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Additional Information
STRONG EXTENSIONS FOR $q$-SUMMING OPERATORS
ACTING IN $p$-CONVEX BANACH FUNCTION SPACES FOR

$1 \leq p \leq q$

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Abstract. Let $1 \leq p \leq q < \infty$ and let $X$ be a $p$-convex Banach function space over a $\sigma$-finite measure $\mu$. We combine the structure of the spaces $L^p(\mu)$ and $L^q(\xi)$ for constructing the new space $S^q_{X,\mu}(\xi)$, where $\xi$ is a probability Radon measure on a certain compact set associated to $X$. We show some of its properties, and the relevant fact that every $q$-summing operator $T$ defined on $X$ can be continuously (strongly) extended to $S^q_{X,\mu}(\xi)$. Our arguments lead to a mixture of the Pietsch and Maurey-Rosenthal factorization theorems, which provided the known (strong) factorizations for $q$-summing operators through $L^q$-spaces when $1 \leq q \leq p$. Thus, our result completes the picture, showing what happens in the complementary case $1 \leq p \leq q$.

Operator and extension and factorization and $p$-convex and $q$-summing.

1. Introduction

Fix $1 \leq p \leq q < \infty$ and let $T: X \to E$ be a Banach space valued linear operator defined on a saturated order semi-continuous Banach function space $X$ related to a $\sigma$-finite measure $\mu$. In this paper we prove an extension theorem for $T$ in the case when $T$ is $q$-summing and $X$ is $p$-convex. In order to do this, we first define and analyze a new class of Banach function spaces denoted by $S^q_{X,\mu}(\xi)$ which have some good properties, mainly order continuity and $p$-convexity. The space $S^q_{X,\mu}(\xi)$ is constructed by using the spaces $L^p(\mu)$ and $L^q(\xi)$, where $\xi$ is a finite positive Radon measure on a certain compact set associated to $X$.

Corollary 5 states the desired extension for $T$. Namely, if $T$ is $q$-summing and $X$ is $p$-convex then $T$ can be strongly extended continuously to a space of the type $S^q_{X,\mu}(\xi)$. Here we use the term “strongly” for this extension to remark that the map carrying $X$ into $S^q_{X,\mu}(\xi)$ is actually injective; as the reader will notice (Proposition 3), this is one of the goals of our result. In order to develop our arguments, we introduce a new geometric tool which we call the family of $p$-strongly $q$-concave operators (see the definition at the beginning of Section 4). The inclusion of $X$ into $S^q_{X,\mu}(\xi)$ turns out to belong to this family. In particular, it is $q$-concave.

If $T$ is $q$-summing then it is $p$-strongly $q$-concave (Proposition 5). Actually, in Theorem 4 we show that in the case that $X$ is $p$-convex, $T$ can be continuously extended to a space $S^q_{X,\mu}(\xi)$ if and only if $T$ is $p$-strongly $q$-concave. This result can be understood as an extension of some well-known relevant factorizations of the operator theory:

I Maurey-Rosenthal factorization theorem: If $T$ is $q$-concave and $X$ is $q$-convex and order continuous, then $T$ can be extended to a weighted $L^q$-space related
to $\mu$ (see for instance [3, Corollary 5]). Several generalizations and applications of the ideas behind this fundamental factorization theorem have been recently obtained (see [1, 2, 4, 5, 9]).

(II) Pietsch factorization theorem: If $T$ is $q$-summing, then it factors through a closed subspace of $L^q(\xi)$, where $\xi$ is a probability Radon measure on a certain compact set associated to $X$; see for instance [6, Theorem 2.13].

Let us explain how the relation of our results with these ones must be understood. The extreme case $p = q$ in Theorem 4 gives the Maurey-Rosenthal type factorization (I), since the $q$-strongly $q$-concave operators are exactly the $q$-concave operators. This is the situation in the well-known case $1 \leq q \leq p$ for which $p = q$ can be assumed, since $p$-convexity of $X(\mu)$ implies $q$-convexity of $X(\mu)$. The factorization space $S^q_X(\xi)$ can be then identified with a weighted $L^q$-space, that is, the measure $\xi$ appearing in its definition can be given by the Dirac’s delta $\delta_w$, where $w$ is the weight function. The other extreme case $p = 1$ gives a Pietsch type factorization (II). In this case the convexity requirement disappears —every Banach lattice is 1-convex—and the 1-strongly $q$-concave operators are defined by a $q$-summing type inequality. Indeed, for an operator acting in a $(K)$-space, $q$-concavity, $q$-summability and 1-strong $q$-concavity are the same thing. More aspects of the asymptotic behavior of $p$-strongly $q$-concave operators will be explained in Remark 4.

We must also say that our generalization will allow to face the problem of the factorization of several $p$-summing type of multilinear operators from products of Banach function spaces—a topic of current interest—, since it allows to understand factorization of $q$-summing operators from $p$-convex function lattices from a unified point of view not depending on the order relation between $p$ and $q$.

As an application, we also prove by using Theorem 4 a kind of Kakutani representation theorem (see for instance [7, Theorem 1.b.2]) through the spaces $S^q_X(\xi)$ for $p$-convex Banach function spaces which are $p$-strongly $q$-concave (Corollary 4).

2. Preliminaries

Let $(\Omega, \Sigma, \mu)$ be a $\sigma$-finite measure space and denote by $L^0(\mu)$ the space of all measurable real functions on $\Omega$, where functions which are equal $\mu$-a.e. are identified. By a Banach function space (briefly B.f.s.) we mean a Banach space $X \subset L^0(\mu)$ with norm $\| \cdot \|_X$, such that if $f \in L^0(\mu)$, $g \in X$ and $|f| \leq |g|$ $\mu$-a.e. then $f \in X$ and $\|f\|_X \leq \|g\|_X$. In particular, $X$ is a Banach lattice with the $\mu$-a.e. pointwise order, in which the convergence in norm of a sequence implies the convergence $\mu$-a.e. for some subsequence. A B.f.s. $X$ is said to be saturated if there exists no $A \in \Sigma$ with $\mu(A) > 0$ such that $f|_A = 0$ $\mu$-a.e. for all $f \in X$, or equivalently, if $X$ has a weak unit (i.e. $g \in X$ such that $g > 0$ $\mu$-a.e.).

