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Design of timber beams in bending according to DB SE Madera (CTE)

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1 Summary

This document shows how to design a solid timber beam in simple bending considering only bending moments according to the Spanish Code: Basic Document Structural Safety: Timber (DB SE M is the acronyms in Spanish)
The resistance condition is applied to a worked example, and a proposed exercise in included in order to involve the student in practising the procedure.

2 Aims

At the end of this document students will be able to design a solid timber beam considering the resistance condition in simple bending proposed by the Spanish code DB SE M.

3 Introduction

3.1 Timber as structural material

Timber is wood used for buildings. Being wood a porous and fibrous structural tissue found in the stems and roots of trees.

Timber is a heterogeneous, hygroscopic, cellular and anisotropic material, and being a biologic material, can be biodegraded by biotic agents (fungus, insects, etc.)

Being timber the oldest building material able to transfer tension and compression forces, it has been used to design beams since old times. It has a very strength to weight ratio, it is relatively easy to fabricate, it does not corrode, and its carbon footprint is quite small in comparison to other building materials such as concrete or steel, as both need a lot of energy to extract the raw materials.

As the growing of tress is positive for the environment, timber is the most sustainable structural material.

Nowadays, timber beams can be made by Solid Timber or Glued Laminated Timber. This document is focus in Solid Timber.

3.1.1 Load-duration classes

The load-duration classes are characterised by the effect of a constant load acting for a certain period of time along the life of the structure. Actions shall be assigned to one of the load-duration classes given in Table 1 for strength and stiffness calculations (corresponds to table 2.2 DB SE M of the Spanish code CTE)

Load duration class	Order of accumulated duration of characteristic load	Load
Permanent	More than 10 years	Self-weight
Long-term	6 months -10 years	storage
Medium-term	1 week – 6 months	Imposed load, snow load when altitude > 1000 m
Short-term	Less than one week	Wind, snow when altitude < 1000 m
Instantaneous		Accidental load, Earthquake

Table 1. Load duration classes

3.1.2 Service classes

Each structural element shall be assigned to one of the service classes given in Table 2, considering the environmental conditions

Service class 1	It is characterised by a moisture content in the materials corresponding to a temperature of 20 ± 2ª C and the relative humidity of the surrounding air only exceeding 65% for a few weeks a year. This service class corresponds, usually to indoors timber structures.
Service class 2	It is characterised by a moisture content in the materials corresponding to a temperature of 20 ± 2ª C and the relative humidity of the surrounding air exceeding 85% for a few weeks a year. This service class corresponds, usually to covered outdoor timber structures like sheds and indoor swimming pools due to the relative humidity.
Service class 3	It is characterised by climatic conditions leading to higher moisture contents than in service class 2

Table 2. Service classes

The influence of load-duration and moisture on strength is considered by a modification factor (see Table 3)

If the combination of load to take into account for the resistance condition includes loads belonging to different load-duration classes, the modification factor to be considered corresponds to the load with the lesser duration.

3.1.3 Load-duration and moisture influences of strength

Modification factors, k_{mod} for the influence of load-duration and moisture content on strength for solid timber are given in Table 3

Service class	Load-duration classes				
	Permanent	Long-term	Medium-term	Short-term	Instantaneous
1	0,60	0,70	0,80	0,90	1,10
2	0,60	0,70	0,80	0,90	1,10
3	0,50	0,55	0,65	0,70	0.90

Table 3. Modification factors for load-duration and class service

3.2 Resistances of timber structural elements

3.2.1 Design resistances

The design value R_d of a resistance (load-carrying capacity) shall be calculated as shown in equation 1:

$$R_d = k_{\text{mod}} \frac{R_k}{\gamma_M}$$ *equation 1* where

R_k is the characteristic value of the corresponding load-carrying capacity. Values can be found in Table 4 and Table 5.

γ_M is the partial factor for a material property, being γ_M for solid timber equal to 1.30

k_{mod} is a modification factor for taking into account the effect of the duration of the load and moisture content, k_{mod} values are shown in Table 3.

