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Additional Information

INTER-FIELD NONLINEAR TRANSFORMATION OF JOURNAL IMPACT INDICATORS: THE CASE OF THE h-INDEX

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ABSTRACT. Impact indices used for joint evaluation of research items coming from different scientific fields must be comparable. Often a linear transformation —a normalization or another basic operation— is considered to be enough for providing the correct translation to a unified setting in which all the fields are adequately treated. In this paper it is shown that this is not always true. The attention is centered in the case of the h-index. It is proved that it that cannot be translated by means of direct normalization preserving its genuine meaning. According to the universality of citation distribution, it is shown that a slight variant of the h-index is necessary for this notion to produce comparable values when applied to different scientific fields. A complete example concerning a group of top scientists is shown.

1. Introduction

Several factors related to research costs have made it necessary for governments to demonstrate that public investment in R&D produces positive economic benefits for society. As a consequence, governments in several European countries - and elsewhere - have introduced quantitative methods for allocating research funding based on outputs and results. There are two main procedures that can be used to measure scientific quality and productivity. The first is based on the peer review of research results; the second on an automatic evaluation based on standard bibliometric indicators. Geuna and Piolatto 2016 have recently published an in-depth comparative analysis of both methods;

see also Hicks 2012. It is easy to understand that automatic evaluation is easier and cheaper to apply, so it is often used.

The so-called h-index for persons and institutions is sometimes present in these evaluation processes, at least in the first steps. For instance, the Spanish agency responsible for research assessment has recently started to use it as a complementary source of information (see Proyectos I+D+i 2018). However, it is well-known that this index is not independent of several relevant aspects as for instance the age of the researchers, the mean number of publications in the scientific field and other important properties of the researchers' profile. Consequently, its use may produce a loss of balance among different scientific fields.

This focuses attention in the nature of the h-index again, and motivates our attempt for providing hte equations of the transformation of the h-index that should be used when researchers from all scientific fields are considered as a whole in evaluation processes. In this paper we will prove that this cannot be solved just by re-scaling the h-index depending on the field, or by considering different scientific subjects separately. As we will show, the problem is that *the h-index means different things depending on the field*. A bit more complicated transformation is needed, but is still a matter of adding some simple computations to the calculus of the index, that can be done easily. We propose a correction in the definition of the h-index of single researchers in order to allow to compare scientific curricula coming from different fields of knowledge. Although this is not probably the best solution to this anomalous indiscriminate use of the h-index, we have to accept the situation as a matter of fact. Let us recall that the problem is of strict mathematical nature: the assumption that establishes that the comparison of the values of the h-index in different fields can be done by a simple normalization is mathematically wrong, exactly in the way that the equation $(a/k)^2 = a^2/k$ is. In this paper we show this fact just doing formal computations, and also with some examples.

We must point out that our purpose does not consider other corrections which are also important. For example, the problems of how to compare h-indices of researchers of different age and how to normalize to the mean number of publications in a concrete scientific field are not

treated. Other matters which involve the practical aspects of citation measurement are also excluded, as the author name disambiguation problem. Many authors share the same last name and first name initials. Furthermore, many authors' publications are listed under several names in databases and this of course affects the measurement (Strotmann and Zhao 2012).

2. Review of the existing scientific literature on the subject

Since the h-index appeared in the scene as a bibliometric indicator, a considerable effort has been made for understanding its behavior and justify its use (see Hirsch 2005 for the original definition and results, and Egghe, 2010 and the references therein for an analysis of its properties and variants). However, after some years of intense research about its main properties and applications —mainly from 2005 to 2009, see van Leeuwen 2008, Alonso et al 2009, Imperial and Rodríguez-Navarro 2007, Bornmann and Daniel 2009, Aoun et al 2013—, some critical voices raised against its indiscriminate use from the researchers in Information Science. Some of them provided other indices or modifications of the original h-index to improve its behavior (Waltman and Van Eck 2012, Rousseau et al 2013, Burrell 2013, Farhadi et al 2013, Schreiber 2013(1), Khan et al 2013). The main criticism comes from the fact that its definition is arbitrary, in the opinion of some researchers. Broadly speaking, this means that the h-index measures exactly what it measures, and its value cannot be understood as any sort of ratio among citations and papers. Moreover, it cannot be clearly associated —in a mathematical way— to any other bibliometric indicator. Actually, other similar indicators having apparently the same meaning produce absolutely different ranks (see Waltman and Van Eck 2012, and Schreiber 2013(2)). We will try to give a clear definition of what "arbitrariness" would mean in this context later on the paper .

