

An integrated model for aircraft routing and crew scheduling: Lagrangian Relaxation and metaheuristic algorithm

Un modelo integrado para el enrutamiento de aeronaves y la programación de la tripulación: Relajación lagrangiana y algoritmo metaheurístico

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Abstract

Airline optimization is a significant problem in recent researches and airline industrial as it can determine the level of service, profit and competition status of the airline. Aircraft and crew are expensive resources that need efficient utilization. This paper focuses simultaneously on two major issues including aircraft maintenance routing and crew scheduling. Several key issues such as aircraft replacement, fairly night flights assignment and long-life aircrafts are considered in this model. We used the flight hours as a new framework to control aircraft maintenance. At first, an integrated mathematical model for aircraft routing and crew scheduling problems is developed with the aim of cost minimization. Then, Lagrangian relaxation and Particle Swarm Optimization algorithm (PSO) are used as the solution techniques. To evaluate the efficiency of solution approaches, model is solved with different numerical examples in small, medium and large sizes and compared with GAMS output. The results show that Lagrangian relaxation method provides better solutions comparing to PSO and also has a small gap to optimum solution.

Keywords: *Aircraft maintenance routing, Crew scheduling, Integer Programming, Lagrangian Relaxation, Particle Swarm Optimization.*

Resumen

La optimización de aerolíneas es un problema importante en investigaciones recientes e industria de aerolíneas, ya que puede determinar el nivel de servicio, el beneficio y el estado de competencia de la aerolínea. Las aeronaves y la tripulación son recursos costosos

que necesitan una utilización eficiente. Este artículo se centra simultáneamente en dos cuestiones principales, incluyendo el enrutamiento de mantenimiento de aeronaves y la programación de la tripulación. En este modelo se consideran varios temas clave, como el reemplazo de aeronaves, la asignación de vuelos nocturnos y los aviones envejecidos. Usamos las horas de vuelo como un nuevo marco para controlar el mantenimiento de las aeronaves. Al principio, se desarrolla un modelo matemático integrado para el enrutamiento de aeronaves y los problemas de programación de la tripulación con el objetivo de la minimización de costos. A continuación, se utilizan como técnicas de solución la relajación lagrangiana y el algoritmo "Particle Swarm Optimization" (PSO). Para evaluar la eficiencia de los enfoques de la solución, el modelo se resuelve con diferentes ejemplos numéricos en tamaños pequeños, medianos y grandes y se compara con la salida GAMS. Los resultados muestran que el método de relajación lagrangiana proporciona mejores soluciones en comparación con PSO y también tiene una pequeña diferencia para una solución óptima.

Palabras clave: *Enrutamiento de mantenimiento de aeronaves, programación de tripulación, Programación entera, Relajación lagrangiana, Particle Swarm Optimization.*

Introduction

Airline companies attempt to minimize costs in the highly competitive airline industry. Airline schedules often undergo different unpredictable problems that not only cause disruptions and delays but also lead to passengers' dissatisfaction as well as other disadvantages for the airline (Ben Ahmad et al., 2017). Over the recent decade, numerous researches have shown a special interest in planning and operations for airlines such as flight scheduling, fleet assignment, maintenance routing, and crew scheduling. Airlines are looking for solutions to these problems in order to reduce their operational costs (Muter et al., 2013). Here, these problems are introduced briefly:

- Schedule design determines the flight schedule that results in the maximum possible profit. This schedule forms the basis of the airline operations (Warburg et al., 2008, Jiang and Barnhart, 2009 and Eltoukhy et al., 2017).
- Fleet assignment is responsible for assigning aircrafts to the scheduled flight legs according to the aircrafts' sizes, costs, and expected profit (Sherali et al., 2006 and Dozic and Kalic, 2015).
- Aircraft routing defines the sequence of flights for each aircraft while trying to cover each flight and satisfy the maintenance requirements (Lacasse-Guay et al., 2010 and Al-Thani et al., 2016).
- Crew scheduling consists of assigning a qualified crew member to each flight as this assignment has to satisfy several rules and regulations (Gopalakrishnan and Johnson, 2005 and Kasirzadeh et al., 2015).