3.2.2 Characteristic resistances

That timber is an anisotropic material means that physic and mechanic properties are directionally dependent. For the resistance of the solid timber, two main directions are considered: parallel to the grain and orthogonal to the grain. Table 3 and Table 4. Coniferous and Poplar show the different strength classes and characteristic values of coniferous and Poplar and deciduous species, respectively.

Strength classes and characteristic values: Coniferous and Poplar

Characteristic strength N/mm²		C14	C16	C18	C20	C22	C24	C27	C30	C35	C40	C45	C50
Characteristic bending strength	$f_{m,k}$	14	16	18	20	22	24	27	30	35	40	45	50
Characteristic tensile strength along the grain	$f_{t,0,k}$	8	10	11	12	13	14	16	18	21	24	27	30
Characteristic tensile strength perpendicular to grain	$f_{t,90,k}$	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
Characteristic compression strength along the grain	$f_{c,0,k}$	16	17	18	19	20	22	22	23	25	26	27	29
Characteristic compression strength perpendicular to grain	$f_{c,90,k}$	2,0	2,2	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	3,1	3,2
Characteristic shear strength	$f_{v,k}$	3,0	3,2	3,4	3,6	3,8	4,0	4,0	4,0	4,0	4,0	4,0	4,0
Characteristic stiffness kN/mm²													
Mean value of modulus of elasticity along the grain	$E_{0,mean}$	7	8	9	9,5	10	11	11.5	12	13	14	15	16
Fifth percentile value of modulus of elasticity along the grain	$E_{0,k}$	4,7	5,4	6,0	6,4	6,7	7,4	7,7	8,0	8,7	9,4	10,0	10,7
Mean value of modulus of elasticity perpendicular to grain	$E_{90,mean}$	0,23	0,27	0,30	0,32	0,33	0,37	0,38	0,40	0,43	0,47	0,50	0,53
Mean value of shear modulus	G_{mean}	0,44	0,50	0,56	0,59	0,63	0,69	0,72	0,75	0,81	0,88	0,94	1,00
Density kg/m³													
Characteristic density	ρ_k	290	310	320	330	340	350	370	380	400	420	440	460
Mean density	ρ_{mean}	350	370	380	390	410	420	450	460	480	500	520	550

Table 4. Coniferous and Poplar

Strength classes and characteristic values: Deciduous[1] species

Characteristic strength N/mm²		D18	D24	D30	D35	D40	D50	D60	D70
Characteristic bending strength	fm,k	18	24	30	35	40	50	60	70
Characteristic tensile strength along the grain	ft,0,k	11	14	18	21	24	30	36	42
Characteristic tensile strength perpendicular to grain	ft,90,k	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Characteristic compression strength along the grain	fc,0,k	18	21	23	25	26	29	32	34
Characteristic compression strength perpendicular to grain	fc,90,k	7,5	7,8	8,0	8,1	8,3	9,3	10,5	13,5
Characteristic shear strength	fv,k	3,4	4,0	4,0	4,0	4,0	4,0	4,5	5,0
Characteristic stiffness kN/mm²									
Mean value of modulus of elasticity along the grain	E0,mean	10	11	12	12	13	14	17	20
Fifth percentile value of modulus of elasticity along the grain	E0,k	8,4	9,2	10,1	10,1	10,9	11,8	14,3	16,8
Mean value of modulus of elasticity perpendicular to grain	E90,mean	0,67	0,73	0,80	0,80	0,86	0,93	1,13	1,33
Mean value of shear modulus	Gmean	0,63	0,69	0,75	075	0,81	0,88	1,06	1,25
Density kg/m³									
Characteristic density	ρk	500	520	530	540	550	620	700	900
Mean density	ρmean	610	630	640	650	660	750	840	1080

Table 5. Deciduous species

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[1] Deciduous trees are those that lose all of their leaves in Autumn.

4 Ultimate Limit States

Timber structures are generally analysed using elastic structural analysis.

