_t	-	Constant which permits to relate the *a priori state estimate* variable with the measurement taken in each time period t.
v_t	-	White noise associated to the error committed when obtaining the measurement in each time period t.
R_t	-	Noise covariance of parameter v_t
K_t	-	Kalman Gainer. Reduces the uncertainty in the *a posteriori state estimate* variable and corrects the *a priori state estimate* variable.

VARIABLES FOR THE SIMPLE KALMAN FILTER

Symbol	Unit	Definition
$\tilde{x}_t$	€/MWh	The *a priori state estimate* variable in each time period t. Collects offering prices in the forecasting step.
$\tilde{P}_t$	-	Predicted (a priori) estimate covariance associated to the previous variable in each time period t.
y_t	€/MWh	Observation of the true state in each time period t.
$\hat{x}_t$	€/MWh	The *a posteriori state estimate* variable in each time period t. Collects corrected offering prices in the updating step.

Symbol	Unit	Definition
$\hat{P}_t$	-	Updated (a posteriori) estimate covariance associated to the previous variable in each time period t.

PARAMETERS FOR THE ENSEMBLE KALMAN FILTER

Symbol	Unit	Definition
M_t	-	Constant matrix which permits to propagate the *initial state estimate* variable matrix into the *a priori state estimate* variable matrix in each time period t.
w_t	-	White noise associated to the error committed when propagating the *initial state estimate* variable matrix in each time period t.
Q_t	-	Noise covariance of parameter w_t.
E_t	-	Parameter which reflects the mean values with respect to each column given matrix in each time period t. Used only with the *a priori state estimate* variable matrix.
A_t	-	Parameter which collects the result of subtracting to the *a priori state estimate* variable matrix the E_t matrix in each time period t.
C_t	-	Parameter which collects the sample covariance matrix regarding the *a priori state estimate* variable in each time period t.
H_t	-	Constant matrix which permits to relate the *a priori state estimate* variable matrix with the simulated measurement matrix in each time period t.
v_t	-	White noise associated to the error committed when obtaining the simulated measurement in each time period t.
R_t	-	Noise covariance of parameter v_t

Symbol	Unit	Definition
$\hat{K}_t$	-	Estimated Kalman Gainer. Reduces the uncertainty in the *a posteriori state estimate* variable and corrects the *a priori state estimate* variable.

VARIABLES FOR THE ENSEMBLE KALMAN FILTER

Symbol	Unit	Definition
$\tilde{x}_t$	€/MWh	The *a priori state estimate* variable matrix in each time period t. Collects offering prices in the forecasting step.
$\tilde{y}_t$	€/MWh	Simulated observation of the true state in each time period t.
y_t	€/MWh	Observation of the true state in each time period t.
$\hat{x}_t$	€/MWh	The *a posteriori state estimate* variable matrix in each time period t. Collects corrected offering prices in the updating step.

ADDITIONAL PARAMETERS FOR THE ENSEMBLE KALMAN FILTER

Symbol	Unit	Definition
d_t	-	Constant matrix which measures the risk taken by the last company in a market clearing in each time period t.
j_t	-	Constant matrix which extrapolates the risk taken by the last company in a market clearing to the other participants in each time period t. This constant will multiply matrix H_t

Nomenclature

Symbol	**Definition**
PCI	Projects of Interest
OMIE	Operator of the Iberian Energy Market
EPEX	The European Power Exchange
GME	Electricity Market Manager (Italy)
LMP	Local Marginal Price
SMP	System's Marginal Price
S	Strategic Producer
O	Rival Producer
PQ (Bus)	Load Bus
ISO	Independent System Operator
KKT	Karush-Kuhn-Tucker
EnKF	Ensemble Kalman Filter
KF	Simple Kalman Filter
PDF	Probability Distribution Function
GPS	Global Positioning System
CPU	Central Processing Unit
MIBEL	Iberian Electricity Market

Contents

CHAPTER 1

Introduction

The current structure of electricity markets is a consequence of the liberalization of energy that took place in the middle nineties when the main framework was monopolistic. The main utilities were owned by the respective states and were vertically-integrated which means that the whole chain of activity regarding electrical power (generation, transmission, distribution, and retail mainly) was driven by the same party. Despite that situation, the monopolies were challenged to change the situation. It is considered as a first step in this process the unbundling between generation and distribution, the competition between producers or even the actual trading and pricing methods which were conducted by the Chicago Boys in Chile during Pinochet dictatorship.

Meanwhile, in that context along Europe, some countries started to adopt this new features in their electricity markets. First notions regarding liberalization were observed in the UK with the release of the England and Wales electricity market in 1990 or in Norway in 1991. USA, Australia and New Zealand followed this first movements in the coming years. Moreover, as a general movement, the construction of the European Union's Internal Energy Market was released with the main purpose of making accessible the benefits of liberalization to citizens and companies in terms of both affordable prices and provision of primary services. It could be said that the most remarkable results, leaving aside the liberalization of gas and electricity markets, were three: the first was the regulation of the energy market in order to be able to monitor the development of the network and markets, to investigate possible abuses and to apply penalties together with the approval and collaboration of member states. This was intended to increase coordination and to improve the transparency of gas and electricity prices that were being charged to final users; The second notable result was the security of supply established by measures such as levelling of interconnections between Member States or balances between offers and demands; Finally, the last significant result was the establishment of Trans-European energy networks identified as projects of common interest (PCI). These projects carried out by the European Commission, are intended to assure the first two points described before [1].

Therefore, the tendency in many of the European countries was to reorganize power markets regulated in a way which results in an increment of competition between companies and efficiency in power supply. Many structures were initially proposed in such a way that the electricity market would be deregulated, being the pool the preferred market (or day-ahead market). One of the reasons was the generation of liquidity in the market as a consequence of obligations and incentives which motivate the majority of available generators to present offers in the market. In these types of markets, there are companies that offer electricity and companies that buy in a continuous framework of competition where the objective of each of them is to maximize their profits from purely strategic bids. Some examples of organized markets can be found in all regions of Europe such as OMIE in the Iberian Peninsula, Nord Pool Spot in the Nordic countries, EPEXSpot in France, Germany and other Central European countries or even GME in Italy among others [2].

Competition is a concept that goes hand in hand with the pool market and inevitably linked to this competitiveness in these electrical markets, this is what is known as the problem of ”potential exercise of market power” exercised by generation companies. This problem is not exclusive to the electricity sector as it is common in all sectors open to competition. But before making an analysis of the different situations that can occur when a market power is carried out, it deserves special attention to clarify that this action is not equivalent to accomplish an abuse of power. When the second option is executed, the use of illegitimate information is presupposed in order to seek its own benefit. Market power would be defined as the ability of a single company, or even a set of them, to modify market prices or quantities offered for their own benefit. Some bid prices per energy block can be altered so that they are below the price levels offered by the competition and thus restrict bids that are above the market price at the equilibrium point. In other words, speculate with the block price to push to the right of the border where bid and demand marry most of the competition, generating more profits. Another possibility is to speculate with the offering of some energy blocks so that they are offered in a lower quantity but at higher prices. [3]

In order to reach a situation where a bid strategy is put together for the purpose of obtaining a greater benefit in the market, a prior action is required: obtaining and processing data from public sources. Without information of what is happening in the system, the speculation in the offers of energy blocks or of the prices of these blocks, will be a very complicated exercise in terms of obtaining continuous good results. The premise is clear: the more valid information can be taken from the market, the more accurate the modelling of the market can be and, as a result, the movements will be more precise as it is desired to get a better return to the benefit of the market. It should be emphasized that all this information should be consulted in public databases, such as, for example, the price of energy in the spot market or even weather forecasts. In addition, these data must be validated through a model and must properly represent fairly the system that is being analyzed.

One of the options that is discussed in this document will be to disclose the offering prices of the rivals in order to define an optimal strategy and thus achieve the maximum performance to the market benefit. Although this option, a priori, is very ambitious and laborious as it would be necessary to estimate all the marginal costs of each of the generating companies that make up the market, nevertheless it can lead to interesting results and a practical application could be feasible. To carry out this first option, a series of mathematical and statistical resources has been used, in addition to techniques already carried out previously by other authors. To be more specific, there have been two different alternatives with some variations that will be presented in the next four chapters of this document.

In chapter 2, a brief review of the work conducted with respect to the objective of the current project will be made. In addition, taking as reference what has already been done, the motivation found to accomplish the work realized will be explained. Finally in this same chapter, the model used for the application of the different methodologies will be exposed, creating two versions of it. In chapter 3 the two methodologies applied in the project will be presented. It will be explained what is the Inverse Optimization, the Kalman Filter and the Ensemble Kalman Filter. In addition, the equations of the model will be presented, due to they are part of the optimization problem, as well as the formulation corresponding to the three alternatives along with graphs, tables and examples. In chapter 4 the obtained results will be exposed. Based on them, an explanation of the behavior in each methodology will be made and, specifically for the application of the Ensemble Kalman Filter, three scenarios will be proposed with changes both in the formulation and market conditions. Finally, in chapter 5, conclusions will be drawn about what has been applied and obtained in the project.

CHAPTER 2
Motivation & Electricity Markets

It is known that few references are publicly available when looking for methods guaranteeing a strategic advantage. Notwithstanding the difficulties, in this chapter, it will be reviewed some of the literature regarding the main scope of this project. Not all the work already done in this field stands with revealing prices though its final purpose is equivalent. That is why this following section contemplates tasks such as optimal offering without the prerequisite of using the same means to generate a strategic advantage, as will be endorsed further on this document. In any case, it there have been found good alternatives in an academic nature and hence, based on this work already done in this leveraging field, it will also be exposed the motivation of how it could be improved in many terms.

In addition, while examining whether a benefit is fulfilled in a system or not, some model is required in order to verify it. Later, it will be seen that all these previous applications relying on optimal bidding, avail of one depending on their desired results. Based on this necessary aspect, the model used in this project will be also presented in this section. Moreover, as the main solution suggested in this proposal is novel, a model used in another application is taken as reference but presenting some streamlining characteristics that bring some integrity on the suggested algorithm's requirements. The elected model will be an electricity market being the outcomes from their clearings in several scenarios a contemplation of different market conditions. Some general aspects regarding the electricity markets will be defined before disclosing the characteristics of the model. The complexity when accurately representing its behaviour is a truism and will not be faithfully represented, this is the reason why a preamble regarding markets' attributes is appended. A complete market composed by three nodes with interconnections and rival players competing will be clarified as a first step. Next, as the application does not require a great detail level under the point of view of this project, this market will be technically shortened even then obtaining reasonable events.

2.1 Literature Review

As it has been well detailed in the introduction part, the main idea of this project is to disclose offer prices corresponding to energy blocks from the different actors that offer electrical power in a given market. It is essential to highlight the delicacy that comes with the issue of predicting prices from direct competitors since in this type of research, carried out in the private spheres by the different electricity producers, it is the opacity of their different methodologies what easily stands out. Due to the extreme importance for a private entity of having information of direct rivals which supposes a competitive advantage in the market, there is little public information on methodologies to accomplish this purpose. Therefore, it is common sense to understand that, in the case that there is an effective alternative to apply a method that allows obtaining results that reflect the behavior of the market to be studied, it is not made public so preventing competitors to take advantage.

It can be said that the main issue that concerns the project is closely linked to obtaining an optimization decision in the strategic offering for each of the bids in the market within each temporary unit. This type of application is practiced in all companies, both those that offer and those that buy energy. Inside this field, there is a long list of references where the main purpose is to obtain a procedure that derives a maximization of the benefits of an energy producer company. These types of techniques have been proposed recently for their application in problems regarding decision making in electrical markets. As an example, a multiperiod network-constrained market-clearing algorithm is considered in [4] to obtain strategic offers based on a bilevel programming model where the upper-level represents the maximization of the benefits of a given company while the lower-level represents the market clearing which corresponds to the formation of the corresponding prices. Similarly, [5] also proposes a bilevel programming model for deriving offers of a strategic generator under uncertainty. In this case it differs from the previous one in that the lower-level represents an economic dispatch problem and thus Lagrange multipliers must be applied to linearize the constraints. Another example is found in [6] where a multi-stage risk-constrained stochastic complementary model is proposed to obtain optimal offering strategy from a wind energy company which participates both in the day-ahead market and in the balancing market. Through the use of mixed-integer linear programming programs, results are obtained from the different models that represent productions of wind-power, market prices, demand's bids and rival's offers.

In order to maximize benefits through an optimal strategic offer, other types of practices apart from optimization problems can be used by private entities. One among many alternatives to achieve this optimal state is when revealing or estimating the offering prices of the different generating units that are in competing with a given company in an electricity market. The set of offering prices assemble what is known as the upward sloping supply curve. If the price of the rivals is predicted with certainty, a very complete model can be achieved through which a maximization of profits would become greater as this results in a real important competitive advantage. Such an example is in [7], where is proposed the revelation of the appointed supply curve by applying a Bayesian Inference approach to a model that represents the uncertainty of the system. The proposed algorithm is based on Markov Chain Monte Carlo and Sequential Monte Carlo methods and the results are really adjusted to market data outputs and therefore representing a real situation. A similar case when revealing offering prices is obtained from [8] whence, instead of applying methods related with inference and advanced statistics, the determination of the optimization of an Inverse Problem is proposed. An electricity market is presented where marginal production costs are known, and through the strong duality theorem becomes the problem to be solved in a linear programming scope, thus turning it into a quick problem in terms of computing. Starting from the basic idea that is applied from the point of view of a private company which is striving in a competitive market and, as a consequence and in principle, has non-public information related to energy quantities and number of offered and demanded accepted blocks, offered prices are revealed from its rivals.

Yet, none of the aforementioned studies provided how to get all the required information in order to insert the required parameters in a model to reflect a real application and then apply each algorithm. Most of them, take as reference data that is supposed to be known when there are no public verified sources if thinking in real world data. This is one of the main drawbacks found in this type of applications: the need for technical data to be able to create a basic model through which results in an approach to a real situation. This issue is difficult to avoid as there is a need of model verification, nevertheless, in this project some alternatives will be presented to deal with this. Moreover, in some applications there are also needed market outcomes as it is, for instance, the accepted energy blocks from both generators and buyers companies. The presented models are composed by a number of technical parameters, i.e. ramping constraint of generators, related to each of the supply energy opponents that are competing in the market. This can result in a recursive problem because the electrical market model which is used, takes as reference, to generate results, these technical data and therefore, when verifying the results with the proposed model again, as a consequence, a limited applicability to a real situation is obtained. As it can be seen, this problem is important in this project and that is why alternatives are proposed in order to avoid the use of information that can not be contrasted and thus be able to present the option of applying the method in a situation that is as close as possible to a realistic environment. Another issue is the idea of revealing offering prices instead of marginal prices. In the project, it will be assumed that, nowadays, a marginal cost is able to be calculated as it is exposed in [9]. However, it is also proposed an alternative preventing the unfamiliarity with the proposed methods.

In this document, the work done in [8] has been reproduced as a starting point. Based on the results obtained, alternatives will be proposed to correct the problems encountered. As already mentioned, there are no references that propose alternatives when revealing offering prices. As a consequence, throughout the project, several methodologies belonging to machine learning have been tested to attack the problem, not having success in all of them. Although it does not appear in this report, apart from the proposed methodologies, the use of unsupervised neural networks, multi-target regression through random forests, polynomial regressions together with recursive filters and even a Simple Kalman Filter in parallel with an Ensemble Kalman Filter has been tested. Filter. Due to the poor results obtained, more attention has been paid to the algorithms presented in this project.

2.2 The Electricity Market

2.2.1 Main Features of Electricity Markets

Electricity is considered an essential resource for the present-day society since it meets the basic needs of both residences and industry. On account of the importance of the supply in a reliable and profitable way, there exist electric markets to carry it out.

There are several types of markets depending on the type of contract through the actors and on the term of the signed contracts between them. Generally, trading is developed through pools or bilateral contracts. Regarding the temporary factor, short-term transactions are carried out in a daily market (day-ahead market) which is made up of producers, retailers and large consumers. This type of markets will be the reference for its modeling in this project. These participants submit bids for the purchase and delivery of electricity for the next day. Specifically, in this type of market there are 24 buying and other selling offers corresponding to each of the hours that make up the day. It should be noted that not all day-ahead markets operate on an hourly basis, being presented in some countries with a greater frequency, such as the example of New Zealand where there are 48 bids a day. All offers depend on a price related to an amount of energy so that the market operator can determine the supply and demand market curves. To carry out this exercise, the selling offers are ordered according to the increasing prices and the buy offers in decreasing direction, this is what is known as the merit order. If the market has interconnections to other electricity markets or nodes in the case of an US market, they will also be taken into account for market compensation and thus get the equilibrium point for each hour. The equilibrium point will dictate the accepted buying and selling offers: the selling offers whose price is not greater than the price at the equilibrium point will be accepted and viceversa, the buy offers whose price is not lower than the price at that point will be accepted.

Finally, it is worth mentioning that there are other types of markets with longer and shorter terms, for instance to match production and consumption at real-time in the latter case. Although these types of markets are outside the scope of this project, it is clear that intra-day, balancing and reserve capacity markets are considered short-term and, options and derivatives long-term contracts are named forward (also called long-term financial contracts) [10].

2.2.2 The day-ahead Market

The pool-type market offers a platform where all producers and consumers meet to establish 24 balance points for a given day, instead of making bilateral contracts between producers and consumers. Although these types of markets are not the most used option in terms of the exchange of basic products, in the case of electricity, the operation of large energy systems works correctly from the beginning of the liberalization of energy so as to maintain the competitiveness and security of supply. There are many variations within this type of market, although fundamentally they work in the following way and with the following actors: [11]

- Generating companies: For a considered period of time, they present quantities of energy at a price. The set of offers made by all the generators will form what is known as the market supply curve. The energy quantities are ordered according to the rising price and are represented as a function of their price.
- Demand: Similar to the generation companies, the demand curve will be formed by offers that specify a certain amount of energy demanded at a given price. Contrary to the previous ranking method, these offers will be ordered according to the decreasing price, with the most expensive buying offers being those closest to the origin of coordinates. In addition, it should be mentioned that the demand is usually very inelastic, this indicates that the variations in the price have a relatively small effect on the quantity demanded of the electricity and therefore can be treated as a fixed value. For this reason, the demand curve is often treated as a vertical line instead of a stepped curve. In the present project, for the inverse optimization problem it will not be necessary to treat it as a straight line because it is computationally feasible. On the other hand, for the rest of the markets used to apply the rest of the alternatives, the demand will be treated inelastically.
- By intersecting the two supply and demand curves, the equilibrium point is obtained so that the offers from generators and retailers that are on the left of said point will be accepted.
- The price corresponding to the equilibrium point is called the system's marginal price (SMP). Generators are paid according to this price for each MWh produced and consumers pay according to this price for each MWh they consume. If considered the congestion component and the marginal component of the price, it will be used the Locational Marginal Cost (LMP).

As can be seen, this type of market does not adopt the pay-as-bid scheme. If this were the case, the prices paid to the winning suppliers will be based on their actual offers, rather than the higher-priced supplier's offer. Yet, this would no occur since the generators would offer energy at a price that would never reflect marginal cost of production per MWh and would present offers very close to the SMP. As there is a great uncertainty regarding the market price in each resolution, the generators slightly increase the price of their production cost to have profits and just enough to avoid the risk of staying out of the market. Here is the reason why, an algorithm that allows a private generating company to know what the market price is going to be, is so important. Nevertheless, the scope in this project is more ambitious by going further and generate an even greater competitive advantage by getting the prices of the closest rivals to the SMP. This not only makes the company increase its profits, but also allows it to push rivals out of social welfare and make their profits go down. It permits total control of the critical area where the market usually resolves.

Hereafter, the electrical markets used in this project will be presented. The first will be a market composed by 3 nodes and with restrictions in the interconnections between them. It is a very complete market that can reflect the results of a real market although all the necessary data for the preparation of the same should be confirmed. Then a second, simpler market will be described in order to generate results with respect to a single market but with more generating companies.

2.2.3 The 3-Node Electricity Market

2.2.3.1 Layout of the Market

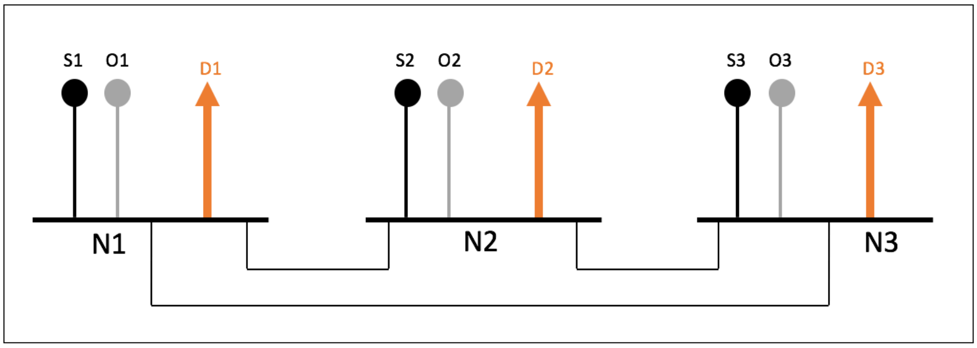

Figure 2.1: 3-Node Electricity Market Overall.

In this section, it will be described the market together with its modeling which will be used for the application of the first alternative, the inverse optimization problem. There are 3 nodes communicated among them through 3 interconnections lines. As in this first perspective it is the strategic company that wants to take advantage, it will be distinguished in the group of energy generating companies the strategic producer (S) and its rival producer (O), presented in the following sections. It will be considered that all the data of the strategic producer in each node is known because the problem is solved from the point of view of the private company and not from a global perspective. Therefore, both offer price and block size are known. In addition, the marginal costs of the rival producer are also known and will be used as the base price to generate their offering price. Each node is composed of a demand and the two generating companies that offer at different prices, creating a direct competition between them. The demands will be different, being node 3 the most demanding one creating imbalances in some periods and thus creating a good sweep in the outputs of the market clearings. For further clarification, figure 2.1 shows a global scheme where the basic structure of the market can be visually understood.