Let $X$ be a saturated B.f.s. For every $f \in L^0(\mu)$, there exists $(f_n)_{n \geq 1} \subset X$ such that $0 \leq f_n \uparrow |f|$ $\mu$-a.e.

Proof. Consider a weak unit $g \in X$ and take $g_n = ng/(1 + ng)$. Note that $0 < g_n < ng$ $\mu$-a.e., so $g_n$ is a weak unit in $X$. Moreover, $(g_n)_{n \geq 1}$ increases $\mu$-a.e. to the constant function equal to 1. Now, take $f_n = g_n|f|1_{\omega \in \Omega : |f| \leq n}$. Since $0 \leq f_n \leq ng_n$ $\mu$-a.e., we have that $f_n \in X$, and $f_n \uparrow |f|$ $\mu$-a.e.

The Köthe dual of a B.f.s. $X$ is the space $X'$ given by the functions $h \in L^0(\mu)$ such that $\int |hf| d\mu < \infty$ for all $f \in X$. If $X$ is saturated then $X'$ is a saturated
B.f.s. with norm \( \|h\|_{X'} = \sup_{f \in B_X} \int |hf| \, d\mu \) for \( h \in X' \). Here, as usual, \( B_X \) denotes the closed unit ball of \( X \). Each function \( h \in X' \) defines a functional \( \zeta(h) \) on \( X \) by \( \langle \zeta(h), f \rangle = \int hf \, d\mu \) for all \( f \in X \). In fact, \( X' \) is isometrically order isomorphic (via \( \zeta \)) to a closed subspace of the topological dual \( X^* \) of \( X \).

From now and on, a B.f.s. \( X \) will be assumed to be saturated. If for every \( f, f_n \in X \) such that \( 0 \leq f_n \uparrow f \) \( \mu \)-a.e. it follows that \( \|f_n\|_X \uparrow \|f\|_X \), then \( X \) is said to be order semi-continuous. This is equivalent to \( \langle \zeta(X') \rangle \) being a norming subspace of \( X^* \), i.e. \( \|f\|_X = \sup_{h \in B_{X'}} \int |hf| \, d\mu \) for all \( f \in X \). A B.f.s. \( X \) is order continuous if for every \( f, f_n \in X \) such that \( 0 \leq f_n \uparrow f \) \( \mu \)-a.e., it follows that \( f_n \to f \) in norm. In this case, \( X' \) can be identified with \( X^* \).

For general issues related to B.f.s.’ see [7], [8] and [10, Ch. 15] considering the function norm \( \rho \) defined as \( \rho(f) = \|f\|_X \) if \( f \in X \) and \( \rho(f) = \infty \) in other case.

Let \( 1 \leq p < \infty \). A B.f.s. \( X \) is said to be \( p \)-convex if there exists a constant \( C > 0 \) such that

\[
\left\| \left( \sum_{i=1}^{n} |f_i|^p \right)^{1/p} \right\|_X \leq C \left( \sum_{i=1}^{n} \|f_i\|^p \right)^{1/p}
\]

for every finite subset \( \{f_i\}_{i=1}^{n} \subset X \). In this case, \( M^p(X) \) will denote the smallest constant \( C \) satisfying the above inequality. Note that \( M^p(X) \geq 1 \). A relevant fact is that every \( p \)-convex B.f.s. \( X \) has an equivalent norm for which \( X \) is \( p \)-convex with constant \( M^p(X) = 1 \), see [7, Proposition 1.d.8].

The \( p \)-th power of a B.f.s. \( X \) is the space defined as

\[
X_p = \{ f \in L^0(\mu) : |f|^{1/p} \in X \},
\]

endowed with the quasi-norm \( \|f\|_{X_p} = \|f|^{1/p}\|_X \), for \( f \in X_p \). Note that \( X_p \) is always complete, see the proof of [8, Proposition 2.22]. If \( X \) is \( p \)-convex with constant \( M^p(X) = 1 \), from [3, Lemma 3], \( \|\cdot\|_{X_p} \) is a norm and so \( X_p \) is a B.f.s. Note that \( X_p \) is saturated if and only if \( X \) is so. The same holds for the properties of being order continuous and order semi-continuous.

3. The space \( S^q_{X_p}(\xi) \)

Let \( 1 \leq p \leq q < \infty \) and let \( X \) be a saturated \( p \)-convex B.f.s. We can assume without loss of generality that the \( p \)-convexity constant \( M^p(X) \) is equal to 1. Then, \( X_p \) and \( (X_p)' \) are saturated B.f.s. Consider the topology \( \sigma((X_p)', X_p) \) on \( (X_p)' \) defined by the elements of \( X_p \). Note that the subset \( B^+_{(X_p)'} \) of all positive elements of the closed unit ball of \( (X_p)' \) is compact for this topology.

Let \( \xi \) be a finite positive Radon measure on \( B^+_{(X_p)'} \). For \( f \in L^0(\mu) \), consider the map \( \phi_f : B^+_{(X_p)'} \to [0, \infty] \) defined by

\[
\phi_f(h) = \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p}
\]

for all \( h \in B^+_{(X_p)'} \). In the case when \( f \in X \) it follows that \( \phi_f \) is continuous and so measurable, since \( |f|^p \in X_p \). For a general \( f \in L^0(\mu) \), by Lemma 2 we can take a sequence \( (f_n)_{n \geq 1} \subset X \) such that \( 0 \leq f_n \uparrow |f| \) \( \mu \)-a.e. Applying the Monotone Convergence Theorem, we have that \( \phi_{f_n} \uparrow \phi_f \) pointwise and so \( \phi_f \) is measurable. Then, we can consider the integral \( \int_{B^+_{(X_p)'}} \phi_f(h) \, d\xi(h) \in [0, \infty] \) and
define the following space:

\[ S^q_{X_p}(\xi) = \left\{ f \in L^0(\mu) : \left( \int_{B^{+}_{X_p}(\xi)} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) < \infty \right) \right\}. \]