Research papers dealing with the h-index after 2012 are rare. After some years of careful analysis an agreement about the —rather strict— limits of a reasonable use was reached among the specialists. As often happens, it started then to live in the "real world" of the

research evaluation (Rousseau et al 2013, Schreiber 2013(2)), affecting the policy of research institutions and the funding of scientific activities. In some countries h-index of researchers is being used together with other parameters for deciding which projects —coming from different disciplines!— will be fund by public administrations and private companies (see for example Rodríguez Navarro and Imperial Ródenas 2009).

Some authors argue that there are much better metrics available for usage in evaluation committees than the original h-index or some modified version, especially if different scientific fields are involved. These options ranges from easy modifications of classical impact indices (Pudovkin and Garfield 2004) to more sophisticated options, e.g. the Mean Normalized Citation Score (MNCS) or the proportion of Top 10 % publications. The reader can find general information and an overview of alternate measures in some recent works of reference, as the DORA report, the Metric Tide 2015 and the Leiden Manifesto 2015. In these documents it can be seen that the most widespread opinion among information scientists is that the h index —or the Journal Impact Factor, JIF— is still used in many evaluation committees. Also, specialists agree that it would be better if the h-index is used in a contextualized manner —which unfortunately is not often the case—, than if some modified h-index is used without contextualization. The recently published paper by Dienes (Dienes 2015) faces also the exposed problem (see also Ayazl and Afzal 2016) from a theoretical point of view, analyzing the coherence and consistency of the index. We also face the study of the translation of the h-index using a theoretical/mathematical method. It seems to be necessary to attend to the meaning and consistency of the bibliometric tools already in use instead of only considering its experimental behavior (see Waltman 2016).

3. Methodology and mathematical tools

Technically, we *do not need to introduce new concepts*, indices or any other mathematical tool. Fortunately, the work made by the researchers in information sciences in the last decade provides a big amount of instruments, that will be enough for our aim, as for example

the index h_α introduced by Van Eck and Waltman 2008. The reader can find a good description of these tools in Alonso et al 2009.

We will analyze the problem form a mathematical point of view. This means that we will deduce the equations of the pertinent changes theoretically, trying to preserve the fundamental meaning of the h-index after transformations. As a main tool, we will assume the (principle of) universality of citation distribution as main theoretical tool. In fact, our correction is a mathematical consequence derived from this assumption. This property was established in 2008 in the paper by Radicchi et al 2008. It has been proven to have a reasonable statistical significance. Broadly speaking, it means that after a re-scaling by dividing its number by the mean of citations for paper in a given discipline, the distribution of citations of the papers does not depend on the scientific field to which they belong. Let c_0^A and c_0^B be the mean of citations in a fixed period of time t of a paper in the scientific field A and a paper in the scientific field B, respectively. It can be explained in probabilistic terms as follows: in the period of time t, the number n of papers in the field A having more than n/C_0^A citations coincides with the number n of papers in the field B having more than n/C_0^B citations. The fact that the h-index should be modified in light of this new bibliometric law was already observed in the original paper by Radicchi et al. However, the transformation explained there is different than the one we propose here. It is based in a sort of normalization inter-fields applied to the normalized citation rate, and its application would indeed improve the behavior of the h-index. In our purpose, this kind of normalization may be applied after some fundamental non-linear changes. We will compare the purpose of Radicchi et al with ours latter on the paper.

3.1. **What does the h-index measure?** Concerning the existing mathematical tools that we will use, let us introduce some definitions. It does not seem to be easy to find an explanation of the meaning of the h-index in terms of, for example, the ratio among the total number of citations of the papers having more than a fixed number of citations and the total number of citations of all the papers of an author, that

clearly would give an easier way of understanding its meaning. A big effort for trying to find this kind of quasi-h-indices with a more clear meaning was done, mainly from 2005 to 2011 (see for instance Egghe and Rousseau 2008, and Van Eck and Waltman 2008). A relevant example of these attempts was the introduction of the so called g-index of Egghe (Egghe 2006 (1) and (2)); the reader can find more variants in Iglesias and Pecharromán 2007, Egghe 2008(2), Van Eck and Waltman 2008, Schreiber 2015 and the references therein. In our opinion, in addition to some interesting properties that it satisfies it has the relevant property that it can be understood in terms of special ratios among citations and published papers —the original basic conceptual elements appearing in bibliometrics—. That is, it can be understood as a "second order" bibliometric indicator, what does not happen in the case of the original h-index.

