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Additional Information

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## Multi-objective optimisation of product quality in the manufacture of Ti-6Al-4V prostheses

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**Abstract:** This paper presents a multi-objective optimisation procedure for optimising the quality of prostheses and manufacturing productivity. The aim of this procedure is to develop machining performance models through a minimal and progressive Design of Experiment (DoE), which models the variables of interest by linear regressions or Surface Response Models (SRMs). The multi-objective optimisation is based on desirability functions, which are defined according to the relative importance of each variable of interest. The procedure was implemented to optimise a process of manufacturing spherical turned components for Ti-6Al-4V hip prostheses with special requirements as regards surface roughness  $R_a$ ,  $R_z$  and geometrical form tolerance.

**Keywords:** titanium alloys; prostheses; multi-objective optimisation; desirability functions; surface roughness; DoE; design of experiments.

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Carlos Vila Pastor is an Associate Professor and the Head of the Department of Industrial Systems Engineering and Design at Jaume I University (Spain). He is also the Co-director of the Master's Degree in Design and Manufacturing. He joined the University in 1993 and since then he has been involved in manufacturing technologies, computer-aided manufacturing and computer integrated manufacturing courses. He has set up the Concurrent Engineering and Collaborative Manufacturing Laboratory and he has been involved in various national research projects around implementing concurrent engineering and product lifecycle management tools. His research interests are machining manufacturing process, concurrent engineering and product lifecycle management.

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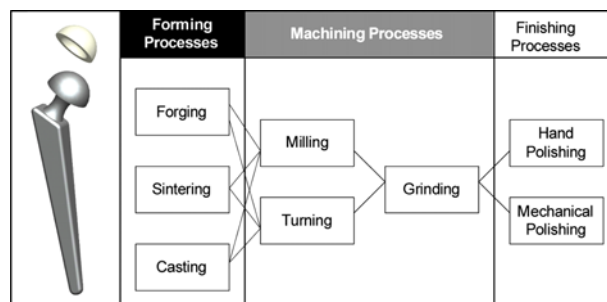
## 1 Introduction

Recent efforts to find biocompatible materials for human bone replacement have led to the synthesis of new metallic and polymeric materials and the emergence of enhanced

manufacturing processes. The results of this evolution have improved the life expectancy and comfort of patients with bone diseases and damage. Moreover, cases of rejection of orthopaedic implants (prostheses) have decreased owing to the biocompatibility achieved with the use of Ultra-High Molecular Weight Polyethylene (UHMWPE), chromium-cobalt alloys, titanium alloys (mostly Ti-6Al-4V) and other state-of-the-art metals, polymers and ceramics.

As Figure 1 shows, manufacturing processes for prostheses made from metals such as titanium alloys involve several forming operations (casting, forging or sintering), machining operations (conventional and alternative) and finishing processes (hand and mechanical polishing) (Balazic et al., 2007).

**Figure 1** Manufacturing route for metallic components of prostheses



The challenge of finding appropriate combinations of quality, productivity and costs has led different research groups to explore and understand processing behaviour through exhaustive experimentation, process modelling and simulation. Because titanium alloys are also being used in the manufacture of aerospace components, research work in the field of machining operations is widespread (Aspinwall et al., 2005; Barry et al., 2001; Ezugwu, 2005; Ezugwu et al., 2005; López de Lacalle et al., 2000; Ohkubo et al., 2000; Yang and Liu, 1999). In these works, the authors conducted comprehensive studies on tool life, cooling techniques, chip formation mechanisms and other factors that influence the machinability of these kinds of engineering materials.

In the field of the manufacture of prostheses, different factors must be studied in addition to those considered in aerospace applications. For example, it is necessary to ensure that highly polished surface finishes or specialised textures are achieved to avoid releasing wear debris or to promote healthy tissue–biomaterial interactions, respectively (Shi, 2008). Adverse effects can include cellular damage, infections, blood coagulations and failure of the implants (Grill, 2003).

Machining operations involving Ti-6Al-4V alloys face a series of difficulties related with low machinability, such as

- a very high temperatures in the tool–workpiece interface
- b localised plastic instability
- c excessive tool wear caused by diffusion and chemical reactivity (Camalaz et al., 2008).

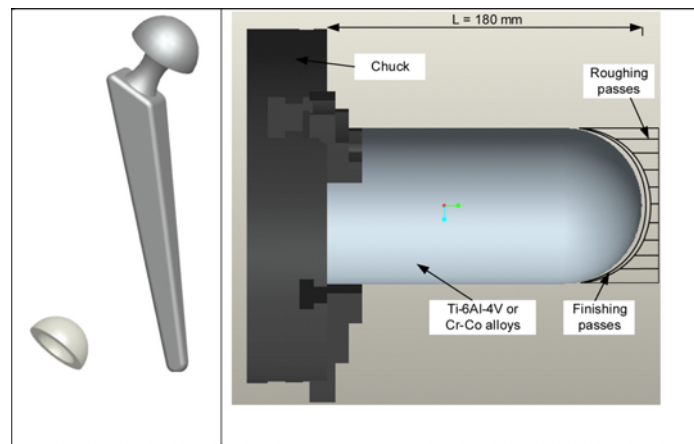
These issues make it difficult to keep surface roughness and geometrical form tolerances under control, thereby resulting in increased processing times and costs during other

manufacturing processes such as grinding or polishing. In addition, the thermal deformations that occur during metal removal in these other processes impose severe restrictions on the capacity to keep final parts within tight form tolerances. A suitable trade-off between prosthesis quality requirements and manufacturing costs should, therefore, be achieved through implementation of optimal cutting conditions.