2.2.3.2 Technical Data from the Grid

By simulating the market, productions and demands accepted blocks in each node will be obtained, resulting in 3 market prices per hour. But the market price will also depend on technical restrictions of the network. All transmission lines between nodes will be identical with a capacity (expressed in terms of active power) of 100 MW and a susceptance of 1000 p.u. The node 1, will be the slack bus or reference bus and therefore its angle will be forced to be the reference (0º) and the angle of the rest of the buses will oscillate between -180º and 180º. Buses or nodes 2 and 3 are considered to be PQ type. In addition, it will be seen later that a high demand peak in node 3 together with the interconnection and ramping limits of the generators will also create unsatisfied demand, a result that in principle is not of interest for the method but that helps understanding on how the market behaves.

2.2.3.3 Strategic and Rival Generating Companies

Leaving aside the technical aspects of the network, the details of the two generating companies will be presented. To facilitate the generation of data and the understanding of the market, the marginal costs of energy production by the strategic producer will be the same in all three nodes. These costs will represent the minimum bid price and, although the cost will be the same in each node, the corresponding offer will not be the same as the added price will vary (see equation 2.1). Even so, both the block size and its marginal price will not vary between nodes.

Nodes	**Block**	**Marginal Cost [€/MWh]**	**Block Size [MW]**
1-3	1	10	100
1-3	2	15	100
1-3	3	20	100
1-3	4	25	100
1-3	5	30	100
1-3	6	35	100
1-3	7	40	100
1-3	8	45	100

Table 2.1: Strategic Producer Data.

In the table 2.1, the data representing this strategic company is collected. On the other hand, the same does not happen with the rival company since its marginal costs will vary depending on the node, although its block size does remain at 100 MW as happens with the strategic company. These prices are not going to be those of supply but the minimum price on which to base the same as before. In the equation 2.1 it is indicated how the offer prices are generated by the two companies. The additive part to the cost will vary for each node, block, hour and day. In the same way, the costs and size of the rival company's block are left in the table 2.2.

B.	**M. C. Node 1 [€/MWh]**	**M. C. Node 2 [€/MWh]**	**M. C. Node 3 [€/MWh]**	**B. Size [MW]**
1	12	15	13	100
2	17	17	16	100
3	20	21	23	100
4	24	22	25	100
5	29	30	27	100
6	33	31	31	100
7	41	38	42	100
8	47	45	48	100

B. = Block. M. C. = Marginal Cost

Table 2.2: Rival Producer Data.

Taking the marginal costs as base values, the following equation will be used when generating offering prices. Being λ^{MC} the marginal cost per MWh, $\lambda^{S/O}_{dtnb}$ the final offering price and Ω_{dtnb} a random number between [0,1]:

$$\lambda^{S/O}_{dtnb} = \lambda^{MC}(1 + 0.1\,\Omega_{dtnb}) \tag{2.1}$$

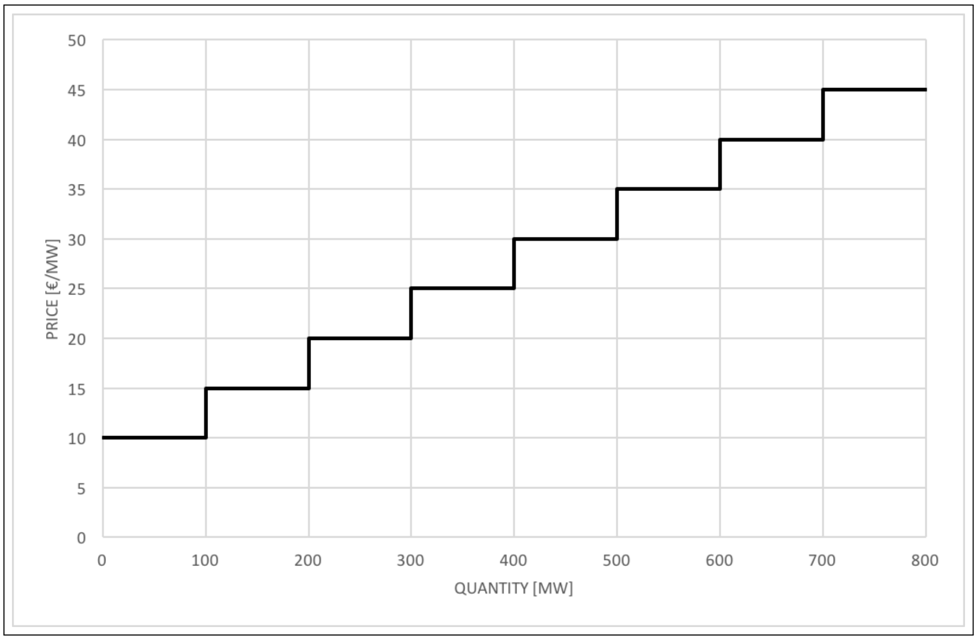

Figure 2.2: Strategic Producer Offer Curve.

The production of the companies are subdivided into 8 energy blocks in both cases. This way of presenting offers is very common in today's electricity markets because it facilitates the versatility in the combinatorial when creating a strategy and thus maximize profits. As blocks are accepted, the marginal price increases in both cases. This order of ascending prices can be seen more graphically in the figure 2.2 for the strategic company and in the figure 2.3 for the rival company.

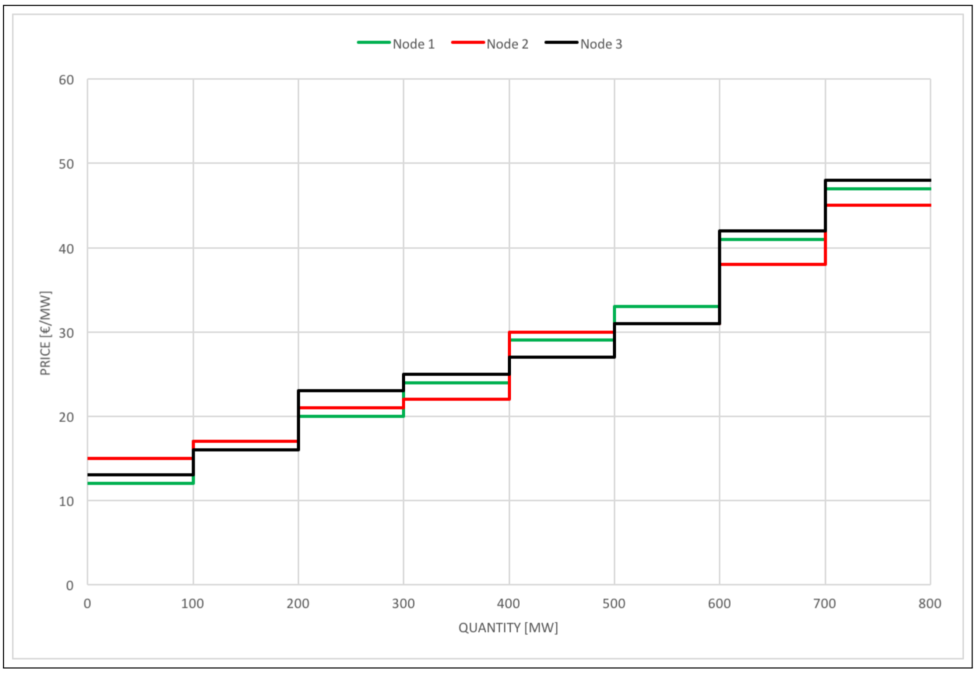

Figure 2.3: Rivals Producer Offer Curves.

In addition to these aspects related to generators, ramping constraints are also taken into account, which will be limited to 1000 MW/h in both directions when increasing and a decreasing the production. The blocks will be used to limit the maximum production, therefore $P_{nb}^{Gmax} = P_{nb}^{Omax} = 100$ MW due to all supply blocks are equal. The initial value of production will be null: $P_{nb}^{Gini} = P_{nb}^{Oini} = 0$ MW, data to be taken into account for the ramping in the first simulation period of every day because they are different auctions.

2.2.3.4 Demand

Since the inverse problem deals with the demand in an elastic way, it is presented by blocks together with the associated prices as occurs in the generating companies. In cases where is desired to treat the demand as a single value, a simple calculation by adding the resulting accepted blocks is done and together with the energy offered the market is cleared and thus it is obtained the market price. As shown in the table 2.3, the blocks are no longer all equal being the last four a half of the rest in terms of size. Although having two more blocks of demand (10 in total), if compared with the number of blocks by the generators, the final computation of energy supply and demand per actor is the same. In addition to the table, its curves have also been illustrated in the figure 2.4.

Block	Price [€/MW]	Block size (N1&N2) [MW]	Block size (N3) [MW]
1	50	100	400
2	45	100	400
3	42	100	400
4	40	100	400
5	35	100	400
6	30	100	400
7	25	50	200
8	20	50	200
9	15	50	200
10	10	50	200

N = Node

Table 2.3: Demand Data.

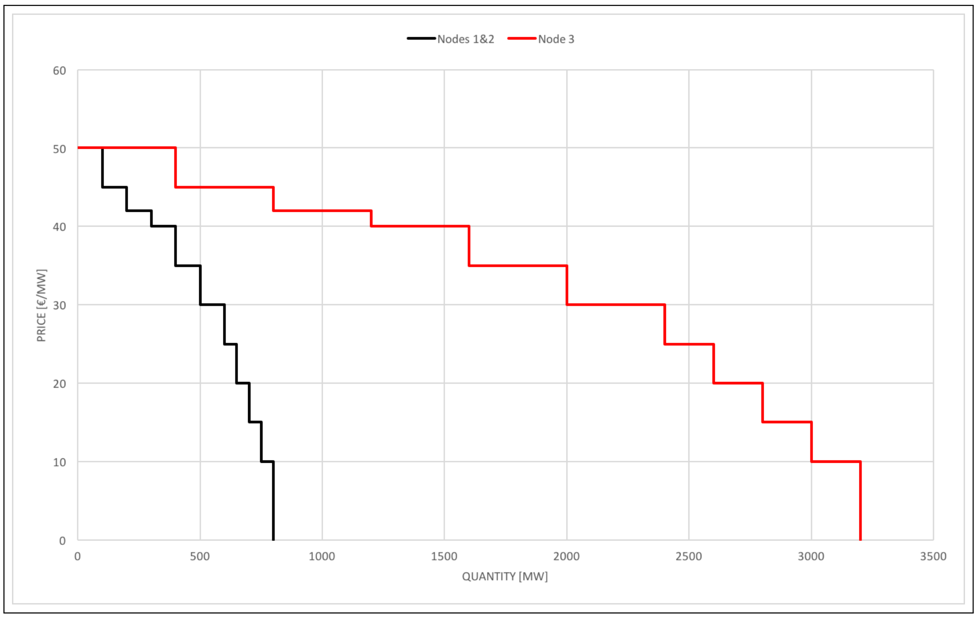

Figure 2.4: Demand Bidding Curves.

It is well known that demand varies according to the time of day, the day of the week and the month of the year. The same energy is not consumed at 2 a.m. that at 13 p.m. and activity in both households and industries is different on a Tuesday compared to a Sunday, for instance. Due to these changes, the electricity tariff also varies in price depending on the hour of the day and on the day of the week. As a result of this behavior, three types of day have been represented with some associated factors. The factors to be multiplied by the demand are represented in the figure 2.5: base, shoulder and peak. Thanks to these three factors, the demand throughout the day varies so that the spoken behaviors are represented. In addition, there are special days where the demand is more demanding or less, that is why the base and peak factors are considered.

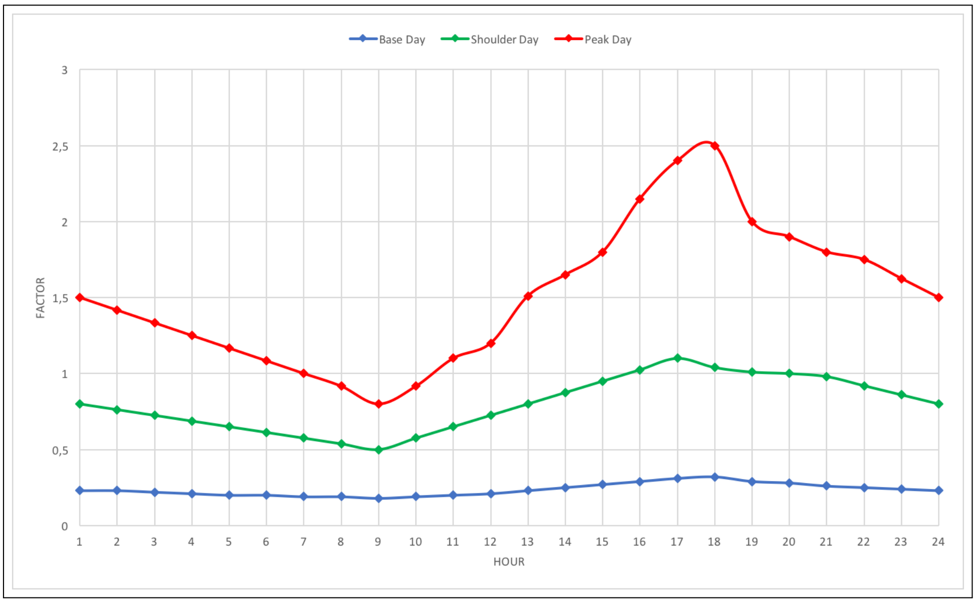

Figure 2.5: Demand factor per hour for three representative days.

2.2.3.5 Equations

As the equations describing this market are part of the inverse problem, the model is left at the reader's disposal in the section 3.1.2. The model corresponds with the primal formulation of the problem. However, in the next section it is left some results from a market clearing in order to have some reference of the order of magnitude of them.

2.2.3.6 Outcomes from Clearings

Even though the formulation regarding market clearing will be presented further in section 3, some results are presented here to have an order of magnitude. By simulating the market in a random period, in this case day 1 and hour 6, some results are obtained as showed below. In node 1 for a demand of 866.66MW the offers reflected in table 2.4 are accepted together with their respective prices. The market price will be 30.46 €/MW in this case and all the demand will be fulfilled thanks to the transmission of 33.33 MW through the interconnection from node 2 to node 1. These results depend on all the parameters that make up the market, a priori unknown, as are the ramp restrictions of generators for instance. Although the results have been presented for one hour in a given node, this market will be simulated for the 30 days that make up a month resulting in several circumstances when applying alternative 1.

Block	Strategic's A. P. [MW]	S. Off. Price [€/MW]	Rival's A. P. [MW]	R. Off. Price [€/MW]
1	100	10,401	100	12,901
2	100	15,168	100	18,214
3	100	20,592	100	21,749
4	100	26,476	100	25,983
5	33,333	30,463	100	29,790
6	0	36,575	0	36,005
7	0	41,174	0	41,687
8	0	48,951	0	48,055

A.P. = Accepted Production; S. Off. Price = Strategic Offering Price; R. Off. Price = Rival Offering Price

Table 2.4: Market Outcomes in Node 1 for day 1 and hour 6.

2.2.4 1-Node Electricity Market

2.2.4.1 Layout of the Market

On account of the analysis that is to be applied in this project implies a single market, it is decided to reduce the previous market to a single node by eliminating their respective connections and varying their components just a little. A market consisting of 9 generating companies with a single demand is proposed, as shown in the figure 2.6.

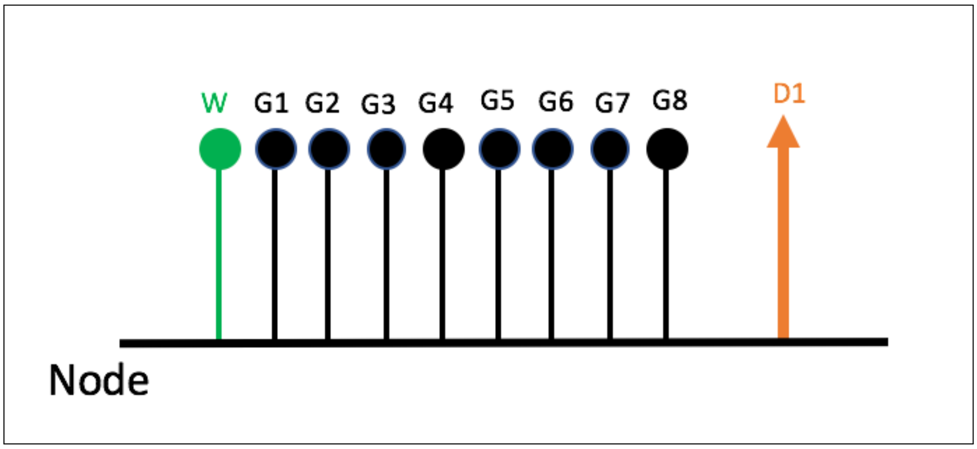

Figure 2.6: 1-Node Electricity Market Overall.

Currently, not all generating companies have an associated marginal cost since their production depends on a natural resource such as it is solar radiation, wind or even tides among others. As a consequence, it is considered convenient to introduce to the market a wind farm with 100 % clean production which will represent all the production methods which have a lack of production cost. This leads to a more realistic supply curve where the first stretch of the curve will be formed by offers at cost 0 and therefore the staggering will begin shifted to the right resulting in lower prices compared to the previous nodes. Figure 2.7 shows a market clearing indicating how the renewable energies are integrated in the forenamed curve. It will be showed later in section 3.1.2, the figure 3.1 where will be presented the case that zero-cost generators are not taken into account and therefore the price is higher if it is compared.

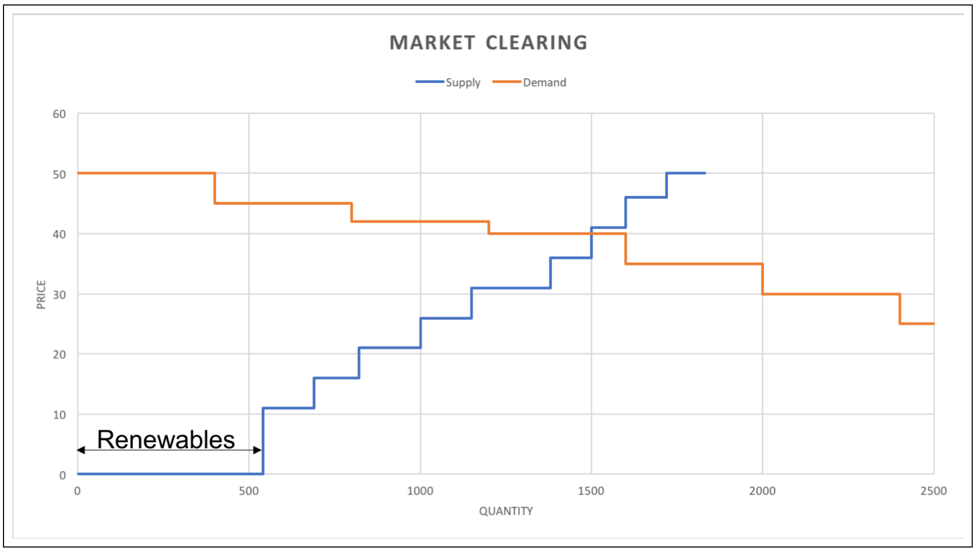

Figure 2.7: Market Clearing for the 1-Node Electricity Market.

2.2.4.2 Generating Companies

As mentioned in the previous section, wind power (W) represents the renewable part of production. On the other hand, each of the remaining 8 generators will represent different production methods such as nuclear, hydroelectric or gas, among others. There are also changes in the way of offering the energy since it has been considered that there is no more than one block per company. In the end, the inclusion of blocks per generator can be considered as entering more generators in the market. That is why companies are presented instead of blocks and, moreover, since the objective is to reveal prices, it is considered irrelevant for a didactic application to distinguish a company from a block since in the end a revealed price will be associated with an offer. Table 2.5 collects the data formulated in the two following points where are described the generation groups:

- Wind Power Company (W): By having a completely renewable production, its marginal cost will be set at zero and therefore it will offer at zero cost its produced energy in the market. As the nominal production capacity of wind turbines is small compared with traditional methods of energy generation, such as a nuclear or coal power plant, 8 blocks of 30 MW will be established, thus being an integration of around 25% of renewable energies in the market. This parameter can be changed depending on the real characteristics of the real market. This Wind Power company will represent the total amount of free-cost energy production as it is only remarkable how is the price pushed in relation with the total renewable energy block. Moreover, it is possible to collect the renewable production of Denmark from public data so this would be a known parameter in a real application.

- Non-Renewable Companies (C1-C8): Together with the Wind Power Generator, eight more non-renewable generators will be found in order to have marginal costs when producing. These companies will be able to offer one block each of which will be of 100 MW. The marginal costs of the blocks will be ascending in all the companies although their prices will be different if a given block is compared. With this, it is achieved variety in the offerings and also each company will represent a method of generating energy. In order to create competitiveness apart from obtaining different market clearings in each of the simulations, to the marginal costs will be added an amount which will represent the profit of each electricity company. It will be assumed that, when a block is available, it will be fully offered, what is known in the electrical market as an all-in, and its starting price will be bounded between the marginal cost and a maximum of five more added euros. That is, if the block of Company C1 has a marginal cost of 10€, then its offering price will oscillate randomly in each time frame between 10 € and 15 €. Overlapping in the offers is achieved and therefore some variety in the market clearing results. With this, it is assumed to be a little closer to the reality of the market behaviour. Finally, the added part to the marginal cost is considerable when compared with the 3-Node Market and therefore is more difficult to reveal its price due to its spectrum. In equation 2.2 it is left how these prices are generated.

