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Archie – A 3D Printed Tool for Teaching Archimedes' Principle

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Abstract
Archimedes' principle can be a complicated idea to grasp for students. To help with the process, we offer a simple 3D printed tool which can be used for teaching Archimedes' principle. A detailed description of the tool is given, as well as basic measurements showing its capabilities and the results to expect by using it. Also, a list of activities is given which shows how Archie can be used in a high school environment. The activities are focused on students discovering Archimedes' principle on their own, while paralleling some of the original statements made by Archimedes about floating objects. A few activities are also offered focused on strengthening and verifying students' gained understanding of the principle using Archie. Archie is a simple 3D-printed tool which can help students achieve a better understanding of Archimedes' principle and how objects behave in liquids.

Keywords: *Archimedes' principle, 3D printing, historical approach, student activities.*

1. Introduction

The subject of sinking and floating is a complex subject spanning from the introduction of the concept of density up to understanding Archimedes' principle. For example, to explore this issue, it took Archimedes two book and nineteen propositions (Archimedes, 2002) and even today its proper understanding is discussed and analyzed (Bierman & Kincanon, 2003), (Graf, 2004). The complexity of the subject can also be seen in students struggling to "apply Archimedes' principle even in very simple situations" (Loverude, Kautz, & Heron, 2003) especially, when the principle is taught through the variation of pressure between the top and bottom surfaces of an object.

An experimental and empirical approach to teaching the concepts can be of great importance to reach a proper understanding of the principles of sinking and floating. Therefore, in this paper, we introduce a unique 3D printed tool to help students discover those principles. The presented tool was given the name Archie after the main idea of the topic, Archimedes' principle. The paper will introduce Archie and its properties as a 3D-printed tool and will present experimental data showing Archie's functionality. Finally, the paper will offer a few simple activities and experiments which can be conducted in a classroom using Archie either for discovering the main ideas behind floating and sinking or to verify and further deepen students' understanding of these ideas.

2. Archie Parameters

Archie is a 3D-printed tool with a variable mass and volume that comes in two main versions, a basic economic version, and a larger full-sized version. Both are composed of two partially hollow cylinders, similar in shape to a measuring cylinder, and a small rubber band. The smaller of the two cylinders has an inner and outer diameters of 27 mm and 30 mm respectively, while the larger one has an inner and outer diameters of 32 mm and 35 mm respectively. This means that the smaller cylinder fits into the larger cylinder leaving a 1 mm gap. Inserting the smaller cylinder into the larger one allows Archie to have a variable volume according to the needs of the user. Also, the diameter of the smaller cylinder was designed so all Euro coins, and many coins in the world, would fit into it, meaning coins can be used as weights to change Archie's mass.

Fig. 1 Left: Archie's smaller and larger cylinders along with a small rubber band. Center: The rubber band wrapped around the top of the smaller cylinder. Right: An Archie with a volume set to 140 ml.

To combine the cylinders into Archie a small rubber band is used. The rubber band is wrapped around the smaller cylinder and then they are inserted together into the larger cylinder (figure 1). The reason for using the rubber band is twofold: the rubber band works as an insulation preventing liquid from pouring into Archie, and it insures the volume of Archie stays fixed after it is changed by the user.

Both cylinders are marked on the outside showing the volume of the cylinder at a given height in ml. When Archie is floating, the marks allow the user to read the value of the volume immersed in liquid. Similarly, the marks help the user discern Archie's total volume for the experiment. When printed, the marks are the same color as Archie, so they may be harder to see for some students. This problem can be easily fixed by coloring the marks using a permanent marker. The marks on the cylinders differ a bit as the least count is 1 ml for the smaller cylinder and 2 ml for the larger cylinder.

The main difference between the two versions of Archie, economic and full-sized, is their size. The economic version, while offering the user a shorter range of experimental data, is smaller and therefore requires less material for printing. The total volumes of its cylinders are 80 and 100 ml, meaning that when combined Archie's total volume can be changed between 110 and 170 ml. The full-sized version has larger total volumes of its smaller and larger cylinder of 110 and 150 ml respectively, so the total volume of a full-sized Archie can range between 160 ml and 250 ml. As opposed to the economic version, the larger version offers a wider range of experimental data but for the cost of more printing material.

Archie offers a few advantages for the subject of floating and swimming. It has a variable mass and volume, meaning it can be used for observing how density depends on those physical quantities. It is also easy to read Archie's immersed volume in a liquid, which makes it an ideal instrument for working with Archimedes' principle in various ways. Another advantage is its low cost and availability for teachers with access to 3D printing.

The files needed for printing both versions of Archie are available on the authors' personal webpage (Wannous & Kováč, 2019). Any future updates or improvements to Archie will also be available there.

3. Experimental Results of Using Archie

To better showcase Archie's capabilities, this section offers sample measurements done using the economic version of Archie. Specifically, three types of experiments are conducted, changing Archie's mass while floating and sinking, changing Archie's volume while floating and sinking and finally examining the tension of a string holding Archie while submerging Archie, all done in water.