## 2 Problem description

This study is focused on machining operations carried out in the manufacture of Ti-6Al-4V prostheses, with special attention being paid to spherical turned components of hip prostheses. The purpose of the procedure described here is to apply a minimal and progressive DoE to obtain machining performance models that can be used to optimise a multi-objective function based on prosthesis quality requirements and manufacturing productivity. For this case study, spherical prosthesis parts are turned as shown in Figure 2. The requirements of these parts are: surface roughness  $Ra$ ,  $0.03 \mu\text{m}$ ; surface roughness  $Rz$ ,  $0.15 \mu\text{m}$ ; spherical form tolerance,  $25 \mu\text{m}$ . Thus, the turning operation should minimise the surface roughness to reduce processing time in the grinding and polishing steps. Spherical form errors should also be minimised, since thermal deformations in the grinding and polishing steps could increase the form error.

**Figure 2** Machining operations carried out in the experimentation (see online version for colours)



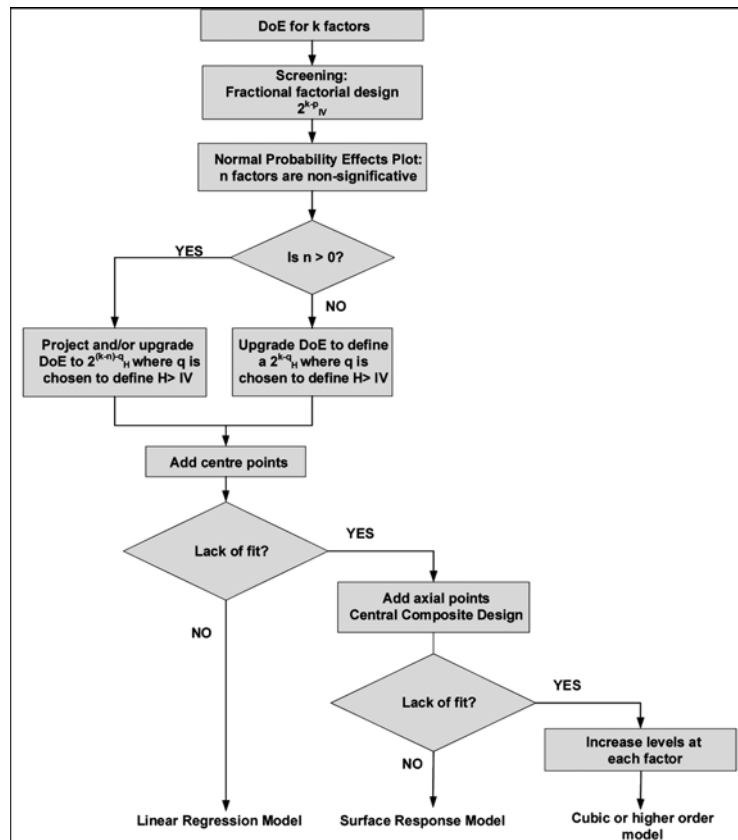
## 3 Multi-objective optimisation methodology

### 3.1 Methodology overview for modelling machining performance variables

Unlike common modelling and optimisation methodologies based on a predefined DoE such as Taguchi orthogonal arrays or surface response methodologies (Mohanasundararaju et al., 2008; Gaitonde and Karnik, 2007), the methodology proposed here is based on a progressive DoE in which the relationships between performance variables such as surface roughness ( $Ra$  and  $Rz$ ), Material Removal Rate (MRR) or geometrical form deviations are captured. The progressive DoE methodology is outlined

in Figure 3. First, a screening experiment is conducted to find out which factors and second-order factors are significant to each performance variable. This screening experiment is a fractional factorial DoE with resolution IV, namely a DoE  $2^{k-p}_{IV}$ , where  $k$  is the factors studied and  $p$  is selected to define the experimentation with resolution IV. Resolution IV refers to experiments where some main effects are confounded with three-level interactions and two-factor interactions are aliased with each other. From this screening DoE, a normal probability plot can be drawn to determine which factors are not significant. Then, if  $n$  factors were considered as non-significant, the initial DoE is projected to a DoE with a higher resolution. Otherwise, an additional DoE should be conducted to increase the initial DoE to a full factorial DoE or to a DoE with a resolution higher than IV to capture all the main effects and two-factor interactions.