Let us endow \( S^q_{X_p}(\xi) \) with the seminorm

\[ \|f\|_{S^q_{X_p}(\xi)} = \left( \int_{B^{+}_{X_p}(\xi)} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \right)^{1/q}, \]

In general, \( \| \cdot \|_{S^q_{X_p}(\xi)} \) is not a norm. For instance, if \( \xi \) is the Dirac measure at some \( h_0 \in B^{+}_{X_p}(\xi) \) such that \( A = \{ \omega \in \Omega : h_0(\omega) = 0 \} \) satisfies \( \mu(A) > 0 \), taking \( f = g\chi_A \in X \) with \( g \) being a weak unit of \( X \), we have that

\[ \|f\|_{S^q_{X_p}(\xi)} = \left( \int_{A} |g(\omega)|^p h_0(\omega) \, d\mu(\omega) \right)^{1/p} = 0 \]

and

\[ \mu(\{ \omega \in \Omega : f(\omega) \neq 0 \}) = \mu(\Omega \Delta \{ \omega \in \Omega : g(\omega) \neq 0 \}) = \mu(A) > 0. \]

If the Radon measure \( \xi \) satisfies

\[ \int_{B^{+}_{X_p}(\xi)} \left( \int_{A} h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = 0 \implies \mu(A) = 0 \]

then, \( S^q_{X_p}(\xi) \) is a saturated B.f.s. Moreover, \( S^q_{X_p}(\xi) \) is order continuous, \( p \)-convex (with constant 1) and \( X \subset S^q_{X_p}(\xi) \) continuously.

**Proof.** It is clear that if \( f \in L^0(\mu), g \in S^q_{X_p}(\xi) \) and \( |f| \leq |g| \) \( \mu \)-a.e. then \( f \in S^q_{X_p}(\xi) \) and \( \|f\|_{S^q_{X_p}(\xi)} \leq \|g\|_{S^q_{X_p}(\xi)} \). Let us see that \( \| \cdot \|_{S^q_{X_p}(\xi)} \) is a norm. Suppose that \( \|f\|_{S^q_{X_p}(\xi)} = 0 \) and set \( A_n = \{ \omega \in \Omega : |f(\omega)| \geq \frac{1}{n} \} \) for every \( n \geq 1 \). Since \( \chi_{A_n} \leq n |f| \) and

\[ \int_{B^{+}_{X_p}(\xi)} \left( \int_{A_n} h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \|\chi_{A_n}\|_{S^q_{X_p}(\xi)} \leq n^{q/p} \|f\|_{S^q_{X_p}(\xi)} = 0, \]

from (1) we have that \( \mu(A_n) = 0 \) and so

\[ \mu(\{ \omega \in \Omega : f(\omega) \neq 0 \}) = \lim_{n \to \infty} \mu(A_n) = 0. \]

Now we will see that \( S^q_{X_p}(\xi) \) is complete by showing that \( \sum_{n \geq 1} f_n \in S^q_{X_p}(\xi) \) whenever \( (f_n)_{n \geq 1} \subset S^q_{X_p}(\xi) \) with \( C = \sum \|f_n\|_{S^q_{X_p}(\xi)} < \infty \). First let us prove that \( \sum_{n \geq 1} |f_n| < \infty \) \( \mu \)-a.e. For every \( N,n \geq 1 \), taking \( A^N_n = \{ \omega \in \Omega : \sum_{j=1}^{n} |f_j(\omega)| > N \} \), since \( \chi_{A^N_n} \leq \frac{1}{N} \sum_{j=1}^{n} |f_j| \), we have that

\[ \int_{B^{+}_{X_p}(\xi)} \left( \int_{A^N_n} h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \|\chi_{A^N_n}\|_{S^q_{X_p}(\xi)} \leq \frac{1}{N^q} \sum_{j=1}^{n} |f_j| \|f_j\|_{S^q_{X_p}(\xi)} \leq \frac{C^q}{N^q}. \]
Note that, for $N$ fixed, $(A_n^N)_{n \geq 1}$ increases. Taking limit for $n \to \infty$ and applying twice the Monotone Convergence Theorem, it follows that
\[
\int_{B_{(X_p)}^+} \left( \int_{\bigcup_{N \geq 1} A_n^N} h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \leq \frac{C^q}{N^q}.
\]
Then,
\[
\int_{B_{(X_p)}^+} \left( \int_{\bigcap_{N \geq 1} \bigcup_{n \geq 1} A_n^N} h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \leq \lim_{N \to \infty} \frac{C^q}{N^q} = 0,
\]
and so, from (1),
\[
\mu \left( \left\{ \omega \in \Omega : \sum_{n \geq 1} |f_n(\omega)| = \infty \right\} \right) = \mu \left( \bigcap_{N \geq 1} \bigcup_{n \geq 1} A_n^N \right) = 0.
\]
Hence, $\sum_{n \geq 1} f_n \in L^0(\mu)$. Again applying the Monotone Convergence Theorem, it follows that
\[
\int_{B_{(X_p)}^+} \left( \int_{\Omega} \left| \sum_{n \geq 1} f_n(\omega) \right|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \leq \lim_{n \to \infty} \int_{B_{(X_p)}^+} \left( \int_{\Omega} \left( \sum_{j=1}^n |f_j(\omega)| \right)^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \lim_{n \to \infty} \left\| \sum_{j=1}^n f_j \right\|_{S_{X_p}^q}(\xi) \leq C^q
\]
and thus $\sum_{n \geq 1} f_n \in S_{X_p}^q(\xi)$.

