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Modulated-nonlinearity in phononic crystals: From extremely linear to effective cubic nonlinear media

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In solids, the constitutive relations between strain and stress lead in most cases to a dominant quadratic nonlinear response. However, there exist some solid materials that exhibit negative quadratic nonlinearity, as occurs in common glasses as borosilicate glass or fused silica. In this work, we present the dynamical response of a phononic crystal with modulated nonlinearity. In particular, we analyze a 1D phononic crystal composed of layers of alternating properties with quadratic nonlinearity, each layer presenting an alternating (positive and negative) nonlinear parameter value. First, we observe that the common distortion produced by the nonlinear steepening can be almost compensated. Thus, the propagation through the phononic crystal can be extended far beyond the shock formation distance, avoiding common saturation effects. Second, we show that in the absence of dispersion the artificial material exhibits weakly-cubic nonlinear effective properties. The structure presented here can be employed to avoid nonlinear distortion in waveguides, or to design metamaterials with effective cubic nonlinear properties using quadratic nonlinear building blocks.

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1. INTRODUCTION

Common homogeneous materials generally present a dominant quadratic nonlinearity for longitudinal waves. In fluids, the material nonlinearity obeys a power-law between the pressure and density fields, which is accurately described by linear and quadratic terms up to second order acoustic perturbations. In most solids, the constitutive relations between strain and stress also lead to a dominant quadratic nonlinearity. However, there exist some solid materials that exhibit negative quadratic nonlinearity. These include common glasses as borosilicate glass or fused silica,[1] where their polycrystalline micro-structure leads to this uncommon dynamical response. Cubic dominant nonlinearity is more unusual to observe in homogeneous materials for longitudinal waves. It can be found in solid materials with mesoscale heterogeneities, arising from local non-linear phenomena[2] or due to hysteretic processes.[3] It is worth noting here that for symmetry considerations, cubic nonlinearity dominates for plane shear waves,[4] even in homogeneous materials described by quadratic nonlinear parameters, i.e., third-order Landau moduli. Other kinds of nonclassical nonlinearity can be found in complex solids, as clapping or sliding processes in granular materials, sand, rocks, or solids with micro-cracks.[5]

In addition, special attention has been paid to design structured artificial materials to manipulate acoustic and elastic waves, e.g., using phononic crystals or metamaterials. On the one hand, in phononic crystals Bragg scattering introduces strong anisotropic dispersion in the propagation, leading to the generation of band-gaps due to spatially modulated media. On the other hand, metamaterials make use of local resonances to modify wave propagation, allowing exotic properties as negative constitutive parameter values. Until recently, the propagation through these artificially structured materials was considered linear. While nonlinear waves in structured periodic media, e.g. in photonic crystals, have been explored in Optics since long time ago,[6] only recently nonlinear propagation in phononic crystals and metamaterials has been considered to study progressive harmonic generation in granular chains,[7] phononic crystals[8,9] or in lattices of coupled nonlinear oscillators with local resonances[10–12] or locally resonant systems.[13] Other phenomena has been also presented as simultaneous self-collimation of first and second harmonic in sonic crystals,[14] nonlinear focusing systems,[15] or non-reciprocal systems as acoustic diodes and rectifiers.[16–19] The generation of solitary waves and nonlinear localized modes has been also reported.[20–22]

In this work, we present the dynamical response of a phononic crystals with spatially modulated quadratic nonlinearity. In particular, we analyze a 1D phononic crystal composed of layers of alternating properties. Each layer presents an alternating (positive and negative) nonlinear parameter value, as shown in Fig. 1. We advance particular dynamical features, as we will describe below. First, we report the extreme mitigation of the nonlinear distortion due to the compensation of the steepening of the wavefront between layers. Second, we observe that this structured material, which is constructed using quadratic nonlinear building blocks, presents effective cubic nonlinearity.