These four problems are often solved sequentially, and the solution to one problem is an input for the next one. Needless to say, this approach decreases computational complexity and time; however, during recent years, many authors have been able to merge these problems (Shao et al., 2017).

In this paper, aircraft routing and crew scheduling problems are considered simultaneously with the aim of cost minimization. Several important issues are taken into account in the proposed model including

aircraft replacement, deadhead flights, fairly distributed night flights and long-life aircrafts. Aircraft replacement allows the crew to handle two sequential flights with two different aircrafts of the same type. This imposes additional costs to airlines. Deadhead flight means crew traveling from a city to another one as a passenger. Another important issue is crew night flights. Night flights must be assigned in a fairly manner to the crew. Moreover, night flights of each crew member should not exceed a predefined value. Some aircrafts have more function and they require exact schedule. Decision making about the utilization level of these aircrafts is an important task. We suggested a separated schedule for long-life aircrafts for better utilization. We also introduced a new framework to control aircraft maintenance that distinguishes this research from the previous ones. In this framework, each aircraft must be checked before reaching a predefined threshold value.

This paper includes the following sections:

In Section 2, the literature related to the maintenance routing and crew scheduling problems is reviewed. Section 3 presents and analyzes a linear model as well as related assumptions and details. Section 4 describes the proposed solution approaches and results. Section 5 includes noticeable results and future suggestions.

Literature review

The airline scheduling problem is considered a vital issue in the airline industry as the effective and robust schedules can generate more profit in this industry. The scheduling process is defined 12 months before operations but the final schedule are not assigned to each aircraft and crew until a few weeks before implementation. Hence, the decision-making process includes four independent stages that need to be solved frequently and sequentially. These stages are schedule planning, fleet assignment, aircraft routing, and crew scheduling (Yu, 1998). The process generates a flight timetable as well as aircraft assignment and crew covering each flight while satisfying all requirements (Shao, 2012). In this paper, we focus on two issues that are maintenance routing and crew scheduling problem. First, we review these issues in the next part.

Crew scheduling Problem

There are more than 1000 flights per day on popular airlines which in turn necessitates scheduling thousands of crew on a daily basis. Crew scheduling is restricted by the rules imposed by national aviation authorities and other airline unions. As the crew cost is the second largest expenditure of an airline after fuel cost, crew scheduling is a complex task (Barnhart and Cohn, 2004). Airline crew scheduling is one of the most important planning problems that was studied by Saddoune et al. (2011). They highlighted that the total cost of crew includes salaries and benefits. Crew scheduling problem assigns crew members to flights while minimizing the total cost and respecting legal restrictions requirements.

As the constrained combinatorial crew optimization problem is NP-hard, solving the crew scheduling in a reasonable computational time is a hard task. Crew scheduling generates a set of feasible pairings with the

aim of minimizing the total crew cost while trying to satisfy the predefined flight schedule as well as the labor union and governmental rules, fleet routes, and the airline's policies (Azadeh et al., 2013). Yen and Birge (2006) discussed a two-stage random schedule considering the scheduled and resource costs of crew. These stages minimized the mentioned costs based on the planes under uncertainty. Crew scheduling was also studied (Borndorfer et al., 2006). They used the column generation method as their solution approach. Their study is based on a set partitioning model.

Zhang et al. (2015) Proposed a two-stage heuristic approach for the integrated recovery problem. They rescheduled the integrated aircraft recovery and flight model applying the traditional multi-commodity. Their model tended to minimize crew disruption. Crew pairing and rostering are noticed as two related problems according to the study of Zeren and Ozkol (2016). They generated all possible crew pairings and assigned them to flights in two stages.

Deveci and Cetin Demirel (2018) Illustrated two sequential stages, namely generation and optimization stage. The first stage included generating all feasible pairings according to the predefined flights. The second stage optimized the total cost by selecting the best subset of the generated pairings while minimizing. This paper investigated the model based on two genetic-based algorithms (GA) variants and a memetic algorithm (MA). The results presented the outperformance of the proposed MA in comparison to GA.