Note that if $f \in X$, for every $h \in B_{(X_p)}^+$ we have that
\[
\int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \leq \|f\|_p \|x_p\|_{(X_p)^\prime} \leq \|f\|_p^p
\]
and so
\[
\int_{B_{(X_p)}^+} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \leq \|f\|_X^q \xi \left( B_{(X_p)}^+ \right).
\]
Then, $X \subset S_{X_p}^q(\xi)$ and $\|f\|_{S_{X_p}^q(\xi)} \leq \xi \left( B_{(X_p)}^+ \right)^{1/q} \|f\|_X$ for all $f \in X$. In particular, $S_{X_p}^q(\xi)$ is saturated, as a weak unit in $X$ is a weak unit in $S_{X_p}^q(\xi)$.

Let us show that $S_{X_p}^q(\xi)$ is order continuous. Consider $f, f_n \in S_{X_p}^q(\xi)$ such that $0 \leq f_n \uparrow f$ $\mu$-a.e. Note that, since
\[
\int_{B_{(X_p)}^+} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) < \infty,
\]
there exists a $\xi$-measurable set $B$ with $\xi \left( B_{(X_p)}^+ \setminus B \right) = 0$ such that
\[
\int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) < \infty
\]
for all $h \in B$. Fixed $h \in B$, we have that $|f - f_n|^p h \downarrow 0$ $\mu$-a.e. and $|f - f_n|^p h \leq |f|^p h$ $\mu$-a.e. Then, applying the Dominated Convergence Theorem, $\int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \leq \int_{\Omega} |f|^p h(\omega) \, d\mu(\omega) \leq \int_{\Omega} |f|^p h(\omega) \, d\mu(\omega)$.
Consider the measurable functions \( \phi, \phi_n : B^+_{(X_p)'} \rightarrow [0, \infty] \) given by

\[
\phi(h) = \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p}
\]

\[
\phi_n(h) = \left( \int_{\Omega} |f(\omega) - f_n(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p}
\]

for all \( h \in B^+_{(X_p)'} \). It follows that \( \phi_n \downarrow 0 \) \( \xi \)-a.e. and \( \phi_n \leq \phi \) \( \xi \)-a.e. Again by the Dominated Convergence Theorem, we obtain

\[
\|f - f_n\|_{S^q_{X_p}(\xi)}^{q/p} = \int_{B^+_{(X_p)'}} \phi_n(h) \, d\xi(h) \downarrow 0.
\]

Finally, let us see that \( S^q_{X_p}(\xi) \) is \( p \)-convex. Fix \( (f_i)_{i=1}^n \subset S^q_{X_p}(\xi) \) and consider the measurable functions \( \phi_i : B^+_{(X_p)'} \rightarrow [0, \infty] \) (for \( 1 \leq i \leq n \)) defined by

\[
\phi_i(h) = \int_{\Omega} |f_i(\omega)|^p h(\omega) \, d\mu(\omega).
\]

for all \( h \in B^+_{(X_p)'} \). Then,

\[
\left\| \left( \sum_{i=1}^n |f_i|^p \right)^{1/p} \right\|_{S^q_{X_p}(\xi)}^q = \int_{B^+_{(X_p)'}} \left( \int_{\Omega} \sum_{i=1}^n |f_i(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h)
\]

\[
= \int_{B^+_{(X_p)'}} \left( \sum_{i=1}^n \phi_i(h) \right)^{q/p} \, d\xi(h)
\]

\[
\leq \left( \sum_{i=1}^n \|\phi_i\|_{L^{q/p}(\xi)} \right)^{q/p}.
\]

Since \( \|\phi_i\|_{L^{q/p}(\xi)} = \|f_i\|_{S^q_{X_p}(\xi)}^{p/q} \) for all \( 1 \leq i \leq n \), we have that

\[
\left\| \left( \sum_{i=1}^n |f_i|^p \right)^{1/p} \right\|_{S^q_{X_p}(\xi)} \leq \left( \sum_{i=1}^n \|f_i\|_{S^q_{X_p}(\xi)}^p \right)^{1/p}.
\]

Take a weak unit \( g \in (X_p)' \) and consider the Radon measure \( \xi \) as the Dirac measure at \( g \). If \( A \in \Sigma \) is such that

\[
0 = \int_{B^+_{(X_p)'}} \left( \int_A h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \left( \int_A g(\omega) \, d\mu(\omega) \right)^{q/p}
\]

then, \( g_{\chi A} = 0 \) \( \mu \)-a.e. and so, since \( g > 0 \) \( \mu \)-a.e., \( \mu(A) = 0 \). That is, \( \xi \) satisfies (1). In this case, \( S^q_{X_p}(\xi) = L^p(gd\mu) \) with equal norms, as

\[
\int_{B^+_{(X_p)'}} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \left( \int_{\Omega} |f(\omega)|^p g(\omega) \, d\mu(\omega) \right)^{q/p}
\]

for all \( f \in L^q(\mu) \).

Write \( \Omega = \bigcup_{n \geq 1} \Omega_n \) with \( (\Omega_n)_{n \geq 1} \) being a disjoint sequence of measurable sets and take a sequence of strictly positive elements \((\alpha_n)_{n \geq 1} \in \ell^1 \). Let us consider the Radon measure \( \xi = \sum_{n \geq 1} \alpha_n g_{\chi \Omega_n} \) on \( B^+_{(X_p)'} \), where \( g_{\chi \Omega_n} \) is the Dirac measure
at \( g\chi_n \) with \( g \in (X_p)' \) being a weak unit. Note that for every positive function \( \phi \in L^0(\xi) \), it follows that \( \int_{B^+(X_p)'} \phi \, d\xi = \sum_{n \geq 1} \alpha_n \phi(g\chi_n) \). If \( A \in \Sigma \) is such that

\[
0 = \int_{B^+(X_p)'} \left( \int_A h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \sum_{n \geq 1} \alpha_n \left( \int_{A \cap \Omega_n} g(\omega) \, d\mu(\omega) \right)^{q/p},
\]

then, \( \int_{A \cap \Omega_n} g(\omega) \, d\mu(\omega) = 0 \) for all \( n \geq 1 \). Hence,

\[
\int_A g(\omega) \, d\mu(\omega) = \sum_{n \geq 1} \int_{A \cap \Omega_n} g(\omega) \, d\mu(\omega) = 0
\]

and so \( g\chi_A = 0 \) \( \mu \)-a.e., from which \( \mu(A) = 0 \). That is, \( \xi \) satisfies (1). For every \( f \in L^0(\mu) \) we have that

\[
\int_{B^+(X_p)'} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) = \sum_{n \geq 1} \alpha_n \left( \int_{\Omega_n} |f(\omega)|^p g(\omega) \, d\mu(\omega) \right)^{q/p}.
\]

