Sound focusing capability of a CO_2 gas-filled cuboid

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Abstract

The ability of focus waves with concave or convex surfaces is well known both in optics and in acoustics. Nowadays, the possibility of beamforming sound with flat lenses is a hot topic because of its application in different areas such as biomedical engineering or non-destructive techniques. In this paper, we propose a gas filled cuboid lens that has a different sound speed than that of the surrounding medium (air in our case) as a beamforming acoustic device. This constitutes an experimental visualization of the capability of sound focusing with flat surfaces lens and allows understanding the corresponding physic phenomenon.

1. INTRODUCTION

It is known that concave or convex surfaces are able to focus waves. From geometrical point of view, it is not possible to focus waves with flat surfaces. However, nowadays it is known that there are artificial materials that exhibit extraordinary properties and can focus although they have flat surfaces.

Acoustic lenses have great utility in science and technology, so it is critical to study this type of lens. This great potential of applicability makes its study a hot topic in the field of acoustics. Traditionally, the focusing mechanisms in acoustics, as in optics, are due to the phenomena of refraction and diffraction.

Although the first acoustic lens was a gas-filled lens that used carbon dioxide (CO_2) and was built in 1852 by Sondhauss [1], there are only a few published works on the idea of using gas-filled lens to focus sound. The scientific literature shows research where a balloon gas-filled [2] was described as an acoustic lens. The lower gas velocity inside the lens and the curvature of the surface produce the focalization. Balloon gas-filled lens were experimentally verify [3]. In this sense, such spherical gas filled balloon lenses mostly used as a teaching device [4] but no cuboid lens was showed. However, due to the ease of implementing flat lenses, in recent years they have become very important.

Nevertheless, different lenses designs with different mechanisms to produce wave focalization have been proposed, such as thin Fresnel diffractive lens for focusing underwater ultrasound [5], phononic crystal lens [6, 7], gradient acoustic lenses using space-coiling structures [8-11], lenses that leverages Fabry–Perot resonances to enhance the transmitted energy [12, 13]. Metamaterial based gas-filled layers lens, used at least two different gases and requires, in addition high precision manufacture of gas layers [14].

Recent studies have shown that cuboid-based lenses with CO_2 interfaces can be developed. C. Rubio *et al.* [15-17] demonstrated that it is possible to use simple design cuboids filled with gas for several applications [15]. Further, another study shows that by changing the gas concentration, the focusing properties change as well due to the sound speed modification [16]. Other modifications applied to this type of lens were those that a pupil was added to achieve an apodization effect [17]. In all these previous cases presented, lenses focused through a change in the refractive index due to the change in sound propagation speeds. Thus, these lenses achieve a focus of energy in non-resonant mode.

In this paper, we present a cuboid filled with CO_2 gas which focusing capability is not from the refraction phenomenon but from the radiation of a membrane. In this case, the cuboid separates the gas medium (air), so we can assume the cuboid as a uniform layer of fluid between two identical fluids.

2. THEORETICAL BACKGROUND

Physical principle of sound focusing by cuboid gas-filled lens consists in the following: when falling on the CO_2 cuboid the plane wave that propagates in air penetrates into CO_2 cuboid. At certain frequencies, standing waves occur, so that the CO_2 vibration will be maximum at the ends of the cuboid. The film on the cuboid sides actuate as a membrane with its own eigenfrequency. When the membrane is excited by the the stading wave of the cuboid at a frequency which coincides with its eigenfrequency, a resonance phenomenon occurs. In such a way, the film induces an acceleration to the surrounding gas (air), in such a way that the film behaves like a radiating membrane, leading to focusing amplification due to resonance phenomenon. In this case, the film, in addition to the function of retaining the CO_2 in a determinated volume, also acts as a radiating membrane.

The problem to be solved is based on the combination of two physical phenomena. The first one is a transmission problem between different layers. On the other hand, we have an enclosure solid structure eigenmode vibration. The transmission of acoustic waves through fluid layer is studied and discussed in detail in [18]. Suppose that a layer of uniform thickness L is between two different fluids and that a plane wave impinges on the border as indicated in Figure 1. The impedances of the media are $Z_1 = \rho_1 c_1$ and $Z_2 = \rho_2 c_2$, where ρ_i and c_i are the density and sound speed in medium i.

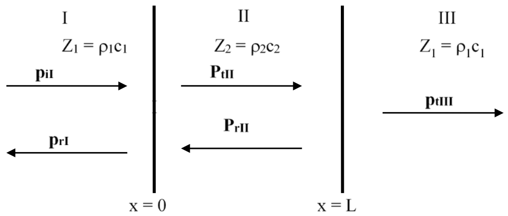

Figure 1. Reflection and transmission of a plane wave normally incident on a layer of uniform thickness.