Aircraft maintenance routing problem

One of the most crucial issues in the airline industry is the aircraft maintenance problem. After assigning aircrafts to the flights, a separate problem can be solved for each aircraft. Most papers seek to find a single rotation for a certain fleet; however, this single rotation does not apply to the entire fleet (Basdere and Bilge, 2014). Belien et al. (2010) Studied a line maintenance routing problem. They discussed the aircraft inspection issue. Aircrafts had to enjoy short inspections between their arrival and departure time in a specific airport.

Masoud Bazargan (2010), in his book titled "Airline Planning and Operations" emphasized that maintenance activities determine the level of success and profit in any airline. Keeping aircrafts safe and being on-time is of high importance in any airline. There are several predefined maintenance programs established by the aircraft manufacturer and the Federal Aviation Administration (FAA). Maintenance programs must be scheduled and operated based on some meticulous procedures and standards.

There are different maintenance checks based on the flights frequencies and durations as well as the aircraft type (Feo and Bard, 1989; Clarke et al, 1997; Basdere and Bilge, 2014). These maintenance checks are as follows:

- When the aircraft flies for 65-125 hours, type A check is done. Also, this check can be repeated per each flight or every week. A check consists of visual inspection of major systems and lasts around eight hours.
- Type B check is operated after every 300-600 hours of flying. This check lasts around 1-3 days.
- Type C and D checks are done once a year or every four years. They can last for about one month.

Al-Thani et al. (2016) Studied the Operational Aircraft Maintenance Routing Problem (OAMRP). The contribution of this paper included two stages. After describing the OAMRP, they proposed an exact model, including polynomial variables and constraints. They used a graph reduction method and valid inequalities to improve the model solubility. They applied a search algorithm for problems in large scales to compute the lower bounds.

Jamili (2017) considered fleet assignment as well as aircraft routing and scheduling. He proposed an integrated model according to the Simulated Annealing approach (SA) that was able to create the best solutions in large-sized problems. Several examples were randomly generated. Reducing delays in the aircraft routing problem was discussed by Yan and Kung (2018). They used the robust optimization method to achieve this aim. Safaei and Hardine (2018) Formulated the new aircraft maintenance routing model considering the capacity, sufficient opportunities for maintenance, available requirements, and demands within all routes. They used an interactive approach to minimize the maintenance misalignment that can accrue between aircraft routing and maintenance schedule. They took advantage of real datasets to validate their model. The results showed a reduction in maintenance misalignment based on the interactive approach. Ben Ahmed et al. (2018) Proposed an integrated model for aircraft routing and crew pairing problems that included polynomial variables and constraints. They searched for the robust routes that not only were cost-effective but also could satisfy the constraints. A general-purpose solver was used to solve the model. They also collected data from major airlines. The computational results of the model admitted its stability and usefulness.

Mathematical model

The contributions of this paper are introducing a new framework to control aircraft maintenance, aircraft replacement, better utilization of long-life aircraft, and fairly distribution of night flights. One of the most important issues distinguishing this research from previous ones is how to control maintenance activities. In previous studies, maintenance was done based on duty period; i.e. check A should be performed before finishing the third working day of an aircraft. In the proposed model, this is defined based on the flying hours of the aircraft which is much more accurate than duty period. In this framework, the aircraft must be checked before reaching a predefined threshold value. After performing maintenance, previous flying hours are ignored and the aircraft can go on the schedule until the threshold is reached again.

In the proposed model, some issues such as reducing aircraft replacement, better and more efficient utilization of long-life aircrafts, maximum crew night flights constraint, fairly distribution of night flights and deadhead flights are considered. The crew may have to make two sequential flights of pre-defined schedule by two different aircrafts for various reasons. This is called the aircraft replacement. Certainly, aircraft replacement causes cost increase. On the other hand, deadhead flights also impose large costs to an airline. The proposed model aim is cost minimization.

Another important issue is utilization of aircrafts. Long-life aircrafts with high flight hours require more exact schedule. Determining the proper utilization level of aircrafts can lead to increase reliability of fleet. In the proposed model, better utilization of long-life aircrafts is considered.