In the first experiment, Archie's mass is measured and then it is set in water, ideally in a measuring cylinder or something similar. Archie's mass and its immersed volume are recorded, Archie's mass is changed using coins and the process is repeated until Archie sinks. As our independent and dependent variables are Archie's mass and its immersed volume respectively, the results to be expected when Archie is floating are as following according to Archimedes' principle:

$$V_i = \frac{1}{\rho} m \tag{1}$$

Where V_i is Archie's volume immersed in water and m is its mass. Sample data from this experiment are given in table 1 and are graphically represented in figure 2. In this sample, we used only Archie's smaller cylinder, therefore its total volume is 80 ml.

Table 1. Archie's immersed volume in water in relation to its mass

m (g)	V_i (ml)	m (g)	V_i (ml)
30	30	66	67
36	36	72	72
42	42	78	79
48	48	84	80
54	54	90	80
60	61	96	80

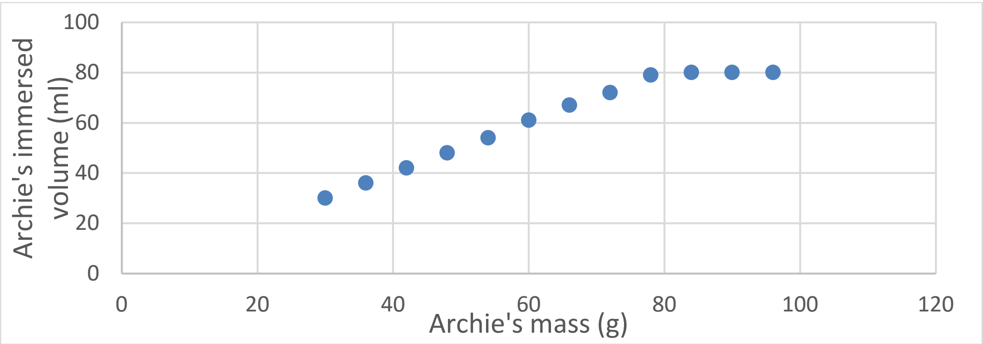

Fig. 2. Archie's immersed volume in water in relation to its mass. When the mass reaches a certain value (80 ml) Archie becomes wholly submerged in water.

As can be seen from the data, when Archie's mass exceeds a certain value, it is wholly submerged under water, meaning its immersed volume is equal to its total volume of 80 ml. As equation (1) represents the case when Archie is afloat, we apply a linear fit only to the data up to 78 g of mass. The linear fit of the data gives the following result:

$$V_i = 1.019m - 0.72 \tag{2}$$

The systematic error in the fit is consistent with the least count of Archie's used part which was 1 ml. The slope when interpreted using equation (1) shows that density of used water is around 981 kg.m^{-3} which differs by only about 2% from the real value and therefore is an acceptable value for a high school physics course.

In the second measurement, Archie’s total volume is changed while its immersed volume is observed. The total volume has an affect on the submerged volume only in the case when the object is sinking, as its totally submerged under water. In the sample data, we increased Archie’s mass using Euro coins to 145 g and then its volume was changed starting with 120 ml up to 160 ml (figure 2).

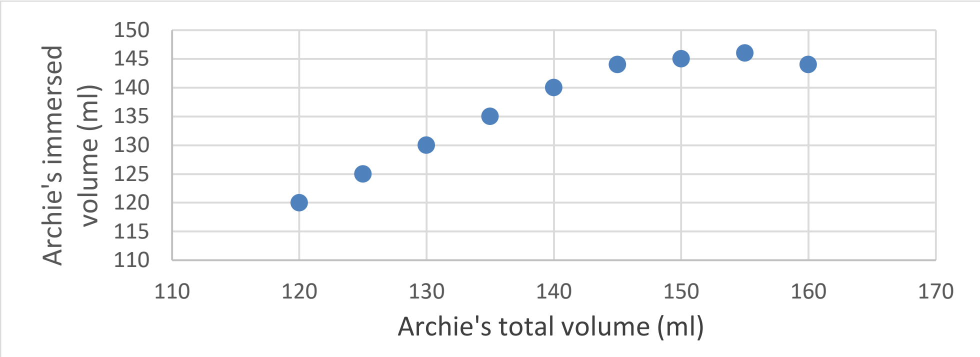

Fig. 2. Archie’s immersed volume in water in relation to its total volume. While Archie is sinking, its immersed volume is its total volume, but when Archie is afloat, its immersed volume is independent of its total volume.

When Archie is afloat its immersed volume has a value around 145 ml, which again agrees with our analysis in equation (1) considering that the density of water is approximately 1000 $kg.m^{-3}$.

