Characterization of between-group inequality of longevity in European Union countries

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Abstract

Comparisons of differential survival by country are useful in many domains. In the area of public policy, they help policymakers and analysts assess how much various groups benefit from public programs, such as social security and health care. In financial markets and especially for actuaries, they are important for designing annuities and life insurance products. This paper presents a method for clustering information about differential mortality by country. The approach is then used to group mortality surfaces for European Union (EU) countries. The aim of this paper is to measure between-group inequality in mortality experience in EU countries through a range of mortality indicators. Additionally, the indicators permit the characterization of each group. It is important to take into account characteristics such as sex; therefore, this study differentiates between males and females in order to detect whether their patterns and characterizations are different. It is concluded that there are clear differences in mortality between the east and west of the EU that are more important than the traditional south-north division, with a significant disadvantage for Eastern Europe, and especially for males in Baltic countries. We find that the mortality indicators have evolved in all countries in such

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a way that the gap between groups has been maintained, both in terms of the differences in mortality levels and variability.

Keywords: mortality surface, PCA, cluster, mortality indicators.

1 Introduction

Interest in health inequalities between European Union (EU) countries and their regions as well as the various social clusters in the EU population is growing (Spinakis et al., 2011; Eurostat, 2016). This is driven by the fact that European and national epidemiological studies highlight a gap between eastern and western, and northern and southern countries and regions of the EU as well as within countries and regions and between socio-economic groups (Mackenbach et al., 1997; Dalstra et al., 2002; Meslé, 2004; Villegas and Haberman, 2014). Indeed, mortality patterns are changing in the west, and the traditional opposition between north and south is undergoing radical transformations (Meslé and Vallin, 2002).

As O'Donnell (2009) comments, survey data and mortality data permit the study of health differences in relation to various socioeconomic dimensions. Mortality rates or survival rates quantify the effect of differences in the daily lives of individuals (life habits, e.g. smoking, nutrition, etc) on mortality and morbidity. We believe that a more focused effort is required in the search for the most appropriate summary measure(s) of health inequality (Spinakis et al., 2011) and consequently mortality indicators.

The problem of determining which mortality indicators influence the characterization of a country group has been considered from different perspectives and statistical methodologies, although the investigation of this issue has not made use of non-parametric techniques, such as "Classification And Regression Trees" (CART) (see Breiman et al. (1984); Qinlan (1993) for an overview). Recently, random forests have been proposed as a methodology. They are an ensemble learning method for classification that operate by constructing a multitude of CART using a different bootstrap sample of the data and a subset of predictors randomly chosen at each node, and then outputting the class which is the mode of the output classes by individual CART (Liaw and Wiener, 2002)

Working in the framework of multi-population mortality models, Li et al. (2004) discuss ways in which the Lee–Carter method can be used for countries with limited mortality data. Li and Lee (2005), Russolillo et al. (2011) and Debón et al. (2011) extend the Lee–Carter framework, modeling a global improvement process together with deviations for small populations. Hatzopoulos and Haberman (2013) present a new common mortality modeling structure to analyse mortality dynamics for a pool of countries. Ahcan et al. (2014) suggest replicating the mortality of a small population by appropriately mixing the mortality data obtained from other populations.

There is a research gap in the characterization of country groups based on mortality indicators and their capacity for discrimination. For this reason, this study also introduces random forests, which allow us to describe groups and rank indicators. The correct characterization of countries is essential to have a clear idea of which indicators Eurostat or other institutions should give the most emphasis to when reporting the statistics. European policy making should focus on the currently alarming phenomenon of widening differences in life expectancy (Mackenbach, 2013). Therefore, this study applies different statistical techniques for the classification and characterization of EU countries on the basis of mortality data.

We present a method for clustering information about differential mortality by country or geographical unit. Then, this approach is used to group mortality surfaces for EU countries. The aim of this paper is to measure the between-group inequalities in mortality in EU countries though synthetic mortality indicators. Additionally, the indicators permit the characterization of each group. We provide separate analyses for males and females in order to detect whether behavior and characterizations vary by sex. Countries are first classified by cluster analysis in order to construct the common structure for the global mortality experience. Then the mortality indicators are analysed to see which better reflect between-group inequalities in mortality. There are many applications for the classification; for example, numerical simulations based on country data illustrate that if mortality improves in the same way for a group, the effect on pension systems could be similar.

The statistical methodology that we propose was developed with the aim of establishing an operating procedure that guides the comparison of mortality for countries, especially in the EU. We divide the analysis into four steps:

1. Clustering the patterns of EU mortality during the period 1990-2010 using a fuzzy cluster algorithm.

2. Describing changes in mortality among the main clusters.

3. Selecting the most important mortality indicators.

4. Characterization of the groups in the EU.

Europe can be divided along many differing lines, for example the pure geographical criteria of east, west, north and south or Mediterranean countries. Mortality differences between these areas have varied at different times, and between-group inequalities in mortality are larger in the east-west direction.

The remainder of the paper is organized as follows. In Section 2, Cluster and Principal Component Analysis (PCA) methods are described in order to derive the European groups. In Section 3 related non-parametric models are introduced specifically to deal with the difficulty of group characterization. In Section 4, using mortality data for the common time range 1990-2010 and for ages from 0 to 109 from the Human Mortality Database, 24 countries are grouped using fuzzy c-means cluster analysis. In this way, males and females are divided into four clusters. After identifying the clusters, the evolution of mortality in these countries between 1990 and 2010 is analysed considering different mortality indicators that measure both their trend and their disparity. Next, random forests and CART are used to select the most important mortality indexes and to implement a coherent classification of the countries based on those indexes such as infant mortality, life expectancy at birth and at age 65, modal age at death, Gini Index at birth, at age 10, 40 and 65 and conditional standard deviation at age 10 calculated for 2010. Finally, in Section 5, the results are discussed and some concluding comments are offered.

2 Cluster Methodology

We present a method for grouping countries according to the distances between their mortality surfaces. A *mortality surface* is a function of the (time t, age x) "Lexis plane", which describes the mortality experience of a population during a given time interval. Examples are: the logarithm of the force of mortality, $\log(\mu_{xt})$, and the logit transformation of the age specific probability of death, $\log(q_{xt}/(1-q_{xt}))$. A mortality surface is drawn when a mortality quantity or its transformation is plotted by age and time period simultaneously in a three-dimensional representation. In this paper we illustrate the methodology using $\text{logit}(q_{xt})$. We propose using $\text{logit}(q_{xt})$ on account of the goodness of fit for mortality models obtained in our previous work Debón et al. (2005) and more recently by Currie (2016). In addition, the cluster and principal components are much more robust and less sensitive to small changes in the data because normality and homoscedasticity are achieved by $\text{logit}(q_{xt})$.

Distances between mortality surfaces are measured and then clustered into country homogeneous groups. In classic cluster analysis each data element belongs to just one cluster. In contrast, in fuzzy clustering, data elements can belong to more than one cluster, each with an associated membership level (Hatzopoulos and Haberman, 2013). This indicates the strength of the association between the data element and a particular cluster. We use a fuzzy version of the kmeans clustering algorithm as well as its online update (Unsupervised Fuzzy Competitive Learning) (Pal et al., 1996) implemented in the e1071 R-package (Meyer et al., 2012).

Cluster analysis is a reasonable approach to separate the countries into groups with similar mortality dynamics. The measurement of similarities between objects in multi-dimensional space is based on Euclidean distances, which results in a distortion of information, causing the effect known in the statistical literature as the curse of dimensionality: "the higher is the space dimensionality, the larger is a space per data point, and therefore less completely and definitively does each feature describe each given object" (Andreev, 2004). To avoid the curse of dimensionality, the most common solution is to reduce the information about countries' mortality surfaces to the most significant features. Multivariate analysis provides methods to identify and summarize joint relationships of variables in large data sets such as for example Principal Component Analysis (PCA). The idea of using PCA to study mortality is not new. For example, the classic model of Lee and Carter (1992) used PCA with parameter estimation proposals, as have various model extensions with more components (Booth et al., 2002; Renshaw and Haberman, 2003a,b,c; Debón et al., 2008a,b; Yang et al., 2010; Mitchell et al., 2013). None of these models deals with missing data or chooses the number of PCs based on cross-validation (CV). In the context of mortality, countries with small populations, especially at young and old ages, could experience zero probabilities of death, which cannot then be log-transformed and in this sense can be considered missing values. In this paper, we estimate PC models using the R-package missMDA (Josse and Husson, 2016), which handles missing values.

Additionally, cluster validity indices are introduced, which assess the average compactness and separation of fuzzy partitions generated by the fuzzy c-means algorithm. There are a number of cluster validation indices available (Rezaee et al., 1998); details can be found in Appendix A.

3 Characterization of Groups

3.1 Mortality Indicators

In order to analyse the behavior of mortality over age and time, both trend indicators and measures to quantify their variability were selected. For each year studied we calculate trend mortality indicators including infant mortality, modal age at death, life expectancy and variability mortality indicators such as Gini index and conditional standard deviation; more details are provided in Debón et al. (2012).

Infant mortality is the probability of death in the first year of life q_{0t}. Mortality in the first year of life is commonly used as a global indicator of population health (Reidpath and Allotey, 2003).

The modal age at death, M_t, is the age associated with the maximum frequency of death in the life table distribution of deaths. The choice of this indicator is justified as it can reflect changes in the probability of death q_{xt} that are not detected with life expectancy (Canudas-Romo, 2008).

The life expectancy for individuals with age x is given by equation (1),

$$e_{xt} = \frac{T_{xt}}{l_{xt}}, \tag{1}$$

where l_{xt} is the hypothetical number of people alive at the beginning of each age interval $[x, x+1)$ for a year t and T_{xt} is the total number of years expected to be lived from age x until the highest attainable age in the year t life table population. The life expectancy indices calculated in Section 4 refer to life expectancy at birth and at 65, e_{0t} and e_{65t}, respectively.