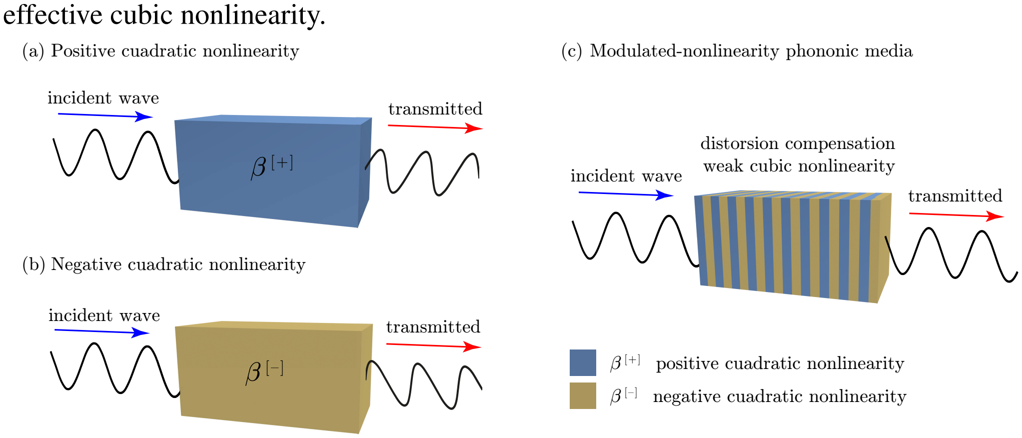

Figure 1: Scheme of the β-modulated acoustic layered media

2. MODEL

For the sake of simplicity, we consider only longitudinal motions in a solid, being the shear displacements not considered due to symmetry reasons. The particle displacement, u_x, in each layer is described by a second-order nonlinear wave equation with quadratic nonlinearity, given in a Lagrangian reference frame by

$$\frac{\partial^2 u_x}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 u_x}{\partial t^2} = \beta(x)\frac{\partial^2 u_x^2}{\partial x^2}, \tag{1}$$

where the nonlinear parameter is given by $\beta = -\left(3/2 + C_{111}/2C_{11}\right)$, and C_{11} and C_{111} are the second and third order elastic constants, that are related to the second-order Lamé and third order Landau parameters as $C_{11} = \lambda + 2\mu$, $C_{111} = 2A + 6B + 2C$, respectively. We excite the system with a harmonic wave at $x = 0$ with a displacement given by $u(0,t) = u_0 \exp(i\omega t)$, being ω the angular frequency and u_0 the excitation displacement. For the sake of simplify, we consider no variations on the density and second order elastic coefficients in both media. Due to the absence of Bragg scattering, the material is considered dispersion-less. Under this assumption, the nonlinear evolution of the waves propagating on each layer can be described by a Burgers equation. Its implicit solution can be written in Eulerian coordinates ξ and in a retarded time-frame $\tau = (t - \xi/c_0)/2\pi$ for the pressure as $p(\xi,\tau) = p_0 \sin\left[\omega\tau + k\xi\beta M/(1+\beta M)\right]$, where $M = v/c_0 = p/\rho_0 c_0^2$ is the Mach number, v the particle velocity, ρ_0 the density and $k = \omega/c_0$. Over a small distance compared to λ, a small distortion of is produced, of the order $k\xi\beta M/(1+\beta M) \to 0$, cause $k\xi \to 0$ and in general $\beta M \ll 1$. Therefore, we define a smallness parameter $\varepsilon = k\xi\beta M/(1+\beta M) \ll 1$. Expanding in series around $\omega\tau$, at the leading order in ε, we obtain the evolution equation for the three lowest harmonic components up to second order

$$\frac{\partial p(\xi,\tau)}{\partial \xi} \approx -\frac{p_0^3 k\beta^2}{4\rho_0^2 c_0^4}\cos(\omega\tau) + \frac{p_0^2 k\beta}{2\rho_0 c_0^2}\sin(2\omega\tau) + \frac{p_0^3 k\beta^2}{4\rho_0^2 c_0^4}\cos(3\omega\tau). \tag{2}$$