**Figure 3** Progressive DoE methodology proposed to capture all significant relationships between cutting parameters and machining performance variables



After obtaining a DoE with a resolution higher than IV (note that a full factorial DoE has a resolution of infinity), an additional DoE should be conducted to find out whether the process being analysed presents a linear behaviour and whether additional factor levels should be included. This additional DoE consists in adding centre points to the previous DoE to obtain the lack of fit of the linear relations. If no lack of fit is obtained, a linear regression is fitted to capture all the significant relations between cutting

parameters and machining performance variables. If a lack of fit is reported, then an additional DoE should be conducted to capture second-order relationships. This additional DoE consists in adding axial points to the experimentation, which upgrades the DoE to a Central Composite Design (CCD). Finally, the CCD may report a lack of fit, which indicates that if higher orders appear in the experimental data, then more levels should be added in the experimentation. If a lack of fit is not reported, an SRM is fitted through the experimental data obtained.

This progressive DoE makes it possible to obtain the significant relationships between cutting parameters and machining performance variables, so the models fitted can be applied in the multi-objective optimisation procedure.

### 3.2 Multi-objective optimisation procedure

The optimisation problem for the case study deals with a multi-objective function, which is composed of several objective functions defined by regression models of performance variables such as surface roughness parameters  $Ra$  and  $Rz$ , the geometrical form deviation model and the MRR. Since these objective functions are conflicting and incomparable, the multi-objective function is defined using the desirability function approach. This function is based on the idea that the optimal performance of a process that has multiple performance characteristics is reached when the process operates under the most desirable performance values (NIST/SEMATECH, 2009). For each objective function  $Y_i(x)$ , a desirability function  $d_i(Y_i)$  assigns numbers between 0 and 1 to the possible values of  $Y_i$ , with  $d_i(Y_i) = 0$  representing a completely undesirable value of  $Y_i$  and  $d_i(Y_i) = 1$  representing a completely desirable or ideal objective value. Depending on whether a particular objective function  $Y_i$  is to be maximised or minimised, different desirability functions  $d_i(Y_i)$  can be used. One useful class of desirability functions was proposed by Derringer and Suich (1980). Let  $L_i$  and  $U_i$  be the lower and upper values of the objective function, respectively, with  $L_i < U_i$ , and let  $T_i$  be the desired value for the objective function. Thus, if an objective function  $Y_i(x)$  is to be maximised, the individual desirability function is defined as:

$$d_i(Y_i) = \begin{cases} 0 & \text{if } Y_i(x) < L_i \\ \frac{(Y_i - L_i)^w}{T_i - L_i} & \text{if } L_i \leq Y_i(x) \leq T_i \\ 1 & \text{if } Y_i(x) > T_i \end{cases} \quad (1)$$

where the exponent  $w$  is a weighting factor that determines how important it is to reach the target value. For  $w = 1$ , the desirability function increases linearly towards  $T_i$ ; for  $w < 1$ , the function is convex and there is less emphasis on the target; for  $w > 1$ , the function is concave and there is more emphasis on the target. If one wants to minimise an objective function instead, the individual desirability function is defined as:

$$d_i(Y_i) = \begin{cases} 1 & \text{if } Y_i(x) < T_i \\ \frac{(Y_i - U_i)^w}{T_i - U_i} & \text{if } T_i \leq Y_i(x) \leq U_i \\ 0 & \text{if } Y_i(x) > U_i \end{cases} \quad (3)$$

The individual desirability functions are combined to define the multi-objective function, which is called the overall desirability of the multi-objective function. This measure of composite desirability is the weighted geometric mean of the individual desirability for the objective functions. The optimal solution (optimal operating conditions) can then be determined by maximising the composite desirability. The individual desirability is weighted by importance factors  $I_i$ . Therefore, the multi-objective function or the overall desirability function to be optimised is defined as:

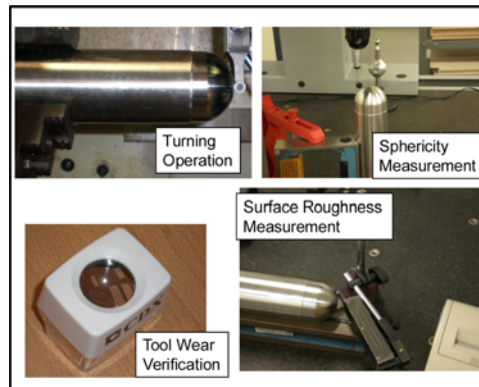
$$D = (d_1(Y_1)^{I_1} d_2(Y_2)^{I_2} \dots d_k(Y_k)^{I_k})^{\frac{1}{(I_1+I_2+\dots+I_k)}} \quad (3)$$

with  $k$  denoting the number of objective functions and  $I_i$  the importance of the  $i$ th objective function, where  $i = 1, 2, \dots, k$ .

#### 4 Experimental set-up

Experimentation was conducted with a CNC lathe (the experimental set-up is shown in Figure 4 and Table 1). The faces of bars of Ti-6Al-4V alloy were contoured for each parameter combination to be tested. Detailed measurements of surface roughness and form deviations were conducted to characterise the process and determine the optimal operating conditions (equipment for measurements is shown in Table 2).