Fairly distribution of night flights has a significant impact on personal life of crew. Also, it has an impact on the efficiency of the airline as it has a significant effect on crew's satisfaction. Fairly distribution of night flights is another issue that is considered in the proposed model. Below, model assumptions are presented:

Model assumptions

Model assumptions are as follows:

- Data is deterministic.
- Each aircraft and crew can have several arrivals and departures to a city in a day.
- The difference in crew night flights should not be higher than a pre-defined level.
- Crew night flights should not exceed the maximum defined value.
- The flying hours of long-life aircrafts should not exceed a certain level.
- Maintenance control is performed based on flying hours.

Mathematical model

At first, sets, indices, parameters and variables are defined:

Sets and indices:

I	Set of cities
i, j	index of city
A	Set of aircrafts
a	index of aircraft
\hat{A}	Set of long-life aircrafts
T	Set of period times (days)
t	index of time period
N	Set of flight rounds. Suppose that a crew starts his duty from city A to B and continues it by flying from B to C. For the flight (A,B), n equals one and for the flight (B,C), n equals two.
n, n'	index of flight round
F	Set of flights $(i, j) \in F$
C	Set of crew
c, c'	index of crew
MA_i	Set of equipped cities for maintenance
O_a	Origin city for aircraft a at the beginning of the planning period

Parameters:

$cost_{ij}$	Travelling cost of an aircraft from i to j
$cost'_{ij}$	Crew cost for travelling from i to j
$cost''_{ij}$	Aircraft replacement cost to perform flight from i to j
$cost'''_{ai}$	Maintenance cost for aircraft a in city i

F'_{\max}	Maximum night flight hours
Q	Maximum difference in night flight hours between crews
U	The upper bound of flying hours for long-life aircrafts
HH_a	Flying hours of aircraft a at the beginning of the planning period
H'_a	Threshold value indicates that maintenance should be performed on aircraft a before reaching it.
Dur_{ij}	Flight time from i to j
L	A big number

Decision variables:

x_{aijnt}	Binary variable indicating whether aircraft a travels from i to j in day t as its n th flight.
y_{cijnt}	Binary variable indicating whether crew c travels from i to j in day t as his n th flight.
z_{cijnt}	Binary variable indicating whether crew c changes the aircraft for travelling from i to j in day t as his n th flight.
v_{aint}	Binary variable indicating whether maintenance is performed on aircraft a in city i after doing its n th flight in day t .
H_{aint}	Flying hours of aircraft a when it arrives at city i after its n th flight in day t .

$$\begin{aligned} & \sum_a \sum_i \sum_j \sum_n \sum_t \text{cost}_{ij} x_{aijnt} + \sum_c \sum_i \sum_j \sum_n \sum_t \text{cost}'_{ij} y_{cijnt} + \sum_c \sum_i \sum_j \sum_n \sum_t \text{cost}''_{ij} z_{cijnt} \\ & + \sum_a \sum_i \sum_n \sum_t \text{cost}'''_{ai} v_{aint} \\ & + \sum_i \sum_j \sum_t \text{cost}'_{ij} \sum_a \sum_n (x_{aijnt} - 1) \end{aligned} \quad (1)$$

$$\sum_a \sum_n x_{aijnt} \geq 1 \quad \forall (i, j) \in F, t \in T \quad (2)$$

$$\sum_c \sum_n y_{cijnt} = 1 \quad \forall (i, j) \in F, t \in T \quad (3)$$

$$\begin{aligned} H_{ai(n-1)t} + Dur_{ij} - L(1 - x_{aijnt}) - Lv_{ai(n-1)t} &\leq H_{ajnt} \\ \forall n > 1, t \in T, i, j \in I: i \neq j, a \in A \end{aligned} \quad (4)$$

$$\begin{aligned} H_{ai(n-1)t} + Dur_{ij} + L(1 - x_{aijnt}) + Lv_{ai(n-1)t} &\geq H_{ajnt} \\ \forall n > 1, t \in T, i, j \in I, a \in A \end{aligned} \quad (5)$$

$$\begin{aligned} Dur_{ij} - L(1 - x_{aijnt}) - L(1 - v_{ai(n-1)t}) &\leq H_{ajnt} \\ \forall n \in N, t \in T, i, j \in I, a \in A \end{aligned} \quad (6)$$