Then, the B.f.s. \( S^q_{X_p}(\xi) \) can be described as the space of functions \( f \in \cap_{n \geq 1} L^p(g\chi_n, d\mu) \) such that \( (\alpha_n 1/q \| f \|_{L^p(g\chi_n, d\mu)})_{n \geq 1} \in \ell^q \). Moreover,

\[
\| f \|_{S^q_{X_p}(\xi)} = \left( \sum_{n \geq 1} \alpha_n \| f \|^q_{L^p(g\chi_n, d\mu)} \right)^{1/q}
\]

for all \( f \in S^q_{X_p}(\xi) \).

4. \( p \)-strongly \( q \)-concave operators

Let \( 1 \leq p \leq q < \infty \) and let \( T : X \to E \) be a linear operator from a saturated B.f.s. \( X \) into a Banach space \( E \). Recall that \( T \) is said to be \( q \)-concave if there exists a constant \( C > 0 \) such that

\[
\left( \sum_{i=1}^n \| T(f_i) \|_E^q \right)^{1/q} \leq C \left( \sum_{i=1}^n |f_i|^q \right)^{1/q} \| X \]

for every finite subset \( (f_i)_{i=1}^n \subset X \). The smallest possible value of \( C \) will be denoted by \( M_q(T) \). For issues related to \( q \)-concavity see for instance [7, Ch.1.d].

We introduce a slightly stronger notion than \( q \)-concavity: \( T \) will be called \( p \)-strongly \( q \)-concave if there exists \( C > 0 \) such that

\[
\left( \sum_{i=1}^n \| T(f_i) \|_E^q \right)^{1/q} \leq C \sup_{(\beta_i)_{i \geq 1} \in B_{eq}} \left( \sum_{i=1}^n |\beta_i f_i|^p \right)^{1/p} \| X \]

for every finite subset \( (f_i)_{i=1}^n \subset X \), where \( 1 < r \leq \infty \) is such that \( \frac{1}{r} = \frac{1}{p} - \frac{1}{q} \). In this case, \( M_{p,q}(T) \) will denote the smallest constant \( C \) satisfying the above inequality. Noting that \( \frac{p}{q} \) and \( \frac{q}{p} \) are conjugate exponents, it is clear that every \( p \)-strongly \( q \)-concave operator is \( q \)-concave and so continuous, and moreover \( \|T\| \leq M_q(T) \leq M_{p,q}(T) \). As usual, we will say that \( X \) is \( p \)-strongly \( q \)-concave if the identity map \( I : X \to X \) is so, and in this case, we denote \( M_{p,q}(X) = M_{p,q}(I) \).

Our goal is to get a continuous extension of \( T \) to a space of the type \( S^q_{X_p}(\xi) \) in the case when \( T \) is \( p \)-strongly \( q \)-concave and \( X \) is \( p \)-convex. To this end we will
need to describe the supremum on the right-hand side of the \( p \)-strongly \( q \)-concave inequality in terms of the Köthe dual of \( X_p \).

If \( X \) is \( p \)-convex and order semi-continuous then

\[
\sup_{(\beta_i)_{i \geq 1} \in B_{ce}} \left\| \left( \sum_{i=1}^{n} |\beta_i f_i|^p \right)^{1/p} \right\|_X = \sup_{h \in B_{(X_p)'}^+} \left( \sum_{i=1}^{n} \left( \int |f_i|^p h \, d\mu \right)^{q/p} \right)^{1/q}
\]

for every finite subset \( (f_i)_{i=1}^{n} \subset X \), where \( 1 < r \leq \infty \) is such that \( \frac{1}{r} = \frac{1}{p} - \frac{1}{q} \) and \( B_{(X_p)'}^+ \) is the subset of all positive elements of the closed unit ball \( B_{(X_p)'} \) of \( (X_p)' \).

**Proof.** Given \( (f_i)_{i=1}^{n} \subset X \), since \( X_p \) is order semi-continuous (as \( X \) so is) and \( (\ell^q/r)^* = \ell^q/p \) (as \( \frac{2}{p} \) is the conjugate exponent of \( \frac{n}{p} \)), we have that

\[
\sup_{(\beta_i)_{i \geq 1} \in B_{ce}} \left\| \left( \sum_{i=1}^{n} |\beta_i f_i|^p \right)^{1/p} \right\|_X = \sup_{(\beta_i)_{i \geq 1} \in B_{ce}} \left\| \left( \sum_{i=1}^{n} |\beta_i f_i|^p \right)^{1/p} \right\|_{X_p} = \sup_{h \in B_{(X_p)'}^+} \left( \sum_{i=1}^{n} \left( \int |f_i|^p h \, d\mu \right)^{q/p} \right)^{1/q}
\]

In the following remark we show a general example of \( p \)-strongly \( q \)-concave operator that can be easily obtained from Lemma 4. In a sense, this operator is the prototype of \( p \)-strongly \( q \)-concave operator.

Suppose that \( X \) is \( p \)-convex and order semi-continuous. For every finite positive Radon measure \( \xi \) on \( B_{(X_p)'}^+ \) satisfying (1), it follows that the inclusion map \( i : X \to S_{X_p}^q(\xi) \) is \( p \)-strongly \( q \)-concave. Indeed, for each \( (f_i)_{i=1}^{n} \subset X \), we have that

\[
\sum_{i=1}^{n} \left\| f_i \right\|_{S_{X_p}^q(\xi)}^q = \sum_{i=1}^{n} \int_{B_{(X_p)'}^+} \left( \int_{\Omega} |f_i(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \leq \xi(B_{(X_p)'}^+) \sup_{h \in B_{(X_p)'}^+} \sum_{i=1}^{n} \left( \int_{\Omega} |f_i(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p}
\]

and so, Lemma 4 gives the conclusion for \( M_{p,q}(i) \leq \xi(B_{(X_p)'}^+)^{1/q} \).