**Figure 4** Experimental set-up and equipment for quality measurements (see online version for colours)



**Table 1** Experimental set-up

<i>Workpieces</i>		
Ti-6Al-4V Alloy	Denomination	ASTM B348-05
	Composition by weight %	Al 6.31, V 4.09, Fe 0.13, C 0.15, N 0.007, O 0.13, Ti Remainder
Geometry		Bars with 60 mm dia
<i>Cutting tool</i>		
ISO denomination		SRDCN2020K10
Insert ISO denomination		RCMT 0502M0



**Table 1** Experimental set-up (continued)

<i>Workpieces</i>	
<i>Cutting tool</i>	
Insert dimensions	Round $d = 5$ mm
Edge preparation rake angle	$\gamma = 15^\circ$
Clearance angle	$\alpha = 7^\circ$
Insert material	Tungsten carbide (WC) without coating
<i>Machine tool</i>	
Model	Lealde
Maximum spindle speed	3200 rpm

**Table 2** Measurement equipment for experimental work

<i>Surface roughness</i>	
Profilometer	MITUTOYO SURFTTEST 301
Measure repeatability	0.02 $\mu\text{m}$
Sampling length and number of spans	$\lambda_c/L = 0.25$ mm, $n = 1$ and 3
<i>3D measurement</i>	
Coordinate Measuring Machine (CMM)	Brown & Sharpe DEAC B3P

## 5 Models of machining performance variables

Following the DoE methodology proposed, a fractional factorial DoE with resolution IV was conducted to identify the significant main effects of the process variables that were analysed, confounding the main effects with three-factor interactions. Four two-level factors were analysed: cutting speed ( $Vc$ ); feed rate ( $f_n$ ); step depth ( $ap$ ); radius of the machining feature ( $R$ ). Table 3 shows the two-level factors analysed in the DoE. Two replicates were conducted in a randomised order to obtain information about the dispersion of surface roughness and geometrical form deviation at each experimental setting. Table 4 shows the experiments conducted and the resulting surface roughness ( $Ra$ ,  $Rz$ ), geometrical form deviation and MRR when turning titanium Ti-6Al-4V alloy. Note that the experimental results were the average of three different measurements.

After conducting the screening experiment, a normal probability plot was drawn for each performance variable to assess the significance of the main effects and to discard the variables which are not important to subsequent experiments. Figure 5 shows the normal probability plot for surface roughness and geometrical form deviation. Results show that the surface roughness parameter  $Ra$  is significantly related to the feed rate and to the interaction of the cutting speed and the depth of cut; the surface roughness parameter  $Rz$  is significantly related to the feed rate, to the interaction of the cutting speed and the depth of cut and also to the interaction of the cutting speed and the feed rate; the geometrical form deviation is not related to any cutting parameter and its behaviour is mostly random. The randomness of the deviation from the spherical form prevents this machining performance variable from being modelled and so this variable is not

considered in the multi-objective optimisation. Furthermore, all the normal probability plots show that the parameter 'radius' is not significant, so it can be removed from subsequent experiments.

**Table 3** Factors and levels analysed in the screening DoE

<i>Cutting speed Vc</i> (m/min)	<i>Feed rate fn</i> (mm/rev)	<i>Step depth ap</i> (mm)	<i>Radius feature R</i> (mm)
40–80	0.05–0.2	0.1–0.3	10–30

**Table 4** DoE conducted and resulting surface roughness and geometrical form error. Workpiece material: Ti-6Al-4V

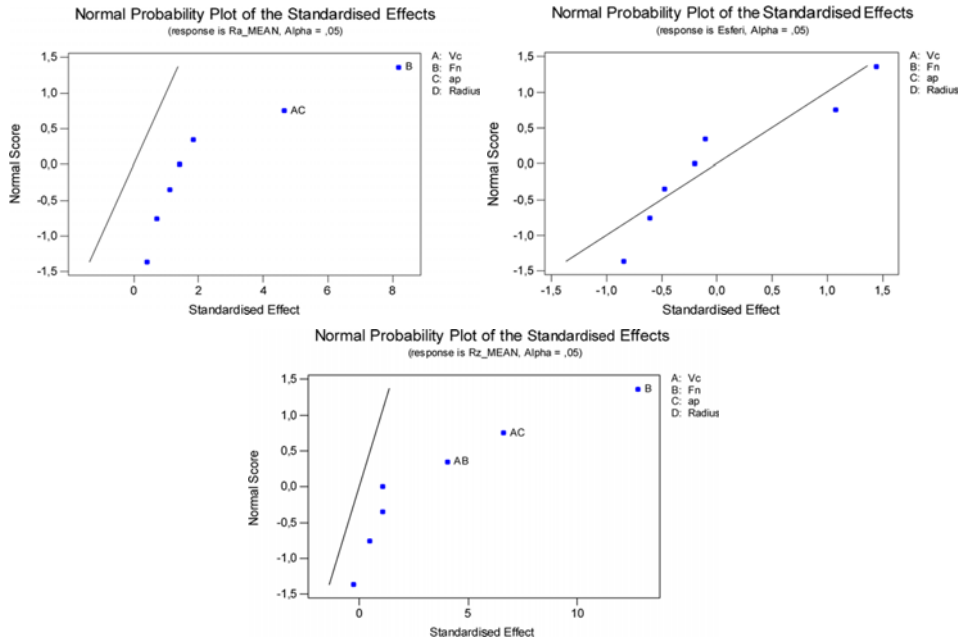
<i>Vc</i> (m/min)	<i>fn</i> (mm/rev)	<i>ap</i> (mm)	<i>Radius</i> (mm)	<i>Ra</i> ( $\mu\text{m}$ )	<i>Rz</i> ( $\mu\text{m}$ )	<i>Form error</i> (mm)	<i>MRR</i> ( $\text{mm}^3/\text{min}$ )
80.00	0.05	0.10	30	0.20	0.90	0.0030	400
40.00	0.05	0.30	30	0.21	1.13	0.0052	600
40.00	0.05	0.30	30	0.20	1.07	0.0060	600
80.00	0.20	0.30	30	0.41	2.07	0.0052	4800
40.00	0.20	0.10	30	0.35	1.63	0.0055	800
40.00	0.20	0.10	30	0.44	1.77	0.0057	800
80.00	0.20	0.30	30	0.51	1.90	0.0053	4800
80.00	0.05	0.10	30	0.19	1.00	0.0046	400
40.00	0.05	0.10	10	0.27	1.43	0.0057	200
40.00	0.20	0.30	10	0.30	1.57	0.0078	2400
80.00	0.05	0.30	10	0.25	1.23	0.0060	1200
40.00	0.20	0.30	10	0.30	1.43	0.0030	2400
80.00	0.05	0.30	10	0.25	1.30	0.0069	1200
80.00	0.20	0.10	10	0.33	1.70	0.0053	1600
80.00	0.20	0.10	10	0.32	1.60	0.0030	1600
40.00	0.05	0.10	10	0.23	1.30	0.0040	200