The incident wave is represented by

$$\mathbf{p_{iI}} = \mathbf{P_{iI}}\, e^{j(\omega t - k_1 x)}$$

where $\mathbf{P_{iI}}$ is the complex pressure amplitude of the incident wave and k is the wavenumber. The reflected wave is represented by

$$\mathbf{p_{rI}} = \mathbf{P_{rI}}\, e^{j(\omega t + k_1 x)}$$

where $\mathbf{P_{rI}}$ is the complex pressure amplitude of the reflected wave.

The waves transmitted and reflected in the fluid II are

$$\mathbf{p_{tII}} = \mathbf{P_{tII}} \cdot e^{j(\omega t - k_2 x)}$$

$$\mathbf{p_{rII}} = \mathbf{P_{rII}}\, e^{j(\omega t + k_2 x)}$$

and the wave transmitted in III is

$$\mathbf{p_{tIII}} = \mathbf{P_{tIII}}\, e^{j(\omega t - k_1 x)}$$

The continuity of the specific acoustic impedance at x = 0 gives

$$\frac{\mathbf{P_{iI}} + \mathbf{P_{rI}}}{\mathbf{P_{iI}} - \mathbf{P_{rI}}} = \frac{Z_2}{Z_1} \frac{\mathbf{P_{tII}} + \mathbf{P_{rII}}}{\mathbf{P_{tII}} - \mathbf{P_{rII}}}$$

and in the same way, in x = L

$$\frac{\mathbf{P_{tII}} \cdot e^{-jk_2 L} + \mathbf{P_{rII}} \cdot e^{jk_2 L}}{\mathbf{P_{tII}} \cdot e^{-jk_2 L} - \mathbf{P_{rII}} \cdot e^{jk_2 L}} = \frac{Z_1}{Z_2}$$

The intensity transmission coefficient τ_I is obtained after some algebraic steps

$$\tau_I = \frac{1}{1 + \frac{1}{4}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\right)^2 sin^2(k_2 L)}$$

Figure 2 shows the calculation of the intensity transmission coefficient for a 0.16 m thick CO_2 layer placed in air. As can be seen, there are frequencies at which a total transmission is obtained, that is to say an intensity transmission coefficient whose value is 1. In this case, Z_1 was air, with $Z_1 = 343*1.21 = 415$ rayls, and Z_2 was CO_2 with $Z_2 = 260*1{,}977 = 514$ rayls.

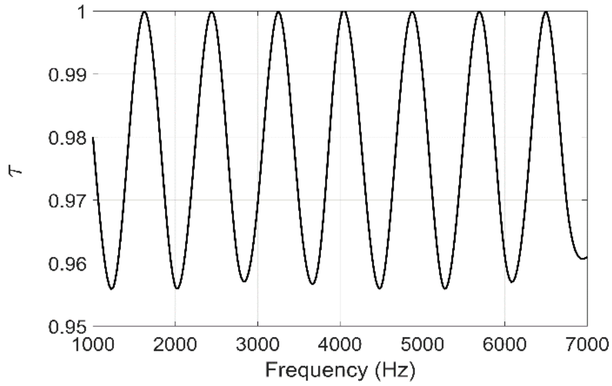

Figure 2. Intensity transmission coefficient for a 0.16 m thick CO_2 layer placed in air

It is possible to obtain total transmission, namely $\tau_I = 1$ when $sin^2(k_2 L) = 0$. This condition is satisfied when $k_2 L = n\pi$ (n = 1, 2 3, …) corresponding to the frequencies $f = \frac{nc_2}{2L}$. These frequencies correspond to the frequencies at which standing waves occur in the fluid layer. In this case, the fluid vibrates with its maximum amplitude at the ends.

Moreover, frequencies that excite resonance modes of the cuboid are considered. Therefore, it is necessary to explain the resonant phenomenon. Previous works have demonstrated the interaction of square and rectangular enclosures in excitation of eigenfrequency [19, 20]. Wu *et al.* [19] proposed a solution based on Bessel functions. There is another way to solve this problem by using Navier's exact solution [21]. In this work we will compare both analytical solutions with the result of the experimental frequency obtained. The free vibration differential equation in rectangular plates is:

$$D\nabla^4 W - \omega^2 \rho h W = 0$$

where *W(x,y)* is a typical mode, h is the plate thickness, $D = Eh^3/12(1-v^2)$ is the bending rigidity governed by the Young's modulus (*E*) and the Poisson's ratio (*v*), ω is the natural frequency and ρ is the mass density. Navier-type solution for mode functions and natural frequencies are,

$$W_{m-n} = A_{m,n} \sin\frac{m\pi x}{L} \sin\frac{n\pi y}{L}, (m, n = 1,2, \ldots)$$
$$k^2 = \left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{L}\right)^2$$