	Number of blocks	**Block size [MW]**	**M. C. [€/MW]**	**Offering Price [€/MW]**
Wind Power (W)	8	30	0	0
Company C1	1	100	10	10-15
Company C2	1	100	15	15-20
Company C3	1	100	20	20-25
Company C4	1	100	25	25-30
Company C5	1	100	30	30-35
Company C6	1	100	35	35-40
Company C7	1	100	40	40-45
Company C8	1	100	45	45-50

M.C. = Marginal Cost

Table 2.5: Market Data.

Again, taking the marginal cost as base values, the following equation will be used when generating offering prices.Being λ^{MC} the marginal cost per MWh, λ^{G}_{dtnb} the final offering price and Ω_{dtnb} a random number between [0,1]:

$$\lambda^{G}_{dtnb} = \lambda^{MC} + (5\,\Omega_{dtnb}) \tag{2.2}$$

2.2.4.3 Other specifications of the Market

The fundamental part of the changes in the supply part between the two markets are reflected in the previous section. As for the demand taking as reference the curve of node 1 and the indices of the three representative days, data is generated for a whole year. Because bid prices sweep a greater range of values, there is a high variability of results when market clearing is done. For the resolution of the market, the same equations will be used as in the previous market with some caveats:

- The demand is treated in a non-elastic way: the values of each block that enters the market will be taken and multiplied by the corresponding factor depending on the type of day. Once said calculation is made, the final result of each block is added and the demand is treated as a single value.

- As there are no nodes, there are no interconnections and therefore there are no capacity restrictions of the transmission lines. There is also no reference node and therefore it is not relevant to force the only node to be the slack bus.

- Since the calculated demand is presented as a single value and also does not have nodes, the equations that describe the market will vary with respect to the formulation of 1-Node Market. These equations are described in the following section.

CHAPTER 3

Methods

Many applications could be considered when getting hidden data from a model given available information. The reviewed literature is an example of how by means of a correct formulation which reflects some behaviour, this exercise can be attained.

By cause of its simplicity and authenticity, the first part of this project will be a reproduction and an analysis carried out in [8]. In this section the Inverse Optimization equations are presented and explained. Together with the algorithm, the formulation of the model, which is a market clearing, is also exposed.

Finally, as an alternative to the previous method, the Ensemble Kalman Filter is explained taking as reference the Simple Kalman Filter. In this last section all steps are analyzed and come along with graphs and representations.

3.1 Inverse Optimization Problem

3.1.1 Introduction

Since the late 1980s, the Inverse Problem has been an object of research in various fields of science. Its application in recent years has been focused more on geophysics, medical imaging or even on traffic balance problems, although other applications of the algorithm are found in other fields such as engineering. Actually, in order to apply an inverse optimization problem, a physical system is required which must be modeled and which also may produce observable values. This type of problem is described as a forward problem because it identifies the values of the observable parameters given the values of the parameters of the model or, in other words, infers in the values of the model parameters given the values of the observed parameters or optimal decision variables [12].

For the presented application in this section, as applied in [8], the model will be an electrical market through which values of the observed parameters will be obtained and will be the results of the market clearing. Therefore, the appointed model will be constituted by a series of equations representing a market clearing, bearing in mind that the same results could be obtained by applying an optimization problem of the economic dispatch type as it is done in [13].

It will be considered a strategic producer that offers electricity in a given market, taking into account that all the parameters related to the named producer are known as a reference company is needed to make a strategy with respect to some competitors. As a priori information, aspects of the network will also be taken into account, such as the technical parameters regarding the interconnection between nodes. Although this information is generally not directly available from the strategic producer, for the application of the problem the marginal production costs of each of the competitors' supply blocks will be used. On the other hand, accepted blocks of generation and demand after each market clearing will be also considered as available data.

Therefore, through this application of the Inverse Optimization Problem, it will be possible to reveal bid prices of competitors that in some of the study periods have been marginal and that, as a consequence, have also had an impact on the results of the market clearing such are prices in each node and both production and demand accepted blocks.

In the following section, the equations of the model to be treated and the subsequent application of the Optimization problem will be presented.

3.1.2 Market Clearing Model

As has been well presented previously, the model to which the parameters are to be inferred is a market clearing. The notation of the problem is left for the reader's knowledge in the chapter symbols. As a clarifying note regarding the symbols used in this section, if a parameter depends on variables and then in the formulation they are not found, it is because their value depends on these forgotten variables. In addition, in one case it will be had a variable that will also behave as a parameter: this will be explained throughout the formulation.

Finally, and before presenting the model, contrary to what is proposed in [12], KKT conditions will not be applied to obtain results. Taking advantage of the fact that the model is linear, the strong theorem of duality will be applied resulting in satisfactory results.

3.1.2.1 Primal Problem Formulation

Considering a market of the pool type, which has a very similar behavior in the different markets of both Europe and the USA, the following linear problem is presented which will seek to maximize the social welfare of the forenamed market. For this purpose, it will be needed an offer and a demand. On the supply side, a set of strategic and rival producers are considered, which will offer different amounts of energy represented by energy blocks at different prices. It should be emphasized that the strategic company will be differentiated in the formulation being all the data known and from the rival company as the final objective is to reveal its hidden information. On the demand side, it will also be represented by blocks of energy that vary in their quantity and price. The maximization problem will allow the basic concept of the electricity market to be applied, which is to order both offers and demands through the merit order ranking, that is: the offers where the price of energy is cheaper and the bids where the price of energy is more expensive, they will be taken as priority in the calculation of the equilibrium point. Figure 3.1 shows how the resolution of a market clearing would be as a consequence of a maximization of the social welfare.

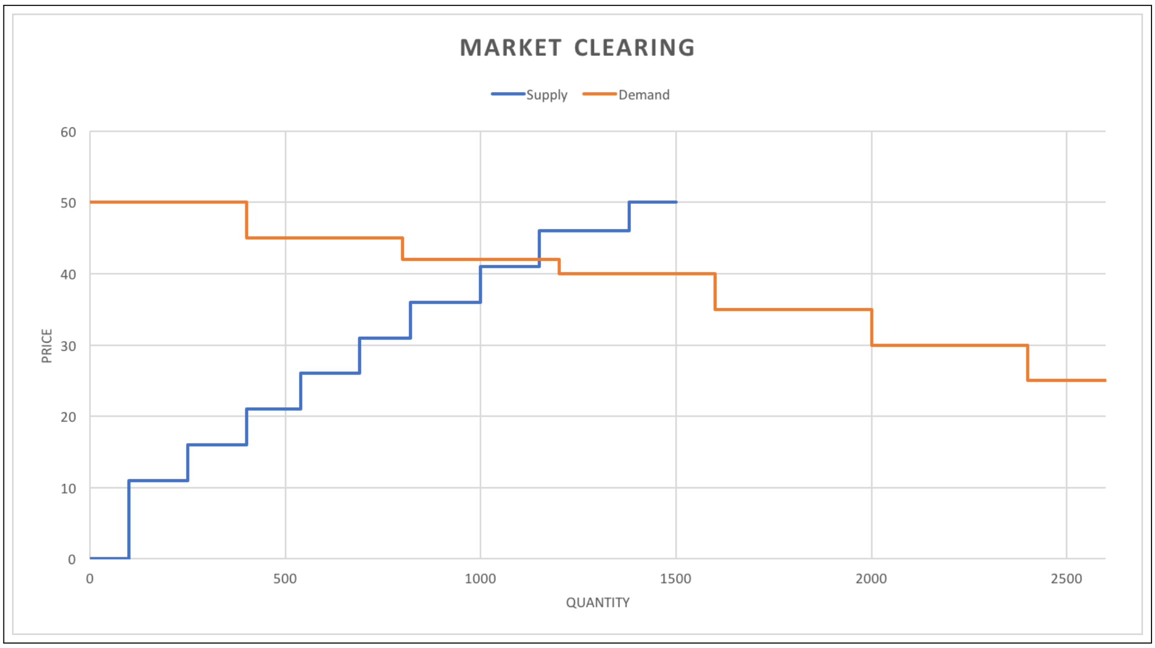

Figure 3.1: Market Clearing.

This equilibrium point will be calculated by the independent system operator (ISO), which will solve the market and fix the quantities accepted by the related actors and the final price. This problem is applied every hour and the formulation is decomposed independently from day to day, which will give pertinent results to 24 time blocks in a single day period. Therefore, problems can be treated in a broken manner but always taking into account that the smallest temporal period is a day as shown by the equations 3.1a - 3.1l.

$$\underset{P^G_{dtnb}, P^O_{dtnb}, P^D_{dtnk}, \delta_{dtn}}{\text{Minimize}} \quad \left[\sum_{tnb} \lambda^G_{dtnb} P^G_{dtnb} + \sum_{tnb} \lambda^O_{dtnb} P^O_{dtnb} \right] - \sum_{tnk} \lambda^D_{dtnk} P^D_{dtnk} \tag{3.1a}$$

subject to:

$$\left[\sum_b P^G_{dtnb} + \sum_b P^O_{dtnb}\right] - \sum_k P^D_{dtnk} = \sum_n B_{nm}(\delta_{dtn} - \delta_{dtm}) : \quad \lambda_{dtn} \quad \forall t, n, b \in \Theta_n \quad (3.1b)$$

$$0 \leq P^G_{dtnb} \leq P^{Gmax}_{nb} : \quad \mu^{Gmin}_{dtnb}, \mu^{Gmax}_{dtnb} \quad \forall t, n, b \quad (3.1c)$$

$$0 \leq P^O_{dtnb} \leq P^{Omax}_{nb} : \quad \mu^{Omin}_{dtnb}, \mu^{Omax}_{dtnb} \quad \forall t, n, b \quad (3.1d)$$

$$0 \leq P^D_{dtnk} \leq P^{Dmax}_{dtnk} : \quad \mu^{Dmin}_{dtnk}, \mu^{Dmax}_{dtnk} \quad \forall t, n, k \quad (3.1e)$$

$$-R^{Gdwn}_n \leq \sum_b P^G_{d1nb} - \sum_b P^{Gini}_{dn} \leq R^{Gup}_n : \quad \mu^{Gdwn}_{d1n}, \mu^{Gup}_{d1n} \quad \forall n \quad (3.1f)$$

$$-R^{Gdwn}_n \leq \sum_b P^G_{dtnb} - \sum_b P^G_{d(t-1)nb} \leq R^{Gup}_n : \quad \mu^{Gdwn}_{dtn}, \mu^{Gup}_{dtn} \quad \forall t > 1, n \quad (3.1g)$$

$$-R^{Odwn}_n \leq \sum_b P^O_{d1nb} - \sum_b P^{Oini}_{dn} \leq R^{Oup}_n : \quad \mu^{Odwn}_{d1n}, \mu^{Oup}_{d1n} \quad \forall n \quad (3.1h)$$

$$-R^{Odwn}_n \leq \sum_b P^O_{dtnb} - \sum_b P^O_{d(t-1)nb} \leq R^{Oup}_n : \quad \mu^{Odwn}_{dtn}, \mu^{Oup}_{dtn} \quad \forall t > 1, n \quad (3.1i)$$

$$B_{nm}(\delta_{dtn} - \delta_{dtm}) \leq P^{max}_{nm} : \quad \nu^{max}_{dtnm} \quad \forall t, n, m \in \Theta_n \quad (3.1j)$$

$$-\pi \leq \delta_{dtn} \leq \pi : \quad \xi^{min}_{dtn}, \xi^{max}_{dtn} \quad \forall t, n \quad (3.1k)$$

$$\delta_{dtn} = 0 : \quad \xi^1_{dt} \quad \forall t, n = 1 \quad (3.1l)$$

The equation to optimize (1a) is the minus social welfare which is the difference between supply and demand. Instead of maximizing the left region a minimizing problem is applied to its right one. The network is represented through a dc linear model, and the power balance at every node is enforced by equation (1b). This means that if there is a surplus when producing or demanding there will be a difference and will result in a power flow through the transmission line between nodes that must respect the saturation limits imposed by the technical aspects in the interconnections. Equations (1c-1e) represent the generation limits of each of the companies, which will be equal to the size of the block that corresponds to them. Indeed, this constraint controls the block size depending on the actor. Equations (1f-1i) define the ramp time restrictions between the initial period and the first period as well as between periods for both the generators and the demands. This constraint deals with generators technical limitations which are considered to be known in this model. Equation (1j) fixes the interconnection in order to not overload the limit capacity of the line. Constraint (1k) indicates that the phase angle of the voltages in each node will be between 180º and -180º and constraint (1l) fixes the angle of the voltage in node 1 to 0 so that we have the bus as a reference and so the rest of angles take values with respect to mentioned bus.

3.1.2.2 Dual Problem Formulation

As can be seen in the previous formulation, each restriction is related to a dual variable. This is because this inverse optimization problem will use the strong duality theorem as a constraint instead of KKT (Karush-Kuhn-Tucker) conditions. It is recalled that the strong duality theorem ensures that if a Linear Programming (Primal) problem has an optimal solution, then the corresponding Dual problem also has an optimal solution, and their respective values in the objective function are identical. Taking this into account, the dual formulation of the previous market clearing is left:

$$\underset{\Xi_d}{\text{Maximize}}$$

$$-\sum_{tnb} P^{Gmax}_{nb}\mu^{Gmax}_{dtnb} - \sum_{tnb} P^{Omax}_{nb}\mu^{Omax}_{dtnb} - \sum_{tnk} P^{Dmax}_{dtnk}\mu^{Dmax}_{dtnk} + \sum_{n}\mu^{Gdwn}_{d1n}(P^{Gini}_{dn} - R^{Gdwn}_{n})$$
$$-\sum_{t>1,n}\mu^{Gdwn}_{dtn}R^{Gdwn}_{n} - \sum_{n}\mu^{Gup}_{d1n}(R^{Gup}_{n} + P^{Gini}_{dn}) - \sum_{t>1,n}\mu^{Gup}_{dtn}R^{Gup}_{n} + \sum_{n}\mu^{Odwn}_{d1n}(P^{Oini}_{dn} - R^{Odwn}_{n})$$
$$-\sum_{t>1,n}\mu^{Odwn}_{dtn}R^{Odwn}_{n} - \sum_{n}\mu^{Oup}_{d1n}(R^{Oup}_{n} + P^{Oini}_{dn}) - \sum_{t>1,n}\mu^{Oup}_{dtn}R^{Oup}_{n} - \sum_{tnm}P^{max}_{nm}\nu^{max}_{dtnm}$$
$$-\sum_{tn}\pi\xi^{min}_{dtn} - \sum_{tn}\pi\xi^{max}_{dtn} \tag{3.2a}$$

subject to:

$$\lambda_{dtn} - \lambda^{G}_{dtnb} + \mu^{Gmin}_{dtnb} - \mu^{Gmax}_{dtnb} + \mu^{Gdwn}_{dtn} - \mu^{Gdwn}_{d(t+1)n} - \mu^{Gup}_{dtn} + \mu^{Gup}_{d(t+1)n} = 0 \quad \forall t < T, n, b \tag{3.2b}$$

$$\lambda_{dtn} - \lambda^{G}_{dtnb} + \mu^{Gmin}_{dtnb} - \mu^{Gmax}_{dtnb} + \mu^{Gdwn}_{dtn} - \mu^{Gup}_{dtn} = 0 \quad t = T \quad \forall n, b \tag{3.2c}$$

$$\lambda_{dtn} - \lambda^{O}_{dtnb} + \mu^{Omin}_{dtnb} - \mu^{Omax}_{dtnb} + \mu^{Odwn}_{dtn} - \mu^{Odwn}_{d(t+1)n} - \mu^{Oup}_{dtn} + \mu^{Oup}_{d(t+1)n} = 0 \quad \forall t < T, n, b \tag{3.2d}$$

$$\lambda_{dtn} - \lambda^{O}_{dtnb} + \mu^{Omin}_{dtnb} - \mu^{Omax}_{dtnb} + \mu^{Odwn}_{dtn} - \mu^{Oup}_{dtn} = 0 \quad t = T \quad \forall n, b \tag{3.2e}$$

$$-\lambda_{dtn} + \lambda^{D}_{dtnk} + \mu^{Dmin}_{dtnk} - \mu^{Dmax}_{dtnk} = 0 \quad \forall t, n, k \tag{3.2f}$$

$$\sum_{m} B_{nm}(\lambda_{dtm} - \lambda_{dtn}) + \sum_{m} B_{nm}(\nu^{max}_{dtmn} - \nu^{max}_{dtnm}) + \xi^{min}_{dtn} - \xi^{max}_{dtn} + (\xi^{1}_{dt})_{n=1} = 0 \quad \forall t, n \tag{3.2g}$$

$$\mu^{Gmin}_{dtnb}, \mu^{Gmax}_{dtnb}, \mu^{Omin}_{dtnb}, \mu^{Omax}_{dtnb}, \mu^{Dmin}_{dtnb}, \mu^{Dmax}_{dtnb}, \mu^{Gdwn}_{dtn}, \mu^{Gup}_{dtn}, \mu^{Odwn}_{dtn}, \mu^{Oup}_{dtn}, \nu^{max}_{dtmn},$$
$$\xi^{min}_{dtn}, \xi^{max}_{dtn}, \xi^{1}_{dt} > 0 \quad \forall t, n, b, k \tag{3.2h}$$

where:

$$\Xi_d = \Big\{\lambda_{dtn}, \mu^{Gmin}_{dtnb}, \mu^{Gmax}_{dtnb}, \mu^{Omin}_{dtnb}, \mu^{Omax}_{dtnb}, \mu^{Dmin}_{dtnb}, \mu^{Dmax}_{dtnb}, \mu^{Gdwn}_{dtn}, \mu^{Gup}_{dtn}, \mu^{Odwn}_{dtn}, \mu^{Oup}_{dtn},$$
$$\nu^{max}_{dtmn}, \xi^{min}_{dtn}, \xi^{max}_{dtn}, \xi^{1}_{dt}\Big\} \quad \text{are the dual variables.}$$

Remark that, contrary to the inverse problems which use as a model a unit commitment optimization problem, this market clearing has the advantage of not considering discrete decisions variables. Taking this into consideration, the formulation becomes linear and thus not convex, helping to derive its associated dual problem. Moreover, it will be required less computation running time if compared and it could be said that it reflects fairly how the current European markets behave from an external point of view.

Both problems primal and dual will be used in the following section when formulating the inverse optimization problem.

3.1.3 Inverse Problem Formulation

For the resolution of the Inverse Problem, a prior market clearing must be carried out, in order to obtain data such as the accepted production of the strategic producer and that of each rival, as well as the accepted demand blocks and the market clearing price in each node. Once this data set is obtained, it is proceeded to the first version formulation of the Inverse Problem:

$$\underset{\Lambda}{\text{Minimize}} \quad \sum_{tnb} |\lambda^{O}_{dtnb} - \lambda^{Oini}_{dtnb}| \tag{3.3a}$$

subject to:

$$(3.1a) = (3.2a) \quad \forall d \tag{3.3b}$$

$$(3.2b) - (3.2h) \quad \forall d \tag{3.3c}$$

where:

$$\Lambda = \Big\{ \lambda^{O}_{dtnb}, \mu^{Gmin}_{dtnb}, \mu^{Gmax}_{dtnb}, \mu^{Omin}_{dtnb}, \mu^{Omax}_{dtnb}, \mu^{Dmin}_{dtnb}, \mu^{Dmax}_{dtnb}, \mu^{Gdwn}_{dtn}, \mu^{Gup}_{dtn}, \mu^{Odwn}_{dtn}, \mu^{Oup}_{dtn},$$

$$\nu^{max}_{dtmn}, \xi^{min}_{dtn}, \xi^{max}_{dtn}, \xi^{1}_{dt} \Big\} \quad \text{are the variables of the above problem.}$$

The main objective of the previous optimization problem is to bring as close as possible the different values of offer prices to an initial estimate given for each price. Note that the initial estimate will change throughout simulations. Restriction (3.3b), as discussed above, applies the strong theorem of duality by forcing the solution vector of the objective function of the primal problem to be equal to the solution vector of the objective function of the dual problem. In addition, and previously solving the problem (3.3a), it has been necessary to make use of solutions of the market clearing problem and thus the restrictions of the primal problem are not necessary to constraint the problem of inverse optimization.

Once the inverse problem has been raised, the last qualification in relation to the formulation must be clarified. The problem must be linearized in order to be able to apply a solver for a linear programming and in this sense reduce computation time. By means of a simple linearization method, the following problem will be considered as the definitive one as exposed in [12]:

$$\underset{\Lambda,\alpha_{dtnb},\beta_{dtnb}}{\text{Minimize}} \quad \sum_{dtnb}(\alpha_{dtnb}+\beta_{dtnb}) \tag{3.4a}$$

subject to:

$$\lambda^{O}_{dtnb}-\lambda^{Oini}_{dtnb}=\alpha_{dtnb}-\beta_{dtnb} \quad \forall d,t,n,b \tag{3.4b}$$

$$\alpha_{dtnb},\beta_{dtnb}\leq 0 \quad \forall d,t,n,b \tag{3.4c}$$

$$(3.3b)-(3.3c). \tag{3.4d}$$

The solution to the previous problem will be collected in the variable λ^{*O}_{dtnb}, which will indicate the offering price of each of our rivals depending on the calculation period and energy block. Analyzing the formulation of the problem, it can be observed that when a supply price that is at the same time marginal in the market clearing and therefore generates a market price in a given node for a given period, the initial estimate made λ^{Oini}_{dtnb} will not influence the result. However, having initial estimates for each of the rival blocks, when a price is not shown, it will be forced to have the optimal value equal to our assumption. Therefore, if there are offers of rivals which are not marginal, that is, if they are part of the accepted production and are always below the equilibrium point whatever the period, it will be forced to an initial estimate value without its value being proven.

3.2 Ensemble Kalman Filter

3.2.1 Introduction

The Ensemble Kalman Filter (*EnKF*) is a recursive filter which is used as a computational technique in order to inference models composed by state variables that change in a given Euclidean space. This structure of space is presented within this technique because for the representation of the different states of the variables it will be needed a vector gathering all the associated values (that correspond to a Probability Density Function) and will be represented in a given region of the mentioned space. As there are recursive transitions within each time step, two dimensions of the Euclidean space will be used to get the final result given by the algorithm and will be used to advance throughout the remaining dimension into the next application forward in time. This behaviour could be arduous to follow without a graphic representation, for this reason figures will come along when exposing equations to clarify each evolution of the variables.