4.1.1 Design of cross-sections subjected to stress in one principal direction

This section applies to straight solid timber of constant cross-section, whose grain runs essentially parallel to the length of the member. The member is assumed to be subjected to stress in the direction of only one of its principal axes (see Figure 1)

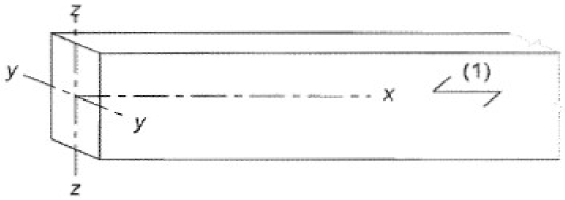

Figure 1. Straight solid timber beam Source: CTE DB SE-M Key: (1) direction of grain

4.1.2 Simple bending

The resistance condition to be satisfied, in simple bending considering only bending moments is that the normal stresses due to bending moment shall be minor than the resistance (Equation 2):

$\sigma_{m,d} \leq f_{m,d}$ Equation 2

Being: $\sigma_{m,d} = \frac{M_{Ed}}{W_{el}}$ Equation 3 the normal stresses due to the bending moment

where $W_{el} = \frac{bh^2}{6}$ Equation 4 is the elastic modulus for a rectangular cross section

And $f_{m,d} = k_{\text{mod}} \frac{f_{m,k}}{\gamma_M}$ Equation 5 the design value of the resistance as has been defined in epigraph 2.2.1

4.1.3 Dimensions of the cross-sections

The dimensions of sawn wood are not standardized in Europe, although it seems that Nordic countries usual dimensions are quite common due to their large market share.

In Spain, the sawn wood of "*pino laricio*" for structural use is generally supplied in squares higher than 100x100 mm, following a range of 100x150, 150x150, 150x200, 200x200 and 200x250. The dimensions for bigger squares vary according to the species.

The most usual cross sections in UK· Are shown in Figure 2.

Table 6 and Table 7 include the values of the second moment of area (Iy) and the elastic modulus ($W_{el,y}$) for the cross-sections in Figure 2 (the reference axis are those in figure 1)