The above two indicators do not provide any information about whether the improvement in mortality rates applies equally to different age groups. The Gini coefficient is the most commonly used statistical index of diversity or inequality in the social sciences and it can also be used as a measure of inequality in length of life (Shkolnikov et al., 2003). The Gini index for individuals with age x_0 derives from the Lorenz curve, which is the curve obtained when we represent the cumulative proportion dead in the life table stationary population on the x-axis, $f_{xt}^{x_0}$,

$$f_{xt}^{x_0} = \frac{l_{x_0t} - l_{xt}}{l_{x_0t}} = 1 - \frac{l_{xt}}{l_{x_0t}}, \tag{2}$$

and the proportion of total life table years lived by those that have died by age x on the y-axis, $g_{xt}^{x_0}$

$$g_{xt}^{x_0} = \frac{T_{x_0t} - T_{xt} - (x - x_0)l_{xt}}{T_{x_0t}}. \tag{3}$$

The Lorenz curve is obtained by joining these points and it is always below the diagonal. One of the most widely used approaches for estimating the Gini index at age x_0 is given in equation (4),

$$G_{x_0 t} = \frac{\sum_{x=x_0}^{(\omega-1)} (f_{xt}^{x_0} - g_{xt}^{x_0})}{\sum_{x=x_0}^{(\omega-1)} f_{xt}^{x_0}}, \tag{4}$$

where ω is the last age observed.

The Gini index summarizes the degree of concentration contained in the Lorenz curve with a single value, and its value varies from 0 (perfect equality) to 1 (perfect inequality). The value 0 is obtained when all individuals die at the same age, while the value 1 is achieved if the entire population (except one) dies at age 0 and one individual lives until the highest attainable age. In keeping with Keyfitz's idea (Keyfitz and Caswell, 2005) that everybody dies prematurely, Mitra (1978) developed the measure the average life expectancy lost due to death and Shkolnikov et al. (2003) initiated a new direction by means of the Gini Index. As Debón et al. (2012) proposed, the contribution of a particular age group x to the life expectancy at birth, $e_{0t} = \frac{T_{0t}}{l_{0t}}$, can be seen as the balance between the contribution to the numerator (years lived) and the denominator (population). Therefore, the proportion dead of the life table stationary population at age x can be obtained by $\Delta f_{xt} = f_{xt} - f_{(x-1)t}$, and the proportion of life table years lived by those who have died by age x can be obtained by $\Delta g_{xt} = g_{xt} - g_{(x-1)t}$, taking first differences of the corresponding expressions (2) and (3) with $x_0 = 0$. We define c_{xt} to be the difference between those proportions, as follows:

$$c_{xt} = \Delta g_{xt} - \Delta f_{xt} = [g_{xt} - g_{(x-1)t}] - [f_{xt} - f_{(x-1)t}], \quad x = 1, \ldots, 109, \tag{5}$$

noting that the differences each sum to 1 as in equations 6 and 7,

$$\sum_{x=0}^{109} (f_{xt} - f_{(x-1)t}) = 1, \tag{6}$$

$$\sum_{x=0}^{109} (g_{xt} - g_{(x-1)t}) = 1. \tag{7}$$

So if the difference in expression (5) is negative the proportion of the population exceeds the corresponding proportion of contributed years, and therefore that age group is deficient and, on the contrary, if the difference is positive.

In addition, the Gini index is an intuitively meaningful measure which satisfies the following three basic properties that have been recommended by Shkolnikov et al. (2003):

1. *population-size independence*, the index does not change if the overall number of individuals changes with no change in proportions of years lived,

2. *scale independence*, the index does not change if everyone's years lived changes by the same proportion, and

3. *Pigou-Dalton condition*, any transfer from an older to a younger individual that does not reverse their relative ranks reduces the value of the index.

The G_0, G_{10}, G_{40} and G_{65} in Section 4 refers to the Gini index calculated at ages 0, 10, 40 and 65, respectively.

There is a growing agreement in the demographic literature that a discussion of differences in population health across various groups and over time should also be based on other measures of mortality dispersion. An example of such an additional measure is the conditional standard deviation $S_{x_0 t}$ invariance over additive change proposed by Edwards and Tuljapurkar (2005) in expression (8),

$$S_{x_0 t} = \sqrt{\frac{\sum_{x=x_0}^{\omega} d_{xt}(x + 1/2 - M_{x_0 t})^2}{l_{x_0 t}}}, \tag{8}$$

where $M_{x_0 t}$ is the life expectancy plus current age, $M_{x_0 t} = e_{x_0 t} + x_0$ and d_{xt} number of death at the beginning of each age interval $[x, x+1)$ for a year t. For present purposes, we focus on Gini index and conditional standard deviation because the first is invariant over proportional translations of the underlying distribution while the second is invariant over additive translations (Edwards, 2010).

The S_{10} in Section 4 refers to conditional standard deviation calculated at age 10.

3.2 Classification by Random Forests

Demographic and actuarial research are frequently faced with handling complex data that include a large number of variables and with the necessity to obtain information, find patterns and identify trends. To our knowledge, CART or random forests has not been used in the characterization of country groups based on mortality indicators. A detailed description of CART can be found in Hastie et al. (2001). Trees can be used for several different types of response data; our model is formally a classification tree. Recently there has been considerable interest in “ensemble learning” — methods that generate many classifiers and aggregate their results (Liaw and Wiener, 2002). Breiman (2001) proposed random forest, which adds an additional layer of randomness to bagging. In addition to constructing each tree using a different bootstrap sample of the data, each node is split using the best among a subset of predictors randomly chosen at that node instead of using all of the variables as in CART. Random forests are among the most popular machine learning methods thanks to their relatively good accuracy, robustness and ease of use.

In standard trees, each node is split using the best split among all the variables. In a random forest, each node is split using the best among a subset of predictors that are randomly chosen at that node. In addition, it is very user-friendly as it only has two parameters (the number of variables in the random subset at each node and the number of trees in the forest), and is usually not very sensitive to their values of these parameters.

The randomForest package provides an R interface for the Fortran programs developed by Breiman and Cutler (available at `http://www.stat.berkeley.edu/users/breiman/`). Liaw and Wiener (2002) provide a brief introduction to the use and features of the R functions. The randomForest package optionally produces an additional piece of information which is a measure of the importance of the predictor variables. The reader is referred to (Liaw and Wiener, 2002) for its definition. In, our application, this principally shows which indicators are appropriate for comparing countries' mortality experience.

4 Analysis of mortality data from the European Union

All of these methods are used to study mortality for males and females, separately, in EU countries. The application is carried out using the language and environment for statistical computing and graphics R (R Core Team, 2015).

4.1 Data

The EU currently consists of 28 countries. However, the Human Mortality Database life tables are only available for the following 24 countries: Austria, Belgium, Bulgaria, The Czech Republic, Denmark, Estonia, Finland, France, Greece, Germany, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, The Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden and The United Kingdom. There are complete data for the above EU countries for the common time range 1990-2010 and for ages from 0 to 110 years old. Period life tables were downloaded from the Human Mortality Database (2015) for each country using HMDHFDplus R-library (Riffe, 2015) in order to explore the clustering made by Meslé and Vallin (2002) for Europe.

Meslé and Vallin (2002) analysed the evolution of mortality using $log(q_{xt})$ for 28 European countries for the period 1965-1995 by means of hierarchical clustering separately for each period t. The countries were classified in the following four groups:

- Northern Europe: Denmark, Finland, West Germany, Ireland, Netherlands, Norway, the United Kingdom, and Sweden.
- Mediterranean Alpine Europe: Austria, Belgium, France, Greece, Italy, Spain and Switzerland
- Central Europe: Bulgaria, the Czech Republic, East Germany, Hungary, Poland, Romania and Yugoslavia. Portugal should is into this group but the authors remove it in order to preserve the geographical continuity of the group.
- Former USSR: Estonia, Latvia, Lithuania, Russia and the Ukraine.

In the first step we select one representative country from each group to check if any difference can be observed between them. The selected countries were Finland, Spain, Poland and Lithuania. Figure 1 shows logit(q_{xt}) Lexis mortality surfaces in a sequential blue scale with labeled contours for the corresponding q_{xt}. Analysing Figure 1, we observe the following:

- In general mortality has improved along the time axis and at all ages in Finland, Spain and Poland regardless of sex. However, Lithuania is the country with the highest levels of mortality for any time and age.
- In the case of males, Spain is the country which presents the lowest values in mortality for any time and age. From around age 18 in 1990, this country experiences an important decrease in mortality that moves to higher ages up to age 35 in 2010.

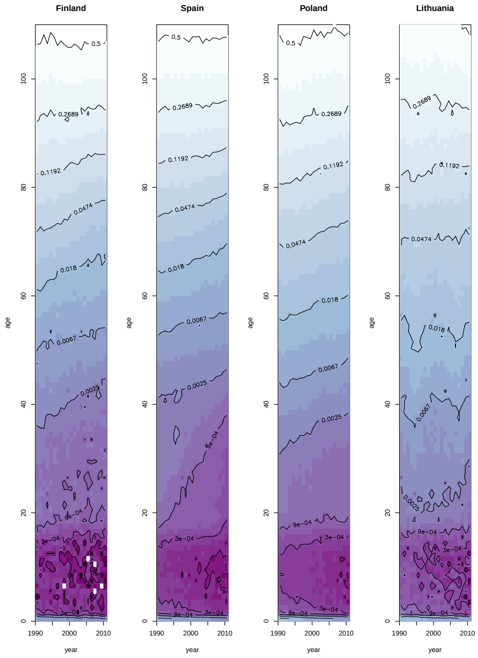
- In the case of females, the differences in mortality are less marked among Finland, Spain and Poland. Spain is again the country with the lowest mortality level. We can see that mortality decreases with the passage of time except for the case of Lithuania. In Lithuania, mortality starts decreasing up to age 60.