3.2. **The h_α index.** Another important definition for our objectives is the h_α index —α being a positive real number—, that is defined to be the maximum number of papers of an author —write h_α— with at least $\alpha \cdot h_\alpha$ citations (see Van Eck and Waltman 2008). In this work some computations have been made for real cases for which the values h_α are compared, and the prevalence of the ordering by the value of h_α of authors when different values of α is analyzed. In fact, it is proven that the ordering is not preserved when α is changed. The idea of the authors is that by choosing the adequate α depending on the field, the h_α-index can provide a better comparison tool of scientists from different fields than the h-index (see Section 2 in Van Eck and Waltman 2008). In a sense, this is the starting point our analysis, but our approximation is more involved since the value of α should depend also on the year of publication. On the other hand, we provide a explicit formula for the computation of the adequate value of α.

Actually, just the assumption of the fact that the h-index must be considered with different reference values depending on the discipline —and sometimes depending even on the particular subfield—, is not enough. The problem is of a slightly different nature. As has been

pointed out by several authors, its definition is *arbitrary* (see the Introduction in Schreiber 2013(2) and the references therein). We may try to fix what does this means in technical terms. *A definition of a measuring index I is arbitrary if: (1) it has no intuitive interpretation in terms of the concepts that are usual in bibliometrics, and (2) when we compare with other measuring index that is supposed to measure a similar property the results are randomly different.* Regarding (1), although the h-index allows an interpretation in temrs of citations and number of publications, it is not clear why this particular relation could be relevant for measuring the success of a set of publication, instead of any other relation among these elements. Regarding (2), as we said above, this happens for example when the h-index is compared with the h_α-index, or with the g-index which meaning can be understood in terms of ratios among meaningful magnitudes.

In any case, let us preserve for the rest of the paper the vague idea that the h-index measures jointly the number of relevant papers published by an author and its total number of publications. Thus, it is a mixed measure of "production" and "quality". This is probably all that we can state about the meaning of the h-index; in fact, this forces to change the direction of our analysis, assuming for the next section that *the h-index measures what it measures*: that is, its value must be classified as primary information, and it must not be understood as a secondary index defined by its relations with other well-established bibliometric magnitudes.

4. Translating the meaning of the h-index from the scientific field A to the scientific field B

Once we assume that there is no explanation of the meaning of the h-index in relational terms, we come back to our aim. Since the use of this index for research evaluation must be accepted as a matter of fact, we need to know how to translate its "meaning" from a discipline that is taken as reference to other disciplines, trying to preserve the characteristics of the information that it contains. Although the election of the reference discipline is arbitrary, we will explain our arguments using the field "Physics" as starting point. We have chosen it due to our

aim of comparing different fields related to biology as example: physics is also a natural science, but not in the group of biological sciences, in an attempt of ensuring "equidistance" to these disciplines. Moreover, Physics was considered in the original arguments provided by Hirsch in his inaugural paper (Hirsch 2005).

It must be said that our arguments are of conceptual nature. We are not providing a wide statistical analysis for proving that the suggested translation are giving comparable values. We give a mathematical proof based in the so called universality of citation distributions, that has been empirically tested in recent years (see Radicchi et al 2008). Therefore, our arguments are supported by the validity of this principle. Actually, in order to translate the "meaning" of the h-index to different fields we assume the next two main facts for proving our result. The first one has been proved, assuming some statistical deviation using real data from the bibliometric studies. The second one is justified by which we explained in the previous section.

- The universality of citation distributions holds. That is, for a fixed period of time, the distribution of citations for the papers in a given discipline A and in other one B is the same when the total number of citations of a paper $C_t(\cdot)$ is divided by the mean of citations in this discipline and in this time period $C_{0,t}^A$. Using the notation in Radicchi et al 2008, we write $C_{f,t}^A(\cdot)$ for the normalized number of citations $C_t(\cdot)/C_{0,t}^A$.
- The h-index is defined and makes sense for the discipline $A = P$ —P =Physics, the field in which it appeared—. Since the election of the discipline of reference is arbitrary, we develop our ideas writing a generic P for the reference scientific field. The mean number of papers published by an author in the fields A and B must be similar. Otherwise, a normalization dividing by this mean number must be done in a second step in the way that will be explained in the last section.