$$\text{Dur}_{ij} + L(1 - x_{ajnt}) + L(1 - v_{ai(n-1)t}) \geq H_{ajnt} \quad \forall n \in N, t \in T, i, j \in I, a \in A \quad (7)$$

$$\text{HH}_a + \text{Dur}_{ij} - L(1 - x_{ajnt}) \leq H_{ajnt} \quad \forall a \in A, j \in I, i = o_a, n = 1, t \in T \quad (8)$$

$$\text{HH}_a + \text{Dur}_{ij} - L(1 - x_{ajnt}) \geq H_{ajnt} \quad \forall a \in A, j \in I, i = o_a, n = 1, t \in T \quad (9)$$

$$-Q \leq \sum_i \sum_{j:(i,j) \in F} \sum_n \sum_t y_{cijnt} - \sum_i \sum_{j:(i,j) \in F} \sum_n \sum_t y_{cijnt} \leq Q \quad \forall c, \acute{c} \in C \quad (10)$$

$$\sum_i \sum_{j:(i,j) \in F} \sum_n \sum_t y_{cijnt} \leq \hat{F}_{\max} \quad \forall c \in C \quad (11)$$

$$\sum_i \sum_{j:(i,j) \in F} \sum_n \sum_t \text{Dur}_{ij} x_{ajnt} \leq U \quad \forall a \in A \quad (12)$$

$$\sum_{\substack{i \in I: \\ (i,j) \in F}} x_{ajnt} \geq \sum_{\substack{i \in I: \\ (j,i) \in F}} x_{aji(n+1)t} \quad \forall j \in I, a \in A, n \in N, t \in T \quad (13)$$

$$\sum_{\substack{j \in I: \\ j \neq i}} x_{ajnt} = 1 \quad \forall a \in A, i = o_a, n = 1, t = 1 \quad (14)$$

$$v_{aint} \leq \sum_{\substack{j \in I: \\ (j,i) \in F}} x_{ajnt} \quad \forall i \in MA_i, a \in A, n \in N, t \in T \quad (15)$$

$$H_{aint} \leq \hat{H}_a \quad \forall a \in A, i \in I, n \in N, t \in T \quad (16)$$

$$H_{aint} \geq 0 \ \& \ x_{ajnt}, y_{cijnt}, z_{cijnt}, v_{aint} \in \{0,1\} \quad (17)$$

The objective function consists of five sections: Travelling cost of aircraft, crew cost, aircraft replacement cost, maintenance cost and deadhead flight cost. Constraint (2) ensures that all flights are covered. Relation (3) shows that a crew is assigned to each flight. Constraints (4) and (5) show that if an aircraft travels from i to j with no maintenance in city i , its total flying hours equals flying hours in i plus flight time from i to j . Inequalities (6) and (7) describe that if an aircraft travels from i to j and maintenance has been performed on it in i , total flying hours (with no maintenance action) equals to the flight time from i to j . Constraints (8) and (9) also indicate the flying hours of an aircraft in its first flight. Constraint (10) ensures that the difference between crew night flights is less than a pre-defined value. Constraint (11) indicates that total crew night flights should not exceed a pre-defined value. Inequality (12) ensures that flying hours of long-life aircrafts should not exceed a certain value. Relation (13) ensures the balance of flow-in and flow-out to a city. Equation (14) expresses that each aircraft should fly from its origin at its first flight. Constraint (15) shows that if maintenance has been performed on an aircraft in a city, it should

visit the city. Equation (16) ensures that maintenance should be performed on an aircraft before reaching a pre-defined threshold. Relation (17) determines the type of decision variables.

Solution approach

In this section, solution approaches applied to solve the model are briefly described.