Table 2. The tension of a string holding Archie while Archie is immersed in water

V_i (ml)	F_t (N)	V_i (ml)	F_t (N)
0	1.46	70	0.76
10	1.36	80	0.68
20	1.25	90	0.55
30	1.16	100	0.46
40	1.07	110	0.36
50	0.97	120	0.29
60	0.86		

The final measurement is a classical measurement used to show buoyant force. Archie is gradually immersed in water while hung on a string. The tension of the string is measured in relation to Archie’s immersed volume, which should lead to showing the buoyant force affecting Archie.

When immersed in the liquid, Archie is hung on a force sensor using a string. The sensor measures the tension of the string, which according to Archimedes’ principle can be calculated as following:

$$F_t = F_g - F_b \quad (3)$$

As our independent variable is the immersed volume of Archie, the resulting relationship would be:

$$F_t = -\rho g \, V_i + F_g \quad (4)$$

Archie is perfect for such an experiment as its mass and volume can be set, so its density is higher than the density of the liquid. The marking of its volume also makes it ideal for such an experiment as the user can easily read its immersed volume with acceptable accuracy. Sample data for the experiment is given in table 2 and is graphically represented in figure 3. Using a linear fit on the measured data we get the following result:

$$F_t = -9890 \, V_i + 1.4625 \quad (5)$$

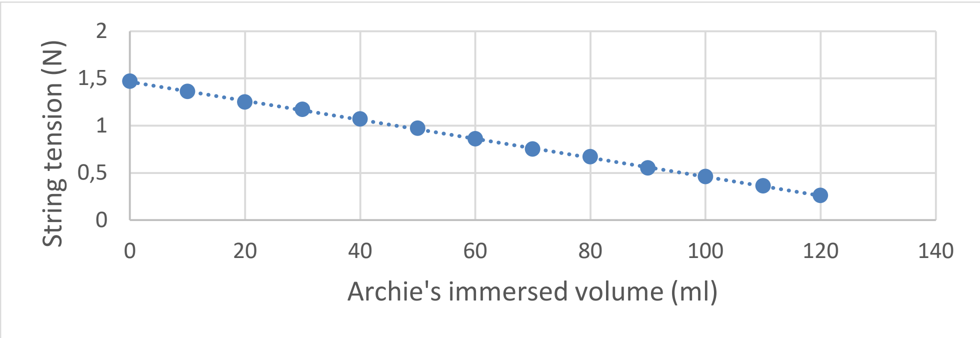

Fig. 3. The tension of a string holding Archie in relation to its immersed volume.

When comparing with equation (4) we can see that the gravitational force affecting Archie according to the fit agrees with the measurement in the table when Archie is not immersed in water. Also, the density of the water according to this measurement is 1008 kg.m^{-3}, which is again an acceptable result for a high school physics course.

4. Classroom Activities with Archie

As we can see from the previous section, Archie is well suited for use in the classroom as it offers the teacher many possibilities. In this section, we offer a few simple activities which can be realized using Archie:

- **Discovering density:** when Archie's mass and volume are changed it changes how it behaves in a liquid. The higher the mass, the more Archie sinks into water, while the higher the total volume, the less Archie sinks. This can lead students to discover a physical quantity connected to floating, i.e. density, which is directly proportional to mass and indirectly proportional to volume.

- **Floating and sinking in liquids:** as an introduction to Archimedes' principle students can discover how the mass and volume of an object affect its immersed volume in a liquid. This is similar to what we did in the first two experiments in the previous section. However, while only water was used in those experiment, we recommend using other liquids as well, e.g. alcohol, so the effect of the density of a liquid can be seen. A similar activity using measuring cylinders has been proposed by Benenson (1975).
- **Buoyancy force:** based on the previous activity, it is obvious there is a relationship between an object's immersed volume in a liquid and its mass. Only the reason for the relationship is missing, which can lead to an analysis of forces and the discovery of buoyant force. That force can be analyzed with the final experiment presented in the previous chapter. The experiment is not novel as it even appears in some textbooks (Giancoli, 2014), although the addition of Archie can make it much easier.
- **The density of an unknown liquid:** Archie can be used to verify and deepen students' understanding of Archimedes' principle. One such activity is measuring the density of an unknown liquid using Archie. This can be easily achieved using the first experiment in the previous section.
- **Mazur's riddle:** Archie can be also used to demonstrate the results of Mazur's riddle: "A boat carrying a large boulder is floating on a lake. The boulder is thrown overboard and sinks. The water level in the lake (with respect to the shore): 1. rises. 2. drops. 3. remains the same" (Mazur, 1997). Using Archie as the boat, a measuring cylinder with water as the lake and coins (weights) as the boulder, it can be shown that the water level drops.

5. Conclusion

As we have seen in the paper, Archie is an appropriate 3D printed tool for the whole subject of sinking and floating. Its design allows it to be able to do many things at once, which in turn makes it a desirable instrument in a physics high school course. Archie is also easily available to many teachers. Its simplicity and availability make it an appropriate instrument that can be used not only by teachers but also by students working on their own or in small groups to discover and observe the principles and concepts behind sinking and floating.

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