Consider a given author who has published a number of papers N in all its professional life. Assume that they are ordered from the one having more citations ($n = 1$) to the one having less citations ($n = N$).

Let us deduce the equations that relate the expected value of the h-index in physics h^P of this author with its h-index h^A in other discipline A for an author that has published the same number of papers in the same years, but in the discipline A.

Each paper $n \in \{n = 1, ..., N\}$ was published t-years ago, that is $t = t(n)$. Using the distribution of citations given for a time t, the expected value of the number of citations of n if it belongs to the discipline A is

$$C_{t(n)}^A(n) = \Big(\frac{C_{t(n)}^A(n)}{C_{0,t(n)}^A}\Big) \cdot C_{0,t(n)}^A = C_{f,t(n)}^A(n) \cdot C_{0,t(n)}^A.$$

Thus, for computing the h^A index, we have to consider the decreasing sequence of values

$$(a_n)_{n=1}^N := (C_{t(n)}^A(n))_{n=1}^N = \big(C_{f,t(n)}^A(n) \cdot C_{0,t(n)}^A\big)_{n=1}^N.$$

Then h^A *is the maximum number k such that $k \leq a_k$.*

Now, let us compute the relation of these equations with the ones appearing in the h^P index, that is, the h-index. The idea is to compare the previous sequence with the expected sequence of numbers of citations for each article in case the N papers were published in the discipline P. It is given by

$$(b_n)_{n=1}^N := (C_{t(n)}^P(n))_{n=1}^N = \big(C_{f,t(n)}^P(n) \cdot C_{0,t(n)}^P\big)_{n=1}^N.$$

Using the principle of universality of citation distribution, we have that for all n, the expectation values of both normalized series are the same, that is

$$C_{f,t(n)}^A(n) = C_{f,t(n)}^P(n).$$

Consequently, we have that the h^A index is given by the maximum value of k such that

$$k \leq C_{f,t(k)}^P(k) \cdot C_{0,t(k)}^A,$$

that is, the maximum value of k such that

$$k \leq C_{f,t(k)}^P(k) \cdot \frac{C_{0,t(k)}^A}{C_{0,t(k)}^P} \cdot C_{0,t(k)}^P = Q_k \cdot C_{t(k)}^P(k),$$

where $C^A_{0,t(k)}/C^P_{0,t(k)} = Q_k$. However, recall that the h^P-index is simply given by the maximum of k such that $k \leq C^P_{t(k)}(k)$.

Thus, we have proved that *the expected value of the h-index depends strongly on the discipline in which the papers are published.* That is, the value of the h^A-index depends on the constants Q_k, that are computed as the ratios among the means of citations of the papers in the disciplines A and P after t years of their publication.

Therefore, the expectation of the number of citations of a paper in a given period of time is not independent of the scientific field. However, the corrected number of citations $C_{f,t}$ is indeed independent of the field, as a consequence of the assumption of the universality of citation distribution. If we want an h-index being valid for all fields we need to define it in terms of this quantity, maybe scaled by a constant that must be the same for all fields. In the field of reference (Physics), we can let the index as it is, using the inequality

$$k \leq C^P_{f,t(k)}(k) \cdot C^P_{0,t(k)}$$

for defining the h^P-index. For any other discipline A, it must be given by an inequality depending on $C^A_{f,t(k)} = C^P_{f,t(k)}$ and a scaling constant $(C^P_{0,t(k)})$ that must be the same for all fields, that is

$$k \leq C^A_{f,t(k)}(k) \cdot C^P_{0,t(k)} = C^A_{f,t(k)}(k) \cdot \frac{C^P_{0,t(k)}}{C^A_{0,t(k)}} \cdot C^A_{0,t(k)} = \frac{C^A_{t(k)}(k)}{Q_k}.$$

Therefore, *the corrected h^A-index h^A_c must be defined as the maximum k such that*

$$k \leq \frac{C^A_{t(k)}(k)}{Q_k},$$

for $Q_k = C^A_{0,t(k)}/C^P_{0,t(k)}$, where the sequence of normalized number of citations of the papers is ordered in decreasing order. That is, the resulting h-index has the formal structure of an h_α-index in the sense of Van Eck and Waltman 2008, but the constant α is different depending on the time that each single paper was published.