Unlike the simple Kalman Filter, the *EnKF* works with an ensemble of vectors approximating a state distribution whatever its nature. In the following sections, it will be seen that in order to make an estimation of a variable, its values must first mature along a propagation and update of states following some model and observations. This representation through ensembles allows a reduction of the dimensions due to the propagation of a small part of the ensemble instead of all the values if it is necessary, making use of the partial covariance matrix of the sample (called sample covariance). When the initial guess of the state variable is propagated or updated, functions that represent its behavior as a function of time are needed and, one of the advantages offered by this method is that it does not require a linear propagation function or a model composed by non-Gaussian distributions, besides that the degree of dimensionality of the variables is not a hindrance when applying this method [14].

This filtering method has been used since the mid 90's when Geir Evensen applied it to the field of geophysics. Since this period, it has had considerable repercussion within the scope of science due to its small formulation and its wide range of application. A quite popular implementation is found in data assimilation processes for meteorological problems, where it must be applied for large amounts of data obtaining good results in terms of computational requirements when compared with other similar methods with more sophisticated objectives [15]. Besides, other applications apart from data assimilation are found. Such one example regarding estimating parameters between known states is suggested in [16] where atmospheric methane concentrations are calculated thanks to reliable observations from anthropogenic and biospheric sources. As commented, not all the applications seek the same outcome. In [17], the *EnKF* is used to adjust a given model by recalculating its previous parameters and perform what is known as history matching. This work has more to do with model validation. Taking as reference these examples, it is identified different applications in several fields of studies that succeeded supporting the versatility when applying this algorithm. Further in the following section, the basic formulation respecting the Kalman Filter is stated as a starting point.

3.2.2 Basic formulation

3.2.2.1 Kalman Filter

Before introducing the *EnKF* equations, it is valuable to understand first how a Simple Kalman Filter (*KF*) works. Point that the mechanism of both the Ensemble and the Simple is the same, being the first used when the systems are high-dimensional and the calculation of the covariance matrix is not computationally feasible. The *KF* is wide used in signal processing and assumes that all the state variables follow a Gaussian distribution. Its equations are split in a prediction and a correction part, being inferred recursively. This means that a process is estimated and then corrected by using a feedback control thanks to some measurements [18]. In the following figure, it is described how the algorithm progress in each time step and which could serve as a reference for the later formulation.

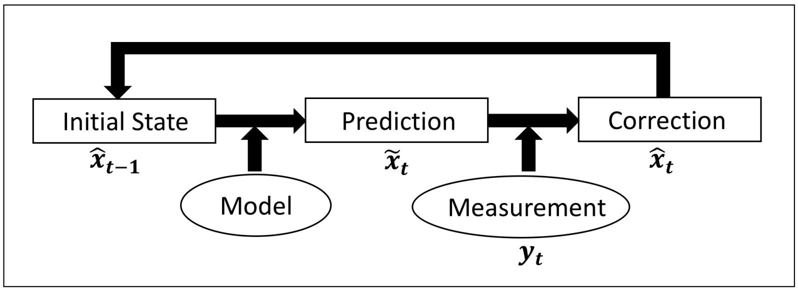

Figure 3.2: The ongoing discrete Kalman Filter cycle.

In a first step, it is assumed that a distribution of the variable is hold which will be called initial estimation of the state variable and will come from the previous time step. If prior information is not available, initial values for its mean and variance are associated without matter their magnitude. Therefore, it is had an initial information of the estate ($\hat{x}_{t-1}$, $\hat{P}_{t-1}$) which will be used throughout a couple of expressions to calculate the first guess of its new estate. For this purpose, it is required a model or a relation of the state between time steps. In equations 3.5 and 3.6 it is suggested to have a linear relation represented by the M_t matrix. This constant will be used to propagate the mean value and the covariance, resulting in $\tilde{x}_t$ and $\tilde{P}_t$. The prediction equations can be formulated as follows:

$$\tilde{x}_t = M_t \, \hat{x}_{t-1} \tag{3.5}$$

$$\tilde{P}_t = M_t \, \hat{P}_{t-1} \, M_t^T + w_t, \quad w_t \sim \mathcal{N}_n(0, Q_t) \tag{3.6a}$$

or

$$\tilde{P}_t = M_t \, \hat{P}_{t-1} \, M_t^T + Q_t \tag{3.6b}$$

Here, the resulting parameter is what is known as a priori estimation of the state vector in the following time period t. As assumed before, there is a linearity between states but this relation will depend on how the state vector behaves along time. Furthermore, the value of M_t should vary in time if the model behaves dynamically. To the resulting covariance, it is added a random error in this first estimation which will follow a normal distribution with a null mean and some Q_t standard deviation (also called white error) adding uncertainty to the resulted distribution. This can be graphically observed in figure 3.4.

As this first step is a prediction, it is required some measurement regarding the parameter to obtain an accurate final distribution as a result of an uncertainty reduction.

Accordingly, this measurement will infer in the *a priori state estimate* together with the Kalman Gainer which will been obtained based on the relation between both values. Being y_t an observation or measurement of the true state and assuming that there is a relationship between this observed value and the first estimation, the equation will be:

$$y_t = H_t\, \tilde{x}_t + v_t, \quad v_t \sim \mathcal{N}_n(0, R_t) \tag{3.7}$$

The matrix H_t will give a linear relationship between both variables along with an associated error. Here is again assumed to have a proportional between variables although it will depend on the nature of the model. This added error will have the same attributes as the one in equation 3.6a, being both of them independent. As y_t is an observation that comes from a measurement, it is supposed that an error is committed when getting the value. As happens with H_t, the process noise covariance Q_t, the measurement noise covariance R_t and the H_t matrix will change over time steps, that is why their subscript indicates a time dependence.

Once the observation is presented, it will serve by means of the Kalman Gainer (or blending factor) to correct the *a priori state estimate* represented by the results in equations 3.5 and 3.6. This constant will bring the mean of the first estimate towards the mean of the observation distribution as well as conceiving a reduction of the resulting final state estimate standard deviation. Assuming another time that an error will be associated and that will be exactly the same held in the previous equation, its formulation will be:

$$K_t = \tilde{P}_t\, H_t^T (H_t\, \tilde{P}_t\, H_t^T + R_t)^{-1} \tag{3.8}$$

This equation is considered to be the first one in the update step and its value will vary depending on how accurate is the a priori estimation. See that if the measurement noise covariance remains null, then the *optimal state estimate* will be equal to the measurement. On the other hand, if the a priori covariance $\tilde{P}_t$ yields to zero then the *optimal state estimate* would be the same the *a priori state estimate*. Here are found the bounds between which the Kalman Gainer is correcting the final result. One last remark, which comes from the last statement, is that if the a priori estimate error covariance is considerable, the value of the gainer will be higher to reduce it (see their proportional relation).

Finally, with the gainer already calculated, the correction of both the covariance error and mean value of the state estimate is achieved by applying the following equations:

$$\hat{x}_t = \tilde{x}_t + K_t(y_t - H_t\, \tilde{x}_t) \tag{3.9}$$

$$\hat{P}_t = (I - K_t\, H_t)\, \tilde{P}_t \tag{3.10}$$

As this method is recursive, after each time step will be a new measurement and therefore the process will be applied again following the presented equations. The notation on the subscripts will vary as an indicator of a step forward.

As the equations describing this process require some graphical guidance to deeply understand how the data flow is evolving along them, some figures are left below. In figure 3.3 it can be seen both predictive and update state along with their corresponding equations. In addition, some representative Gaussian distributions are plotted in figure 3.4 which will help to appreciate how the covariance errors are minimized. As appointed at the beginning of this section, the main application of this recursive filter is on signal processing as are the GPS systems. In this type of applications what is wanted, apart from an accurate result, is the reduction of the uncertainty that is already presented in the *a priori estimation*. In figure 3.4, this predicted state estimation has a *PDF* that follows a normal distribution with a mean value of 7 and a standard deviation of 1. Depending on the application, the value of the standard deviation may be acceptable or not but always is a good exercise to reduce it as it means that the distribution has lower error covariance and thus more credibility. Then, assuming that the measurement represent a lower erratic value and thanks to the correction conducted by the effect of the Kalman gainer, what results is an *optimal state estimate* with a lower error covariance if compared with the initial predicted state estimate. Even if the uncertainty in y_t is larger than the first predicted estimate which means that its associated R is large, the value of the Kalman gainer will absorb this particularity in each time step and the error will be mitigated along updates [19].

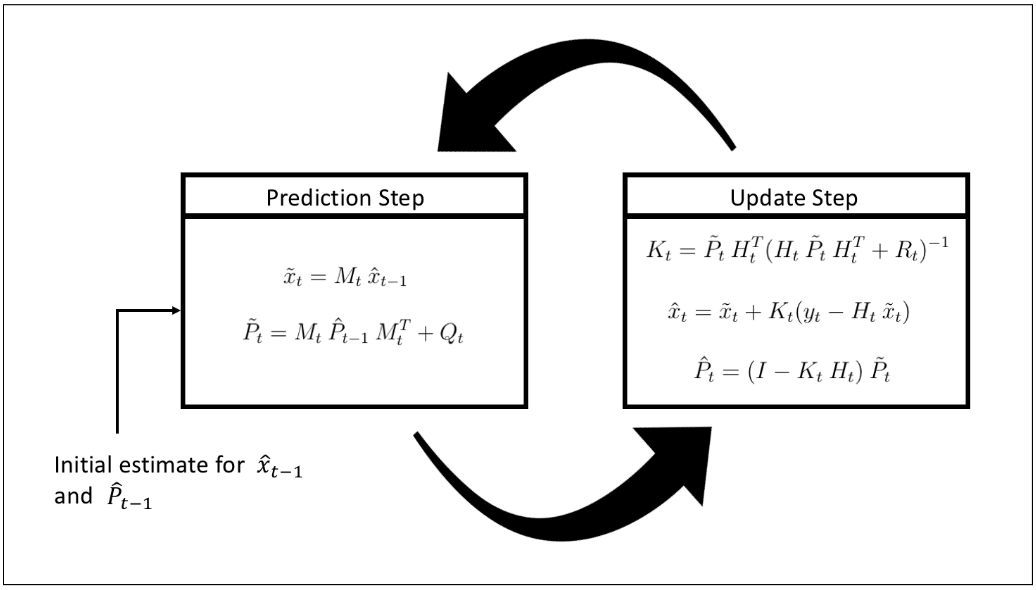

Figure 3.3: Steps in the *KF* cycle.

Figure 3.4: Optimal State Estimator Algorithm.

Some problems can come up depending on how the measured value is obtained. In these sample applications, it is supposed that a sensor takes some values about the position however, some interpretation is required when the measurement is a single value with a null R_t. This type of drawback is presented when extrapolating this algorithm to the purpose of this project and it will be analyzed properly. In any case, the Ensemble Kalman Filter will be exposed firstly taking as reference the algorithm above.

3.2.2.2 The Ensemble Kalman Filter

As took place with the *KF*, the Ensemble Kalman Filter behaviour is also based on the Bayesian update problem which uses the Bayes theorem. This implies that a *PDF* of a posteriori state variable can be obtained from a priori *PDF* taking into account its likelihood function. Bayesian update problem or Bayesian inference, calculates the a posteriori probability according to Bayes' theorem:

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)} \tag{3.11}$$

It is also often said that the Ensemble Kalman Filter is a consequence of the Kalman Filter if considering Monte Carlo approximations which rely on random sampling to obtain numeric results. By analyzing the following formulation, it will be appraised that both Bayesian Inference and Monte Carlo approximation have influence on the algorithm [20]. On the part of Bayesian Inference, as seen in the previous section, it will be the observation the distribution which will behave as the likelihood function to infer the previous guess. Monte Carlo methods will be applied when desired to generate ensembles from a given distribution. Thanks to this method, a simulation of results accounting for variability of its factors or inputs will be reached. Setting aside this first introduction, in the following paragraphs the formulation of this algorithm is presented.

From this point, the Gaussian distributions for each state variable will be formed from numbers which are known as realizations and that, when gathered, compose an ensemble. This means that the mean and variance of each *PDF* is no longer taken into account to calculate shifted values neither error covariance matrices. Instead, matrices will collect ensembles and the forecasting and update steps will be realized with them attaining to a great variability. Therefore, the initial state estimate will be constituted by a matrix which its row's dimension (n) will represent the number of individual initial states estimate $\hat{x}_{t-1}^{(i)}$ and its column's dimension (N) the number of samples gotten from each state variable. For instance, if it is had four temperatures in four different locations with a hundred samples for each one, the dimension of the matrix will be (4, 100). A fair representation could be the following:

$$\hat{x}_{t-1} = \begin{bmatrix} \hat{x}^{11} & \hat{x}^{12} & \hat{x}^{13} & \dots & \hat{x}^{1n} \\ \hat{x}^{21} & \hat{x}^{22} & \hat{x}^{23} & \dots & \hat{x}^{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{x}^{N1} & \hat{x}^{N2} & \hat{x}^{N3} & \dots & \hat{x}^{Nn} \end{bmatrix}_{t-1} \tag{3.12a}$$

$$\hat{x}_{t-1}^{(ij)} \sim \mathcal{N}_n(\hat{\mu}_{t-1}, \hat{\Sigma}_{t-1}), \quad i = 1, 2, ..., N. \quad j = 1, 2, ..., n. \tag{3.12b}$$

The previous matrix will be named the *initial state estimation ensemble*. The values conforming the columns of the matrix are directly related with the distribution depicted in equation 3.12b and therefore, the covariance error it is implicit in the values and that is the reason why only one equation is required. As it was proceeded with the *KF*, the *a priori state estimation* will be calculated by relating the presented ensembles with a given relation matrix, M_t also in this case, and with some added error w_t. The following formulation reflects the relationship.

$$\tilde{x}_t = M_t\, \hat{x}_{t-1} + w_t, \quad w_t \sim \mathcal{N}_n(0, Q_t) \tag{3.13a}$$

$$\tilde{x}_t^{(ij)} \sim \mathcal{N}_n(\tilde{\mu}_t, \tilde{\Sigma}_t), \quad i = 1, 2, ..., N. \quad j = 1, 2, ..., n. \tag{3.13b}$$

The resulting matrix will have the same dimensions as the initial guess matrix, therefore the dimension of M_t must be (N, N). Moreover, this relation matrix is important in the formulation as with its diagonal the individual propagation of the variables can be controlled. However, what is more, thanks to its square dimension every state variable can be correlated with each other when tuning properly the upper and lower triangles of the matrix.

Continuing with the formulation, as this is a forecast that must be inferred considering a measurement y_t in each time step, an update based on this value will be carried out. It is worthy to mention that two options are applicable when updating the first predicted ensemble: stochastically or deterministically. In this project, the updating will be done in a stochastic point of view as it is more logical for the nature of the model. To justify this decision, some differences between them are exposed below. Furthermore at the end of this section an example of the algorithm will be showed with a deterministic update to fully understand the main disadvantage.

When applying deterministic updates:

- There is no need of generating a simulated observation. The simulated observation is an initial guess of the measurement that will be corrected afterwards by the real one. This simulated parameter helps to control the shape of the final *PDF* which is valuable in the case of this project. Moreover, the observations in the application will not represent all the state variables collecting only one measurement per time step. Therefore, this matrix will also complete the estimation of all the variables without really seeing them. In next chapter all this formulation features will be properly explained.
- Before obtaining posterior draws of the state variable, it is required to apply two more steps called standardize and "unstandardize". This additional part comes also with a Trochowski decomposition when computing results with large ensembles, leading this alternative to be computationally unfeasible when applying to a large number of time steps which is the case.

Following the stochastic update and taking into consideration the result of equation 3.13a, the covariance sample matrix is calculated. Remind from the introduction part of this section that, the representation of the states by means of ensembles permits propagation making use of the partial covariance matrix of the sample. Hence, all realizations will be taken into account when reaching $\tilde{x}_t$ but not with the covariance matrix calculation as it is recommended an approximation which makes the algorithm CPU viable. From the *a priori state estimation ensemble*, the mean value of each column is determined and thus conforming the matrix $E_t(\tilde{x})$ as can be observed:

$$E_t(\tilde{x}_t) = \frac{1}{N} \sum_{i=1}^{N} \tilde{x}^{ij} \tag{3.14}$$

This matrix will contain the average value for each of the individual state variable represented in each column and its dimension will be (1, n). As it is desired to subtract each related mean value to all the realizations that form an state ensemble, the resulted E_t matrix will be resized to a (N, n) dimension as can be seen in equation 3.15 through multiplying in the left side by an eye matrix. Additionally in this equation, the mentioned subtraction is done obtaining the A_t matrix that will be used to calculate the covariance sample matrix S_t as can be seen in equation 3.16. Note that the values of this covariance matrix will be used as the C_t matrix in order to compute hereinafter the Kalman Gainer.

$$A_t = \tilde{x}_t - (e_{Nx1} E_t(\tilde{x}_t)) \tag{3.15}$$

$$S_t = \frac{A_t A_t^T}{N-1} = C_t \tag{3.16}$$

Here, the last step to update the initial guess is presented. One such difference from the *KF* is considered in this algorithm. Instead of using the *a priori predicted state estimate* ensemble $\tilde{x}_t$ as a reference to infer its value with the measurement, a simulated observation is calculated previously from this parameter. This step can be done or not depending on how important is the measurement for the process. Indeed, is identical to calculating the a priori matrix assuming a couple of associated errors. In any case and following all theoretical aspects from the formulation, it is showed in the next equation how this simulated measurement is obtained.

$$\tilde{y}_t = H_t \tilde{x}_t - v_t, \quad v_t \sim \mathcal{N}_{m_t}(0, R_t) \tag{3.17}$$

Note that this gainer, apart from minimizing the error R_t, it also minimizes the error committed when simulating previously the measurement. Once the relation between $\tilde{y}_t$ and $\tilde{x}_t$ is known and, taking into consideration the covariance sample matrix, the Kalman Gainer can be calculated. This gainer has a special approximation as the dimension of the matrices are huge becoming in unfeasible simulations. This is the reason why instead of calculating the standard Kalman gain, it will be replaced by an estimated Kalman gainer based on the forecast ensemble which formulation is done in this way:

$$\hat{K}_t = C_t H_t^T (H_t C_t H_t^T + R_t)^{-1} \tag{3.18}$$

Once all the variables for the last equation are attained, the update step will be as presented in equation 3.19a with the associated distribution given in 3.19b. See that the covariance error matrix of this *a posteriori state estimate* ensemble is calculated from the other covariance error matrices as showed in 3.19c. This equation is useful when calculating variances separately and it is used in the deterministic update.

$$\hat{x}_t = \tilde{x}_t + \hat{K}_t (y_t - \tilde{y}_t) \tag{3.19a}$$

$$\hat{x}_t^{(ij)} \sim \mathcal{N}_n(\hat{\mu}_t, \hat{\Sigma}_t), \quad i = 1, 2, ..., N. \quad j = 1, 2, ..., n. \tag{3.19b}$$

$$var(\hat{x}_t) = var(\tilde{x}_t) + var(\hat{K}_t \tilde{y}_t) - 2cov(\tilde{x}_t, \hat{K}_t \tilde{y}_t) = \tilde{\Sigma}_t + \hat{K}_t H_t \tilde{\Sigma}_t - 2\hat{K}_t H_t \tilde{\Sigma}_t = \hat{\Sigma}_t \tag{3.19c}$$

Finally, and after presenting all the equations that represent this algorithm, it is valuable to gather all the expressions by steps so the correct understanding of the application can be achieved. Thus, and as done with the *KF*, the steps in the EnKF cycle are left above. Moreover, an example using deterministic updating is left to the reader in order to justify numerically the decision done previously.

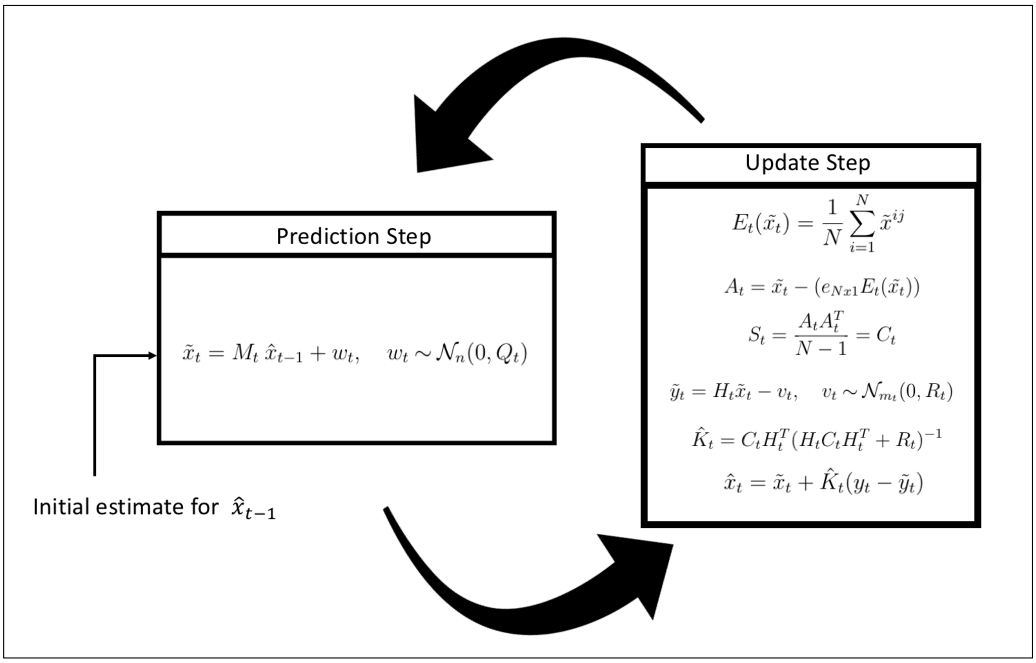

Figure 3.5: Steps in the *EnKF* cycle.