As  $n = 1$ , the screening DoE is then projected from the initial  $2^{(4-1)}_{IV}$  to a full factorial  $2^3$ . After the DoE projection, additional experimental runs in the centre points of the design were added to ensure that other effects such as quadratic or higher factor interactions are not significant. For this purpose, the DoE was upgraded by adding three centre points to check for the lack of fit. The experimental results are shown in Table 5.

The Analysis of Variance (ANOVA) with the addition of the centre points was analysed for surface roughness  $Ra$  and  $Rz$  variables. Tables 6 and 7 show the ANOVA results. For  $Ra$ , the lack of fit was considered to be negligible, since the 'curvature' at the centre points was not significant ( $p$ -value of 0.531). So, the  $Ra$  was correctly defined by linear relationships between the cutting parameters. However, the ANOVA for the  $Rz$  parameter showed a significant lack of fit, since the curvature of the linear relations at the centre points was significant ( $p$ -value of 0.026).

**Figure 5** Normal probability plot for surface roughness  $Ra$ ,  $Rz$  and geometrical form error.

The factor ‘radius’ is not significant (see online version for colours)



**Table 5** Additional experiments. Centre points

$Vc$ (m/min)	$fn$ (mm)	$ap$ (mm)	Radius (mm)	$Ra$ ( $\mu\text{m}$ )	$Rz$ ( $\mu\text{m}$ )	$MRR$ ( $\text{mm}^3/\text{min}$ )
60	0.125	0.2	30	0.30	1.50	1500
60	0.125	0.2	30	0.35	1.50	1500
60	0.125	0.2	30	0.32	1.67	1500

**Table 6** ANOVA for  $Ra$  to check for the lack of fit after adding centre points

<i>Analysis of Variance for Ra (coded units)</i>						
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main effects	4	0.091762	0.0905500	0.0226375	20.09	0.000
2-Way interactions	3	0.029950	0.0299500	0.0099833	8.86	0.004
Curvature	1	0.000474	0.0004744	0.0004744	0.42	0.531
Residual error	10	0.011267	0.0112667	0.0011267		
Pure error	10	0.011267	0.0112667	0.0011267		
Total	18	0.133453				

To model the  $Rz$  variable, additional experiments should, therefore, be included in the DoE to acquire those additional effects. To this end, several axial points were added to upgrade the previous DoE to a CCD, where quadratic and two-factor interaction effects can be evaluated. The results of the additional experiments are reported in Table 8. The SRM obtained from the CCD correctly models the  $Rz$  variable, since the  $p$ -value of the lack-of-fit test presented in the ANOVA table (Table 9) is higher than 0.05.

**Table 7** ANOVA for  $R_z$  to check for the lack of fit after adding centre points

<i>Analysis of Variance for <math>R_z</math> (coded units)</i>						
<i>Source</i>	<i>DF</i>	<i>Seq SS</i>	<i>Adj SS</i>	<i>Adj MS</i>	<i>F</i>	<i>P</i>
Main effects	4	1.23353	1.17862	0.29466	26.49	0.000
2-Way interactions	3	0.42992	0.42992	0.14331	12.89	0.001
Curvature	1	0.07561	0.07561	0.07561	6.80	0.026
Residual error	10	0.11122	0.11122	0.01112		
Pure error	10	0.11122	0.11122	0.01112		
<i>Total</i>	<i>18</i>	<i>1.85027</i>				