4.2 PCA with missing data for EU countries

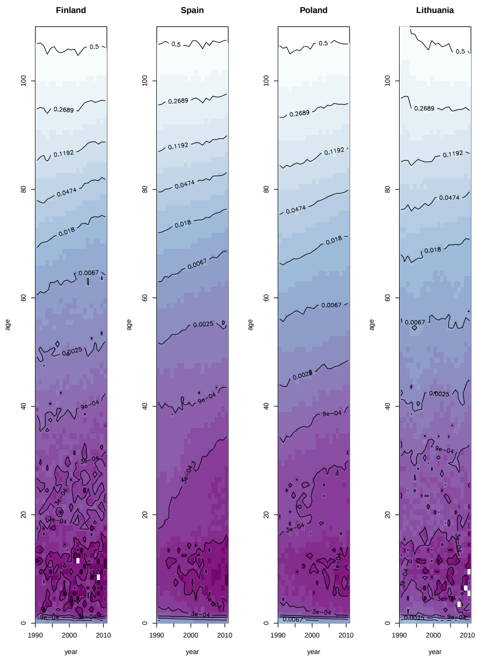
In our method, for each country, each age group in each year is considered to be a separate variable for males and females, so each country is represented by $110 \times 21 = 2310$ variables for each sex. The dimensionality is reduced by using only the first few principal components (PCs). Findings are obtained for males and females separately, using the same methodology for both sexes. The percentages of missing values in the observations (due to the application of the log function to zero death rates at some ages) are 0.05% and 0.09% for males and females respectively, and they occur at different ages.

Principal component analysis (PCA) was invented in 1901 by Pearson (1901), and a good survey on the subject can be found in the book by Jackson (2005). The method is mostly used as a tool for reducing dimensionality in data variables while losing as little information as possible. PCA is a statistical procedure that uses an orthogonal transformation to convert the correlated original variables into a set of linearly uncorrelated variables called principal components (PC). The number of PCs is less than or equal to the number of original variables. These PCs which are linear combinations of the original variables are defined in such a way as to have the largest possible variance. PCA can be carried out by the eigenvalue decomposition of a the data covariance (or correlation) matrix or the singular value decomposition of the data matrix. It can be shown that the eigenvalues are the variances explained by each principal component, and are constrained to decrease monotonically from the first principal component to the last.

Once the reduction of dimensionality is achieved, we focus on a few principal components rather than many variables. Several criteria have been proposed to determine how many PCs should be investigated and how many should be ignored. One common criterion is to include all those PCs up to a predetermined total percentage of explained variance, such as 90%. Recently a class of "objective" cross-validation methods have been developed to determine this quantity (see Josse and Husson (2012)). The optimal number of components can be defined as the minimum number of components which accounts for the maximum possible variance, and we are going to use one of these methods based on a generalized cross-validation procedure (GCV).

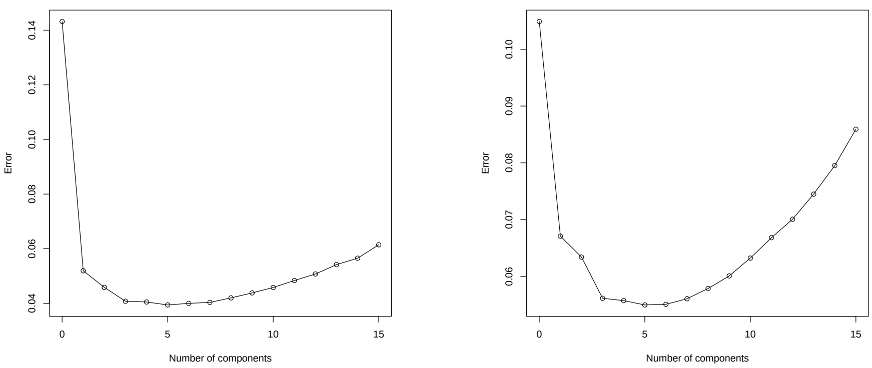


(a) Males



(b) Females

Figure 1: Mortality Lexis surfaces logit(q_{xt}) for Finland, Spain, Poland and Lithuania.



(a) Males

(b) Females

Figure 2: Cross-validation of PCA applied to mortality surfaces (the logit transformation of the period life tables) for the time range 1990-2009 and ages 0 to 109 for 24 European countries.

In this section, PCA on an incomplete dataset is performed using the missMDA R-package. The first step consists of estimating the number of dimensions that will be used in the regularized iterative PCA algorithm using the $estim\_ncpPCA$ command. Figure 2 represents the prediction error for different numbers of dimensions calculated by GCV for males and females, respectively. The error for the model without any components corresponds to a reconstruction of the data for logit(q_{xt}) (for further details, see on page 8 Josse and Husson (2016)). Cross-validation and its GCV approximation have a well-marked minimum of five components for both sexes (Figure 2).

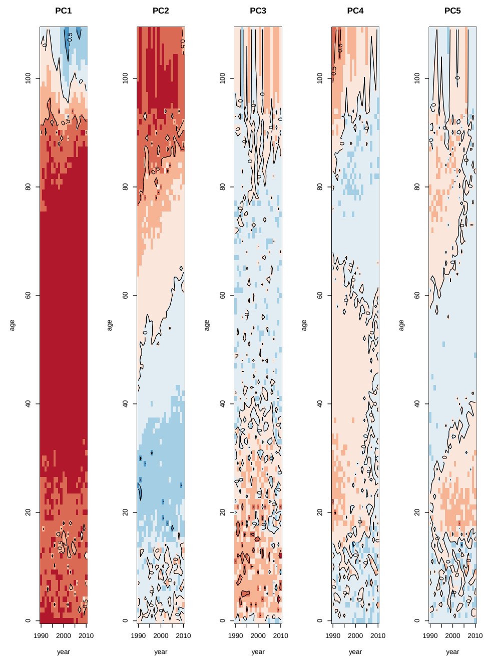
We have achieved the goal of reducing the dimension of the 2310 original variables to five components, which account for approximately 85.35% and 80% of the variance for males and females, respectively. We use the scores from the defined components, which are the coordinates regarding the 24 countries in each of the five components. These five components can be interpreted as a summary of the original variables. Figure 3 shows the coefficients that relate the principal components to each of the logit transformations of probabilities of death for each age and year for males and females respectively. As can be seen for males (Figure 3(a)) the first component is mainly related to ages 0-85 for all years, and component 2 is related to ages 80-109 for all years. With the inclusion of this second component, the explained variability was 75%. The remaining components account for only 10%, and do not have a clear interpretation. Similarly Figure 3(b) for females, shows a first component that is related to ages 0-100 for all years, and a second that is inversely related to high ages for all years. In the case of females, with the inclusion of this second component, the explained variability is 64%. The remaining components account for 34%, and do not have a clear interpretation.

Then, in the second step a regularized iterative PCA algorithm with 5 dimensions is performed to obtain a complete dataset. Prior to clustering, the data are completed with techniques to fill in any missing values. An alternative approach would be non-parametric smoothing of the whole age-period matrix using the R-package MortalitySmooth (Camarda, 2012). Although this is a good choice, we propose working with the logit of crude rates and only imputing the missing values; therefore, for this purpose PCA is more suitable than smoothing. Finally, these two complete datasets from the 5 dimensions of the PCA are used in the next section to cluster countries.

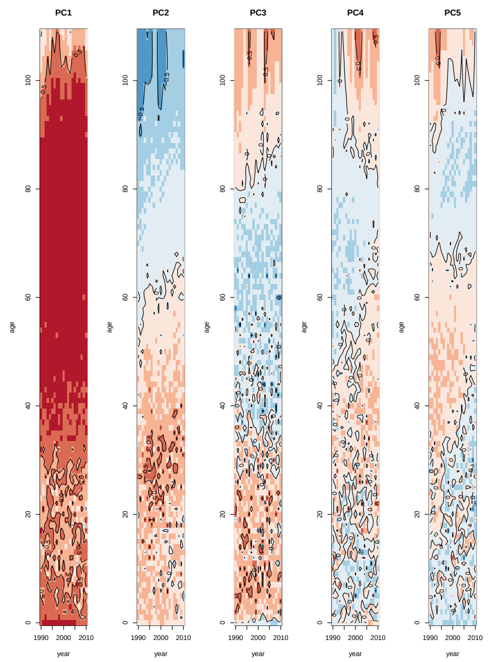
4.3 Clustering of the countries of EU using Principal Components

We use the methods to explore whether previously undefined groups may exist in the dataset. Cluster analysis is a data exploration tool for dividing the EU countries mortality dataset into "natural" clusters (groups), using the corresponding 5 PCs obtained in Section 4.2 for males and females separately. We want to gather the 24 countries of the dataset into a number of clusters which would correspond to different mortality profiles. First, we perform a hierarchical classification on the principal components of each country Figure 4(a) and Figure 4 (b) for males and females, respectively. In addition, fuzzy clustering generalizes the partition clustering methods by allowing a country to be partially classified into more than one cluster. Each country's cluster-membership is distributed among the four clusters, summing to one (Table 3 shows the values in Appendix B). Figure 5 shows the four clusters where countries are ordered by the maximum country's cluster-membership. Comparing the membership-values in Table 3, where the highest value appears in bold, and the cluster in Figure 4(a) for males; it can be seen that a similar composition of the clusters is indicated for males.