Now let us prove our main result.
If $T$ is $p$-strongly $q$-concave and $X$ is $p$-convex and order semi-continuous, then there exists a probability Radon measure $\xi$ on $B^+_{(X_p)'}$, satisfying (1) such that

$$
\|T(f)\|_E \leq M_{p,q}(T) \left( \int_{B^+_{(X_p)'}} \left( \int_{\Omega} |f(\omega)|^p h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h) \right)^{1/q}
$$

for all $f \in X$.

**Proof.** Recall that the topology on $(X_p)'$ is $\sigma((X_p)', X_p)$, the one which is defined by the elements of $X_p$. For each finite subset (with possibly repeated elements) $M = (f_i)_{i=1}^m \subset X$, consider the map $\psi_M : B^+_{(X_p)'} \rightarrow [0, \infty)$ defined by

$$
\psi_M(h) = \sum_{i=1}^m \left( \int_{\Omega} |f_i|^p h \, d\mu \right)^{q/p} \text{ for } h \in B^+_{(X_p)'}.
$$

Note that $\psi_M$ attains its supremum as it is continuous on a compact set, so there exists $h_M \in B^+_{(X_p)'}$, such that $\sup_{h \in B^+_{(X_p)'}} \psi_M(h) = \psi_M(h_M)$. Then, the $p$-strongly $q$-concavity of $T$, together with Lemma 4, gives

$$
\sum_{i=1}^m \|T(f_i)\|_E^q \leq M_{p,q}(T)^q \sup_{h \in B^+_{(X_p)'}} \sum_{i=1}^m \left( \int_{\Omega} |f_i|^p h \, d\mu \right)^{q/p} 
$$

$$
\leq M_{p,q}(T)^q \sup_{h \in B^+_{(X_p)'}} \psi_M(h) 
$$

$$
= M_{p,q}(T)^q \psi_M(h_M).
$$

(3)

Consider now the continuous map $\phi_M : B^+_{(X_p)'} \rightarrow \mathbb{R}$ defined by

$$
\phi_M(h) = M_{p,q}(T)^q \psi_M(h) - \sum_{i=1}^m \|T(f_i)\|_E^q
$$

for $h \in B^+_{(X_p)'}$. Take $B = \{\phi_M : M \text{ is a finite subset of } X\}$. Since for every $M = (f_i)_{i=1}^m, M' = (f'_i)_{i=1}^m \subset X$ and $0 < t < 1$, it follows that $t \phi_M + (1-t) \phi_{M'} = \phi_{M''}$ where $M'' = (t^{1/q} f_i)_{i=1}^m \cup ((1-t)^{1/q} f'_i)_{i=1}^m$, we have that $B$ is convex. Denote by $C(B^+_{(X_p)'}$ the space of continuous real functions on $B^+_{(X_p)'}$, endowed with the supremum norm, and by $A$ the open convex subset $\{\phi \in C(B^+_{(X_p)'}) : \phi(h) < 0 \text{ for all } h \in B^+_{(X_p)'}\}$. By (3) we have that $A \cap B = \emptyset$. From the Hahn-Banach separation theorem, there exist $\xi \in C(B^+_{(X_p)'}')$ and $\alpha \in \mathbb{R}$ such that $\langle \xi, \phi \rangle < \alpha \leq \langle \xi, \phi_M \rangle$ for all $\phi \in A$ and $\phi_M \in B$. Since every negative constant function is in $A$, it follows that $0 \leq \alpha$. Even more, $\alpha = 0$ as the constant function equal to 0 is just $\phi_{(0)} \in B$. It is routine to see that $\langle \xi, \phi \rangle \geq 0$ whenever $\phi \in C(B^+_{(X_p)'}$ is such that $\phi(h) \geq 0$ for all $h \in B^+_{(X_p)'}$. Then, $\xi$ is a positive linear functional on $C(B^+_{(X_p)'}$ and so it can be interpreted as a finite positive Radon measure on $B^+_{(X_p)'}$. Hence, we have that

$$
0 \leq \int_{B^+_{(X_p)'}} \phi_M d\xi
$$
for all finite subset $M \subset X$. Dividing by $\xi(B_{(X_p)}^+)$, we can suppose that $\xi$ is a probability measure. Then, for $M = \{f\}$ with $f \in X$, we obtain that

$$\|T(f)\|_E^q \leq M_{p,q}(T) \int_{B_{(X_p)}^+} \left( \int_{\Omega} |f(\omega)|^{p} h(\omega) \, d\mu(\omega) \right)^{q/p} \, d\xi(h)$$

and so (2) holds.

Actually, Theorem 4 says that we can find a probability Radon measure $\xi$ on $B_{(X_p)}^+$ such that $T: X \to E$ is continuous when $X$ is considered with the norm of the space $S^q_{X_p}(\xi)$. In the next result we will see how to extend $T$ continuously to $S^q_{X_p}(\xi)$. Even more, we will show that this extension is possible if and only if $T$ is $p$-strongly $q$-concave.

Suppose that $X$ is $p$-convex and order semi-continuous. The following statements are equivalent:

(a) $T$ is $p$-strongly $q$-concave.
(b) There exists a probability Radon measure $\xi$ on $B_{(X_p)}^+$ satisfying (1) such that $T$ can be extended continuously to $S^q_{X_p}(\xi)$, i.e. there is a factorization for $T$ as

$$X \xrightarrow{i} S^q_{X_p}(\xi) \xrightarrow{\bar{T}} E$$

where $\bar{T}$ is a continuous linear operator and $i$ is the inclusion map.

If (a)-(b) holds, then $M_{p,q}(T) = \|\bar{T}\|$. 