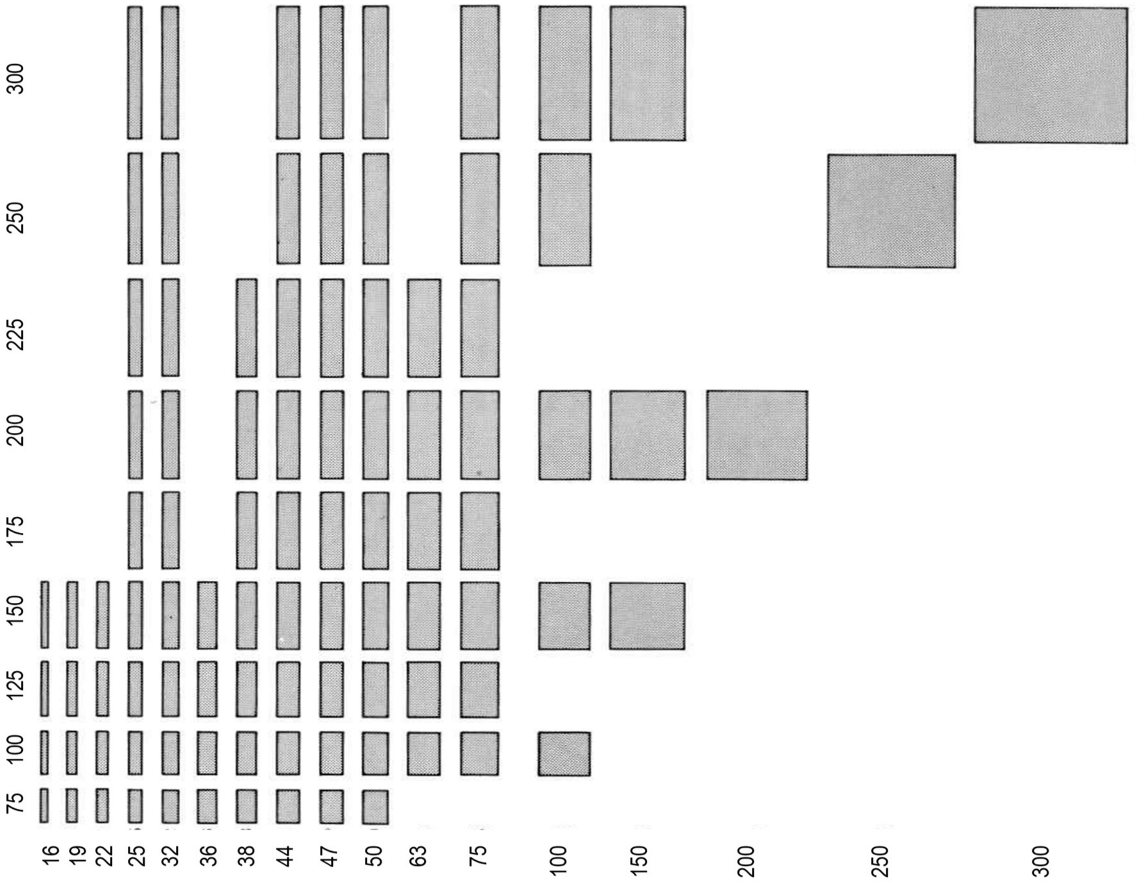

Figure 2. UK usual solid timber cross-sections . source: http://software.trada.co.uk

I_y x 10^6 mm^4	Height(h) in mm								
Breadth(b) in mm	**75**	**100**	**125**	**150**	**175**	**200**	**225**	**250**	**300**
16	0.56	1.33	2.60	4.50					
19	0.67	1.58	3.09	5.34					
22	0.77	1.83	3.58	6.19					
25	0.88	2.08	4.07	7.03	11.17	16.67	23.73	32.55	56.25
32	1.13	2.67	5.21	9.00	14.29	21.33	30.38	41.67	72.00
36	1.27	3.00	5.86	10.13					
38	1.34	3.17	6.18	10.69	16.97	25.33	36.07		
44	1.55	3.67	7.16	12.38	19.65	29.33	41.77	57.29	99.00
47	1.65	3.92	7.65	13.22	20.99	31.33	44.61	61.20	105.75
50	1.76	4.17	8.14	14.06	22.33	33.33	47.46	65.10	112.50
63		5.25	10.25	17.72	28.14	42.00	59.80		
75		6.25	12.21	21.09	33.50	50.00	71.19	97.66	168.75
100		8.33		28.13		66.67		130.21	225.00
150				42.19		100.00			337.50
200						133.33			
250								325.52	
300									675.00

Table 6. Second moment of area of the cross sections

$W_{el,y}$ x 10^3 mm^3	Height(h) in mm								
Breadth(b) in mm	75	100	125	150	175	200	225	250	300
16	15.00	26.67	41.67	60.00					
19	17.81	31.67	49.48	71.25					
22	20.63	36.67	57.29	82.50					
25	23.44	41.67	65.10	93.75	127.60	166.67	210.94	260.42	375.00
32	30.00	53.33	83.33	120.00	163.33	213.33	270.00	333.33	480.00
36	33.75	60.00	93.75	135.00					
38	35.63	63.33	98.96	142.50	193.96	253.33	320.63		
44	41.25	73.33	114.58	165.00	224.58	293.33	371.25	458.33	660.00
47	44.06	78.33	122.40	176.25	239.90	313.33	396.56	489.58	705.00
50	46.88	83.33	130.21	187.50	255.21	333.33	421.88	520.83	750.00
63		105.00	164.06	236.25	321.56	420.00	531.56		
75		125.00	195.31	281.25	382.81	500.00	632.81	781.25	1,125.00
100		166.67		375.00		666.67		1,041.67	1,500.00
150				562.50		1,000.00			2,250.00
200						1,333.33			
250								2,604.17	
300									4,500.00