When performing hierarchical clustering for females as shown in Figure 4(b), the dendrogram suggests a grouping of countries into six groups; six is an excessively large partition and has two groups with just one country (respectively Bulgaria and Luxembourg). In addition, in the case of females for period 1965-1995, there is no large difference between the patterns for females in terms of mortality in the Baltic area according to Meslé and Vallin (2002) however, fuzzy clustering detects differences for this area. Therefore,

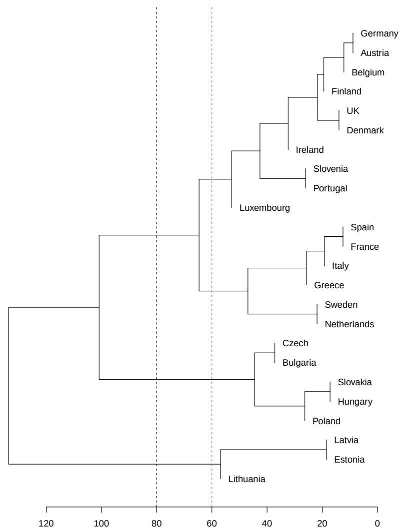


(a) Males

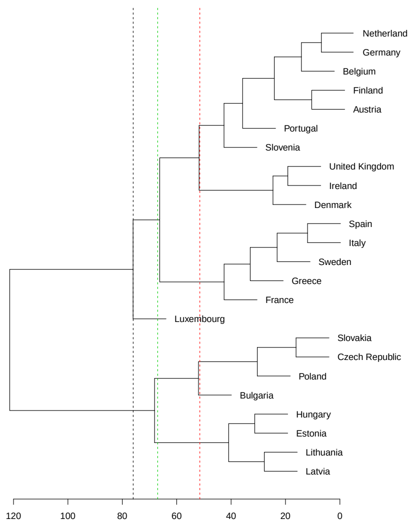


(b) Females

Figure 3: Loadings of each logit(q_{xt}) in the corresponding PC.

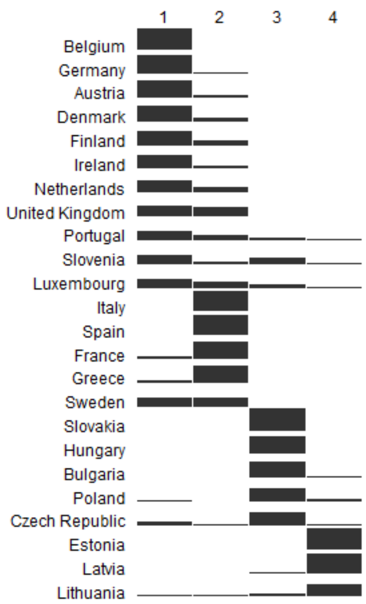


(a) Males

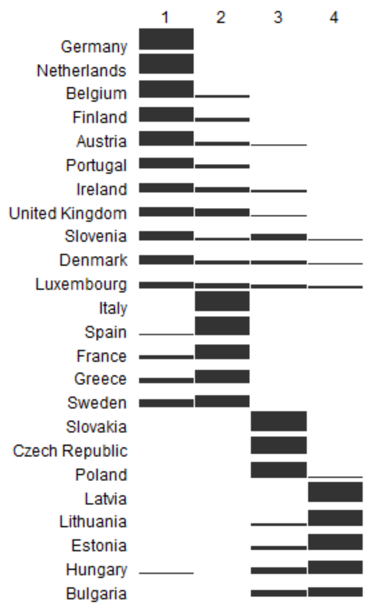


(b) Females

Figure 4: Dendrogram of the hierarchical clustering of the countries.



(a) Males



(b) Females

Figure 5: The membership values of the EU countries to the clusters.

the fuzzy membership-values in Figure 5 show four groups for females, the same groups as for males. As can be seen, Slovenia is the only eastern country that is classified in a western group for both sexes. Slovenia is located south of Austria and to the east of Italy, close to the west, which makes membership of an east group more ambiguous than for the other countries.

The period analysed in this work is 1990-2010, during which time there was a significant expansion of the EU. Most eastern European countries joined the EU in 2004. These countries, which are in Eastern Europe, tend to have a lower life expectancy because of social, political and economic factors, as explained in detail by Leon (2011). There is a consensus that one of the most important influences on life expectancy trends in the former Soviet countries has been hazardous alcohol consumption (Leon, 2011). Following the dissolution of the Soviet Bloc, these countries underwent a transition period before joining the EU in 2004, when they had to modify different criteria in order to be accepted by the EU member countries. All of these changes had a positive impact on health, less importantly for males than females and especially in the Baltic countries where the mortality levels are higher. Therefore, even though these countries have improved in this regard, there is still a significant gap between eastern and western countries, which themselves undergo the traditional south-north partition.

In addition, as was the case in the previous studies of differences in age-at-death distributions (Edwards and Tuljapurkar, 2005; Tuljapurkar and Edwards, 2011), we have eliminated differences in infant and child mortality by truncating the life table at ages like 10 verifying that the results of the fuzzy cluster do not change. We use the results of the fuzzy technique as it is not merely descriptive but provides an idea of which countries are more difficult to classify, besides indicating where they can move. We also analyse the division of the western countries where the Mediterranean countries together with Sweden are different from the rest for both males and females. We use a comparative level plot (Minton et al., 2017) to visualise mortality differences of each cluster with respect to the total EU in Figure 6, specifically we have obtained the difference between the mean of the logit transformation for the group minus the mean for the EU. Four different mortality patterns can be observed in Figure 6. The Baltic countries show, on average, the worst mortality values and the Mediterranean cluster the best ones, especially for the intermediate ages in men. Meslé and Vallin (2002) found that the gap between northern and southern countries was closing, but, in contrast, Figure 6 shows the Mediterranean countries of Europe have an advantage over their northern counterparts. Meslé (2004) analysed the trend of mortality in Central and Eastern European from 1965 until 2000. The paper concluded that the situation of Baltic countries was still uncertain in 2000, but it was possible that these countries would soon resume sustainable progress in life expectancy. However, we have seen that mortality in adult ages has not improved.

The next step is the description of a country group based on the mortality indicators and showing the difficulty of characterizing them with more classical statistical techniques.

4.4 Trends in Mortality Indicators from 1990 to 2010

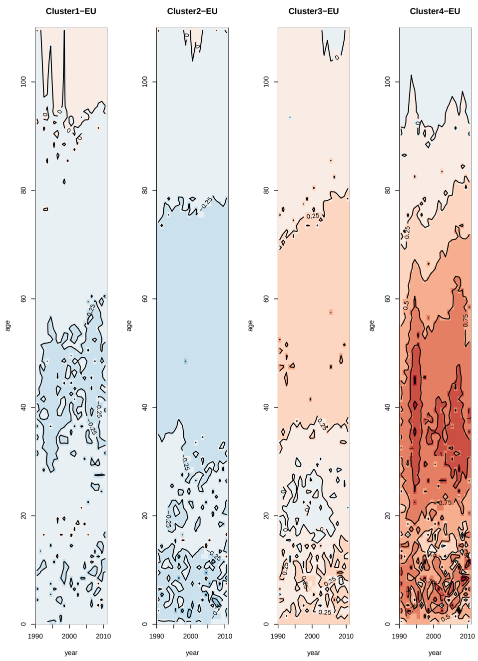
The numerical characteristics that are used to describe the groups identified in the cluster analysis are: life expectancy, life expectancy at age 65, the mode, infant mortality, the Gini coefficient at birth, age 10, age 40 and age 65 and conditional standard deviation at age 10. All these indicators have been calculated for 1990 and 2010, the beginning and the end of the time period in order to test the evolution of the indicators over time.

The description of the groups with all of the indicators are shown in Tables 4, 5, 6 and 7 for males and females in 1990 and 2010, respectively which are placed in Appendix C. The second group for males has a higher life expectancy at birth and at 65 years of age and a higher mode. This happens in the group where most of countries are western Mediterranean countries while the eastern European countries (third group) have a lower life expectancy, especially Estonia, Latvia and Lithuania (fourth group), these countries are the worst off in terms of mortality rates both compared to other eastern countries as well as throughout the EU. For females, we can see that the first and second groups have a higher life expectancy at birth and at 65 years of age. Eastern European countries show a poorer performance in general for all of the indicators and for both sexes, and this is especially true for the Baltic countries.

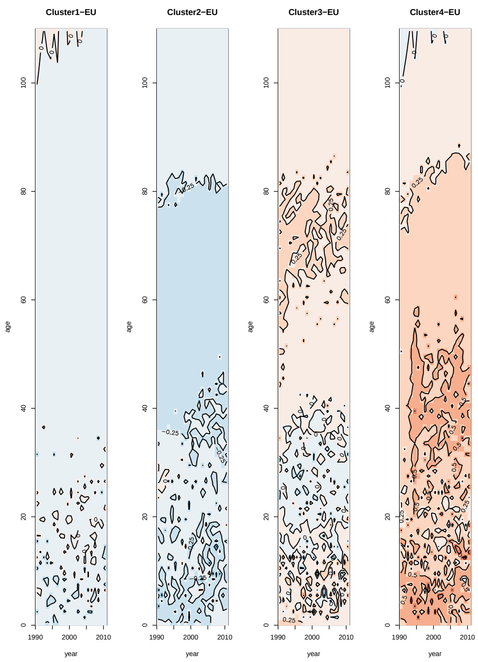
In general, and independent of sex, changes can be seen in the different indicators from 1990 to 2010, comparing Table 4 vs Table 5 and Table 6 vs Table 7 for males and females, respectively. In order to analyse whether these changes can be considered statistically significant at 1% level, the t-Student test has been applied for paired data at 2010 and 1990 for each indicator. Table 1 shows the value of the t-statistic and the p-value for each sex. The results show that:

- In both sexes, there has been a significant improvement on average in infant mortality, life expectancy at birth, life expectancy at age 65 and in the mode, as well as,

- a significant reduction in the average variability of mortality measured with the index G_0 and S_{10}.



(a) Males



(b) Females

Figure 6: Comparative level plots of Mortality Lexis surfaces logit(q_{xt}) for clusters.