Summing up, the *discipline-independent h-index h^A_c* must be computed using the following *algorithm.*

(i) Consider the list of publications indexed by $k = 1, ..., N$ of a given author and write for each of them the weighted number of citations given by

$$w(k) := \frac{C_{t(k)}(k) \cdot C^P_{0,t(k)}}{C^A_{0,t(k)}},$$

where k indicates a publication of the list, $C_{t(k)}(k)$ its number of citations since it was published —that is, $t(k)$ is the number of years since its publication—, $C^P_{0,t(k)}$ and $C^A_{0,t(k)}$ are the means of citations by article after $t(k)$ years since their publication in Physics —or the scientific field that we choose as reference—, and in the discipline A, respectively.

(ii) Write the resulting list of numbers in decreasing order, for getting the decreasing sequence of real numbers $(w(n))^N_{n=1}$.

(iii) The h^A_c-index is the maximum number k that satisfies that $k \leq w(k)$.

5. Some assumptions for an easy computation of the discipline-independent h-index, and an example

Fix a scientific field A. The following situations, that simplify the arguments and the calculations, may happen.

- The h-index does not depend on the discipline only if $Q_k = 1$ for all k up to a reasonable deviation. That is, the ratio among the mean of citations depending on the number of years since the publication t in the disciplines A and P is equal to 1 for all t. Recall that the mean number of papers published by an author in both fields A and P must be similar; otherwise, a second step will be needed for normalizing the resulting indices.

- The ratios Q_k, $k = 1, ..., N$, are constant for all k and equals $Q \in \mathbb{R}^+$. This means that the form of the curve of the citations per article $C_{0,t}$ versus t is similar in both disciplines A and P, but the sizes are different and depend on Q. The discipline independent h-index is in this case an h_α-index, where α is given by $1/Q$.

- In the list of publications of a given author there are papers from different fields $A_1, A_2, A_3, \ldots$. Then the algorithm for computing the corrected h-index must be the same, but the numbers $w(k)$ must be defined as

$$\frac{C^{A_i}_{t(k)}(k) \cdot C^P_{0,t(k)}}{C^{A_i}_{0,t(k)}}$$

  when the paper indexed by k is classified to belong to the discipline A_i, $i = 1, \ldots, I$.
- If all the means of citations by paper in the fields involved satisfy that

$$C^A_{0,t} \leq C^P_{0,t}$$

  for all t, then $h^A_c \geq h$.

Let us provide now an example. It is well known that a standard author that publishes in the field MATHEMATICS —actually in any field of pure mathematics— has a smaller value of h-index that a researcher in, for example, PHYSICS. The dynamics of citations in these big fields of science are absolutely different. Depending on the particular field of mathematics and physics, the ratio of citations in both disciplines may be even $1/10$ or more. It is not our aim to explain the scientific/sociological reasons for this to hold in this paper, but we must assume it as a matter of fact.

The following Table 1 shows a real case. We have chosen a researcher in ALGEBRA=A that started to publish in 2001, and we have computed its total number of citations and the h-index. We want to compare the value of the h-index —the one following the original definition— and the value of the h^A_c-index, adapted taking as a reference the field of NUCLEAR PHYSICS=P. We have found the total number of published papers of the chosen researcher in Web of Science, as well as the number of citations to each paper. We have also obtained in it the mean value of citation per paper of the set of all the papers published each year in the field ALGEBRA for obtaining the values of $C^A_{0,t}$. The same has been done in the field NUCLEAR PHYSICS for obtaining the values of $C^P_{0,t}$. The reader can find a table of this kind of

citation means by discipline in the paper by Iglesias and Pecharromán 2007.

	Papers	Citations/paper	$C_{0,t}^A$	$C_{0,t}^{NP}$	Weighted citations/paper
2001	1	19	11.6	21.25	34.80
2002	3	$1, 4, 35$	10.66	25.64	$2.41, 9.62, 84.18$
2003	4	$2, 3, 6, 19$	10.61	31.97	$6.02, 9.04, 18.08, 57.25$
2004	1	3	9.22	19.93	6.48
2005	4	$1, 6, 11, 12$	8.41	16.96	$2.02, 12.10, 22.18, 24.20$
2006	1	1	8.34	23.61	2.83
2007	3	$3, 7, 11$	7.47	15.62	$6.27, 14.64, 23.00$
2008	2	$1, 3$	6.89	15.23	$2.21, 6.63$
2009	2	$1, 3$	5.93	10.19	$1.72, 5.16$
2010	2	$2, 3$	5.26	10.74	$4.08, 6.13$
2011	1	3	4.25	9.69	6.84
2012	1	1	3.14	7.93	2.53
2013	0	0	2.3	7.17	0
2014	2	$1, 1$	1.24	2.96	$2.39, 2.39$
2015	0	0	0.34	0.79	0

TABLE 1. Number of citations and weighted citations.