where A_n is a constant to be determined by the use of the orthogonal characteristic of vibration modes and L is the edge length of the square plate. On the other hand, through Wu *et al.* [19] we can consider the cube enclosures as fully supported but not clamped type. It is ruled out that enclosures are fully clamped due to the film is not rigidly fastened at the edges. Accordingly, the general solution for the vibration modes of solid rectangular plates as Wu *et al.* [19] stated is,

$$W_n(x,y) = \sum_{m=-\infty}^{\infty} [A_n J_{n-m}(kx) J_m(ky) + B_n I_{n-m}(kx) I_m(ky)] \sin\left(\frac{m\pi}{2}\right)$$

where B_n is a constant, J_n is the Bessel function and I_n is the first kind modified Bessel function. Considering the boundary conditions for a free-vibration square enclosure fully simply supported with edge length of L,

$$W(x=0)=0; W(x=L)=0;\ W(y=0)=0; W(y=L)=0;$$
$$\frac{\partial^2 W}{\partial x^2}\Big|_{x=0;x=L}=0;\ \frac{\partial^2 W}{\partial y^2}\Big|_{y=0;y=L}=0;$$

It is possible to state that the vibration mode function could be rewritten as follows,

$$\begin{aligned} W_{n-m} = (A_n\{J_{n-m}(kx) + J_{n-m}[k(L-x)]\}\{J_m(ky) \\ + J_m[k(L-y)]\} \\ + B_n\{I_{n-m}(kx) + I_{n-m}[k(L-x)]\}\{I_m(ky) \\ + I_m[k(L-y)]\}) \sin\frac{m\pi}{2}\cos\frac{n\pi}{2} \end{aligned}$$

where m and n are odd and even numbers for nontrivial solutions. In the cuboid shape $m = \frac{n}{2}$ and

$$\det\begin{vmatrix} J_{n-m}(kL) & I_{n-m}(kL) \\ J''_m(kL) & I''_m(kL) \end{vmatrix} = 0$$

where J''_m and I''_m are the second derivatives of the Bessel function and the first kind modified Bessel function, respectively.

3. EXPERIMENTAL SET-UP

Experimental measurements were carried out in an anechoic chamber 8 × 6 × 3 m^3 in dimensions, simulating free field conditions. To keep the CO_2 gas inside the cuboid, a 0.16 m edge cubic structure that was covered with a thin plastic film. The cuboid was hung from a frame to avoid the ground effect, as can be seen in Figure 3. Due to the possibility of radiation from the thin plastic film, a prepolarized free-field microphone was placed on the opposite side to that of incidence. The microphone was 1/2 ″ Type 4189 B&K and was located at a distance of 0.11 m from the cuboid. A GENELEC 8040A directional sound source emitting continuous white noise, located 1 m from the cuboid, was used to consider that the wave impacting the cuboid could be considered as a plane wave. A three-dimensional robotic measurement system was used to position and acquire the signal. With this system, it was possible to sweep a 3D grid of acquisition points located on any path within the chamber.

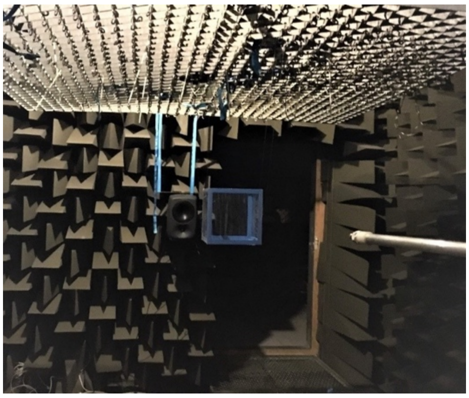

Figure 3. Experimental set-up. A hanging CO_2 filled cuboid in the anechoic chamber