However, the index G_{65} shows a significant increase only in the case of males. An ANOVA has been applied to detect whether the intensity on average of

Table 1: Paired t-student test between 1990 and 2010

	Males		Females	
Indicator	t-statistic	p-value	t-student	p-value
q_0	-10.41	0.0000	-10.71	0.0000
M	11.18	0.0000	12.02	0.0000
e_0	18.49	0.0000	21.50	0.0000
e_{65}	14.59	0.0000	20.53	0.0000
G_0	-8.05	0.0000	-10.67	0.0000
G_{10}	-2.20	0.0380	-2.04	0.0530
G_{40}	3.13	0.0046	2.10	0.0467
G_{65}	3.72	0.0011	0.24	0.8100
S_{10}	-5.74	0.0000	-8.73	0.0000

the previous changes was different according to the cluster in the measures of tendency of mortality and of dispersion. Only the statistical tests for improvement in life expectancy at age 65 and in infant mortality, both for males, are significant at 1%. Afterwards, it has been verified by means of the multiple comparison test of Least Significant Difference (LSD) intervals (Hayter, 1986), that the groups differed, concluding that the magnitude of the improvement in the e_{65} of cluster 4 is significantly smaller than those for clusters 2 and 1. In the case of infant mortality q_0, cluster 1 presents a significantly greater improvement than clusters 2 and 3. Regarding the non-statistical significance of differences in the measures of variability in mortality, it can be concluded that there is no statistical evidence to conclude that the differences in mortality variability between clusters have changed over time.

However, we would like to highlight that the fourth group have the lowest life expectancy and the highest Gini Index for males. These results are in line with Vaupel et al. (2011) who have shown that the countries benefiting from the longest life expectancies are those that have succeeded in reducing disparities in how long individuals live by averting early deaths. In order to compare the four clusters Figures 7 and 8 show the contribution of each age to life expectancy at birth (equation (5)) for each cluster on average. These contributions show positive values for advanced ages and negative values for infant and medium ages in all four of the clusters. The positive values indicate that the difference between the proportion of deaths and the proportion of years lived by individuals in this age group is in favor of the latter and, therefore, that their contribution is greater than for infant and medium ages.

Maximum positive contributions occur at older ages in females than in males (vertical lines). This fact is consistent with the greater life expectancy of females. The maximum contributions occur particularly around age 61 for western countries and age 54 for eastern ones in 1990 for males. These ages move to 63 and 57 in 2010 for western and eastern countries, respectively. Meslé and Vallin (2002) pointed out that in southern countries there was significant mortality around the age of 20 due to traffic accidents especially for males. We notice that this level of mortality is decreasing, because the valley for that age group in Figure 7 for the cluster 2 is decreased greatly in Figure 8. The contribution deficit for the intermediate ages is greater for males than females and especially high in the Baltic countries, which in 2010 refers to the ages between 40-60 years.

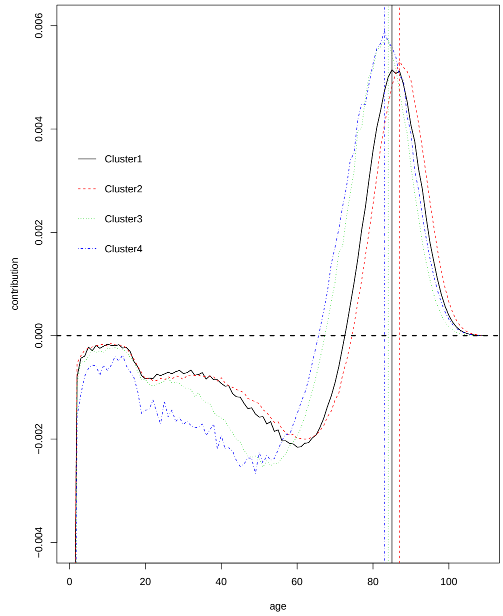
This fact is well observed in the evolution of the logit transformations of mortality death probabilities for males with ages 40 and 60 years in Figure 9; the evolution over time for males does not decreases for cluster 4 as it does for the rest of the clusters. For females, the evolution is smoother.

The next step is the characterization of a country group based on the most important mortality indicators using random forests and CART for the most recent period.

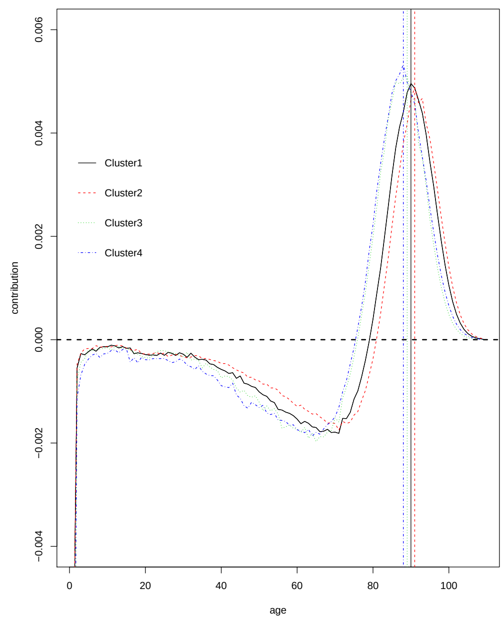
4.5 Random Forest for Mortality Indicators in 2010

The numerical characteristics that are used to create the tree and thereby characterize the groups identified in the cluster analysis are those in the above Section 4.4 calculated for 2010, the most recent year in the sample. Therefore, having these clusters that group mortality trends in terms of period, age and cohort effects, we try to simplify the discrimination between the groups by means of some recent widely-used period mortality indicators as is done by many national and international statistical agencies.

The first step is to balance the sample so that the techniques do not excessively favor the classification in the larger groups. The randomForest package is used as we mentioned at the end of subsection 3.2 to produce a measure of the importance of the mortality indicator in 2010. Figure 10 shows each mortality indicator on the y-axis, and their importance on the x-axis and indicates how important each variable is in classifying the countries. The most important mortality indicators are at the top and an estimate of their importance is given by the position of the dot on the x-axis. The mean decrease in the Gini coefficient (MeanDecreaseGini) is a measure of how each mortality indicator contributes to the classification of a country. In the random forest, each time a particular indicator is used to split a node, the Gini coefficient for the child nodes are calculated and compared

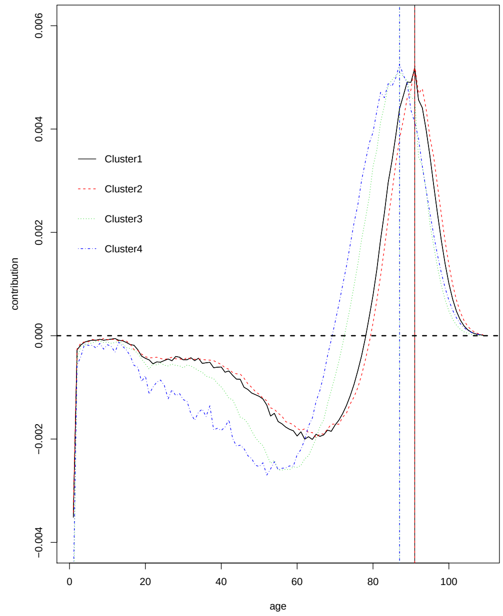


(a) Males

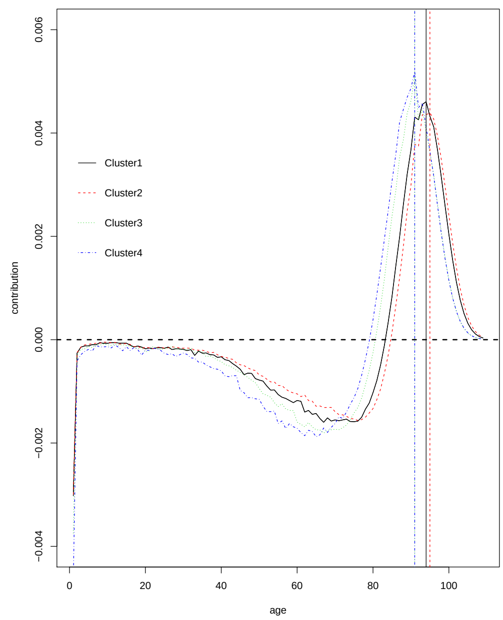


(b) Females

Figure 7: Contribution to life expectancy at birth of each age in 1990 for each cluster on average.

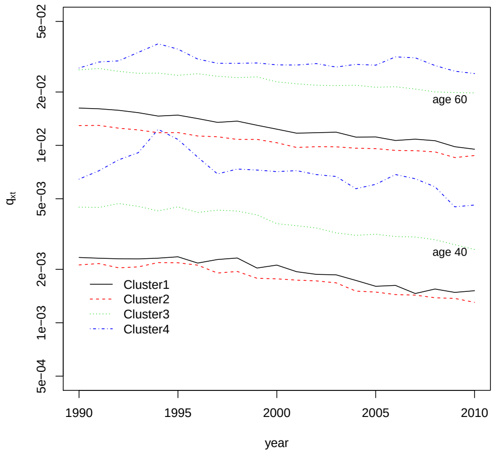


(a) Males

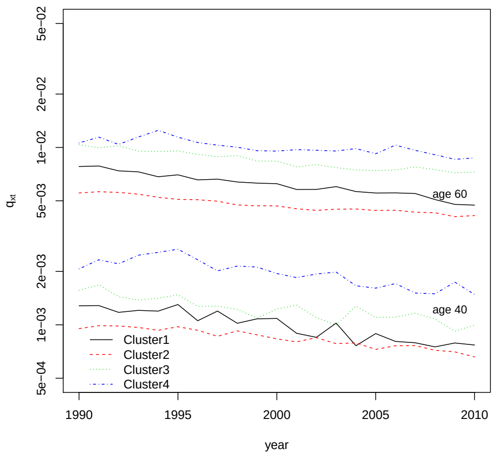


(b) Females

Figure 8: Contribution to life expectancy at birth of each age in 2010 for each cluster on average.