Example: Forecast a given temperature

Statement of the problem

- Temperature at time t_1 is T_1 and then at t_2=t_1+Δt is T_2, being T_2=ΔtT_1+T_1
- The *a posteriori estimate* at time t_1 will follow this normal distribution: $\mathcal{N}(T_{u,\,t=t1}, \sigma_{u,\,t=t1})$. This distribution is represented given a number of ensembles which provides a sample. In figure 3.6 are produced some representative ensembles although it will be more in a realistic case. From these points, the normal distribution will be drawn as it can be observed with its mean and standard deviation.
- The a priori estimate at time t_2 will follow a normal distribution: $\mathcal{N}(T_{p,\,t=t2}, \sigma_{p,\,t=t2})$

Example: Forecast a given temperature

- The observation will also follow a likelihood normal distribution being seen at $t = t_2$: $\mathcal{N}(T_o, \sigma_o)$
- Finally, the updated distribution will be named the *a posteriori estimate* according to this normal distribution: $\mathcal{N}(T_{u,\,t=t2}, \sigma_{u,\,t=t2})$

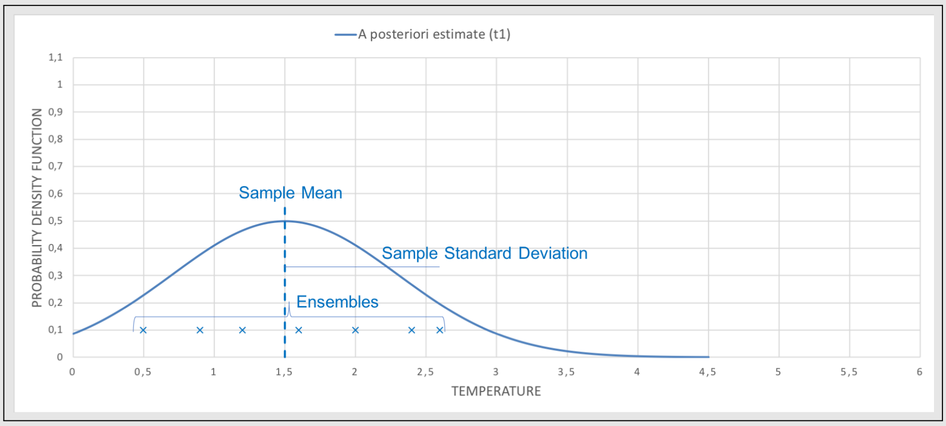

Figure 3.6: A posteriori state estimate in t=t_1.

Step 1: Calculation of the a priori state estimate

Then, first step will be to advance each element to the following time period given a linear relation of temperatures in this case: $T_{2,n}$= L($T_{1,n}$) being n the number of ensembles. This step can be visualized in figure 3.7.

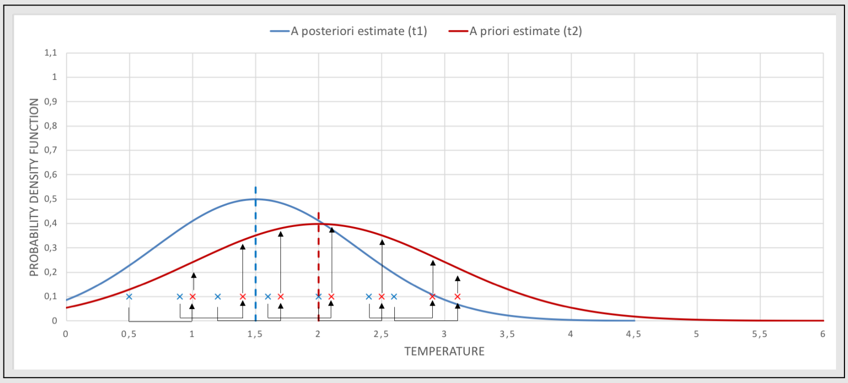

Figure 3.7: A priori state estimate in t=t_2.

See that, as well as performed in the *a posteriori estimate*, from the ensembles generated, a sample is given and therefore a new Gaussian distribution will be fitted to this sample.

Example: Forecast a given temperature

Step 2: Calculation of the a posteriori state estimate

Recall that with an stochastic update, previous to the computation of the posterior *PDF*, it was possible to generate a simulated measurement from the *a posteriori estimate* when applying equation 3.17. But, as this is a deterministic update example, there will not be a H_t neither a R_t matrix. Hence, the observation likelihood distribution is gotten as shown in figure 3.8 and assimilating this observation, a shift will be applied to the ensembles as a first step.

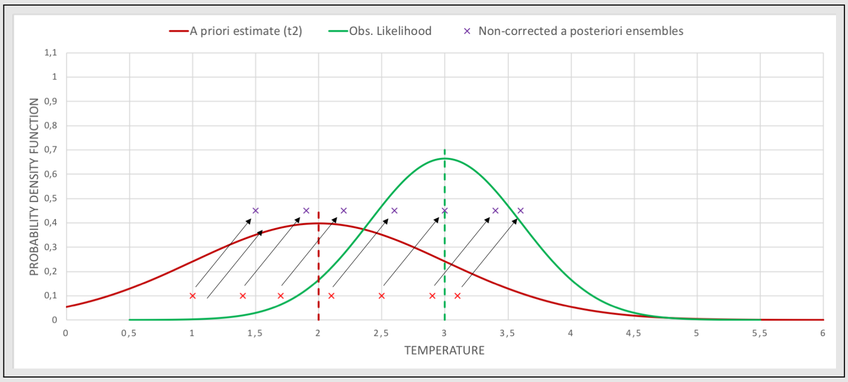

Figure 3.8: Non-corrected a posteriori state estimate ensembles.

Then the variance must be adjusted applying basic statistics as follows:

$$\sigma_u = \sqrt{(\sigma_p^{-2} + \sigma_o^{-2})^{-1}} \tag{3.20}$$

When applying the formula to the ensembles, a linear contract is achieved which must be equal to the a posteriori variance. This result is displayed in figure 3.9.

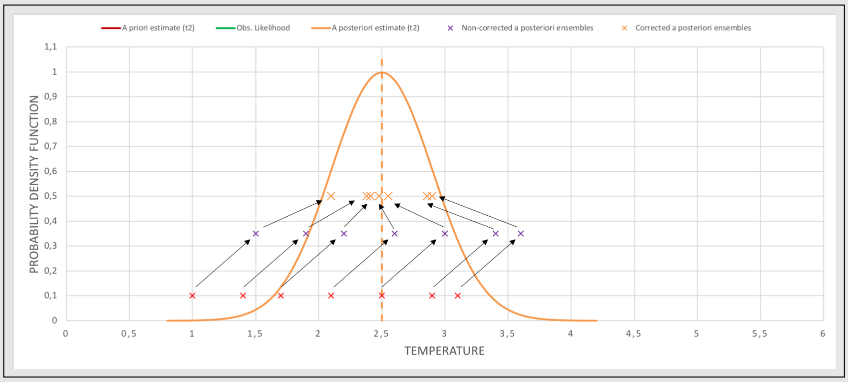

Figure 3.9: A posteriori state estimate.

Example: Forecast a given temperature

Comments and conclusions

Better results must be obtained if a stochastic update is applied. Instead of separating mean and variances from the sample, equations are applied directly to each ensemble. Much more relations are done between single values and consequently a greater collection of results is obtained from which a *PDF* is possible to be drawn. When applying the equations from this point of view, there is no need of applying equation 3.20 and less computation time is required.

This example updates in a deterministic way with a "standardize-unstandardize" calculation perspective, and it is solved according to this alternative to show that if compared with the stochastic update is worthless when relieving the scope of this project. The relation between ensembles in the model presented in last section is not linear and thus the contraction done in the last part of this example becomes really arduous.

Moreover, the simulated observation is not presented in the deterministic update and will be a key parameter in the strategy proposed further in this document.

3.2.3 Implementation for revealing prices

As commented in the prior sections, this algorithm is versatile and if tuned properly can be applied to different purposes. Moreover, the case of study when revealing prices is novel and therefore some of the variables must be treated in the best way, according to their attributes to acquire logical results. One such example are the observations, in compliance with the formulation an error is added to the value assuming a disturbance in the measurement sensor. But what if the nature of the observation is discrete and 100% correct as it can be a market price? This will be studied in the following section comparing results when simulating under different assumptions.

Another issue which has to do with the formulation is the propagation of the ensembles. It is clear that computation becomes easy in case of calculating estimated state variables shifting means and updating variances. Nevertheless, a great order of magnitude is advisable when generating realizations and this is achieved by relating all the realizations of the ensembles.

These two issues will be taken into consideration regarding the formulation. Along next chapter, all changes or assumptions made will be explained.

CHAPTER 4

Results

Once the methodology is presented, the results obtained will be shown. In the case of the Inverse Optimization Problem, a simple equation that will serve as a filter to obtain marginal costs will be presented prior to the results. In the case of the Ensemble Kalman Filter, some steps prior to the methodology will be carried out in order to have an initial state of the variables. Once exposed, the algorithm will be applied to three different cases. Because the current application of this method varies from the one applied in this project, it requires special treatment to some parameters to obtain results. Since this is part of the strategy adopted, it has been considered appropriate to separate additional equations from classical formulation: if the equations exposed in section 3 change, said changes will be explained.

4.1 Inverse Optimization

In this section, the results obtained from the first methodology are exposed. The Inverse Optimization is applied to the 3-Node Market and therefore prices from three rival competitors, one per node, are analyzed by means of tables and graphs. Furthermore, at the end of this section, some conclusions will be drawn from the outcomes.

4.1.1 Revealing marginal costs

When the formulation is applied, a set of prices per block, node and producer is approached. These prices that each company offers, will be equal or higher to their marginal cost as financial losses when producing want to be averted. As a consequence, and taking as inputs these sets of outcoming prices, the following equation will be used for each rival's block in all three nodes when estimating marginal costs.

$$\lambda_{nb}^{\hat{Otrue}} = \underset{\forall d,t}{\text{Min}} \left\{ \lambda_{dtnb}^{*O} \right\} \; \forall n,\; b \tag{4.1}$$

Hence, this equation will yield to the minimum price given all the associated values of a an energy block during all the simulation period which, in this case, is 30 days.

The assumption of offering always above the marginal price is not a certain representation of what really happens in real markets as this value could be lower to guarantee that an energy block is fully accepted, for instance. Nevertheless, as the model is controlled and determines the offering values to be always greater, this is not a problem when solving. Anyhow, a parallel solution to this issue could be employed if happens calculating the average offered price instead.

Taking into consideration the data provided in section 2 regarding the electricity market model and the formulation left in section 3, the linear inverse problem is applied. Once rival producers' offer curves λ_{dtnb}^{*O} are sifted by equation 4.1, the rival producers'marginal costs curves $\lambda_{nb}^{\widehat{Otrue}}$ per block and node are obtained. These results are gathered in table 4.1 as can be identified below.

b	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
λ_{1b}^{Otrue}	12,000	17,000	20,000	24,000	29,000	33,000	41,000	47,000
$\lambda_{1b}^{\widehat{Otrue}}$	-	17,022	20,122	24,030	29,039	33,051	-	-
$\lambda_{1b}^{\widehat{Omax}}$	-	18,486	21,964	26,361	31,789	36,093	-	-
λ_{2b}^{Otrue}	15,000	17,000	21,000	22,000	30,000	31,000	38,000	45,000
$\lambda_{2b}^{\widehat{Otrue}}$	15,219	17,016	21,040	22,004	30,064	31,056	-	-
$\lambda_{2b}^{\widehat{Omax}}$	16,497	18,491	23,082	24,105	32,838	33,993	-	-
λ_{3b}^{Otrue}	13,000	16,000	23,000	25,000	27,000	31,000	42,000	48,000
$\lambda_{3b}^{\widehat{Otrue}}$	-	16,330	23,012	25,005	27,006	-	-	-
$\lambda_{3b}^{\widehat{Omax}}$	-	17,186	25,281	27,44	29,284	-	-	-

Table 4.1: Estimated and True Marginal Costs in [€/MWh].

The results of the table are split into three graphical plots representing clearings in all three nodes taking into account all technical aspects from producers and also technical aspects from the grid. These are left below in figures 4.1, 4.2 and 4.3.

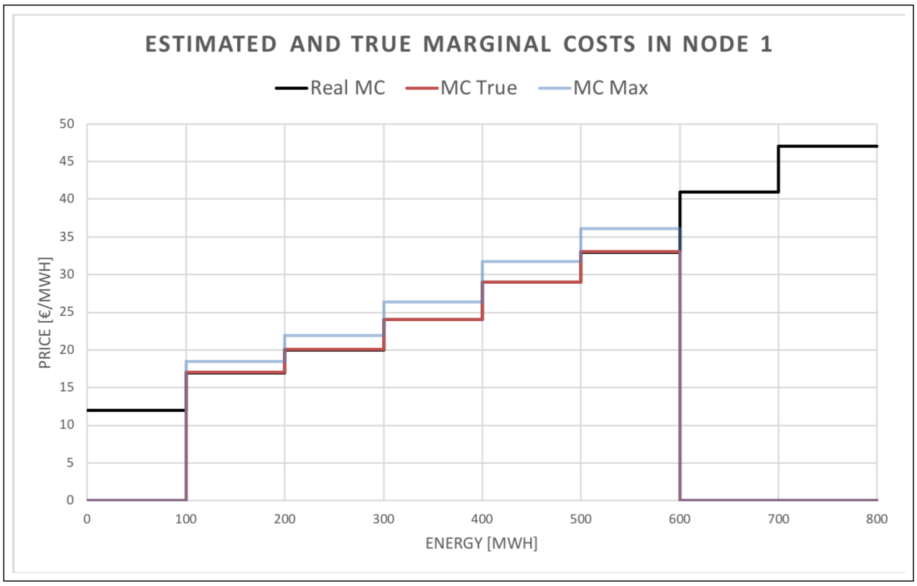

Figure 4.1: Estimated Marginal Costs: Node 1.

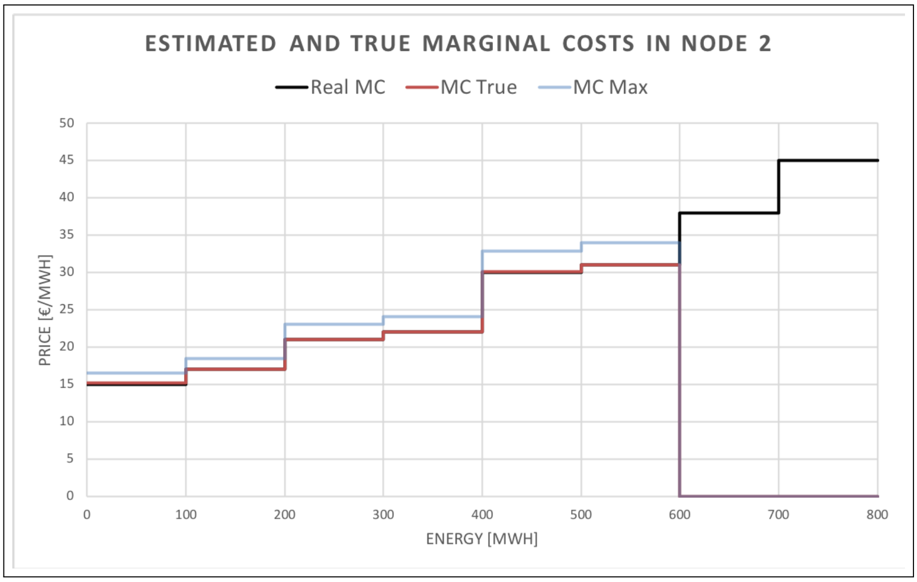

Figure 4.2: Estimated Marginal Costs: Node 2.

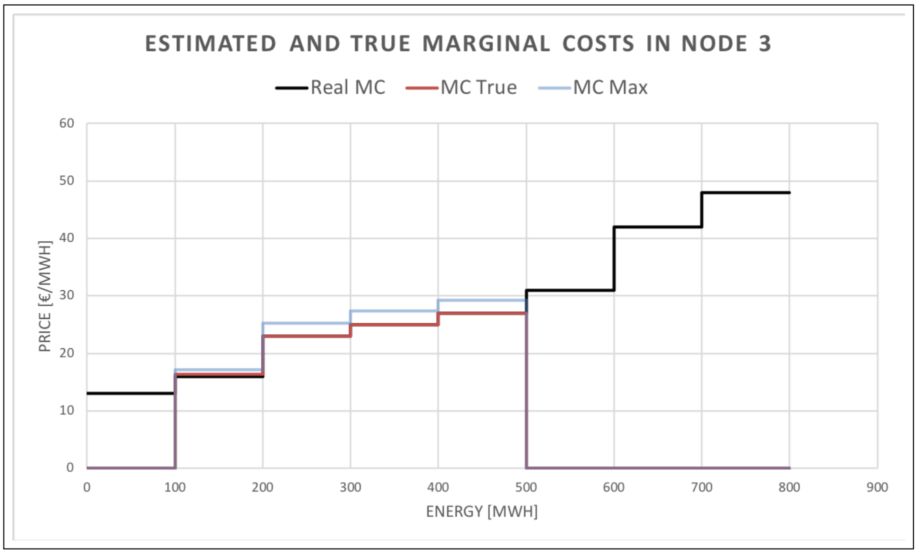

Figure 4.3: Estimated Marginal Costs: Node 3.

All the estimated marginal costs are compared with their associated real value to verify whether the result is accurate or not. This real values come from the previous information that it is had from competitors and used as an input in the market clearing optimization problem. Moreover, the maximum value of each block is also referenced so the spreading of the prices along 30 day can be observed.

The ramping constraints for the generation and demand sides together with the interconnection power limits, produce that not all the prices have been marginal. This results in hidden information which is never reached and thus can not be represented as a part of the final solution even though it is had knowledge of their true values. Some hidden prices have more importance than others from a strategic point of view. The case of not reveling the lower blocks' price could not be a problem due to, with those prices, is difficult to employ market power and push competitors close to the market price beyond the clearing point. The simulation has been extended to a longer time period without improving the number of revealed prices, concluding that the restrictions are the causers.

Analyzing all numerical results of the table 4.1, it can be observed that the estimated marginal cost of the prices that at least once have been a market price (or marginal) and thus had set the corresponding LMP in a given node, are close to their real values. The logic of taking the minimum number indicates a properly functionality when estimating the marginal costs as they are the base number to generate offerings.

Once the outcomes from the optimization problem are discussed, some conclusions are going to be exposed. Being critical and taking into account the scope of this project, some aspects will be considered as decisive contributions and other will be stated as challenges to be covered by the following methodology.

4.1.2 Conclusions

This first methodology has positive features when applying to a given market. The formulation is simple since the optimization problem takes as reference the market clearing problem used in the proposed model. Moreover, the linearization becomes elegant being the strong duality theorem applied successfully in the constraints. This results in low simulation time as it is not computationally demanding if compared with other alternatives that apply an optimization approach.

On the other hand, some aspects that can been improved to achieve the offering prices are also encountered. Many are found and, for that reason, a list is presented:

- First, some prices are not marginal at any simulation period and therefore they are left as unknown variables though they could be estimated by a simple polynomial regression, for instance. Being analytic, it is appropriate as there exist some prices that are allocated in the lower part of the offering curve and thus will not become in a threat from a strategy scope. Despite this, estimating prices from competitors that are not in any market clearing should be a positive study. If obtained, higher prices beyond the equilibrium point will be known and will result in the limited offering price when executing power market.

- Another issue is encountered regarding all the required data in order to apply this methodology. Both accepted offering and demanding blocks, block size, initial production level for each day or even ramping constraints, between others inputs, are required to carry out the proposed formulation. The problem with this amount of information is that, probably is not going to be accessible to a company as it will be considered a strategic advantage given from a competitor as well as having access to private information from the market TSO. One such proposal could be to simplify the market in terms of required information but generating a faithful supply and demand curve which give prices according to the real studied market.

- Finally, the last drawback presented is that, in the end, the offering prices are not calculated but used further on. Marginal prices are revealed instead of offering prices. Indeed, these offering prices are used as a whole gathered by blocks to determine their minimum value and thus obtain the associated marginal cost.

The scope of this project wants to go one step forward. Alternately of using hidden data in the algorithm, only market prices will be taken into account as inputs. This is, from public prices, revealing hidden parameters inside a model as are the offering prices. It is clear that technical data regarding generating companies is needed but only for the model to get market outputs. This will only deal with constructing accurate curves when compared with a real world market. One consequence when facing a problem with multiple hidden variables that are calculated only when one is observable is that, some assumptions will be implied within some logical range. One such example could be to assign to the energy blocks a quantity small enough to detect a change in the price when market clears. All the assumptions will be explained in the following section where the second alternative is applied.

4.2 Ensemble Kalman Filter

In this section, it will be suggested the second alternative taking as reference the second methodology exposed in section 3. Gaussian distributions will come up when representing prices instead of discrete values. Moreover, with this second option, apart from reaching hidden variables, it is wanted to offer remedies to the weak points presented in the conclusions of the previous approach.

Before applying the algorithm, some computing and parameter tuning is required for the correct functioning of the overall formulation. To explain the adjustments realized in some EnKF parameters, a natural convergence to logical decisions will be conducted by the document as a problem solving when some setback is presented. Drawing from the premise that only public information together with obvious knowledge that has a common private company, some extra studies in addition to the EnKF are carried out. As a first step, is proposed a guessing of the amount of companies with their corresponding marginal costs for the studied market.

4.2.1 First estimation

Before applying *EnKF* equations, some initial values for the ensembles that will conform the *initial state estimate* variable are required. These ensembles could be set to zero and along simulations propagate to their corresponding values. However, as market prices are presented since first day, is common sense to take prices regarding the early days and therefore construct initial distributions for the generators with marginal costs.