Table 7. Elastic modulus of the cross sections

5 Practical example

Given the beam AB in Figure 3, designed with a solid timber D60 200 x 250 cross section, considering that it is part of an indoor floor, it is requested to check the resistance condition considering that the design value of the uniformly distributed surface load to be bore is 7 kN/m², being this load a combination of permanent loads and imposed loads

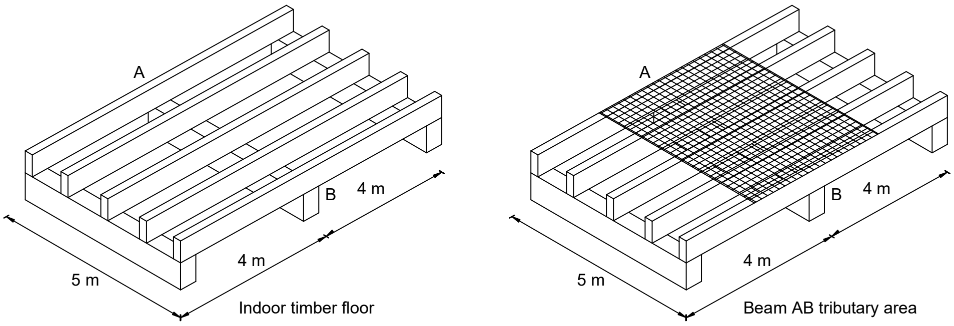

Figure 3. Beam to be designed

1. Structural model: Considering the tributary area on the right in Figure 3, the load per unit length (line load) to consider on AB beam is:

$$q_d = 7\ kN/m^2\ x 4\ m = 28\ \ kN/m$$

Figure 4 shows the structural model to be considered:

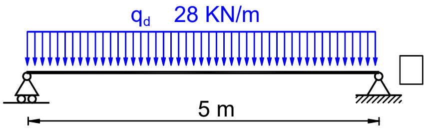

Figure 4. Simple beam

2. Internal forces:

$$V_{max} = \frac{q_d \cdot L}{2} = \frac{28 \cdot 5}{2} = 70\ kN$$

$$M_{max} = \frac{q_d \cdot L^2}{8} = \frac{28 \cdot 5^2}{8} = 87.5\ kN \cdot m$$

Internal forces diagrams (Figure 5)

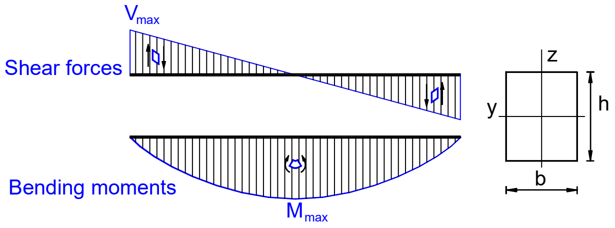

Figure 5. Shear forces and bending moments diagrams

3. Mechanical properties of the cross section

$$W_{el} = \frac{b \cdot h^2}{6} = \frac{200 \cdot 250^2}{6} = 2.083.333\ mm^3$$ Equation 4

4. Resistance condition in bending, considering Equation 2

$$\sigma_{m,d} \le f_{m,d}$$

Being $\sigma_{max} = \frac{M_{max}}{W_{el}} = \frac{87.5 \cdot 10^6\ N \cdot mm}{2.083.333\ mm^3} = 42\ N / mm^2$ as seen in Equation 3

And $f_{m,d} = k_{mod} \cdot \frac{f_{m,k}}{\gamma_M}$ Equation 5

where $k_{mod} = 0.8$ (service class 1) taking into account that when the combination of loads to take into account for the resistance condition includes loads belonging to different load-duration classes, the modification factor to be considered corresponds to the load with the lesser duration, which in this case is the imposed load (medium term load)