(a) Males



(b) Females

Figure 9: Evolution of the logit transformations of mortality death probabilities for each cluster on average.

to that of the original node. Indicators that result in nodes with higher purity have a higher decrease in Gini coefficient. Figure 10 shows that the measures of central tendency, such as life expectancy and, life expectancy at age 65 are the most important as well as the dispersion represented by the conditional standard deviation at age 10, S_{10}, while the Gini index regarding the dispersion in mortality is less important for both sexes.

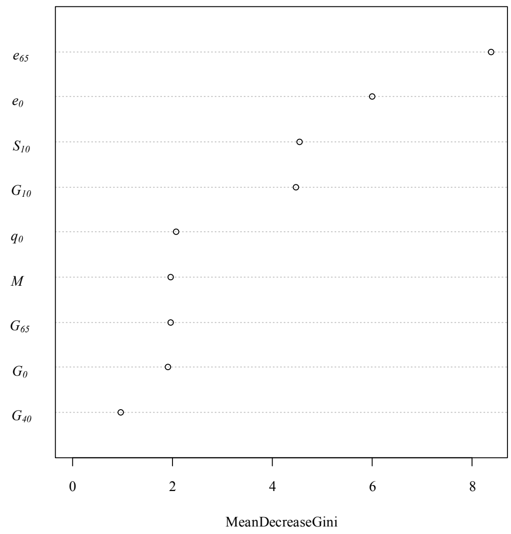
Figure 11 shows the resulting tree, and the interpretation of this tree is straightforward. The top circle (root node) in Figure 11 contains all of the countries (11 countries in each of the groups). This node is split based on the value of the principal separation variable. The first bifurcation represents the first mortality indicator selected by the procedure. When a split occurs, the subsamples, also called nodes, end up either in a circle or in a rectangular box. The rectangular boxes are referred to as terminal nodes and the circles as non-terminal nodes. Terminal nodes do not split further, while non-terminal nodes do. Figure 11 shows a country that although belonging to group 2 ends up being poorly classified in group 1; this country is Greece which according to Table 7 has a life expectancy at birth of 83.18 and therefore slightly less than 83.36.

5 Conclusions

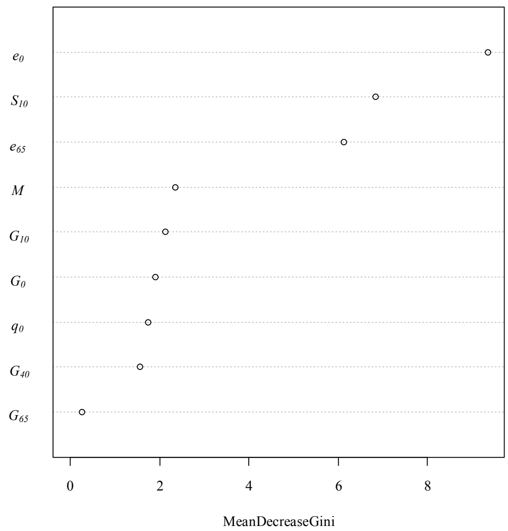
We present a methodology to explore the dynamics of mortality in European Union countries and, in turn, detect the most important differences in mortality between countries using measurements and techniques such as principal components, fuzzy clustering, CART and random forest.

Logit transformations are applied to achieve normality and homoscedasticity conditions, which results in much more robust results and less sensitivity to small changes in the data. Principal components analysis has been applied to the logit transformations of the death probabilities; this technique allow us to compress the information, accounting for more than 80% of the total variability.

Next, fuzzy cluster analysis is applied to the principal components to divide this dataset into "natural" groups and to explore whether previously undefined groups may exist in the dataset. Although Europe can be divided along many differing lines, for example, the pure geographical criteria, and the mortality experience of these areas has varied at different times, it is concluded that between-group inequality in mortality is larger along the east-west divide than the rest. It should be borne in mind that the results provided by the fuzzy technique are not merely descriptive but also provide insight into which countries are more difficult to classify, as well as giving us an idea of

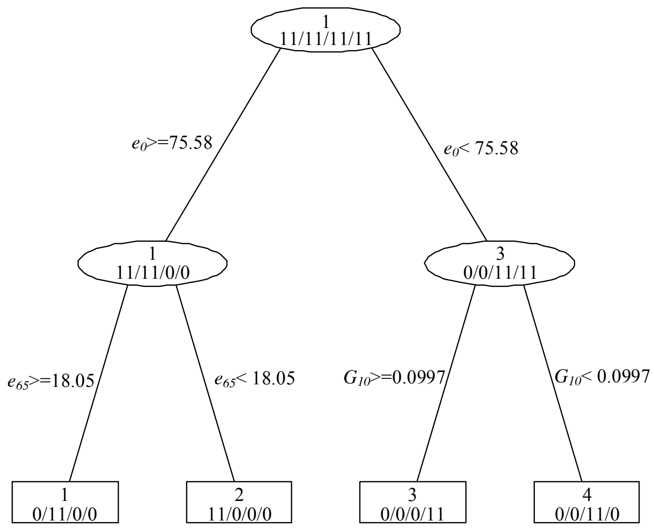


(a) Males

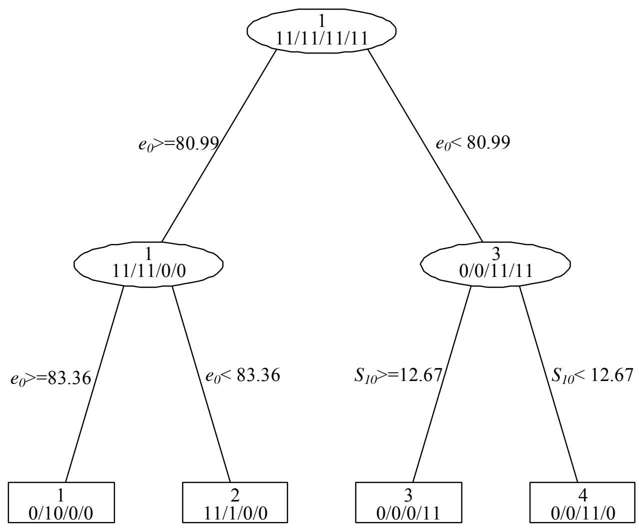


(b) Females

Figure 10: Mortality index importance for 24 European countries.



(a) Males



(b) Females

Figure 11: Classification tree model of size 4 terminal nodes (3 split) for the classification of the countries. The values in each node are the prediction for the cluster (1, 2, 3 or 4).

where they can be placed.

Therefore, mortality information was gathered for 24 EU countries from 0 to 109 years of age, for the period 1990-2010, and 24 EU countries were classified by fuzzy c-means cluster analysis, letting the technique detect the most relevant divisions. As a result, for males and females, one cluster is formed by Mediterranean European countries, a second by northern European countries, a third by the Baltic States and a fourth by eastern European countries.

The clustering method is based on the similarities between mortality surfaces that shows principal mortality time trends. Our results show four clusters just as in Meslé and Vallin (2002) who used a dendrogram to cluster countries for each year separately. However, our results show that the passage of time has changed the composition of some of these groups. Unlike the classification proposed by Meslé and Vallin (2002), Sweden is classified within the group of Mediterranean countries, Austria, and Portugal in the group of northern Europe. In line with the conclusions of Meslé and Vallin (2002), we note that there is still a marked gap between the countries of the east and the west that has not disappeared. More recently, Hatzopoulos and Haberman (2013) classified 35 countries into west and east clusters, but we only consider European countries, and both clusters are divided in a different way, which in our case gives additional divisions.

Classical statistical techniques have generally highlighted a significant improvement in terms of the trend in increasing lifetime and a significant decline in mortality rates. However, there have been no significant changes in variability between groups. Therefore, there is not enough evidence to conclude that between-country differences in measures of within-country disparity are shrinking. On the other hand, the contribution of different ages to life expectancy is unbalanced, with contributions proportionately larger for older ages, and this draws attention to the deficiency experienced by males around 60 years.

Given that with the above techniques it is not easy to discriminate between groups, we have used random forest to implement a coherent selection for the classification of countries based on mortality indicators in 2010 and CART for classification. Using random forests, we identify mortality indicators that best discriminate between groups. We then use these indicators to describe group patterns by using CART.

The CART model is distinguished by several aspects that provide better practical results than other classical techniques. The final results of using tree methods for classification or regression are summarized in a series of (usually few) logical if-then conditions. The interpretation of results summarized in a tree is very simple. This simplicity is useful not only for purposes of rapid classification of new observations, but can also often yield a much simpler "model" by explaining why observations are classified or predicted in a particular group. In addition, there is no implicit assumption underlying the relationships between the predictor variables and the dependent variable.

It should be noted that the mortality indicators that best characterize these groups of countries are life expectancy at birth, life expectancy at age 65, the Gini Index and the conditional standard deviation at age 10. The rest of the indicators do not provide information that helps characterize the groups, as they have a more homogeneous behavior in the countries of the European Union for the period analysed. Western European countries have higher life expectancy at birth than eastern countries for males and females, with a higher life expectancy at age 65, specifically for the Mediterranean countries. Within countries that make up the East-cluster, there is a group which have a higher dispersion than the other countries in the East-group, represented by the Gini index and the conditional standard deviation at age 10 for males and females respectively. In addition, they also show the least longevity among the countries of the European Union. Therefore, different institutions that develop or use information on mortality should not only analyze trend measures but also measures of variability as they help to differentiate the mortality patterns of countries.

Finally, we comment on how this study might be extended. As noted earlier, it would be very useful to complement this work with others that consider the social, cultural, economic and political spheres, in order to find the factors that might influence changes in longevity due to the geographical location of the country. Furthermore, it is important to understand the similarities between countries in these areas in order to group them, as was done in this study. In the analysis, Slovenia tends to blend into the western and eastern groups. The indicators also reflect this, and so care should be taken when placing it in a group. Slovenia should be studied in the next few years, in order to see if its behavior is more like eastern or western Europe and thus classify it more clearly. We also note that EU countries from which no information was available, are located in eastern Europe, and so their patterns may be similar to those clustered in that area. A possible extension is to apply this methodology on cause of death data which is a major structural element that would possibly lead to clearer clusters.