First matter to solve will be calculating the marginal costs of each company. Here, the advantage is that few companies are given in electricity markets nowadays since the big generators are those that occupy great stretch of the offer. One such example could be *Ørsted* in the Nordpool Spot Market or even *Iberdrola* in the MIBEL. With this statement, there is the chance for a private company to have knowledge regarding number and type of energy generation as well as nominal powers.

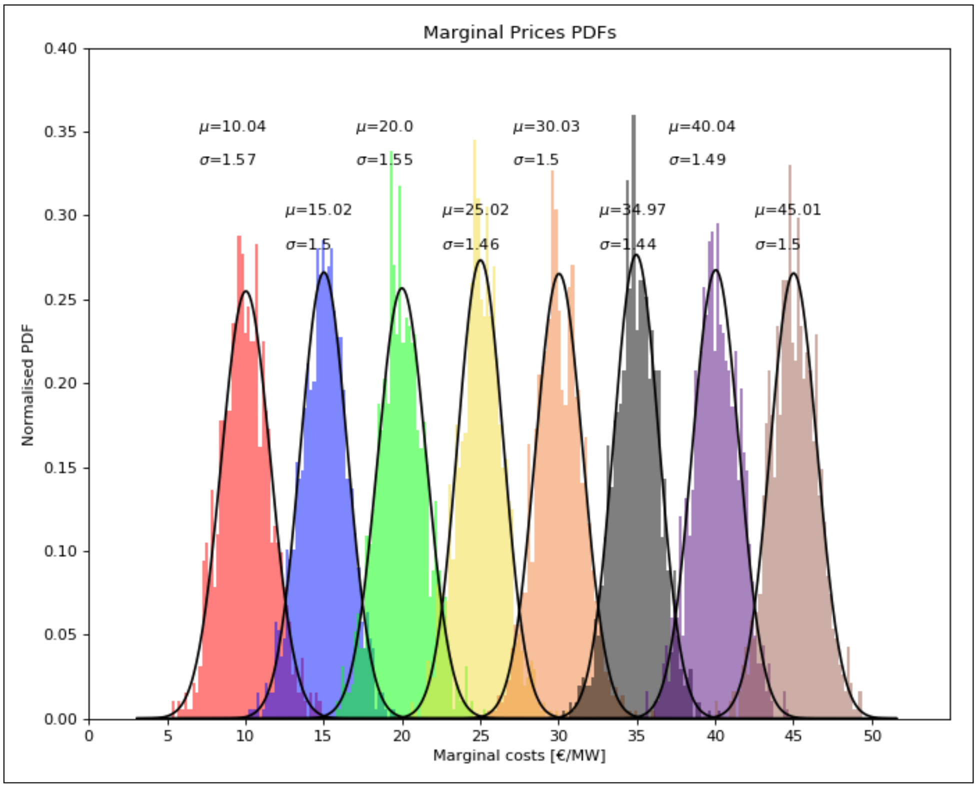

Figure 4.4: Estimated Marginal Costs.

Taking this into account, the strategy will be to calculate marginal costs of the companies and then when taking prices from the market have some basic argument to accomplice price with energy block. As commented in previous sections and according to [9], it is possible to estimate the marginal cost of a company depending on the type of generation, country, years of service among other factors.

In a first estimate, it will be assumed that the marginal costs have been calculated within certain error limits with some consistency. Gaussian distributions are used in the formulation due to the amount of realizations composing the ensembles and therefore, this marginal cost estimations will also follow this type of representation as showed in 4.4.

In this project, this first step of researching companies and calculating generation costs is not done as it is out of the scope in addition to being focused to a realistic application. Anyhow, it is a good starting point when a great power market can be exerted and few companies are direct rivals and susceptibles of being pushed out the market. Instead, it will be supposed that this previous calculation has been done within some margin error and thus the graph above will avail this stage.

As can be observed, the graph indicates the mean value of each marginal cost which is the same used in the model. Together with this value, a standard deviation is identified with each distribution representing the error in cost estimations. Remark that in this implementation, there exist the chance of striking with lower prices if compared with their marginal cost.

Now, initial estimations are calculated added to acquaintance regarding the amount the companies in the market. Taking as reference these distributions a filter will be employed when a new market price appears. This supposes that when updating the ensembles given an observable value, the associated market value will be the responsible of changing the distribution of the marginal costs ensembles. This update will be analyzed further in next section.

4.2.2 Initial distributions

The simulation cycle will be of one year. This long period will permit the formulation's parameters to be updated throughout different demand factors as well as letting the algorithm to converge properly. Due to this fact, prices regarding first month will be used as observations to construct prior ensembles once they are associated to each company. Market prices will be collected from clearings and then gathered according to each company.

Some market prices can be between two marginal costs. As it is assumed that the price can be lower that the actual generating cost, in these special cases, these prices will serve to update distributions immediately preceding or immediately following. The decision making procedure is that if a price is around and below the mean value a company, this will be used to update both distributions. Some representative situations are left in table 4.2.

Market Price [€/MWh]	G10	G15	G20	G25	G30	G35	G40	G45
20.984	✗	✗	✓	✗	✗	✗	✗	✗
14.444	✓	✓	✗	✗	✗	✗	✗	✗
27.781	✗	✗	✗	✓	✗	✗	✗	✗
39.458	✗	✗	✗	✗	✗	✓	✓	✗
46.785	✗	✗	✗	✗	✗	✗	✗	✓

Table 4.2: Classifying Market Prices.

In the table, the name of the generators end with two numbers representing mean values of their corresponding marginal costs. Once all prices from a month of clearings are collected and classified by companies, the distributions are made up and ensembles can be drawn from them. Thus, this ensembles will form the *initial estate estimated ensemble* which will act as the initial conditions. As an example of the resulting distributions from this exercise, figure 4.5 is left below.

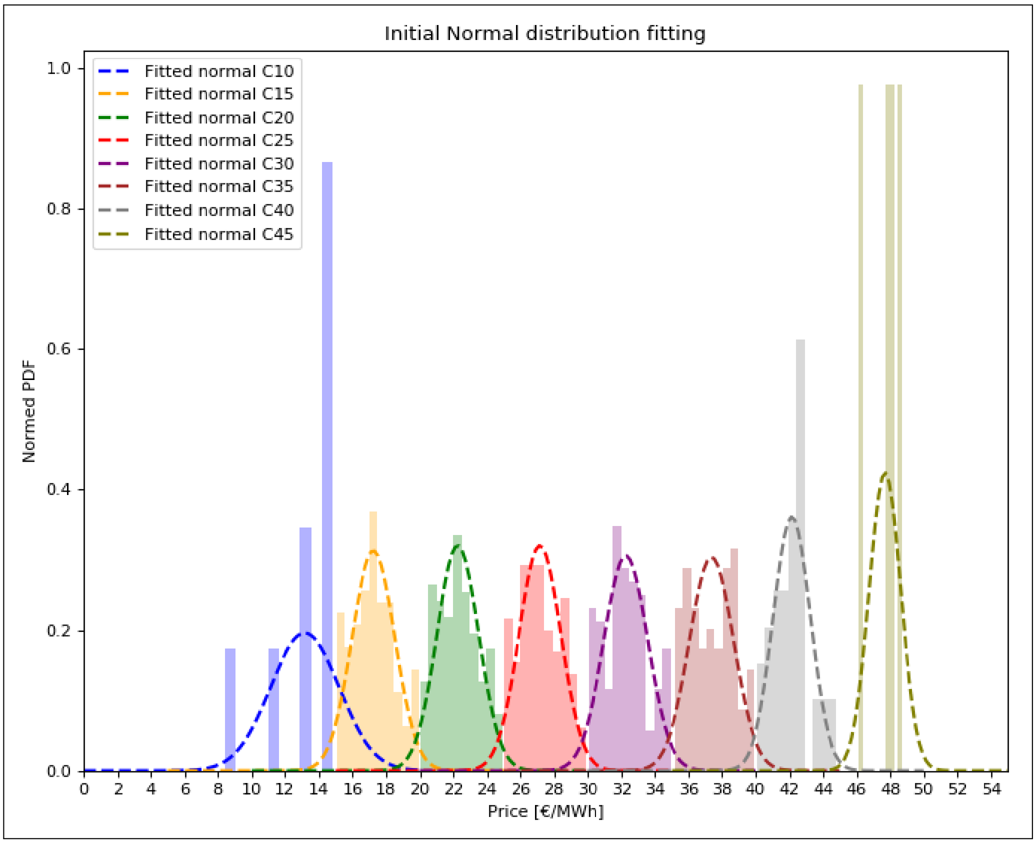

Figure 4.5: Initial distributions from 1-Month Data.

Recalling the matrix form of this variable and giving some numbers from the distributions above, the initial state will be similar to the one presented in equation 4.2. Here, as the number of companies already calculated are 8, this will be the dimension of the columns (n=8). Moreover, as it is required a huge amount of realizations, the number of ensembles per company will be set at N=10^6. See that the realizations composing the ensembles of each company have grown if compared with their initial costs.

$$\hat{x}_{t-1} = \begin{bmatrix} \hat{x}^{11} & \hat{x}^{12} & \hat{x}^{13} & \dots & \hat{x}^{1n} \\ \hat{x}^{21} & \hat{x}^{22} & \hat{x}^{23} & \dots & \hat{x}^{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{x}^{N1} & \hat{x}^{N2} & \hat{x}^{N3} & \dots & \hat{x}^{Nn} \end{bmatrix}_{t-1} = \begin{bmatrix} 10,975 & 16,006 & 21,853 & \dots & 47,026 \\ 13,643 & 16,978 & 21,041 & \dots & 48,533 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 11,188 & 16,559 & 20,093 & \dots & 44,375 \end{bmatrix}_{t-1} \tag{4.2}$$

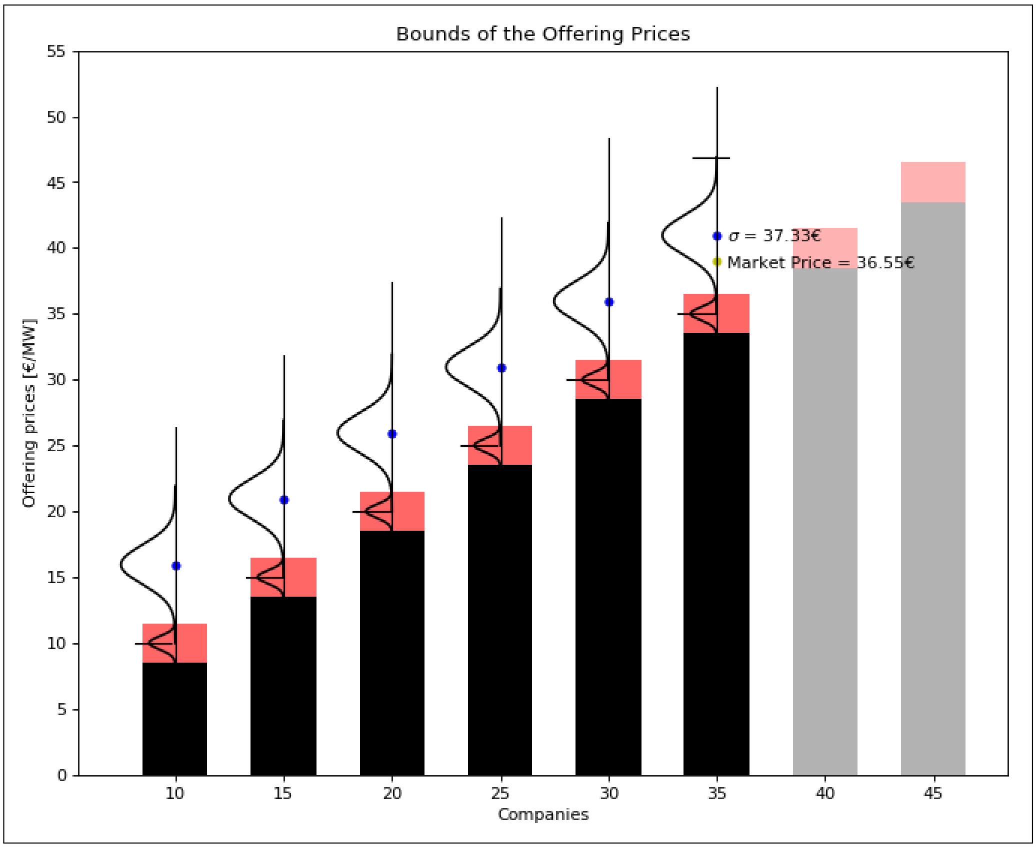

Figure 4.6: *PDFs* of each company.

One such representation of the Gaussian distributions for each marginal cost in addition to the distributions for the offering prices is given in figure 4.6. Each column corresponds to a company and both representations appointed are added to a fixed base price represented by a black bar. As an example, a market price of 36.55€/MWh has been associated to the company with a mean marginal cost of 35€/MWh. See that the mean value of the offering price distribution is higher even though appending a lower value.

This figure will help to place the offering prices separately from the marginal cost. However, and as seen in equation 4.2 only a *PDF* will be used to generate ensembles for each company.

4.2.3 Revealing offering prices

This first step will be done independently of the conditions presented in the market. It has been checked that is the best starting point for the algorithm and will help convergence into good results in less time. In this section, the revealed prices for each company will be presented by means of tables and graphs as a guidance for the presentation of the solutions.

Three scenarios will be presented as some problems regarding the *EnKF* parameters will appear in some of them and therefore retrieved in the following. Firstly, a static model for the *EnKF* will be used to reveal prices from the 1-Node Electricity Market model. Then the results obtained will be compared with the ones achieved in a dynamical model, which will be examined in second scenario. Finally, taking as reference the formulation from the dynamic model, extremely conditions in demand will be applied to check the performance of the algorithm. These conditions will become in great changes of market outcomes and will permit analyze versatility.

4.2.3.1 Scenario 1: Static *EnKF* model

As commented, the algorithm will be applied to the 1-Node Market which has three representing days affecting to the amount of demanded energy per block and hour. With this scenario, it is wanted to show how does work the formulation without changing the parameters. The results of this application will give an idea of what to tune properly inside the *EnKF* model of the *EnKF* in order to fully attach the behaviour of the hidden variables in the market model.

For this reason, some values are needed for the parameters. Before updating an ensemble by means of the measurement (market price), a prior guess is required propagated from the initial known state of previous time step. As the evolution of the offering prices for each company follow a stochastic behaviour, it has no sense to apply any relation between them. This will be equivalent to implement to the matrices, used in the equations, the complexion given in table 4.3.

M_t	Identity matrix
Q_t	Zero matrix
H_t	Identity matrix
R_t	Zero matrix

Table 4.3: *EnKF* Parameters.

The simulation time in this scenario is less than a year due to the meaninglessness of the results given a number of iterations, consequently 2500 hours out of 8760 will be enough instead.

One of the issues identified in the results is that there is no correlation between offering prices. When a market price comes up, it is associated to a company and hence an ensemble will be updated meanwhile the others remain static. It should happen that when last company inside the market clearing is taking some risk in terms of price rising over its marginal cost, something similar would happen with companies that offer energy blocks at lower prices. This problem could improve accuracy when guessing hidden prices.

Secondly, and as a consequence of the prior issue above, when a company offers an energy block which is not marginal and therefore no information is presented to update the corresponding ensemble, the median value of the distribution remains static and a decrease of its standard deviation is presented. Truthfully, all standard deviations converge to zero in less than 700 step periods, being more pronounced in those companies which present more prices that are marginal. It should be highlighted that equation 3.19a reduces covariance error and thus uncertainty. This fact together with null white errors converts this static alternative to be applicable only in small periods or even inapplicable.

One example is given with company *C1*. In figure 4.7, the statistics of the distribution for six time periods are left by means of box plots where data is represented through quartiles. Here, it can be observed the decrease in the standard deviation due to the lack of observations for this company. Moreover, the median remains fairly static as commented. Finally, it can be observed that in these periods and for this company, five of the real offering prices are inside the limits of the distribution. Nevertheless, then it will be observed that the accuracy when estimating will also decrease along time resulting in a small success percentage.

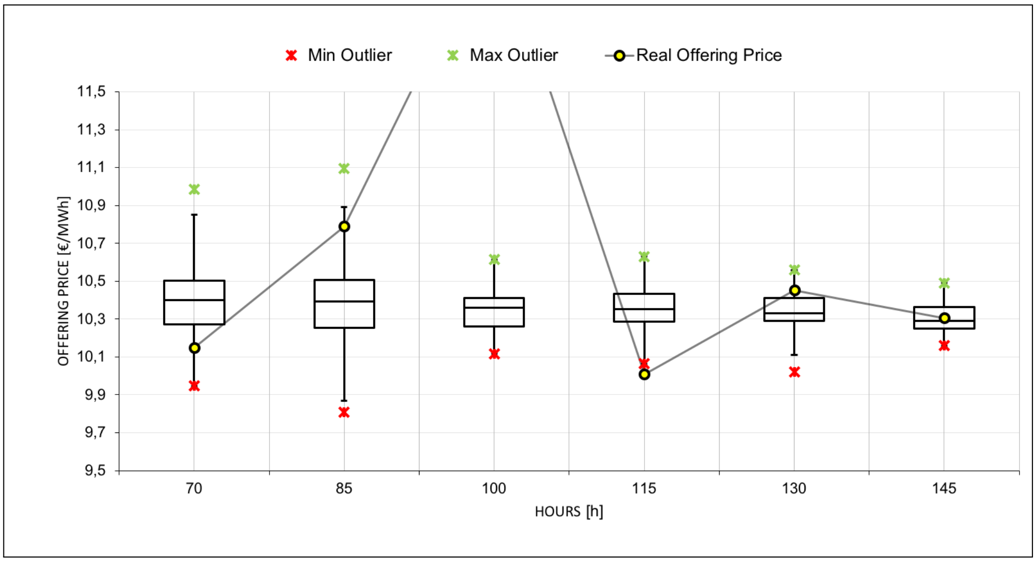

Figure 4.7: Box-Whiskers plot for Company C1.

Labels	**70**	**85**	**100**	**115**	**130**	**145**
Min	9,945	9,807	10,116	10,063	10,022	10,159
Q1	10,270	10,252	10,262	10,285	10,291	10,251
Median	10,400	10,392	10,359	10,354	10,331	10,291
Q3	10,503	10,507	10,412	10,433	10,412	10,362
Max	10,985	11,093	10,613	10,629	10,559	10,490
***IQR* (Interquartile Range)**	0,232	0,255	0,149	0,147	0,120	0,111
Real Offering Price	10,148	10,789	12,390	10,008	10,451	10,305

Table 4.4: Data Regarding Figure 4.7.

Together with the figure above, table 4.4 gives values to the main parts of the box representations. This table gives also numerical justification as the Interquartile Range decreases along time periods. With this problem, the distributions will be converted into discrete guesses and therefore the formulation by means of Gaussian Distributions will have nonsense.

To end with this section, some statistics regarding the market behaviour and the algorithm performance are left below. Company *C1* has been analyzed though could be the less important company when competing as it is always inside the market. In order to justify that the behaviour seen in this company is exhibited in the others, figure 4.8 is exposed.

The blue bar represents when an offer is inside the market clearing. The red bar produces when the offer is inside the distribution this is, within the maximum and minimum parts of the box no matter how many times is inside the market. Finally, the green bar indicates the number of times that a price is marginal along simulations.

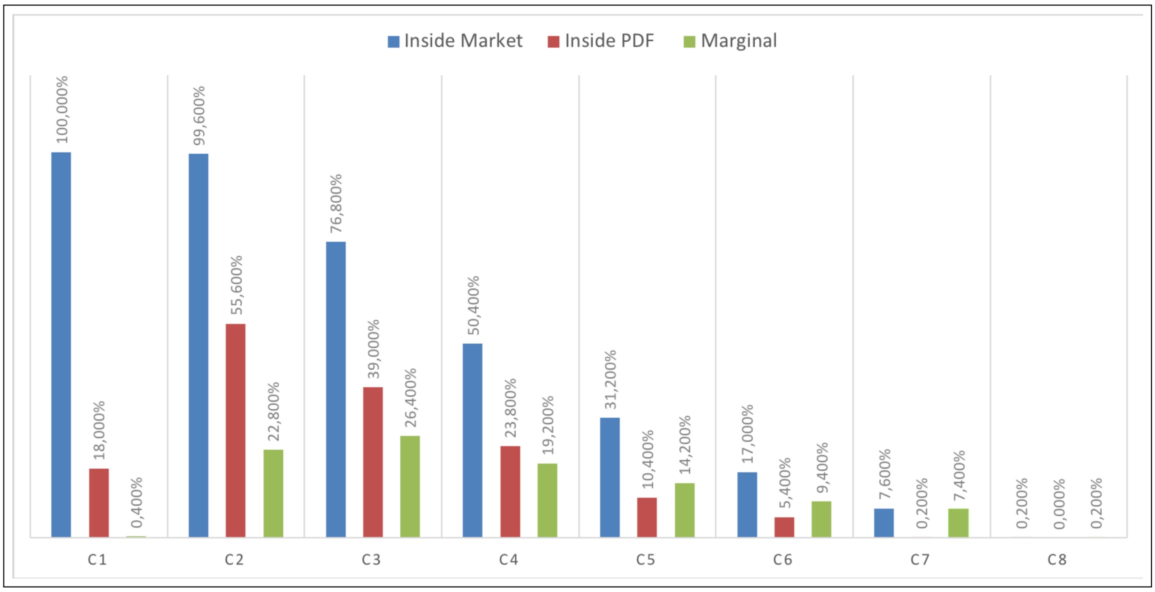

Figure 4.8: Statistics for Scenario 1.

Regarding the market behaviour, a decrease in the blue bar indicates that the demands are not extreme and thus collect medium values. This is the reason why last three companies have small participation. Here, companies *C2*, *C3* and *C4* become important when being monitored as they set more than the 60% of the market prices.