Here, the first therm in the RHS describes the self action on the fundamental component, and the second and third therms describe the rate of second and third harmonic generation, respectively. To describe the propagation in the n-th layer, of length $d\xi$, we can write a recurrence relation at each position $\xi^n = (n)d\xi$ as $p^{n+1}(\xi^{n+1},\tau) = p^n(\xi^n,\tau) + dp^n(\xi,\tau)$. This allows us to integrate the field at position $\xi = Nd\xi$ as

$$p_N(\xi,\tau) = \sum_{n=1}^{N}\left[-\frac{p_0^3 k\beta_n^2}{4\rho_0^2 c_0^4}\cos(\omega\tau) + \frac{p_0^2 k\beta_n}{2\rho_0 c_0^2}\sin(2\omega\tau) + \frac{p_0^3 k\beta_n^2}{4\rho_0^2 c_0^4}\cos(3\omega\tau)\right] d\xi. \tag{3}$$

On the one hand, if the sign of the nonlinear parameter is alternated, $\beta_n = \beta_0(-1)^n$, the second harmonic is given by

$$p_N(Nd\xi, 2\omega\tau) = \frac{p_0^2 k}{2\rho_0 c_0^2} d\xi \sin(2\omega\tau)\sum_{n=1}^{N}\beta_0(-1)^n = 0. \tag{4}$$

As $\sum_{n=1}^{N}(-1)^n = 0$, second harmonic vanishes. Note the sign of the nonlinear parameter controls the phase of the generated second harmonic. Then, the second harmonic field generated in one layer mixes with the second harmonic generated out of phase in the following layer. In the dispersion-less case, if the layers are of equal length at the end of a lattice step the coherent sum of second harmonic fields is absent. On the other hand, as $\sum_{n=1}^{N}(-1)^{2n} = N$, the third harmonic is given by

$$p_N(Nd\xi, 3\omega\tau) = \frac{p_0^3 k\beta_0^2}{4\rho_0^2 c_0^4}(Nd\xi)\cos(3\omega\tau). \tag{5}$$

Thus, third harmonic becomes linearly cumulative with distance. It is worth noting that it is cubic in amplitude and proportional to β_0^2. This result shows that alternating the sign of the nonlinear parameter does not compensate completely the distortion, however, the energy cascade processes are limited and only the third order nonlinear processes cumulates, which are less intense.

3. RESULTS

We set the layer thickness as $a = 2\lambda$, with λ the wavelength. Sound speed was set to $c_0 = 1500$ m/s and $\rho_0 = 1000$ kg/m^3, and we exited the system using a monochromatic wave of frequency $f_0 = 1$ MHz with amplitude $p_0 = 1$ MPa. A crystal of 35 alternating layers of $a = 3$ mm. The the sign of the nonlinear parameter of each layer alternates as $\beta_1 = -\beta_2 = \beta_0 = 5$, and the total size of the crystal corresponds with the shock formation distance 10.7 cm. Fig. 2 shows the results of the harmonic evolution using the theory, a numerical recursive integration, and the full-wave simulation of the nonlinear wave Eq. (1). First, both theory and simulation agree for the first harmonic. The fundamental component transfer energy to second harmonic in positive $(+\beta_0)$ layers but when nonlinearity is inverted in the following layer $(-\beta_0)$energy distortion is compensated and energy is returned back to first harmonic. As a result, the first harmonic amplitude remains almost if compared with homogeneous media constant. Second, the second harmonic grows linearly in every half-layer, but due to sign of the nonlinear parameter the generated field presents opposite phase than the one generated in the preceding layer. As a result, distortion is compensated and at the end ef each layer both numerical integrations agree with theory. Finally, third harmonic accumulates with distance. However, here it can only be produced as a second-order cascade process and, as the amplitude of the second harmonic remains low, the rate of energy pumping to third harmonic frequency is limited. If compared with homogeneous media, which present a discontinuity at $\sigma = 1$, the harmonic content in the β-modulated media is low and the wave profiles are slightly distorted at this distance.