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Appendixes

A Cluster validity indices

In this study, the validity measures used are: xie.beni, fukuyama.sugeno, partition.coefficient and partition.entropy. A short description of these follows,

- xie.beni (xb): This index is a function of the data set and the centroids of the clusters. Xie and Beni (1991) explained this index by writing it as a ratio of the total variation of the partition and the centroids and the separation of the centroids vectors. The minimum values of this index under comparison support the best partitions.

- fukuyama.sugeno (fs): This index consists of the difference between two terms, the first combining the fuzziness in the membership matrix with the geometrical compactness of the representation of the data set via the prototypes, and the second the fuzziness in the row of the partition matrix with the distance from the ith prototype to the grand mean of the data. The minimum values of this index also propose a good partition (Fukuyama and Sugeno, 1989).

- partition.coefficient (pc): An index which measures the fuzziness of the partition but without considering the data set itself. It is a heuristic measure since it has no connection to any property of the data. Maximum values imply a good non-fuzzy partition.

- partition.entropy (pe): A measure that provides information about the membership matrix without considering the data itself. Minimum values imply a good crisp partition.

The cluster validity indices can be independently used in order to evaluate and compare fuzzy clustering partitions or even to determine the number of clusters in a data set. Table 2 shows the values of these cluster validity measures where the optimal values of the indices appear in bold for males and females respectively. Therefore, two or four is indicated as the optimal number of clusters for all the indices as they correspond to the maximum partition coefficient (pc) and minimum entropy (pe), xie.beni (xb) and fukuyama.sugeno (fs). These indices highlight the difference between east and west again, which surpasses any other division and the differences between the Baltic and Mediterranean countries for both males and females.

Table 2: Results of the validation indices using data for EU countries for males and females.

	number of clusters					
	Male			Female		
index	2	3	4	2	3	4
xb	**0.01**	0.03	0.02	0.06	0.03	**0.02**
fs	-22183.30	-16026.60	**-23464.68**	**-19140.4**	-15298.9	-17195.9
pc	**0.80**	0.58	0.59	**0.74**	0.56	0.54
pe	**0.34**	0.72	0.76	**0.37**	0.75	0.85

B Fuzzy clustering

Table 3: The membership values of the EU countries to the clusters.

	Male				Female			
Country	1	2	3	4	1	2	3	4
Austria	**0.80**	0.17	0.03	0.01	**0.68**	0.20	0.09	0.03
Belgium	**0.96**	0.03	0.01	0.00	**0.79**	0.16	0.03	0.02
Bulgaria	0.07	0.04	**0.77**	0.12	0.10	0.06	0.38	**0.47**
Czech Republic	0.21	0.10	**0.61**	0.09	0.08	0.04	**0.81**	0.07
Denmark	**0.73**	0.21	0.04	0.02	**0.42**	0.24	0.21	0.13
Estonia	0.01	0.01	0.04	**0.93**	0.07	0.04	0.19	**0.70**
Finland	**0.67**	0.25	0.05	0.02	**0.69**	0.21	0.07	0.03
France	0.17	**0.78**	0.03	0.02	0.24	**0.67**	0.05	0.04
Germany	**0.86**	0.10	0.02	0.01	**0.97**	0.02	0.01	0.00
Greece	0.18	**0.76**	0.04	0.02	0.27	**0.63**	0.06	0.04
Hungary	0.05	0.03	**0.82**	0.10	0.07	0.04	0.30	**0.59**
Ireland	**0.62**	0.22	0.11	0.05	**0.49**	0.28	0.16	0.07
Italy	0.09	**0.90**	0.01	0.01	0.08	**0.90**	0.01	0.01
Latvia	0.02	0.02	0.07	**0.90**	0.03	0.02	0.07	**0.88**
Lithuania	0.12	0.11	0.20	**0.58**	0.08	0.05	0.14	**0.73**
Luxembourg	**0.43**	0.30	0.18	0.09	**0.33**	0.27	0.23	0.17
Netherlands	**0.60**	0.31	0.07	0.03	**0.89**	0.08	0.02	0.01
Poland	0.11	0.07	**0.63**	0.19	0.10	0.04	**0.74**	0.12
Portugal	**0.45**	0.30	0.17	0.09	**0.55**	0.27	0.11	0.07
Slovakia	0.01	0.00	**0.98**	0.01	0.03	0.02	**0.90**	0.05
Slovenia	**0.43**	0.16	0.33	0.08	**0.44**	0.16	0.31	0.09
Spain	0.09	**0.89**	0.01	0.01	0.12	**0.84**	0.02	0.01
Sweden	**0.45**	**0.45**	0.07	0.03	0.36	**0.57**	0.04	0.02
United Kingdom	**0.50**	0.46	0.03	0.01	**0.46**	0.38	0.10	0.06

C Description of the clusters

Table 4: The characterization of a country group based on the mortality indicators in 1990 for males

	q_0	M	e_0	e_{65}	G_0	G_{10}	G_{40}	G_{65}	S_{10}	cluster
Austria	0.0085	80.00	72.24	14.35	0.1010	0.0911	0.1175	0.1385	14.47	1
Belgium	0.0092	80.00	72.71	14.32	0.1001	0.0877	0.1102	0.1375	14.15	1
Denmark	0.0086	78.00	72.03	13.99	0.0971	0.0856	0.1168	0.1446	14.04	1
Finland	0.0056	78.00	70.94	13.74	0.0993	0.0970	0.1217	0.1406	15.05	1
Germany	0.0080	80.00	71.91	13.94	0.0962	0.0866	0.1135	0.1349	14.13	1
Ireland	0.0091	80.00	72.13	13.35	0.0886	0.0754	0.1010	0.1341	13.17	1
Luxembourg	0.0076	72.00	72.30	14.19	0.0981	0.0901	0.1164	0.1408	14.39	1
Netherlands	0.0082	79.00	73.82	14.38	0.0889	0.0760	0.1044	0.1409	13.09	1
Portugal	0.0123	79.00	70.77	14.21	0.1219	0.1049	0.1175	0.1349	15.61	1
Slovenia	0.0097	79.00	69.77	13.26	0.1006	0.0900	0.1227	0.1384	14.56	1
United Kingdom	0.0090	80.00	72.85	14.00	0.0930	0.0803	0.1066	0.1431	13.53	1
mean	**0.0087**	**78.64**	**71.95**	**13.97**	**0.0986**	**0.0877**	**0.1135**	**0.1389**	**14.20**	
France	0.0085	81.00	72.74	15.54	0.1136	0.1051	0.1308	0.1433	15.49	2
Greece	0.0096	81.00	74.66	15.62	0.1045	0.0899	0.1121	0.1439	14.14	2
Italy	0.0091	81.00	73.64	15.05	0.1021	0.0908	0.1128	0.1446	14.32	2
Spain	0.0083	82.00	73.43	15.47	0.1124	0.1023	0.1197	0.1432	15.22	2
Sweden	0.0068	81.00	74.81	15.31	0.0916	0.0824	0.1078	0.1363	13.55	2
mean	**0.0084**	**81.20**	**73.85**	**15.40**	**0.1048**	**0.0941**	**0.1166**	**0.1422**	**14.54**	
Bulgary	0.0164	77.00	68.08	12.75	0.1171	0.0944	0.1239	0.1335	14.98	3
Czech Republic	0.0126	75.00	67.55	11.63	0.0949	0.0813	0.1154	0.1323	13.96	3
Hungary	0.0165	75.00	65.16	12.05	0.1133	0.0989	0.1373	0.1384	15.50	3
Poland	0.0213	75.00	66.26	12.41	0.1222	0.0982	0.1319	0.1406	15.38	3
Slovakia	0.0140	75.00	66.53	12.09	0.1047	0.0914	0.1320	0.1400	14.84	3
mean	**0.0161**	**75.40**	**66.72**	**12.19**	**0.1104**	**0.0928**	**0.1281**	**0.1370**	**14.93**	
Estonia	0.0137	70.00	64.69	12.01	0.1239	0.1105	0.1363	0.1443	16.41	4
Latvia	0.0143	68.00	64.14	11.99	0.1262	0.1119	0.1378	0.1445	16.53	4
Lithuania	0.0111	73.00	66.39	13.27	0.1255	0.1162	0.1511	0.1583	16.75	4
mean	**0.0130**	**70.33**	**65.07**	**12.42**	**0.1252**	**0.1129**	**0.1417**	**0.1490**	**16.56**	