- **Lagrangian relaxation approach**

Lagrangian relaxation method is a heuristic technique that has been widely used to solve mathematical model specifically integer programming problems. Held and Karp (1970) developed this method to optimizing problems. Lagrangian Relaxation method is based on relaxation of complicated constraints and considering them in objective function. It is expected that solving this sub-problem is easier than the main problem due to the removal of some constraints and the expansion of the feasible region. For each constant value of the Lagrangian coefficients, the solution of the sub-problem is a lower bound for the main problem (in minimization problems). To solve the Lagrangian problem, a lower bound (LB) is determined and, as a result, upper bound (UB) is obtained for the main problem. If the difference between the lower and upper bounds is less than the predetermined value, the algorithm ends and the answer is reached. Otherwise, the algorithm runs until a certain iteration in which Lagrange coefficients must be updated. For this purpose, a step length (k) for each iteration is calculated by the following equation:

$$k = \theta \frac{UB - LB^*}{\sum_{i=1}^n (b_i - a_i x^*)^2}$$

In this paper, subgradient method is used. Subgradient is an iterative method for solving convex minimization problems. Computational and theoretical properties of this method are completely discussed in Held et al. (1974) and particularly in Goffin (1977). The algorithm steps are as follows:

Step 1. Calculate the upper bound (UB) and initial Lagrange coefficients vector λ and set $LB^* = -\infty$.

Step 2. Solve the relaxed problem and determine x^* and modified LB.

Step 3. If $UB > LB^*$, set $LB = LB^*$.

Step 4. $\lambda^{(t)}$ is calculated by $\lambda^{(t-1)} + k\{b - Ax\}$ where k is determined as follows:

$$k = \theta \frac{UB - LB^*}{\sum_{i=1}^n (b_i - a_i x^*)^2}$$

Step 5. If there is no improvement after successive iterations in the best bound, then $\theta = \frac{\theta}{2}$.

Step 6. Go to Step 2.

Among constraints, the 1st and 14th constraints increase complexity significantly. So, they are chosen for relaxation. Sub-problem is written as follows:

$$\begin{aligned} \text{Min} \quad & \{\sum_a \sum_i \sum_j \sum_n \sum_t \text{cost}_{ij} x_{aijnt} + \sum_c \sum_i \sum_j \sum_n \sum_t \text{cost}'_{ij} y_{cijnt} + \sum_c \sum_i \sum_j \sum_n \sum_t \text{cost}''_{ij} z_{cijnt} + \\ & \sum_a \sum_i \sum_n \sum_t \text{cost}'''_{ai} v_{aint} + \sum_i \sum_j \sum_t \text{cost}'_{ij} \sum_a (x_{aijnt} - 1)\} + \text{Landa1} (1 - \sum_a \sum_n x_{aijnt}) + \\ & \text{Landa2} (\sum_{\substack{j \in I: \\ j \neq i}} x_{aijnt} - 1) \\ \text{S.t.} \quad & (2) - (13), (15) - (17) \end{aligned}$$

- **Particle Swarm Optimization algorithms**

Particle swarm optimization (PSO) is a metaheuristic approach to optimize the problems with few or no assumptions. This approach searches very large spaces to find the best solution. PSO as a computational approach can more iteratively optimize the problem to improve the candidate solution. There are populations of candidate solutions as dubbed particles move in the search space based on simple mathematical formulae over the particle's position and velocity. If local best known position effects on movement of particle however is continued the best known positions in the search-space which are updated as better positions are found by other particles. Moving the swarm toward the best solutions is expectable.

Numerical examples

To evaluate the efficiency of solution approaches, nine numerical examples in small, medium and large sizes are generated. All problems are solved using GAMS as well as Lagrangian Relaxation method and PSO algorithm. Some assumptions are determined for all examples. The long-life aircrafts should not be used more than 30 hours. Maximum night flights are defined as 50 hours and the difference in night flight between the crew should not exceed 40 hours. As discussed above, flying hours of aircrafts are considered for controlling maintenance operation and at the beginning of the planning period the previous flying hours of the aircraft is ignored. Each aircraft should initiate from its origin airport. Input data of numerical examples is shown in Table 1.1.

Table 1.1. Numirecal Examples Data

	No. of cities	No. of aircrafts	No. of long-life aircrafts	No. of crew	No. of maintenance stations	Threshold value
1	5	3	1	3	3	150
2	8	4	1	5	3	150
3	11	7	2	7	5	350
4	18	10	3	14	5	350
5	18	10	3	20	9	350
6	23	10	3	20	9	350
7	30	10	3	20	9	350
8	35	16	3	26	12	350
9	40	16	3	26	12	350

All numerical examples were solved by GAMS, Lagrangian Relaxation and PSO algorithm. We considered 500 iterations to solve the model using Lagrangian Relaxation method. Also a proposed mathematical model was coded in Matlab software to read the input data. Then the model solved and identified the optimal solution as output. Furthermore, the gap between PSO and Lagrangian relaxation and GAMS were calculated.