As the reader can see in Table 1 the number of citations per paper is smaller than the weighted number of citations, which justifies the fact that the corresponding h-indices are different. By using this table we can see that the usual h-index is 7, while the corrected index h_c^A is 9.

Let us finish this section with some comments on the correction proposed by Radicchi et al 2008. The original motivation of the present paper is in fact the universality of citations distribution investigated in this paper, in which it is proved that the normalized number of citations in each scientific field $c_f = c/c_0$ explained at the beginning of the paper has a distribution independent of the research field. The correction of the h-index that they propose —the h_f-index— is given by comparing c_f with the reduced rank r/N_0 —where r is the rank number of the papers of an author and N_0 the mean number of publications in the

discipline—. Although the starting point is the same, there are two main differences with our approach. The first one is that we have tried to provide an index that is still possible to understand in terms of "number of published papers with more than a number of citations". This meaning is lost if the h_f-index is used, and the same happens if the normalization proposed in the next section is done; in return, they allow a better comparison among authors. The second reason is that our index is computed by correcting year per year the contribution of the papers of an author published in the year. The ratio between average numbers of citations among different disciplines depends on the year, so the final result is more accurate.

6. Further normalizations: considering the mean number of papers published in each research field

Once the necessary "non-linear transformation component" of the h-index for comparing among different scientific fields has been introduced, further linear changes may be also taken into account. In our previous arguments, no reference to the fact that every research field have a different publication rate has been made. However, it is a matter of fact that the mean number of publications per author and year varies a lot depending on the scientific field. This fact must be considered if the h-index is used for research evaluation, since the "prestige" of an author —in case we accept to measure it by means of the impact of its publications— depends on the total number of papers that he has published, and this fact must be reflected in the value of the impact index.

The way of introducing this effect in the definition of a new corrected h-index is in this case very easy, since it is given by a linear transformation. If the number of publications per year and author is N_A in the reference field A and N_B in a field B, it is enough to re-scale the h-index provided for B by N_A/N_B for comparing both of them. This is a consequence of the fact that as bigger is the number of papers, the greater is the numbers of papers with more than n citations. From our point of view, this does not mean that it must be introduced in the definition of the h-index, since if we do that then it looses completely

the original meaning: it cannot be understood any more as a corrected impact measure that provide a balance among number of citations and a number of published papers. However, if the final purpose is to find an evaluation index for comparing among different fields, the linear transformation

$$h_B \to h_B^* := \frac{N_A}{N_B} \, h_B$$

must be obviously done. Thus, in the example of a researcher in ALGEBRA explained in the previous section, the final modified index is equal to 9; it must be divided by the mean number of published papers in a year by a researcher in ALGEBRA and compare with the h-index of the researcher on NUCLEAR PHYSICS when it is divided by the mean number of published papers per author in this discipline. The greatest number has the better h-index type indicator, but this number cannot be interpreted as a "number of papers with more than such a number of citations" any more.

Other transformations that should be taken into account is the normalization by the mean number of authors per paper–that also depends on the scientific field—, and can be done in a similar way. Just dividing by the mean number in each field provides the desired inter-discipline index.

7. An example: High-level Spanish scientists in the fields of Microbiology and Biotechnology

The aim of this section is to show an application of the corrected h-index for a concrete group of scientist doing research in a particular field of science. Our aim is twofold. First, we want to prove that the distributions of the standard h-index for scientist belonging to different —but related— scientific fields do not coincide in general, even after normalization by the mean number of citations per paper in each field. Secondly, we want to show that the distribution defined by comparing the standard h-index of one of them and the corrected h-index computed with our algorithm fits better than in the previous case.

Using the Web of Science classification, we selected first three fields of research and we considered a group of high level scientist in each

of them. In particular, we chose the fields of MICROBIOLOGY, BIOTECHNOLOGY AND APPLIED MICROBIOLOGY (BIOTECHNOLOGY for short) and ECOLOGY. For the selection of the samples, we followed the list of names appearing in the webpage http://indice-h.webcindario.com for these fields, checking the outputs of Web of Science and using them for the computations explained in what follows. Below (Table 2) we show the number of scientist of the top group that has an h-index bigger or equal than the values fixed in the left column.