**Table 8** Additional experiments to upgrade the previous DoE to a Central Composite Design

<i>Vc (m/min)</i>	<i>fn (mm)</i>	<i>Ap (mm)</i>	<i>Radius (mm)</i>	<i>Ra (µm)</i>	<i>Rz (µm)</i>	<i>MRR (mm<sup>3</sup>/min)</i>
60	0.200	0.2	30	0.32	1.70	2400
60	0.125	0.3	30	0.34	1.60	2250
80	0.125	0.2	30	0.23	1.13	2000
40	0.125	0.2	30	0.22	1.10	1000
60	0.050	0.2	30	0.22	1.13	600
60	0.125	0.1	30	0.24	1.27	750
60	0.200	0.2	30	0.32	1.70	2400
60	0.125	0.3	30	0.35	1.80	2250
80	0.125	0.2	30	0.24	1.23	2000
40	0.125	0.2	30	0.24	1.23	1000
60	0.050	0.2	30	0.20	1.03	600
60	0.125	0.1	30	0.25	1.50	750

**Table 9** ANOVA for  $R_z$  to check for the lack of fit after adding axial points

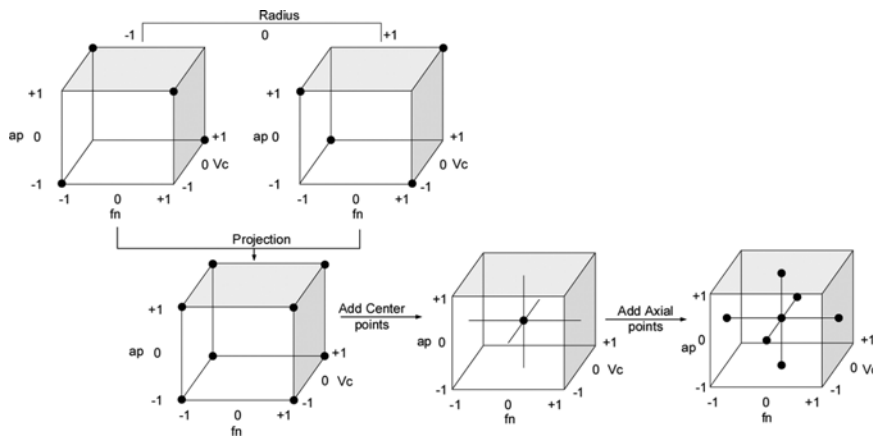
<i>Analysis of Variance for <math>R_z</math></i>						
<i>Source</i>	<i>DF</i>	<i>Seq SS</i>	<i>Adj SS</i>	<i>Adj MS</i>	<i>F</i>	<i>P</i>
Regression	9	2.37317	2.37317	0.263686	26.22	0.000
Linear	3	1.57619	1.57664	0.525546	52.25	0.000
Square	3	0.36737	0.36737	0.122458	12.18	0.000
Interaction	3	0.42961	0.42961	0.143203	14.24	0.000
Residual error	21	0.21121	0.21121	0.010058		
Lack-of-fit	5	0.09675	0.09675	0.019350	2.70	0.059
Pure error	16	0.11447	0.11447	0.007154		
<i>Total</i>	<i>30</i>	<i>2.58439</i>				

As the second-order regression is significant and no lack of fit was reported, the  $R_z$  variable was modelled as a second-order regression, namely as an SRM. Since the final DoE was upgraded to a CCD, the surface roughness  $R_a$  was also fitted by an SRM.

However, as was reported previously, the  $Ra$  model could be accurately fitted by a first-order model since the curvature error was not important.

Figure 6 summarises the progressive DoE conducted in the prosthesis-manufacturing operation that was analysed. The final regressions for each machining performance variable and their adjusted coefficients of determination  $R_{adj}^2$  are shown in Table 10. Note that the MRR is an analytical equation and so it was not necessary to conduct a regression.

**Figure 6** Summary of the progressive DoE conducted to model  $Ra$ ,  $Rz$ , and geometrical form errors

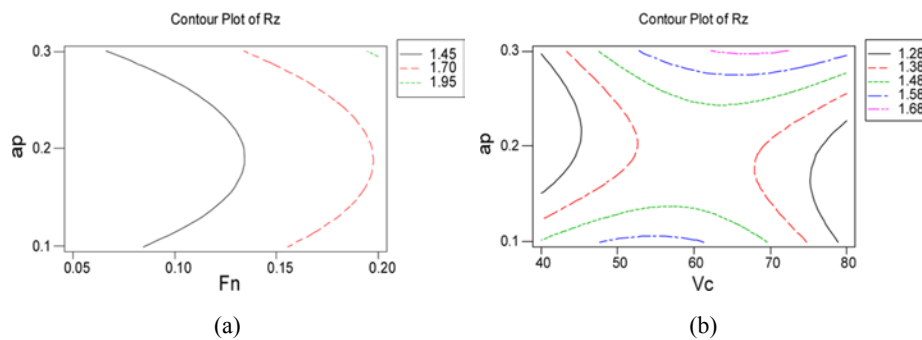


**Table 10** Regressions for each performance machining variable and their adjusted coefficient of determination  $R_{adj}^2$

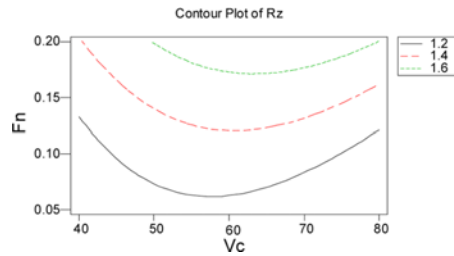
$MRR = 1000 Vc ap fn$	(1)
$Ra = 0.069 fn + 0.04125 Vc ap \quad R_{adj}^2 = 72.1\%$	(2)
$Rz = 0.27 fn - 0.217 Vc^2 + 0.21 ap^2 + 0.085 Vc fn + 0.139 Vc ap \quad R_{adj}^2 = 88.3\%$	(3)

Figures 7–9 show the relation between each machining performance variable and each significant cutting parameter.