It is worth noting here that for short distances the rate of grow for the i-th harmonic is dependent on $\beta_n^{(i-1)}$. Therefore, when theory is applied to high-order harmonics in the β-modulated media we observe that even harmonics vanishes as $\sum_{n=1}^{N}(-1)^{n(i-1)} = 0$ for $i = 2, 4, 6, \ldots$, while odd harmonics became cumulative with distance as $\sum_{n=1}^{N}(-1)^{n(i-1)} = 1$ for $i = 3, 5, 7, \ldots$

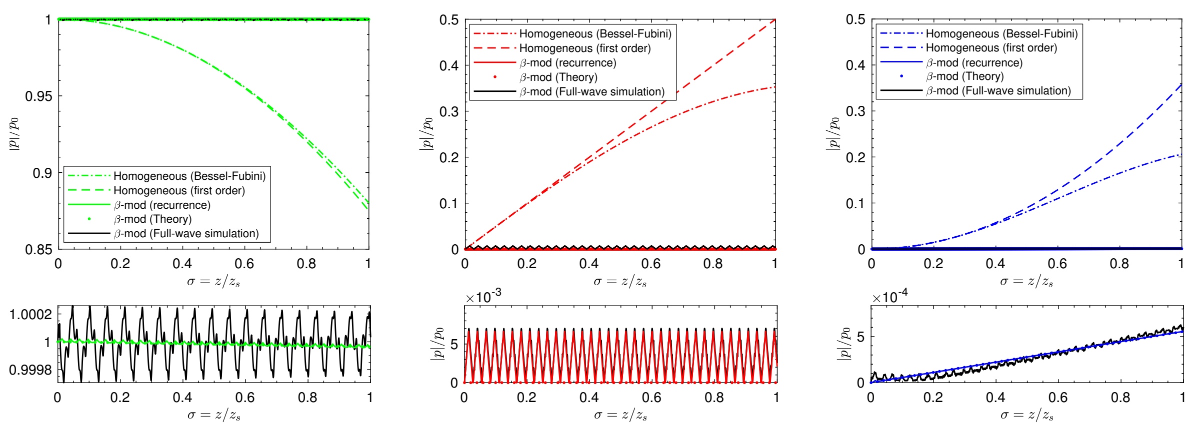

Figure 2: (left) Evolution of the amplitude of fundamental component, (center) second and (right) third harmonic.

4. CONCLUSIONS

The configuration proposed, opens a way to obtain cubic nonlinearity for the longitudinal waves from a layered distribution of quadratic nonlinear materials. Such structure can be designed also using solid composites in the micro scale to obtain new ways of management of acoustic waves. This include low distortion waveguides for intense perturbations where the acoustical saturation effects can be avoided and, therefore, high amplitude waves can be propagated beyond the shock formation distance. In the more general case including dispersion, the system offers the possibility of designing new tunable nonlinear materials where phase velocity and effective nonlinearity can be modified to achieve exotic configurations, while the structure is made of layers of common materials.

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REFERENCES

[1] M. F. Hamilton, D. T. Blackstock, *et al.*, *Nonlinear acoustics*, vol. 237. Academic press San Diego, 1998.

[2] G. D. Meegan Jr, P. A. Johnson, R. A. Guyer, and K. R. McCall, "Observations of nonlinear elastic wave behavior in sandstone," *The Journal of the Acoustical Society of America*, vol. 94, no. 6, pp. 3387–3391, 1993.

[3] V. Aleshin, V. Gusev, and V. Zaitsev, "Propagation of initially bi-harmonic sound waves in a 1d semi-infinite medium with hysteretic non-linearity," *Ultrasonics*, vol. 42, no. 1, pp. 1053–1059, 2004.

[4] S. Catheline, J.-L. Gennisson, M. Tanter, and M. Fink, "Observation of shock transverse waves in elastic media," *Physical review letters*, vol. 91, no. 16, p. 164301, 2003.

[5] V. Nazarov and A. Radostin, *Nonlinear acoustic waves in micro-inhomogeneous solids*. John Wiley & Sons, 2015.

[6] J. Armstrong, N. Bloembergen, J. Ducuing, and P. Pershan, "Interactions between light waves in a nonlinear dielectric," *Physical review*, vol. 127, no. 6, p. 1918, 1962.

[7] V. Sánchez-Morcillo, I. Pérez-Arjona, V. Romero-García, V. Tournat, and V. Gusev, "Second-harmonic generation for dispersive elastic waves in a discrete granular chain," *Physical Review E*, vol. 88, no. 4, p. 043203, 2013.