As the reader can see, the number of scientist is not the same in each group, due to the local nature of the sample (Spain) and the particularities of each one of the considered fields. This makes necessary a first normalization to the number of researchers. Due to the similar characteristics of the scientific subjects, we assume that no normalization by the mean number of papers published in each year by each author is necessary. Our interest is to analyze if the distribution of the h-index in the three categories is similar. In order to do this, we will use these dates to obtain a model for the h-index in each discipline, using the basis of functions $\{1, x, x^2, 1/x\}$ to fit the distribution functions. The result is shown in Table 3, and the corresponding functions are represented in Figures 1 and 2.
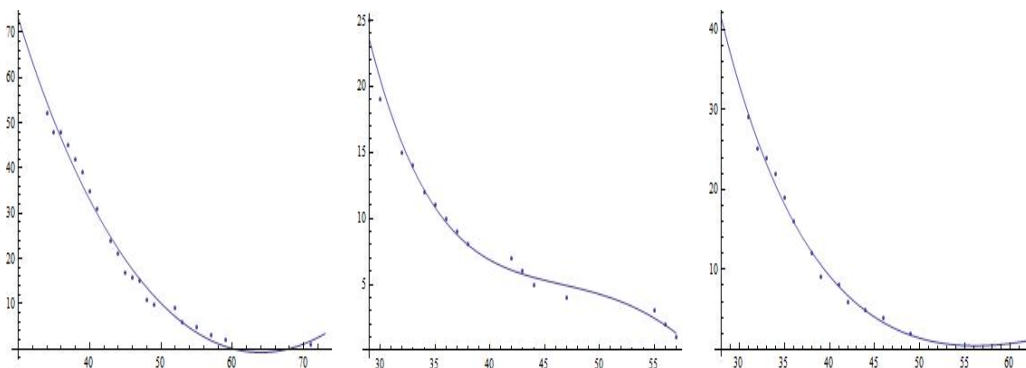

FIGURE 1. h-indices and model functions for Microbiology, Biotechnology and applied microbiology, and Ecology.

Now, we center our attention in comparing the fields MICROBIOLOGY and BIOTECHNOLOGY. Our aim is to show that the model

h-index	MICROBIOLOGY	BIOTECHOLOGY	ECOLOGY
29	–	25	–
30	–	19	–
31	–	–	29
32	–	15	25
33	–	14	24
34	52	12	22
35	48	11	19
36	48	10	16
37	45	9	–
38	42	8	12
39	39	–	9
40	35	–	–
41	31	–	8
42	–	7	6
43	24	6	–
44	21	5	5
45	17	–	–
46	16	–	–
47	15	4	–
48	11	–	–
49	10	–	2
52	9	–	–
53	6	–	–
55	5	3	–
56	–	2	–
57	3	1	–
59	2	–	–
62	–	–	1
71	1	–	–

TABLE 2. Number of researchers in each scientific field for each value of h-index.

for the distribution of the second discipline constructed according to the comparison with the first one fits better with its distribution. For simplicity, assume that the comparison parameters Q_k appearing in the corrected h-index are constant, and so they do not depend on k. That is, the corrected h-index for an author in BIOTECHNOLOGY is given by the biggest value of articles of the author k satisfying that

$$k \leq Q_k \cdot C(k),$$

MICROBIOLOGY
$1.8056 + 49.8133/x - 0.0933487x + 0.000824343x^2$

BIOTECHNOLOGY
$-20.6851 + 329.457/x + 0.449006x - 0.00327353x^2$

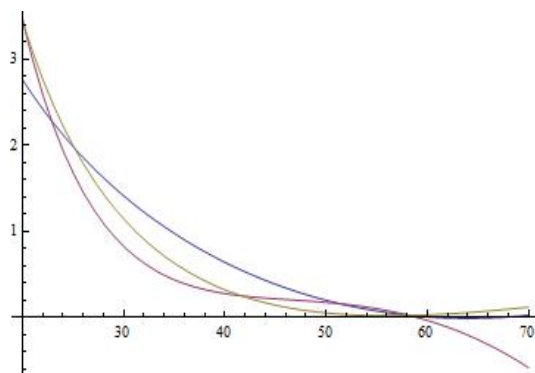| ECOLOGY |
| --- |
| $-7.57217 + 185.118/x + 0.0941547x - 0.00031574x^2$ |

TABLE 3. Distribution models for each discipline.