$\gamma_M = 1{,}30$ for solid timber

$f_{m,k} = 60\ N / mm^2$ for D60 strength class

Substituting: $f_{m,d} = 0.80 \cdot \frac{60}{1.30} = 36.92\ N / mm^2$

As $\sigma_{m,d} = 42\ N/mm^2 > f_{m,d} = 36.92\ N/mm^2$ **it fails**

The beam is designed with a new cross section. Considering the resistance condition in bending, it is obtained the initial size of the beam:

$$\sigma_{max} = \frac{M_{max}}{W_{el}} = \frac{87.5 \cdot 10^6 \ N \cdot mm}{W_{el}} \leq f_{m,d} = 36.92 \ N / mm^2$$ as seen in *Equation 3*

$$W_{el} \geq 2\,369\,989 \ mm^3$$

The first cross section that fulfils this condition in table 7 is 250x250 with $W_{el} = 2604.17 \times 10^3$

Therefore, it is checked this new cross-section:

Resistance condition in bending:

$$\sigma_{max} = \frac{M_{max}}{W_{el}} = \frac{87.5 \cdot 10^6 \ N \cdot mm}{2\,604\,170 \ mm^3} = 33.59 \ N / mm^2$$ as seen in *Equation 3*

Being $\sigma_{max} = 33.59 \ N/mm^2 > f_{m,d} = 36,92 \ N/mm^2$ Beam AB will be designed with a D 60 250x250 cross section.

6 Conclusions

After reading this document and analysing the worked example, students will be able to:

Classify the structural element to be designed considering the load-duration classes and the service classes.

Obtain the Modification factor value, k_{mod} that considers the influence of load-duration and moisture content on strength for solid timber.

Design a solid timber beam considering the resistance condition in bending proposed by the Spanish code DB SE M

7 Proposed exercise

In order to practice the design process for solid timber beams, it is requested to design one of the joist of the timber floor in Figure 1, with the same timber grade.

The structural model and internal forces diagrams are those in Figure 6.

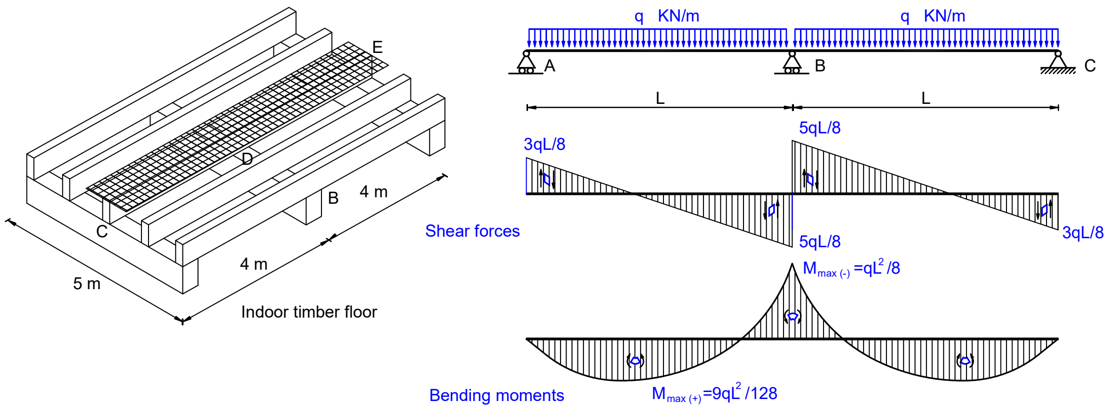

Figure 6 Internal forces for the beam to be designed

8 Bibliography

[1] MINISTERIO de la VIVIENDA: “Documento Básico Seguridad Estructural, Madera”, Código Técnico de Edificación. Ministerio de Fomento. 2006. http://www.codigotecnico.org/web/ *(in Spanish)*

9 Proposed exercise solution

The tributary area for the joist is 1.25 m, therefore, the line load value to be considered in the design of the joist is equal to 8.75 kN/m being the maximum bending moment, at point B equal to 17.5 KNm

Considering the resistance condition given by *equation 3*, the W_{el} needed is equal to 473 997 mm^3.
According to table 7, the elastic modulus of a 47x250 mm cross section is W_{el} = 489 580 mm^3.
Therefore, the joists will be designed with a D60 47x250 cross section.