Moreover and according to the graph, along the 2500 time periods, only 10 observations (0,4%) are given to update the ensemble of *C1*. This is the reason why the predicted distributions are accurate only in the 18% of the times. As commented before, these results could not be a problem when exert market power as this is the first company which is always supplying energy. However, when paying attention to company *C3* which is the one with greater marginal offering prices and thus must be the most consistent with accurate guesses, it can be seen that this is not met.

In the end, blue and green bars are representative regarding the market behaviour as well as indicators of how well the algorithm must perform. Taking an overall view on red bars it can be concluded that the accuracy with static parameters is not enough as the most factual *PDF* is encountered with *C2* representing a 55,6% of correct guesses. There exist even the detail from company *C5* until *C8* than red bars are smaller than greens. This is because at certain time period, the measurement is only correcting the a priori guess distribution by shifting its mean value and the variance is almost zero, which is therefore insufficient.

In the following section, taking as reference this scenario, some changes are applied in order to reach solid results. It is clear that when taking an observation from the model, has a discrete nature and no noise is presented. Despite this, and recalling the formulation, some advantage could be taken when using in a correct approach the simulated observation. Finally and to justify that the main problem of an *EnKF* model without associated noises is not feasible, figure 4.11 is left below showing that the decrease of the standard deviation becomes more critical when more marginal prices are associated to the offering prices of a company.

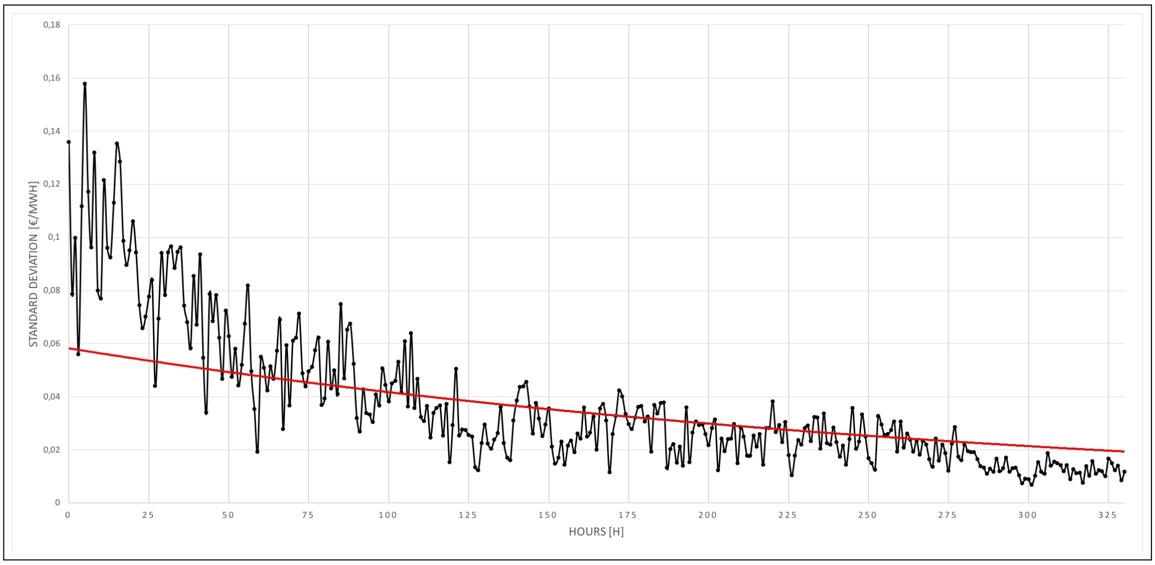

Figure 4.9: Evolution the Standard Deviation for Company C3.

4.2.3.2 Scenario 2: Dynamic *EnKF* model

In this section, some changes regarding the parameters of the *EnKF* will be tested. Again, the algorithm will be applied to the 1-Node Market which has three representing days affecting to the amount of demanded energy per block and hour. With this scenario, it is wanted to show how does work the formulation making the values of the parameters observation-dependent, and then checking if it is fully attach the behaviour of the hidden variables in the market model. Finally and before starting with the formulation, comment that a dynamic model will be forced as a consequence of nature changing along periods of the observations.

As a first step and following the reasons exposed in the previous section, as the offering prices behave in a stochastic manner and thus it is not predictable which could be the forecasted state, the *a priori state estimate* will be the same as the *optimal state estimate* in the previous time step. Hence, values for the first two parameters are known an presented in table 4.5.

M_t	Identity matrix
Q_t	Zero matrix

Table 4.5: *EnKF* Forecasting Parameters.

This will result in calculating the covariance sample matrix with the values of the *optimal state estimate* matrix in *t-1*, which makes sense if there is not a prediction.

Next step will be to set properly the simulated observation matrix exposed in equation 3.17. Its value will depend on the *optimal state estimate* matrix as well as parameters H_t and v_t. The strategy applied in this project will be the following:

- When a new market prices comes out the market, as did with the initial distributions, it is associated with a company. In these steps it will be only supposed that offerings are greater than marginal costs. At the end of the section, special cases which price could be lower will be studied.

- Once the market price is classified, the difference between this value and the mean value of the associated ensemble is calculated thus resulting in variable d_t:

$$d_t = \lambda_t^M - E_t(\tilde{x}_t^{i,j*}) \tag{4.3}$$

  Here, the asterisk next to j indicates that the mean value will be only calculated from the selected company. See that this variable can be either positive or negative, and will be used to calculate both H_t and v_t.

- In order to calculate matrix H_t, it will be supposed that the last company inside the market clearing is taking some risk in terms of price rising. This means that a shift to the mean values of all the companies in each ensemble must be done. To infer proportionally, a new constant is presented:

$$j_t = 1 + \frac{d_t}{E_t(\tilde{x}_t^{i,j*})} \tag{4.4}$$

  Taking as reference this constant, matrix H_t will be a diagonal one which entries will be identical and equal to j_t:

$$H_t = j_t * \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{4.5}$$

- In addition, some white error v_t must be added to avoid decrease in standard deviations. As a second hypothesis, it will be supposed that the risk taken by companies with lower marginal costs should be lower. Therefore, to cover this uncertainty and taking as reference the value of d_t, a value for v_t will be given so that the distribution ranges from the shifted mean value until the previous one. To fulfill this premise, the following equation will be applied:

$$v_t = \tilde{\Sigma}_t - d_t^2 \tag{4.6}$$

- With these determined parameters, the value of the simulated observation can be calculated and thus follow with the formulation as stated in section 3.

These steps have been exposed considering only the situations when the offering price is greater that its corresponding marginal cost. However, as proceeded with the initial distributions, this offered price could be lower. It is known that some energy generating companies implement this action when is cheaper for them not to stop generating rather than being out of the market clearing. Therefore, since there is a base factor that generates small demands, when this case occurs together with a market price immediately below the marginal cost of any of the companies, this special case will be considered. Here, d_t will be negative and as a consequence j_t less than one. It is not a case that has a high occurrence, but must be taken into account because in the current electricity market occurs as it may be the case of a nuclear power plant.

Finally and before commenting the results, remark that the real observation has a discrete nature and thus must be treated. By reason of this fact, y_t will collect mean values from the simulated observation columns of the matrix being the market price in the associated company. This is done in this way to avoid minimizing the variances created with variable d_t.

As was done in the previous scenario, the figure 4.10 is left with six representative boxes of the distributions that represent the forecasts made in six simulation periods. Together with the graph, the corresponding values are left as a guide in table 4.6. In the graph, data corresponding to March 21 has been considered, taking into account that the simulation begins on January 1. In this case, it has been opted for the company *C4* although later it will be analyzed the precision of the results for all others. However, unlike the previous scenario, looking at the graph it can be observed that the real offering prices are within the presented distributions.

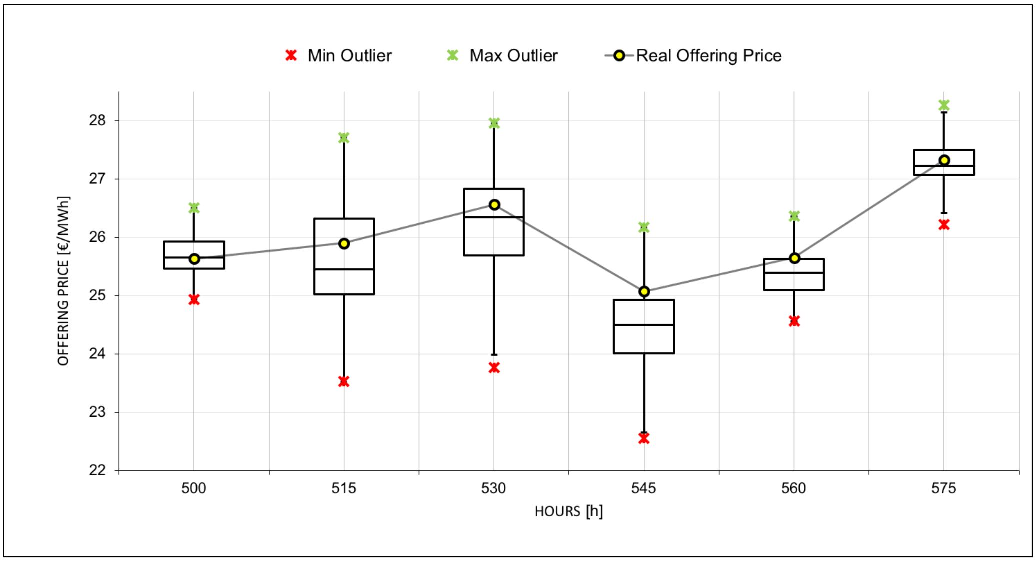

Figure 4.10: Box-Whiskers plot for Company C4.

Labels	**500**	**515**	**530**	**545**	**560**	**575**
Min	24,932	23,524	23,767	22,550	24,559	26,220
Q1	25,460	25,020	25,691	24,012	25,097	27,068
Median	25,650	25,455	26,338	24,494	25,396	27,225
Q3	25,924	26,325	26,828	24,924	25,627	27,499
Max	26,500	27,711	27,958	26,165	26,357	28,267
***IQR* (Interquartile Range)**	0,463	1,305	1,136	0,912	0,530	0,431
Real Offering Price	25,629	25,899	26,562	25,072	25,651	27,326

Table 4.6: Data Regarding Figure 4.10.

Continuing with the analysis of the results of the table, it can be seen that the values of the median indicates changes in prices between simulations. It no longer remains almost static as it could be detected in the previous scenario and it is also a potential indicator that the shift through the mean done by j_t is being applied correctly. In addition, it can be observed that due to the special cases of low demand and prices immediately below the marginal cost, the one mediated in the period 545 is below said cost. As it has been said is a possible indicator, because it would really be the average value of the distribution which should indicate such a situation, although checking in the database the average is also below and therefore would be representative of this special case.

Considering more closely the inter-quartile ranges, it can be observed that the respective values do not decrease along simulations. In fact, observing the values, can be seen that ranges between 0.5 and 1.3 approximately in the presented sampling. Small values can indicate two different cases: the first is that the price change with respect to the previous value is small, and the other is that the company's offer in that period of time has a marginal character. In the cases of the graph, both in the first reference (500) and in the last two (560 and 575) the cause is due to be establishing the market price. It is already possible to anticipate justifiably that the offers of *C4* are marginal almost 1 out of 3 occasions. By means of these data, it can be concluded that the error with the reduction of the standard deviation has been corrected without increasing its value excessively and without any justification.

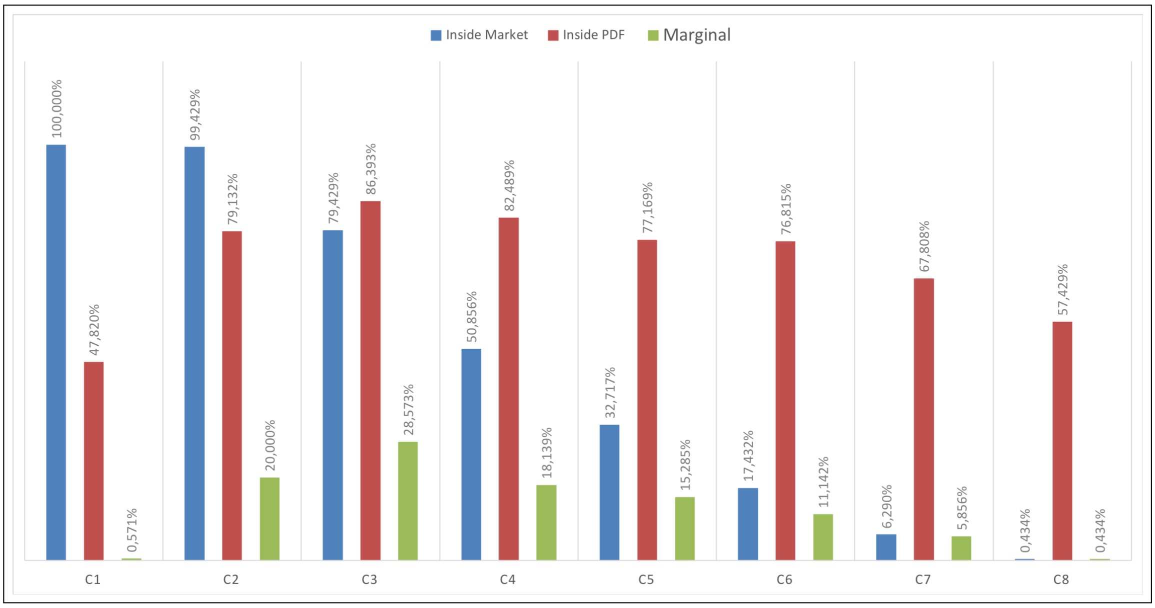

Figure 4.11: Statistics for Scenario 2.

As has been done in the previous scenario, the statistics are left with respect to the results of the simulations of this scenario. The percentages of market participation (blue bar) are almost identical to the previous occasion because the market is the same and so are the demand factors. For the same reasons, the percentages that indicate the marginal character of the prices (green bar) are also practically identical to the previous case.

Assessing now the percentage of success or what is the same, in how many times the true price is within the proposed distribution, the general conclusion is that it has increased considerably if it is compared. In the cases where the companies fix market prices, the percentage is higher, standing at around 80 %. This correlation existed in scenario 1, but due to the changes made in the simulated observation, the effectiveness of the prediction has been increased.

On the occasions when the price is not marginal, results in a potential problem with the application of this alternative due to the lack of references and as a consequence there is a very small percentage of belonging to the distribution. However it can be seen an improvement because without having almost observations for *C1* and *C8* (50 and 38) over 1 year, the success rate is around 50 %. This percentage could be improved, but is considered unimportant because they do not occupy a strategic position to apply market power along market clearings: the first enters inside the market 100 % of the times and the last only a 0.434 %.

Analyzing again the inter-quartile range, in some occasions it can be thought that the range is high and that this can lead to a high number of success because the distributions cover a wide variety of prices. As a consequence of this fact, it is seen the necessity to carry out another study where it can be justified to which part of the box the real offer price belongs. In the table 4.7 are reflected the probabilities that has a predicted price to be in the different regions of the box.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Min-Q1	4,760%	8,464%	8,708%	6,008%	6,510%	7,532%	6,425%	5,232%
IQR	29,380%	46,820%	52,201%	52,054%	46,143%	48,044%	38,146%	32,136%
Q3-Max	13,680%	23,848%	25,484%	24,426%	24,516%	21,238%	23,237%	20,061%
Out	52,180%	20,868%	13,607%	17,511%	22,831%	23,185%	32,192%	42,571%

Table 4.7: Probability of appearance by rank for all offering prices.

From the table, it can be concluded that for the companies with the greatest precision in terms of forecasting, their real prices tend to be in the inter-quartile range. It can be seen special cases for the companies *C7* and *C8*, where the probability of being between the maximum and the third quartile is high and comparable with being between the first and third quartile.

Finally, in the table 4.8 the probability that, given a real price that belongs to the predicted distribution, is within each proposed range is left as a decision-making tool. This table would help in cases where the standard deviations are greatly enlarged due to a large price difference between periods and therefore create uncertainty when accurately predicting a price. On the other hand, it should be noted that these sudden changes in prices do not have a very high frequency, although over such a long period of simulation it may appear occasionally.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Min-Q1	4,760%	8,464%	8,708%	6,008%	6,510%	7,532%	6,425%	5,232%
IQR	29,380%	46,820%	52,201%	52,054%	46,143%	48,044%	38,146%	32,136%
Q3-Max	13,680%	23,848%	25,484%	24,426%	24,516%	21,238%	23,237%	20,061%

Table 4.8: Probability of appearance by rank of the prices that are within the distribution.

To end the section and as a guide, it is left the equivalence between a box diagram and a normal distribution in the figure 4.12 due to observing a statistical distribution is more common in comparison with a box diagram. The graphic comparison between these two representations can be a useful tool to understand the box-whiskers diagram. This type of diagrams has been chosen for the presentation of results due to the fact that they take less space and, therefore, are particularly useful for comparing distributions among several groups or data sets.

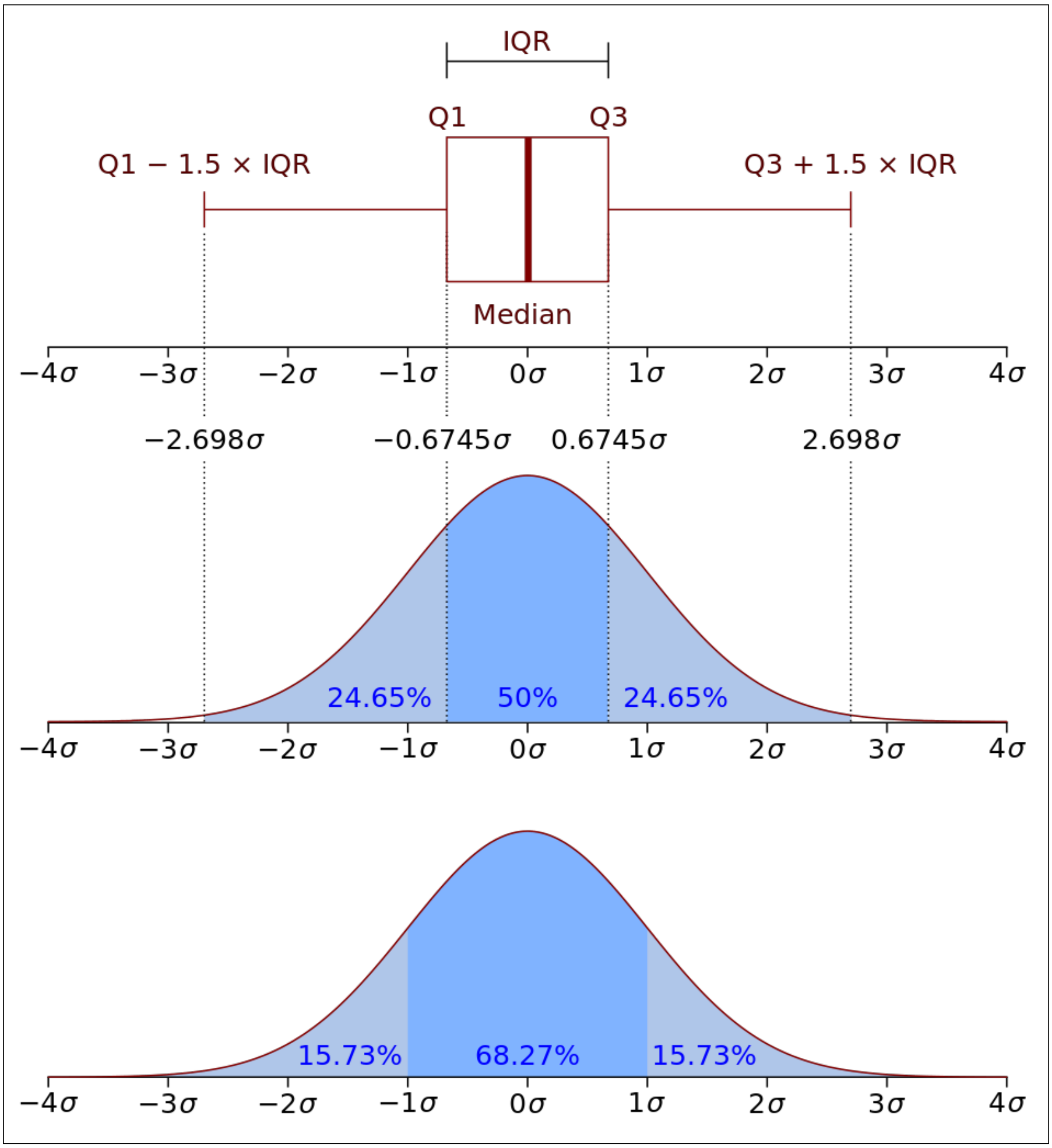

Figure 4.12: Box-plot vs *PDF* of a Normal Distribution.[21].

4.2.3.3 Scenario 3: Dynamic *EnKF* model with seasonal demand

In this section, the last scenario is presented where the same formulation presented in the previous case will be applied to verify its robustness in special situations. It is known that sometimes there are strong fluctuations in demand that can be both predictable and not due to different factors. Some of them can be the failure of a set of generators of a power plant, a saturated interconnection or a special day of the year where consumption is high. In addition, it will also serve to measure performance when there is a transition between seasons of the year.

For the reasons given, this section has been created where the market is going to be subjected to abrupt changes in terms of demand. For this, the demand factors used in the 1-Node Market have been taken into account and have been altered. The figure 4.13 reflects the changes made in these factors.