FIGURE 2. Comparison of the three models (after transformation).

where $C(k)$ is the number of citations of the paper and Q_k is the ratio C^B/C^M among the mean number of citations in the field BIOTECH-NOLOGY and in the field MICROBIOLOGY. An estimate of these values taken directly from WOS gives $C^B = 16.2$ and $C^M = 24,5$. Therefore, the corrected h^B index is given by the maximum of the articles k such that

$$k \le 16.2/24.5C(k) = 0.66 \cdot C(k).$$

In the sequel we will prove that this correction is enough for having a good fitting among the distribution of the h-index for MICRO-BIOLOGY and for the corrected one for BIOTECHNOLOGY. Let us compute the top part of the distribution of the h^B-index for BIOTECH-NOLOGY.

h^B-index	N. authors	h^B-index	N. authors
32	25	45	11
33	24	46	10
34	23	50	9
35	21	54	8
36	20	55	6
37	19	56	5
39	17	65	4
40	14	74	3
41	13	75	2
43	12	76	1

TABLE 4. Number of researchers in each scientific field for each value of the corrected h^B-index in the field BIOTECHNOLOGY.

The results are shown in Table 4. In Figure 3 we compare the distribution models created for top scientist in both fields. According to our ideas, the distribution for the h-index and for correction h^B must coincide. Indeed, it can be seen that the h-indices corrected using our procedure fit better and for a bigger range of values than the original one for inter-fields comparison.
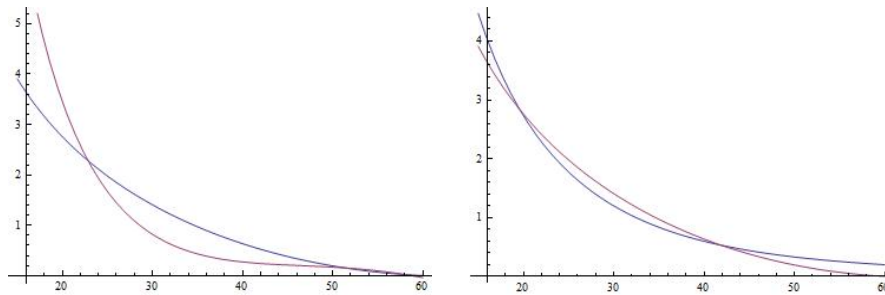

FIGURE 3. Comparison of both models: original h-indices (left) and corrected h-indices (right).

8. **Conclusions**

Although many applications of the h-index for research assessment have been proved to be arbitrary, we must accept that the h-index is being used for this aim by many scientific agencies. We have shown that, based in a well-established biliometric principle that has been recently proved —the principle of universal citation distributions—, the crude application of the h-index computed using the original definition is introducing a big distortion in the scientific evaluation based on the couple production/quality.

It is difficult to understand the meaning of the h-index in relational terms. Its nature cannot be explained in terms of other easy-to-understand and well-established bibliometric indicators. Although it provides information about the balance among number of papers and number of citations, it is not clear why this nymber must be relevant for evaluating a set of papers. But if we assume so we must accept also some modifications in order to translate the information given by the h-index among different scientific fields. We have found that the h-index for the discipline A for comparing with the h-index for the discipline P must be *the maximum number of papers n such that their normalized numbers of citations*

$$\frac{C(k) \cdot C_{0,t(k)}^{P}}{C_{0,t(k)}^{A}},$$

are bigger or equal than n, where $C(k)$ is the number of citations of the paper k, $C_{0,t(k)}^{P}$ and $C_{0,t(k)}^{A}$ are the mean number of citations of a paper published $t(k)$ years ago in the disciplines P and A, respectively.

Further normalizations are also possible, in case that the evaluation committee considers for example that the mean number of papers in the fields that have been compared is meaningfully different and this fact must be taken into account. In this case, the transformation is just given by dividing the corrected index by the mean number of published papers in the field, and so it is of different nature than the changes explained above. The same can be done for considering for example an index being independent of the mean number of authors per paper

in the field. It must be pointed out that these final changes in the h-index are given by an essentially different type of transformation than the main one explained in the first part of the paper.

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