Proof. (a) $\Rightarrow$ (b) From Theorem 4, we get that there is a probability Radon measure $\xi$ on $B_{(X_p)}^+$, satisfying (1) such that $\|T(f)\|_E \leq M_{p,q}(T)\|f\|_{S^q_{X_p}(\xi)}$ for all $f \in X$.

Given $0 \leq f \in S^q_{X_p}(\xi)$, from Lemma 2, we can take $(f_n)_{n \geq 1} \subset X$ such that $0 \leq f_n \uparrow f$ $\mu$-a.e. Then, since $S^q_{X_p}(\xi)$ is order continuous, we have that $f_n \to f$ in $S^q_{X_p}(\xi)$ and so $(T(f_n))_{n \geq 1}$ converges to some element $e$ of $E$. Define $\bar{T}(f) = e$.

Note that $\bar{T}$ is well defined, since if $(g_n)_{n \geq 1} \subset X$ is such that $0 \leq g_n \uparrow f$ $\mu$-a.e., then

$$\|T(f_n) - T(g_n)\|_E \leq M_{p,q}(T)\|f_n - g_n\|_{S^q_{X_p}(\xi)} \to 0.$$ 

Moreover,

$$\|\bar{T}(f)\|_E = \lim_{n \to \infty} \|T(f_n)\|_E \leq M_{p,q}(T) \lim_{n \to \infty} \|f_n\|_{S^q_{X_p}(\xi)} = M_{p,q}(T)\|f\|_{S^q_{X_p}(\xi)}.$$ 

For a general $f \in S^q_{X_p}(\xi)$, writing $f = f^+ - f^-$ where $f^+$ and $f^-$ are the positive and negative parts of $f$ respectively, we define $\bar{T}(f) = \bar{T}(f^+) - \bar{T}(f^-)$. Then, $\bar{T}: S^q_{X_p}(\xi) \to E$ is a continuous linear operator extending $T$. Moreover $\|\bar{T}\| \leq M_{p,q}(T).$
Thus, we obtain from Lemma 4 that in Theorem 4 show some inclusions among the spaces of $\xi$ so. Then, from Theorem 4, there exists a probability Radon measure $\mu$ that for a

$$T(f_n^+ - f_n^-) = T(f_n^+) - T(f_n^-) \rightarrow \tilde{T}(f^+) - \tilde{T}(f^-) = \tilde{T}(f)$$

in $E$. Hence,

$$\|\tilde{T}(f)\|_E = \lim_{n \rightarrow \infty} \|T(f_n^+ - f_n^-)\|_E$$

$$\leq M_{p,q}(T) \lim_{n \rightarrow \infty} \|f_n^+ - f_n^-\|_{S_{X_p}^q(\xi)}$$

$$= M_{p,q}(T)\|f\|_{S_{X_p}^q(\xi)}.$$

(b) $\Rightarrow$ (a) Given $(f_i)_{i=1}^n \subset X$, we have that

$$\sum_{i=1}^n \|T(f_i)\|_E^q = \sum_{i=1}^n \|\tilde{T}(f_i)\|_E^q \leq \|\tilde{T}\|^q \sum_{i=1}^n \|f_i\|_{S_{X_p}^q(\xi)}^q$$

$$= \|\tilde{T}\|^q \sum_{i=1}^n \int_{B_{(X_p)'}^q} \left( \int_{\Omega} |f_i(\omega)|^p h(\omega) d\mu(\omega) \right)^{q/p} d\xi(h)$$

$$\leq \|\tilde{T}\|^q \sup_{h \in B_{(X_p)'}^q} \sum_{i=1}^n \left( \int_{\Omega} |f_i(\omega)|^p h(\omega) d\mu(\omega) \right)^{q/p}.$$ 

Thus, we obtain from Lemma 4 that $T$ is $p$-strongly $q$-concave with $M_{p,q}(T) \leq \|T\|$. 

The definition of the norm of the spaces $S_{X_p}^q(\xi)$ and the characterization given in Theorem 4 show some inclusions among the spaces of $p$-strongly $q$-concave operators. Indeed, for a $p$-convex Banach function space $X$, a suitable probability measure $\xi$ and real numbers $p \leq q_1 \leq q_2$, Hölder’s inequality gives the inclusion $S_{X_p}^{q_2}(\xi) \subseteq S_{X_p}^{q_1}(\xi)$. Therefore, if $q_1 \leq q_2$ and $T$ is $q_1$-concave, then it is also $q_2$-concave.

The structure of the spaces $S_{X_p}^q(\xi)$ also allows to understand the asymptotic behavior of the factorization when $q \rightarrow \infty$. In this case, the norm in the space $S_{X_p}^q(\xi)$ for a given function in $X$ tends to the norm in $X$ when $q$ increases, in the sense that the $L^q(\mu)$-norm of a bounded function tends to the $L^\infty(\mu)$-norm. Note also that for this asymptotic behavior the $p$-convexity of $X$ does not play any role, so it can be assumed to be the trivial 1-convexity.

A first application of Theorem 4 is the following Kakutani type representation theorem (see for instance [7, Theorem 1.b.2]) for B.f.s.’ being order semi-continuous, $p$-convex and $p$-strongly $q$-concave.

Suppose that $X$ is $p$-convex and order semi-continuous. The following statements are equivalent:

(a) $X$ is $p$-strongly $q$-concave.

(b) There exists a probability Radon measure $\xi$ on $B_{(X_p)'}^+$ satisfying (1), such that $X = S_{X_p}^q(\xi)$ with equivalent norms.