**Figure 7** Relation between surface roughness  $Rz$  and cutting parameters. Contour plots of  $Rz$  when: (a) cutting speed is 60 m/min; (b) feed rate is 0.125 mm/rev; (c) depth of cut is 0.2 mm (see online version for colours)

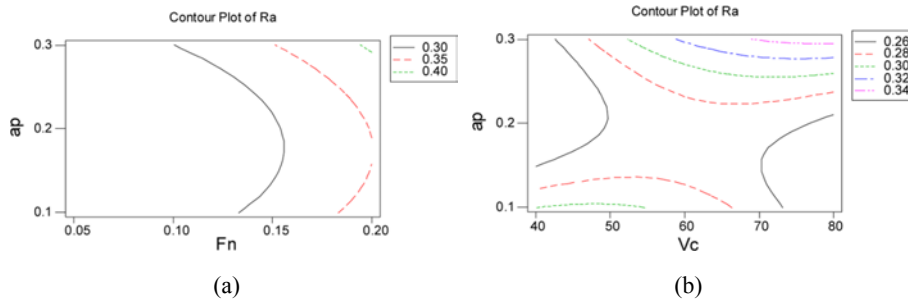


**Figure 7** Relation between surface roughness  $Rz$  and cutting parameters. Contour plots of  $Rz$  when: (a) cutting speed is 60 m/min; (b) feed rate is 0.125 mm/rev; (c) depth of cut is 0.2 mm (see online version for colours) (continued)



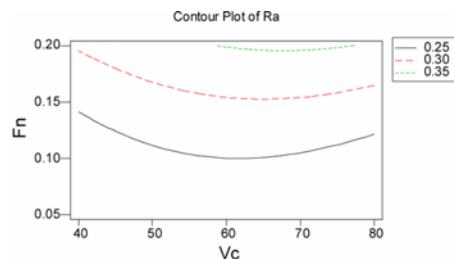
(c)

**Figure 8** Relation between surface roughness  $Ra$  and cutting parameters. Contour plots of  $Ra$  when: (a) cutting speed is 60 m/min; (b) feed rate is 0.125 mm/rev; (c) depth of cut is 0.2 mm (see online version for colours)



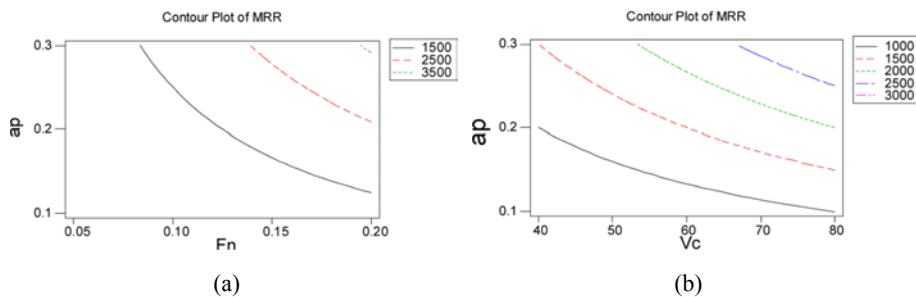
(a)

(b)



(c)

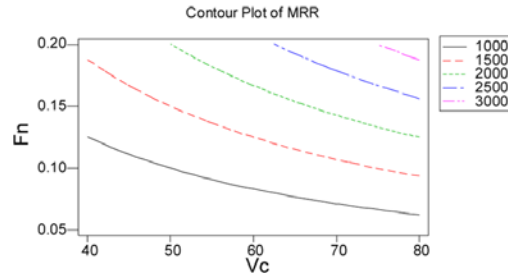
**Figure 9** Relation between MRR and cutting parameters. Contour plots of MRR when: (a) cutting speed is 60 m/min; (b) feed rate is 0.125 mm/rev; (c) depth of cut is 0.2 mm (see online version for colours)



(a)

(b)

**Figure 9** Relation between MRR and cutting parameters. Contour plots of MRR when: (a) cutting speed is 60 m/min; (b) feed rate is 0.125 mm/rev; (c) depth of cut is 0.2 mm (see online version for colours) (continued)



(c)

## 6 Multi-objective optimisation

The desirability functions were considered to be linear ( $w = 1$ ) and the coefficients of importance were chosen to keep all the machining performance variables at the same level of importance. Therefore, the importance factors  $I_1$ ,  $I_2$  and  $I_3$  (which are related to MRR, surface roughness  $Ra$  and surface roughness  $Rz$  respectively) were chosen as 1. The upper bounds, lower bounds and target values for each desirability function were obtained experimentally from the machining data after conducting the progressive DoE. The coefficients defined for each desirability function are shown in Table 11.