4. RESULTS AND DISCUSSION

Figures 4(a) and 4(b) show the experimental results for the normalized sound intensity ($I/I_{incident}$) at 2431 Hz and 3251 Hz respectively. The effect of sound wave focusing is clearly visible for both frequencies. These frequencies correspond to the frequencies at which standing waves occur inside the CO_2 cuboid and can be calculated from the expression $f = \frac{nc_2}{2L}$. In our case, the side of the cuboid is L = 0.16 m and the velocity of the sound in CO_2 is c_{CO2} = 260 m/s, so for n = 3 the frequency at which standing waves occur inside the cuboid is 2437.5 Hz and for n = 4, the frequency is 3250 Hz. Analytical membrane resonance values are compared with experimental ones in Table 1. The study of free-vibrations on rectangular plates has been a field of study of interest. These experimental results allow comparing the Navier-type solution with the Bessel function method. Navier's analytical solution is more complex than Bessel's one. This traditional calculation method proposed in the 19th century [22], was one of the first methods in obtaining a solution based on trigonometric series. On the other hand, Bessel function method is direct, simpler and, as stated by Wu *et al.* [19], it is also a highly accurate method. In Table 1 it can be seen how the solution obtained with the Bessel method is closer to the experimental one. Therefore, it can be affirmed that the Navier-type method, despite being a classic method of resolution, is relegated by the Bessel method. These standing wave frequencies are in good agreement with experimental ones. Figure 4 shows two normalized planes for both 3rd order (Fig.4a) and 4th order (Fig.4b) experimental cases respectively. We found 2 main differences between both planes of normalized intensity. Focal size is one of these differences and the other is the amount of energy transmitted through the cuboid. The maximum normalized intensity for the 3rd order case is around 8 a.u, in the 4th order case is 15 a.u. As the energy intensity is directly proportional to the frequency, higher intensity levels are obtained at higher resonance frequencies. Figure 5 shows the eigenvalues solutions, that is, how the shape of the energy is distributed in resonance modes. As mentioned above, at these frequencies the CO_2 inside the cuboid vibrates with its maximum amplitude at the ends of the cuboid, causing maximum energy transmission.

Table 1. Analytical eigenfrequencies in Bessel and Navier-type solutions comparison with experimental resonance frequency of the cuboid enclosure.

Order	Bessel Solution (Hz)	Navier-type Solution (Hz)	Experimental (Hz)
3	2367	3032	2432
4	2972	3389	3255

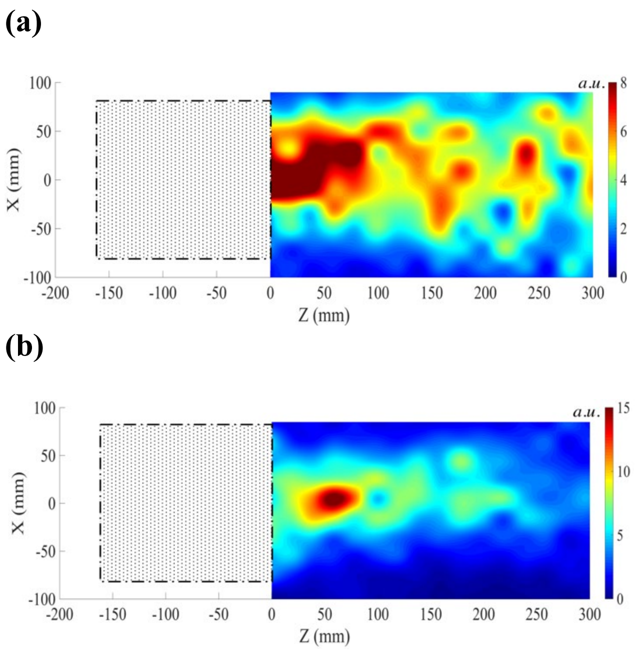

Figure 4. Experimental normalized sound intensity ($I/I_{incident}$) maps in arbitrary units (a.u.) for (a) 2432 Hz (b) 3255 Hz**.**

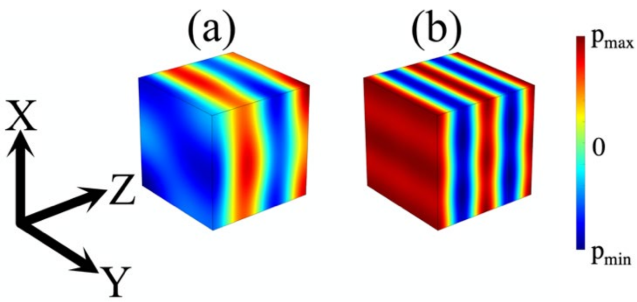

Figure 5. CO_2 vibration modes inside the cuboid for (a) n = 3 and (b) n = 4.

5. CONCLUSIONS

Devices with concave or convex surfaces are well known as a potential lens as in optics as in acoustics. Flat surfaces are not well recognized with capability of focus wave. The article shows how a flat device can be used as a lens using standing waves that are created in a wavescaled gas layer.

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AUTHOR CONTRIBUTION

Constanza Rubio: Conceptualization, Investigation, Formal analysis, Writing - original draft, Writing - review & editing, Supervision, Project administration. **Daniel Tarrazó-Serrano:** Conceptualization, Investigation, Formal analysis, Visualization, Writing - original draft, Writing - review & editing review & editing. **Oleg V. Minin:** Conceptualization, Investigation, Formal analysis, Supervision, Writing - original draft, Writing - review & editing. **Antonio Uris:** Conceptualization, Investigation, Formal analysis, Writing - original draft, Writing - review & editing. **Igor V. Minin:** Conceptualization, Investigation, Formal analysis, Supervision, Writing - original draft, Writing - review & editing.

Conflict of Interest: The authors declare that they have no conflict of interest.

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