Proof. (a) $\Rightarrow$ (b) The identity map $I: X \rightarrow X$ is $p$-strongly $q$-concave as $X$ is so. Then, from Theorem 4, there exists a probability Radon measure $\xi$ on $B_{(X_p)'}^+$.
satisfying (1), such that I factors as

![Diagram showing factors I and S^q_{X_p}(\xi)]

where \( \tilde{I} \) is a continuous linear operator with \( \|\tilde{I}\| = M_{p,q}(X) \) and \( i \) is the inclusion map. Since \( \xi \) is a probability measure, we have that \( \|f\|_{S^q_{X_p}(\xi)} \leq \|f\|_X \) for all \( f \in X \), see the proof of Proposition 3. Let \( 0 \leq f \in S^q_{X_p}(\xi) \). By Lemma 2, we can take \( (f_n)_{n \geq 1} \subset X \) such that \( 0 \leq f_n \uparrow f \) \( \mu \)-a.e. Since \( S^q_{X_p}(\xi) \) is order continuous, it follows that \( f_n \to f \) in \( S^q_{X_p}(\xi) \) and so \( f_n = \tilde{I}(f_n) \to \tilde{I}(f) \) in \( X \). Then, there is a subsequence of \( (f_n)_{n \geq 1} \) converging \( \mu \)-a.e. to \( \tilde{I}(f) \) and hence \( f = \tilde{I}(f) \in X \).

For a general \( f \in S^q_{X_p}(\xi) \), writing \( f = f^+ - f^- \) where \( f^+ \) and \( f^- \) are the positive and negative parts of \( f \) respectively, we have that \( f = \tilde{I}(f^+) - \tilde{I}(f^-) = \tilde{I}(f) \in X \).

Therefore, \( X = S^q_{X_p}(\xi) \) and \( \tilde{I} \) is de identity map. Moreover, \( \|f\|_X = \|\tilde{I}(f)\|_X \leq \|\tilde{I}\| \|f\|_{S^q_{X_p}(\xi)} = M_{p,q}(X) \|f\|_{S^q_{X_p}(\xi)} \) for all \( f \in X \).

(b) \( \Rightarrow \) (a) From Remark 4 it follows that the identity map \( I: X \to X \) is \( p \)-strongly \( q \)-concave.

Note that under conditions of Corollary 4, if \( X \) is \( p \)-strongly \( q \)-concave with constant \( M_{p,q}(X) = 1 \), then \( X = S^q_{X_p}(\xi) \) with equal norms.

5. \( q \)-SUMMING OPERATORS ON A \( p \)-CONVEX B.F.S.

Recall that a linear operator \( T: X \to E \) between Banach spaces is said to be \( q \)-summing \( (1 \leq q < \infty) \) if there exists a constant \( C > 0 \) such that

\[
\left( \sum_{i=1}^{n} \|Tx_i\|_E^q \right)^{1/q} \leq C \sup_{x^* \in B_X^*} \left( \sum_{i=1}^{n} |\langle x^*, x_i \rangle|^q \right)^{1/q}
\]

for every finite subset \( (x_i)_{i=1}^{n} \subset X \). Denote by \( \pi_q(T) \) the smallest possible value of \( C \). Information about \( q \)-summing operators can be found in [6].

One of the main relations between summability and concavity for operators defined on a B.f.s. \( X \), is that every \( q \)-summing operator is \( q \)-concave. This is a consequence of a direct calculation which shows that for every \( (f_i)_{i=1}^{n} \subset X \) and \( x^* \in X^* \) it follows that

\[
\left( \sum_{i=1}^{n} |\langle x^*, f_i \rangle|^q \right)^{1/q} \leq \|x^*\|_{X^*} \left( \sum_{i=1}^{n} |f_i|^q \right)^{1/q} \|X\|.
\]

see for instance [7, Proposition 1.d.9] and the comments below. However, this calculation can be slightly improved to obtain the following result.

Let \( 1 \leq p \leq q < \infty \). Every \( q \)-summing linear operator \( T: X \to E \) from a B.f.s. \( X \) into a Banach space \( E \), is \( p \)-strongly \( q \)-concave with \( M_{p,q}(T) \leq \pi_q(T) \).
Proof. Let $1 < r \leq \infty$ be such that $\frac{1}{r} = \frac{1}{p} - \frac{1}{q}$ and consider a finite subset $(f_i)_{i=1}^n \subset X$. We only have to prove

$$\sup_{x^* \in B_{X^*}} \left( \sum_{i=1}^n |\langle x^*, f_i \rangle|^q \right)^{1/q} \leq \sup_{(\beta_i)_{i \geq 1} \in B_{r'}} \left\| \left( \sum_{i=1}^n |\beta_i f_i|^p \right)^{1/p} \right\|_X.$$ 

Fix $x^* \in B_{X^*}$. Noting that $\frac{2}{p}$ and $\frac{q}{p}$ are conjugate exponents and using the inequality (4), we have

$$\left( \sum_{i=1}^n |\langle x^*, f_i \rangle|^q \right)^{1/q} = \sup_{(\alpha_i)_{i \geq 1} \in B_{r'/p}}\left( \sum_{i=1}^n |\alpha_i | \langle x^*, f_i \rangle|^p \right)^{1/p} = \sup_{(\beta_i)_{i \geq 1} \in B_{r'}}\left( \sum_{i=1}^n |\langle x^*, \beta_i f_i \rangle|^p \right)^{1/p} \leq \sup_{(\beta_i)_{i \geq 1} \in B_{r'}} \left\| \left( \sum_{i=1}^n |\beta_i f_i|^p \right)^{1/p} \right\|_X.$$ 

Taking supremum in $x^* \in B_{X^*}$ we get the conclusion. \(\square\)

From Proposition 5, Theorem 4 and Remark 4, we obtain the final result.

Set $1 \leq p \leq q < \infty$. Let $X$ be a saturated order semi-continuous $p$-convex B.f.s. and consider a $q$-summing linear operator $T : X \to E$ with values in a Banach space $E$. Then, there exists a probability Radon measure $\xi$ on $B_{(X_p)^*}$ satisfying (1) such that $T$ can be factored as

$$X \xrightarrow{\tilde{T}} E \xrightarrow{i} S_{X_p}^q(\xi)$$

where $\tilde{T}$ is a continuous linear operator with $\|\tilde{T}\| \leq \pi_q(T)$ and $i$ is the inclusion map which turns out to be $p$-strongly $q$-concave, and so $q$-concave.

Observe that what we obtain in Corollary 5 is a proper extension for $T$, and not just a factorization as the obtained in the Pietsch theorem for $q$-summing operators through a subspace of an $L^q$-space.

References


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