**Table 11** Coefficients defined for each desirability function

Desirability function	Lower bound	Target value	Upper bound	Importance factor	Weighting factor	Goal
$Ra$	–	0.19 $\mu\text{m}$	0.51 $\mu\text{m}$	1	1	To be minimised
$Rz$	–	0.90 $\mu\text{m}$	2.07 $\mu\text{m}$	1	1	To be minimised
MRR	200 $\text{mm}^3/\text{min}$	4800 $\text{mm}^3/\text{min}$	–	1	1	To be maximised

Then, the desirability functions are defined as follows:

- MRR desirability function

$$d_1(\text{MRR}) = \frac{\text{MRR} - 200}{4800 - 200} \quad (4)$$

- Desirability function of  $Ra$

$$d_2(Ra) = \frac{Ra - 0.51}{0.19 - 0.51} \quad (5)$$

- Desirability function of  $Rz$

$$d_3(Rz) = \frac{Rz - 2.07}{0.9 - 2.07} \quad (6)$$

The multi-objective function or the overall desirability function to be optimised is:

$$D = (d_1(\text{MRR})d_2(Ra)d_3(Rz))^{\frac{1}{(1+1+1)}} \quad (7)$$

constrained to:

$$40 \text{ m/min} \leq Vc \leq 80 \text{ m/min} \quad (8)$$

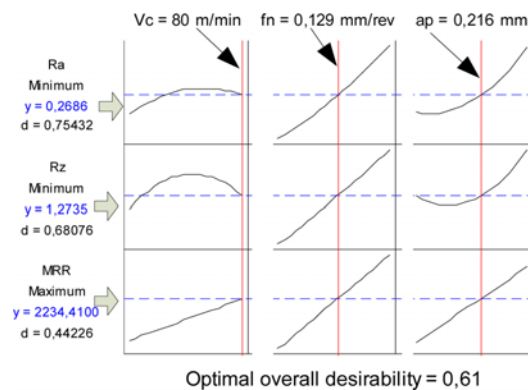
$$0.05 \text{ mm/rev} \leq fn \leq 0.2 \text{ mm/rev} \quad (9)$$

$$0.1 \text{ mm} \leq ap \leq 0.3 \text{ mm}. \quad (10)$$

Maximising the overall desirability function gives the optimal cutting parameters that optimise the prosthesis-manufacturing operation. The estimated optimal value of the overall desirability function  $D$  was 0.61, and the optimal cutting conditions were  $Vc = 80 \text{ m/min}$ ,  $fn = 0.129 \text{ mm/rev}$  and  $ap = 0.216 \text{ mm}$ . Under these cutting conditions, the estimation of each machining performance variable was:  $Rz = 1.27 \mu\text{m}$ ,  $Ra = 0.27 \mu\text{m}$  and  $\text{MRR} = 2234 \text{ mm}^3/\text{min}$ . The graphical results after the optimisation and the value of each desirability function under the optimal cutting conditions are shown in Figure 10.

To validate the optimisation procedure and the machining performance models, two experiments were conducted under the optimal cutting conditions. Both experiments presented machining performance values that were very close to the expected ones. The first experiment showed surface roughness values of  $Ra = 0.26 \mu\text{m}$  and  $Rz = 1.25 \mu\text{m}$  and an overall desirability value of  $D = 0.62$ , whereas the second experiment showed  $Ra = 0.25 \mu\text{m}$ ,  $Rz = 1.15 \mu\text{m}$  and  $D = 0.65$ . The expected optimal values obtained from the optimisation procedure were  $Ra = 0.27 \mu\text{m}$ ,  $Rz = 1.27 \mu\text{m}$  and  $D = 0.61$ . Lastly, the deviation from the experimental tests can be explained by the limited accuracy of the machining performance models, since the accuracy of the models that were fitted were 72.1% and 88.3% for  $Ra$  and  $Rz$ , respectively.

**Figure 10** Result of optimising the overall desirability function. Optimal cutting parameters:  $Vc = 80 \text{ m/min}$ ;  $fn = 0.129 \text{ mm/rev}$ ;  $ap = 0.216 \text{ mm}$  (see online version for colours)



## 7 Conclusions

In the field of the manufacture of prostheses, different factors must be considered to be able to optimise cutting parameters. Specific surface roughness targets and form



tolerances should be met to promote healthy tissue–biomaterial interactions. This paper has presented a multi-objective optimisation procedure for optimising the quality of prostheses and manufacturing productivity. The aim of this procedure is to develop machining performance models through a minimal and progressive DoE, which models the variables of interest as linear regressions or SRM. The multi-objective optimisation is based on desirability functions, which are defined according to the relative importance of each variable of interest. The procedure was implemented to optimise a manufacturing process of spherical turned components for Ti-6Al-4V hip prostheses with special requirements as regards surface roughness  $R_a$ ,  $R_z$  and geometrical form tolerance.

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