Table 5: The characterization of a country group based on the mortality indicators in 2010 for males

	q_0	M	e_0	e_{65}	G_0	G_{10}	G_{40}	G_{65}	S_{10}	cluster
Austria	0.0044	86.00	77.67	17.69	0.0937	0.0902	0.1206	0.1449	13.89	1
Belgium	0.0042	85.00	77.38	17.40	0.0925	0.0888	0.1186	0.1424	13.82	1
Denmark	0.0036	82.00	77.12	16.91	0.0842	0.0823	0.1178	0.1418	13.33	1
Finland	0.0026	83.00	76.72	17.31	0.0952	0.0973	0.1260	0.1443	14.55	1
Germany	0.0038	84.00	77.45	17.27	0.0869	0.0842	0.1189	0.1424	13.45	1
Ireland	0.0041	83.00	78.27	17.70	0.0912	0.0873	0.1106	0.1396	13.59	1
Luxembourg	0.0041	84.00	77.95	17.35	0.0841	0.0819	0.1120	0.1370	13.22	1
Netherlands	0.0040	86.00	78.78	17.56	0.0797	0.0745	0.1051	0.1382	12.50	1
Portugal	0.0026	86.00	76.74	17.18	0.0913	0.0926	0.1278	0.1388	14.20	1
Slovenia	0.0021	83.00	76.28	16.57	0.0837	0.0862	0.1233	0.1443	13.75	1
United Kingdom	0.0047	85.00	78.37	17.98	0.0936	0.0884	0.1185	0.1461	13.66	1
mean	**0.0036**	**84.27**	**77.52**	**17.36**	**0.0887**	**0.0867**	**0.1181**	**0.1418**	**13.63**	
France	0.0039	86.00	78.04	18.59	0.1022	0.1006	0.1369	0.1504	14.63	2
Greece	0.0040	84.00	77.89	18.13	0.0997	0.0973	0.1263	0.1482	14.40	2
Italy	0.0035	86.00	79.49	18.31	0.0837	0.0803	0.1089	0.1395	12.90	2
Spain	0.0032	85.00	79.02	18.37	0.0875	0.0852	0.1223	0.1466	13.34	2
Sweden	0.0027	86.00	79.52	18.20	0.0805	0.0791	0.1044	0.1346	12.81	2
mean	**0.0035**	**85.40**	**78.79**	**18.32**	**0.0907**	**0.0885**	**0.1197**	**0.1439**	**13.62**	
Bulgary	0.0109	79.00	70.31	13.71	0.1024	0.0890	0.1271	0.1390	14.44	3
Czech Republic	0.0028	83.00	74.43	15.34	0.0838	0.0851	0.1201	0.1444	13.84	3
Hungary	0.0054	78.00	70.58	13.94	0.0886	0.0876	0.1394	0.1543	14.34	3
Poland	0.0053	79.00	72.16	14.99	0.0993	0.0986	0.1399	0.1577	15.09	3
Slovakia	0.0063	79.00	71.73	14.04	0.0903	0.0858	0.1248	0.1456	14.10	3
mean	**0.0061**	**79.60**	**71.84**	**14.40**	**0.0929**	**0.0892**	**0.1303**	**0.1482**	**14.36**	
Estonia	0.0042	78.00	70.83	14.23	0.1003	0.1008	0.1321	0.1511	15.36	4
Latvia	0.0055	77.00	67.43	13.16	0.1060	0.1078	0.1520	0.1749	16.11	4
Lithuania	0.0053	81.00	67.55	13.76	0.1102	0.1141	0.1544	0.1629	16.57	4
mean	**0.0050**	**78.67**	**68.61**	**13.72**	**0.1055**	**0.1075**	**0.1462**	**0.1630**	**16.01**	

Table 6: The characterization of a country group based on the mortality indicators in 1990 for females

	q_0	M	e_0	e_{65}	G_0	G_{10}	G_{40}	G_{65}	S_{10}	cluster
Austria	0.0071	85.00	78.88	17.93	0.0901	0.0745	0.1042	0.1273	12.45	1
Belgium	0.0068	86.00	79.34	18.54	0.0964	0.0813	0.1098	0.1337	12.95	1
Denmark	0.0064	84.00	77.73	17.82	0.0971	0.0865	0.1265	0.1502	13.57	1
Finland	0.0057	83.00	78.88	17.69	0.0864	0.0759	0.1017	0.1317	12.58	1
Germany	0.0061	84.00	78.42	17.58	0.0887	0.0760	0.1047	0.1309	12.64	1
Ireland	0.0072	84.00	77.73	17.05	0.0888	0.0755	0.1072	0.1417	12.67	1
Luxembourg	0.0073	85.00	78.23	17.93	0.0992	0.0811	0.1125	0.1333	13.06	1
Netherlands	0.0062	85.00	80.09	18.96	0.0926	0.0783	0.1090	0.1365	12.62	1
Portugal	0.0095	83.00	77.80	17.43	0.1025	0.0797	0.1020	0.1278	12.97	1
Slovenia	0.0065	85.00	77.70	16.91	0.0862	0.0735	0.1027	0.1295	12.51	1
United Kingdom	0.0069	85.00	78.51	17.85	0.0934	0.0800	0.1158	0.1528	12.95	1
mean	**0.0069**	**84.45**	**78.48**	**17.79**	**0.0929**	**0.0784**	**0.1087**	**0.1360**	**12.81**	
France	0.0063	86.00	80.98	19.95	0.0999	0.0857	0.1115	0.1319	13.07	2
Greece	0.0094	84.00	79.64	18.14	0.0915	0.0694	0.0938	0.1315	11.91	2
Italy	0.0072	87.00	80.25	18.89	0.0912	0.0749	0.1030	0.1340	12.31	2
Spain	0.0069	86.00	80.55	19.20	0.0949	0.0775	0.0996	0.1291	12.48	2
Sweden	0.0054	86.00	80.39	19.03	0.0882	0.0775	0.1068	0.1343	12.53	2
mean	**0.0070**	**85.80**	**80.36**	**19.04**	**0.0931**	**0.0770**	**0.1029**	**0.1322**	**12.46**	
Czech Republic	0.0092	82.00	75.42	15.25	0.0866	0.0697	0.0997	0.1286	12.39	3
Poland	0.0167	83.00	75.29	16.12	0.1119	0.0784	0.1114	0.1367	13.09	3
Slovakia	0.0102	81.00	75.39	15.68	0.0949	0.0753	0.1069	0.1312	12.87	3
mean	**0.0120**	**82.00**	**75.36**	**15.69**	**0.0978**	**0.0745**	**0.1060**	**0.1322**	**12.78**	
Bulgaria	0.0123	81.00	74.81	15.24	0.1008	0.0737	0.0978	0.1229	12.76	4
Estonia	0.0098	80.00	74.90	15.71	0.1060	0.0835	0.1106	0.1394	13.58	4
Hungary	0.0133	82.00	73.79	15.37	0.1076	0.0853	0.1184	0.1339	13.83	4
Latvia	0.0105	81.00	74.52	15.64	0.1087	0.0866	0.1150	0.1406	13.87	4
Lithuania	0.0098	85.00	76.15	16.88	0.1089	0.0892	0.1213	0.1441	13.91	4
mean	**0.0111**	**81.80**	**74.83**	**15.77**	**0.1064**	**0.0837**	**0.1126**	**0.1362**	**13.59**	

Table 7: The characterization of a country group based on the mortality indicators in 2010 for females

	q_0	M	e_0	e_{65}	G_0	G_{10}	G_{40}	G_{65}	S_{10}	cluster
Austria	0.0035	90.00	83.13	21.02	0.0817	0.0740	0.1046	0.1292	11.89	1
Belgium	0.0030	90.00	82.66	20.90	0.0861	0.0818	0.1149	0.1377	12.59	1
Denmark	0.0032	87.00	81.33	19.60	0.0809	0.0776	0.1159	0.1488	12.45	1
Finland	0.0019	90.00	83.24	21.16	0.0809	0.0784	0.1081	0.1306	12.24	1
Germany	0.0031	90.00	82.62	20.59	0.0799	0.0741	0.1065	0.1305	11.98	1
Ireland	0.0032	87.00	82.77	20.60	0.0812	0.0758	0.1101	0.1434	12.09	1
Luxembourg	0.0028	89.00	83.18	21.16	0.0832	0.0783	0.1118	0.1344	12.23	1
Netherlands	0.0035	90.00	82.73	20.75	0.0827	0.0759	0.1116	0.1379	12.10	1
Portugal	0.0026	87.00	83.04	20.84	0.0806	0.0760	0.1025	0.1275	12.07	1
Slovenia	0.0030	87.00	82.62	20.47	0.0790	0.0711	0.1041	0.1301	11.72	1
United Kingdom	0.0041	90.00	82.35	20.59	0.0890	0.0814	0.1153	0.1466	12.58	1
mean	**0.0031**	**88.82**	**82.70**	**20.70**	**0.0823**	**0.0768**	**0.1096**	**0.1361**	**12.18**	
France	0.0032	90.00	84.68	22.74	0.0933	0.0867	0.1214	0.1398	12.62	2
Greece	0.0036	85.00	83.18	20.83	0.0788	0.0707	0.0970	0.1231	11.61	2
Italy	0.0029	90.00	84.49	21.89	0.0784	0.0722	0.1032	0.1331	11.54	2
Spain	0.0030	90.00	85.01	22.40	0.0801	0.0727	0.1034	0.1287	11.50	2
Sweden	0.0024	88.00	83.47	21.01	0.0763	0.0724	0.1023	0.1358	11.72	2
mean	**0.0030**	**88.60**	**84.17**	**21.77**	**0.0814**	**0.0749**	**0.1055**	**0.1321**	**11.80**	
Czech Republic	0.0024	86.00	80.64	18.76	0.0732	0.0703	0.1022	0.1303	11.94	3
Poland	0.0044	86.00	80.47	19.23	0.0856	0.0782	0.1152	0.1390	12.59	3
Slovakia	0.0051	84.00	79.15	17.84	0.0805	0.0700	0.1020	0.1281	12.06	3
mean	**0.0039**	**85.33**	**80.09**	18.61	**0.0798**	**0.0728**	**0.1065**	**0.1325**	**12.19**	
Bulgaria	0.0083	83.00	77.25	16.92	0.0926	0.0759	0.1029	0.1200	12.74	4
Estonia	0.0025	86.00	80.55	19.22	0.0846	0.0801	0.1127	0.1382	12.74	4
Hungary	0.0050	86.00	78.34	17.88	0.0858	0.0790	0.1205	0.1373	12.91	4
Latvia	0.0052	83.00	77.42	17.79	0.1012	0.0938	0.1308	0.1546	14.17	4
Lithuania	0.0045	87.00	78.75	18.58	0.0945	0.0883	0.1242	0.1347	13.60	4
mean	**0.0051**	**85.00**	**78.46**	**18.08**	**0.0917**	**0.0834**	**0.1182**	**0.1370**	**13.23**	

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