[8] Y. Yun, G. Miao, P. Zhang, K. Huang, and R. Wei, "Nonlinear acoustic wave propagating in one-dimensional layered system," *Physics Letters A*, vol. 343, no. 5, pp. 351–358, 2005.

[9] N. Jiménez, A. Mehrem, R. Picó, L. M. García-Raffi, and V. J. Sánchez-Morcillo, "Nonlinear propagation and control of acoustic waves in phononic superlattices," *Comptes Rendus Physique*, vol. 17, no. 5, pp. 543–554, 2016.

[10] A. Mehrem, N. Jiménez, L. Salmerón-Contreras, X. García-Andrés, L. García-Raffi, R. Picó, and V. Sánchez-Morcillo, "Nonlinear dispersive waves in repulsive lattices," *Physical Review E*, vol. 96, no. 1, p. 012208, 2017.

[11] V. Sánchez-Morcillo, N. Jiménez, J. Chaline, A. Bouakaz, and S. Dos Santos, "Spatio-temporal dynamics in a ring of coupled pendula: Analogy with bubbles," in *Localized Excitations in Nonlinear Complex Systems*, pp. 251–262, Springer, 2014.

[12] J. Chaline, N. Jiménez, A. Mehrem, A. Bouakaz, S. Dos Santos, and V. J. Sánchez-Morcillo, "Macroscopic acousto-mechanical analogy of a microbubble," *The Journal of the Acoustical Society of America*, vol. 138, no. 6, pp. 3600–3606, 2015.

[13] J. Zhang, V. Romero-García, G. Theocharis, O. Richoux, V. Achilleos, and D. J. Frantzeskakis, "Second-harmonic generation in acoustic waveguides loaded with an array of side holes," *Acta Acustica united with Acustica*, vol. 104, no. 2, pp. 235–242, 2018.

[14] N. Jiménez, R. Picó, V. J. Sánchez-Morcillo, L. M. García-Raffi, K. Staliunas, *et al.*, "Nonlinear self-collimated sound beams in sonic crystals," *Physical Review B*, vol. 92, no. 5, p. 054302, 2015.

[15] C. M. Donahue, P. W. Anzel, L. Bonanomi, T. A. Keller, and C. Daraio, "Experimental realization of a nonlinear acoustic lens with a tunable focus," *Applied Physics Letters*, vol. 104, no. 1, p. 014103, 2014.

[16] B. Liang, B. Yuan, and J.-c. Cheng, "Acoustic diode: Rectification of acoustic energy flux in one-dimensional systems," *Physical review letters*, vol. 103, no. 10, p. 104301, 2009.

[17] B. Liang, X. Guo, J. Tu, D. Zhang, and J. Cheng, "An acoustic rectifier," *Nature materials*, vol. 9, no. 12, pp. 989–992, 2010.

[18] N. Boechler, G. Theocharis, and C. Daraio, "Bifurcation-based acoustic switching and rectification," *Nature materials*, vol. 10, no. 9, pp. 665–668, 2011.

[19] A. Cebrecos, N. Jiménez, V. Romero-García, R. Picó, V. Sánchez-Morcillo, and L. García-Raffi, "Asymmetric propagation using enhanced self-demodulation in a chirped phononic crystal," *AIP Advances*, vol. 6, no. 12, p. 121601, 2016.

[20] N. Sugimoto, M. Masuda, J. Ohno, and D. Motoi, "Experimental demonstration of generation and propagation of acoustic solitary waves in an air-filled tube," *Physical review letters*, vol. 83, no. 20, p. 4053, 1999.

[21] D. H. Yong and R. J. LeVeque, "Solitary waves in layered nonlinear media," *SIAM Journal on Applied Mathematics*, vol. 63, no. 5, pp. 1539–1560, 2003.

[22] J. Zhang, V. Romero-García, G. Theocharis, O. Richoux, V. Achilleos, and D. J. Frantzeskakis, "Bright and gap solitons in membrane-type acoustic metamaterials," *Physical Review E*, vol. 96, no. 2, p. 022214, 2017.