According to Table 1.2, Lagrangian Relaxation provides an efficient lower bound in all examples. Comparison of results shows that Lagrangian Relaxation method can be ideally used for solving this problem. As shown in Table 1.2, GAMS cannot solve large scale problems. In this case, Lagrangian Relaxation method can be used as an efficient method. Figure 1.1 also confirms the efficiency of Lagrangian Relaxation method for solving this model.

Table 1.2. Objective function in GAMS, Lagrangian Relaxation and PSO

Number of Examples	GAMS	Lagrangian Relaxation	Gap	Particle Swarm Optimization	Gap
1	1993000	1968811	1.2	2011400	0.923
2	3284000	3264277	0.6	3304501	0.624
3	9232000	8505301	7.8	9313000	0.877
4	12456000	11228829	9.8	13000415	4.37
5	24471000	22139073	9.5	26702317	9.112
6	35613000	33101081	7.05	39764000	11.656
7	53455000	53300054	0.28	58450345	9.344
8	NA	57212255	-	63500450	-
9	NA	66511495	-	72450760	-

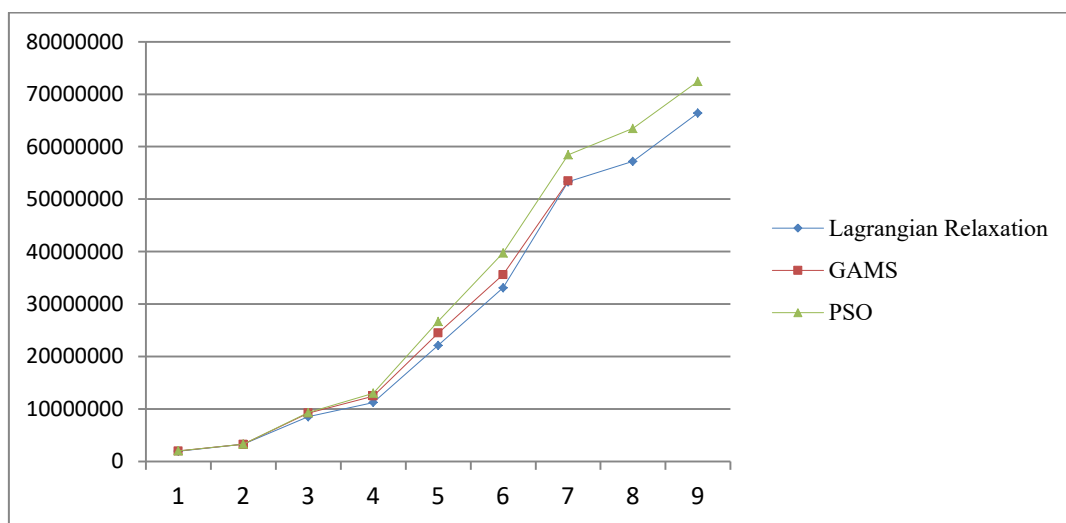


Figure1.1 Comparison of results.

Conclusion

This paper presents an integrated model for aircraft routing and crew scheduling problems. The main contribution of the paper is controlling maintenance activities based on the flying hours of the aircraft instead of duty period. The model also includes aircraft replacement, better utilization of long-life aircraft, deadhead flights and fairly distribution of night flights. Model includes constraints to ensure better utilization of aircrafts as well as fairly distribution of night flights. Also, the proposed model leads to decrease in replacement aircrafts and deadhead costs. Some numerical examples were generated in small, medium and large sizes and solved using GAMS as well as Lagrangian Relaxation method and PSO algorithm. Lagrangian relaxation solution has given a small gap and suggests efficient lower bounds. One of the important assumptions is defining the problem in deterministic situations which can be considered in non-deterministic ones in future researches.

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