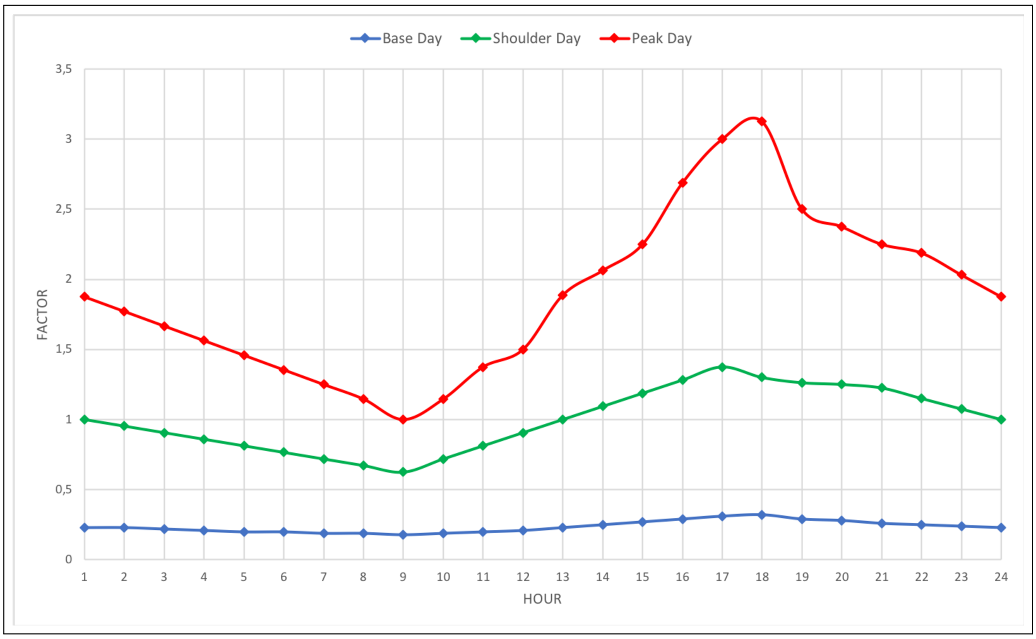

Figure 4.13: Demand Factors for Scenario 3.

Looking at the figure, it can be seen that the factors with respect to a day of the base type remain unchanged. It is interesting to have a reducing factor in order to create both smooth and sharp jumps. Regarding the factors from the Shoulder and Peak days, its value has been raised in order to increase the demands in the market. Because the study of this section is based on having knowledge of how the formulation of scenario 2 behaves when the demands are extreme between them, the simulation has also been opted for over a year.

Finally and before commenting on the results, discuss that the randomness regarding the choice of the factor is no longer with respect to the day but to the time. In the previous cases, a day factor was chosen and applied to the following 24 hours. In this case the factor can jump between the different days throughout the hours, being in the hour 1 a base day and in the hour 2 a peak day, for instance.

Having into consideration the new parameters added to the basic formulation of *EnKF*, it can be anticipated that creating changes in demand should not change the achievement obtained previously. As it has gone up, companies with higher marginal costs will have more opportunities to enter the market and therefore the observations will be more distributed between them. The statistics obtained in this scenario are shown below.

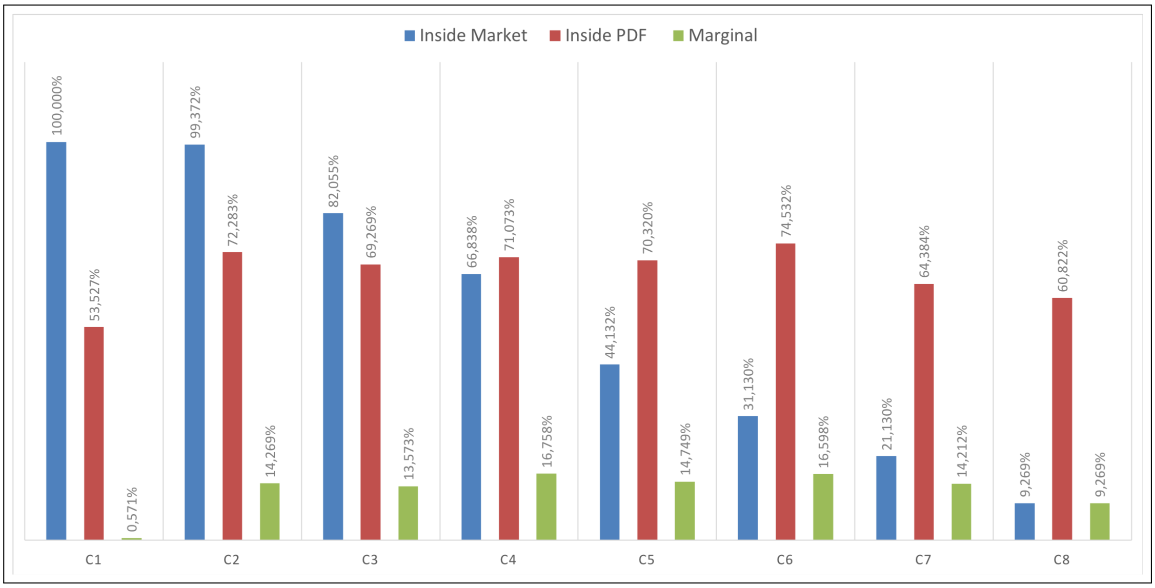

Figure 4.14: Statistics for Scenario 3.

Taking into account the results of figure 4.14, it can be seen that the percentages of success of the predicted distributions fall a bit with respect to the previous results. The reason is none other than the distribution of marginality in the offering prices. In the previous case, it were the companies *C2*, *C3* and *C4* the ones which set prices more frequently and that is why it was presented a greater success in the forecasts.

Another curious fact that can be found in the graph is that the strategy suggested in scenario 2 offers similar success percentages for the extreme companies *C1* and *C8*. The percentage of success of the first company is completely consistent with the strategy and presents a value close to 50 %. While if the last one is analyzed, it offers a practically similar result because from the overall achievement a 9.269 % corresponds due to its marginality. Therefore, it can be concluded that the more observations is had, the more chances will be presented in order to succeed in forecasting.

As this scenario shows events with very similar percentages in comparison to the previous one, instead of a box plot it is chosen to represent some part of the results as shown in the figure 4.15. This chart shows the market prices, the real prices of the offerings from *C3* and a pair of bands that correspond to the extremes values of the interval $[\mu + 3\sigma, \mu - 3\sigma]$ of the forecasted distribution had for this company each time step (this interval covers approximately 99.74 % of the distribution).

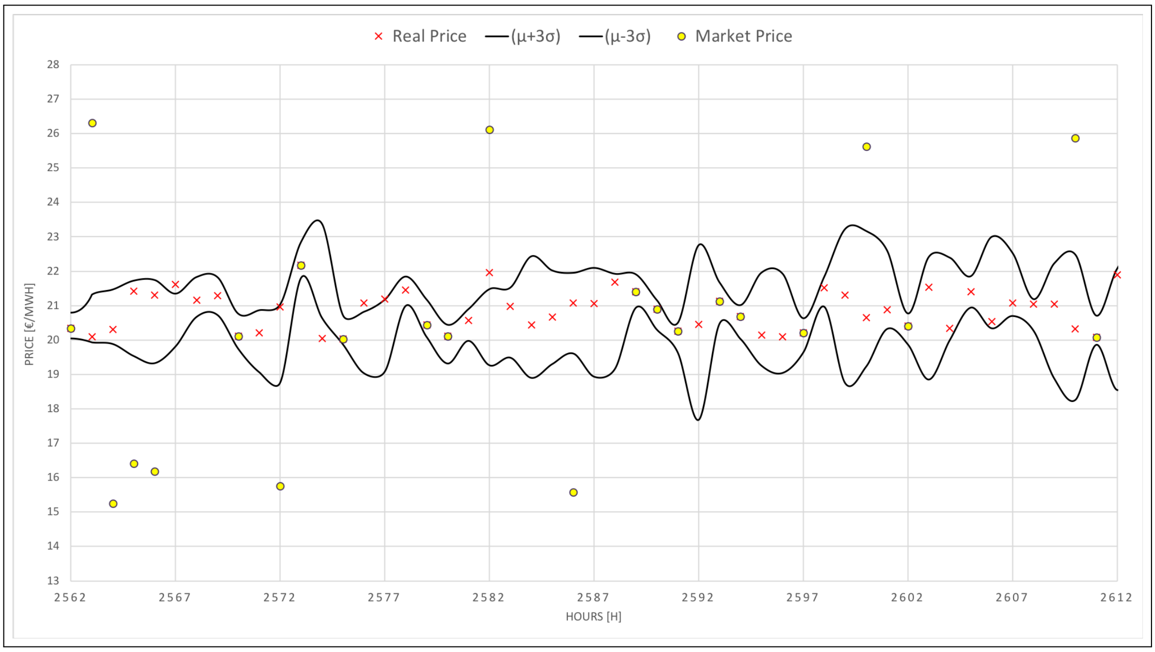

Figure 4.15: Forecasted Distribution for Company *C3*.

Taking advantage of this type of graph and taking it as a reference will permit understanding more clearly how the system works globally. Seeing the first market prices of the graph it can be seen that there is a correlation between its increase from their associated marginal cost and along the bands. When these prices separate from the base values same behaviour is presented with the representative bands of other companies. In addition, the graph corroborates the fact that when a price is marginal the bands become narrow due to there is full knowledge of the value as it is an observation. Quite the opposite happens when the price is hidden as they spread out.

As it has been opted in the previous scenario, the table 4.9 is left where the probabilities of a predicted price of being in the different regions of the distribution are reflected. Since the distributions used in the graph are Gaussian, the intervals are changed. However, very similar values in comparison to the previous ones are obtained as the ranges used in this table are almost aligned with the ranges of the box-whiskers used in the previous table. See the figure 4.12 as reference. Again, a forecast tendency to be around the interval closer to the average is observed. Belonging to the distribution area where there is less uncertainty is an indicator that the shift that is made to the rest of the prices hidden in the model is appropriate.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
(μ-3σ,μ-σ)	5,114%	7,567%	6,887%	5,330%	6,069%	7,028%	6,019%	5,673%
(μ-σ, μ+σ)	32,886%	42,768%	41,854%	44,851%	42,047%	46,617%	36,219%	34,035%
(μ+σ, μ+3σ)	15,527%	21,949%	20,528%	20,892%	22,204%	20,888%	22,145%	21,114%
Out	46,473%	27,717%	30,731%	28,927%	29,680%	25,468%	35,616%	39,178%

Table 4.9: Probability of appearance by rank for all prices.

Again, along with the previous table, the 4.10 is attached where it can be seen the probability that, belonging to a real price to the predicted distribution, is in each of the three regions.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
(μ-3σ,μ-σ)	5,114%	7,567%	6,887%	5,330%	6,069%	7,028%	6,019%	5,673%
(μ-σ, μ+σ)	32,886%	42,768%	41,854%	44,851%	42,047%	46,617%	36,219%	34,035%
(μ+σ, μ+3σ)	15,527%	21,949%	20,528%	20,892%	22,204%	20,888%	22,145%	21,114%

Table 4.10: Probability of appearance by rank of the prices that are within the distribution.

To finish with this section and to show visually how the forecasts of the offering prices would be presented in a closing of a market clearing, the figure 4.16 is exposed. Distributions are presented by box-whisker diagrams because they are more visual for the presentation of these type of results.

In the specific case of the figure, there is a market close where the company *C3* sets the price. Due to this fact, its small standard deviation can be observed again. It could be said that in this scenario, the price revelations of the first four companies are of relevant interest in comparison with the rest. In particular, the prices of these companies are within the proposed distributions.

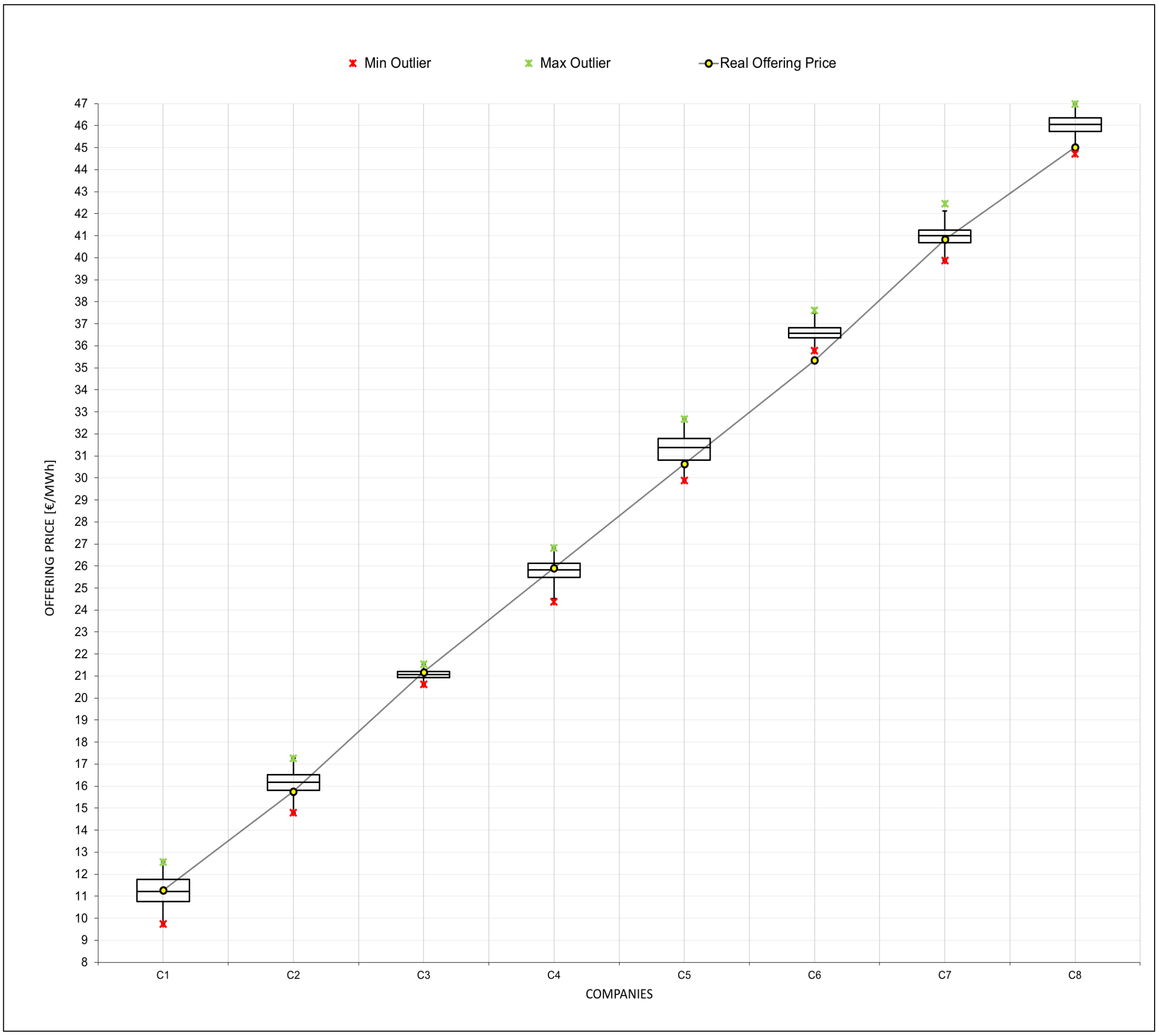

Figure 4.16: Forecasted distributions for a given Market Clearing.

CHAPTER 5

Conclusion

In this project, the novel application of an algorithm in the field of study has been proposed for the disclosure of offering prices in a spot market. Taking as reference the work done in [8], it has been reproduced obtaining practically the same results and its performance has been improved by the application of an Ensemble Kalman Filter.

With respect to the implementation of the inverse Optimization problem, some details have already been concluded in the results section. It has been seen that the resolution is quick and simple although it has also been seen that there are some drawbacks. The blocks that are not marginal in any period are hidden in the model and are unattainable with the proposed formulation. It also requires a lot of information to be able to solve the optimization problem because it uses the market model in the restrictions of the main problem. Finally, and probably the most outstanding problem with respect to the project's objective, is that it does not disclose offering prices of the rivals but their marginal prices, which is therefore irrelevant.

Seeing the potential problems found in the first methodology, they have been overcome with the implementation of *EnKF*. As in the Inverse Optimization Problem, a series of necessary hypotheses have been considered. It is assumed that a private company is able to estimate with certain error the marginal costs of its competitors. Actually, with an approximate estimate would be more than enough because when applying the *EnKF*, being a recursive filter, the real prices converge to their correct values. In the proposal of this project, a reduced model of an electric market is used because it would be practically impossible in the development period to find and calculate all the marginal costs.

The proof that the first proposal has been improved is found in the results obtained for the three scenarios. It can be seen that, with a good strategy that encompasses supply values following a certain logic of market behavior, high success is achieved. In addition, it has been tested with different demand profiles with some distance between them and the performance has been maintained. Finally, another example of performance is that the pricing of the offers in the model is done in a random way without following any risk pattern and it is always greater than the marginal cost. Even considering the ignorance of the behavior of the market prices, the percentage of success of the companies of interest is around 80 %.

But like all the methodologies applied in the field of engineering, there are potential problems that must be prevented or detected when the results are not as expected.

With the application of *EnKF* one can have the problem of cutting the demand curve too much and distributing the marginality between blocks too much. As seen in the results section, the lower the marginality, the lower the percentage of success. A possible solution to this problem could be to divide the supply curve into blocks that are really of interest to the private entity. With this, prices would only be revealed when the market price was associated with the most potential rivals, while if the market price is set by a rival that is not of interest, the algorithm does not apply. With this proposal, in addition, a regression could be made with the revealed prices and build the full supply curve without revealing all the prices of the blocks. There is also the possibility that the market throws a series of prices with very small increases or decreases. This would lead to a decrease in the standard deviation in chain and lower the accuracy obtained. Faced with this problem, it is possible to make use of the additional parameters proposed in the scenario 2 strategy in order to control the propagation of the same.

Making a global critique about the results obtained, it can be conclude that the *EnKF* is a very powerful tool to reveal prices if it is tuned properly. In fact, if a given private company manages to calculate offer prices, it could be applied to a real market. It is worth noting that said applicability is of special interest to those producers that have a significant market share and therefore are able to exercise their market power.

CHAPTER 6

Conclusión

En este proyecto, se ha propuesto la aplicación novedosa de un algoritmo en el campo de estudio para la divulgación de precios de oferta en un mercado spot. Tomando como referencia el trabajo realizado en [8], se ha reproducido obteniendo prácticamente los mismos resultados y se ha mejorado su rendimiento mediante la aplicación de un Ensemble Kalman Filter.

Con respecto a la implementación del problema de optimización inversa, algunos detalles ya se han concluido en la sección de resultados. Se ha visto que la resolución es rápida y simple aunque también se ha visto que hay algunos inconvenientes. Los bloques que no son marginales en ningún período se ocultan en el modelo y son inalcanzables con la formulación propuesta. También requiere mucha información para poder resolver el problema de optimización porque usa el modelo de mercado en las restricciones del problema principal. Finalmente, y probablemente el problema más destacado con respecto al objetivo del proyecto, es que no revela los precios de oferta de los rivales sino sus precios marginales, lo cual resulta irrelevante.

Al ver los posibles problemas encontrados en la primera metodología, se han superado con la implementación del *EnKF*. Al igual que en el problema de optimización inversa, se han considerado una serie de hipótesis necesarias. Se supone que una empresa privada puede estimar con cierto error los costos marginales de sus competidores. En realidad, con una estimación aproximada sería más que suficiente porque al aplicar el *EnKF*, que es un filtro recursivo, los precios reales convergen a sus valores correctos. En la propuesta de este proyecto, se usa un modelo reducido de un mercado eléctrico porque sería prácticamente imposible en el período de desarrollo encontrar y calcular todos los costos marginales.

La prueba de que la primera propuesta se ha mejorado se encuentra en los resultados obtenidos para los tres escenarios. Se puede ver que, con una buena estrategia que abarca los valores de suministro siguiendo una cierta lógica del comportamiento del mercado, se logra un gran éxito. Además, se ha probado con diferentes perfiles de demanda con cierta distancia entre ellos y se ha mantenido el rendimiento. Finalmente, otro ejemplo de desempeño es que el precio de las ofertas en el modelo se realiza de forma aleatoria sin seguir ningún patrón de riesgo y siempre es mayor que el costo marginal. Incluso teniendo en cuenta el desconocimiento del comportamiento de los precios de mercado, el porcentaje de éxito de las empresas de interés ronda el 80 %.

Pero como todas las metodologías aplicadas en el campo de la ingeniería, existen problemas potenciales que hay que prevenir o detectar cuando los resultados no son los esperados. Con la implementación del *EnKF* se puede tener el problema de seccionar demasiado la curva de demanda y repartir demasiado la marginalidad entre bloques. Como se ha visto en el apartado de resultados, a menor marginalidad menor porcentaje de acierto. Una posible solución a este problema podría ser el dividir la curva de oferta en bloques que realmente sean de interés para la entidad privada. Con esto, solo se revelarían ofertas cuando el precio de mercado estuviera asociado a los rivales más potenciales, mientras que si el precio de mercado lo fija un rival que no es de interés, el algoritmo no se aplica. Con esta propuesta, además, se podría realizar una regresión con los precios revelados y construir la curva de oferta al completo sin revelar todos los precios de los bloques. También cabe la posibilidad de que el mercado arroje una serie de precios con incrementos o decrementos muy pequeños. Esto conllevaría a una disminución de la desviación estándar en cadena y bajaría la precisión obtenida. Ante este problema, se puede hacer uso de los parámetros adicionales propuestos en la estrategia del escenario 2 con el fin de controlar la propagación de la misma.

Haciendo una critica global acerca de lo obtenido, el *EnKF* es una herramienta para revelar precios muy potente si se ajusta adecuadamente. De hecho, si la una compañía privada dada consigue calcular precios de oferta, se podría aplicar a un mercado real. Bien cabe destacar, que dicha aplicabilidad es de especial interés para aquellos productores que tienen una importante cuota de mercado y, por lo tanto, pueden ejercer su poder de mercado.

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