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Additional Information

Using Integer Linear Programming to minimize the embodied CO_2 emissions of the opaque part of a façade

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Abstract

Buildings are responsible for about 36% of the CO_2 emissions in Europe but there is a significant potential to reduce these emissions. This paper deals with the embodied CO_2 emissions of the opaque part of a façade, which include the life cycle of any material used in its construction: excavation, processing, construction, operation, maintenance, demolition and waste or recycling.

With the aim of minimizing such embodied CO_2 emissions, an Integer Linear Programming problem is presented, in which CO_2 emissions are minimized depending on other parameters involved in the construction of the façade, like the maximal thermal transmittance allowed by current legislation, thickness of the wall, budget, availability of materials for the different layers of the wall, etc.

The paper also shows a case study based on a constructive solution for the opaque part of the envelope defined by up to six layers, with more than 1.1 million possible combinations. This case study considers seventy scenarios depending on maximal allowed thermal transmittances and thickness intervals for five different technologies applied to the structural element of the wall. Results show that an adequate selection of materials can reduce the embodied CO_2 emissions of the opaque part of the envelope up to 78.5% for similar values of transmittance and thickness.

Keywords: Embodied CO_2 emissions, thermal transmittance, façade, integer linear programming, minimization.

1. Introduction

Among other targets, the 2030 climate and energy framework of the European Commission [1] has at least 40% cut in greenhouse gas emissions (from 1990 levels), and at least 32.5%

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improvement in energy efficiency by 2030. Moreover, in November 2018, the European Commission presented a long-term strategic vision, which contains the aim of cutting al least 80% in greenhouse gas emissions (from 1990 levels) by 2050 [2]. On the other hand, buildings account for 36% of the EU's CO_2 emissions and 40% of its energy consumption [3]. Therefore, the building sector is a key sector to achieve the EU's climate and energy targets, while becoming more sustainable in terms of use of natural resources, energy consumption connected to construction and operation, use of environmentally damaging materials, generation of waste, the reuse of materials and building parts, and the mitigation of greenhouse gas emissions, the latter especially in urban areas [4].

As buildings become more energy efficient, the relative proportion of embodied energy and associated carbon emissions arising during the building lifecycle increases [5]. But as environmental imperatives require reduction of building carbon footprints, highly insulated building envelopes have become more common. The increased insulation levels reduce the operational energy demand, but the additional embodied energy investment can increase the buildings' overall environmental impact [6].

Odum et al. [7] introduced the concept of energy as the direct and indirect needed quantity of solar energy to obtain a product by a given process or to renew a consumed resource. It is basic concept in the Life Cycle Assessment (LCA), which is a widely recognized and accepted method for the assessment of burdens and impacts throughout the lifecycle of a building [8]. LCA evaluates all resource inputs, including energy, materials and water, in order to calculate the environmental impacts of a building at either the material, product or whole building level. There exist several LCA databases and inventories available worldwide as the University of Bath's Inventory of Carbon and Energy (ICE) [9], the Ecoinvent database [10] or the Casaclima/Klimahaus database [11].

Due to errors in the estimation or wrong assumptions about life time and/or transportation, values can be quite different or even show high uncertainty [12]. A standardization of LCA in order to take into account the differences on the energy mix as well as the overall building design is suggested by Chastas et al. [13]. Another crucial aspect to consider is the reuse/recycling rate of the waste [14,15].

It is important to highlight how the primary energy consumed in construction represents between 10% and 30% of the energy required for its functioning during the lifecycle of the building. The embodied energy in a building can be estimated by taking into account the weight of the required materials, the data base information and a pre-dimensioning through a program that can calculate quantities and estimate environmental costs [16]. Five main materials (cement, sand, coarse aggregate, hollow concrete blocks and reinforcement bars) are responsible for the main part of the embodied energy and CO_2 emissions [17]. There is a clear and feasible possibility of reducing the embodied energy and CO_2 in the design phase as well as in the construction phase, through a careful selection or substitution of sustainable building materials [18,19]. Furthermore, thermal diffusivity can provide useful information for evaluating the thermal mass behavior, the speed of energy interchange and accumulation capacity.

Nomenclature

n number of layers of the façade

m_i number of materials available for layer *i*

r_{j_i} number of thicknesses available for material *j* of layer *i*

$k_{i,j,k}$ number of *kg* that weighs 1m^2 of material *j* with type of thickness *k* available for layer *i*

$kco2_{i,j}$ number of *kg* of embodied CO_2 for each *kg* of material *j* available for layer *i*

$t_{i,j,k}$ thickness corresponding to material *j* with type of thickness *k* available for layer *i* (*m*)

$c_{i,j,k}$ cost of placing in layer *i* 1m^2 of material *j* with type of thickness *k* available for layer *i* (€)

$ct_{i,j,k}$ time of placing in layer *i* 1m^2 of material *j* with type of thickness *k* available for layer *i* (*s*)

$mc_{i,j,k}$ maintenance cost for 1 m^2 of material *j* located in layer *i* with type of thickness *k*, given a period of time (€)

U_{max} maximum thermal transmittance allowed for the opaque part of the façade ($Wm^{-2}K^{-1}$)

B_{max} maximum budget to construct 1m^2 of the opaque part of the façade (€)

T_{min} minimum thickness allowed for the façade (*m*)

T_{max} maximum thickness allowed for the façade (*m*)

CT_{max} maximum allowed construction time for 1m^2 of the opaque part of the façade (s)

MC_{max} maximum allowed maintenance cost for 1m^2 of the opaque part of the façade, given a period of time (€)

h_{int} standard internal conductivity ($Wm^{-2}K^{-1}$)

h_{ext} standard external conductivity ($Wm^{-2}K^{-1}$)

λ_j thermal conductivity of material *j* ($Wm^{-1}K^{-1}$)

$x_{i,j,k}$ value 1 if layer *i* is made with material *j* and type of thickness *k*, and 0 otherwise

LCA Life Cycle Assesment

LP Linear Programming

ILP Integer Linear Programming

MILP Mixed Integer Linear Programming

On the other hand, more and more researchers use Linear Programming (LP) [20,21] to solve optimization problems in the field of building management, and particularly in problems related to environmental impact, energy consumption or energy efficiency. For instance, very recently, Brütting et al. [15] use LCA and Mixed Integer Linear Programming (MILP) in order to minimize the environmental impact comparing the reuse of construction waste for the production of structural elements with structural elements made with new materials. Mehrjerdi and Rakhshani [22], Eshraghi et al. [23], Wang et al. [24] and Zhang et al. [25] also use MILP to minimize energy supply costs in problems involving, among others, different energy storage systems. Moreover, Salandin et al. [26] use Integer Linear Programming (ILP) to minimize the cost of the refurbishment of a façade in order to improve its energy efficiency.

Although they are not so recent, it is also worth mentioning other papers that use LP in the field of energy and buildings: Bojic and Trifunovic [27] use LP to improve the thermal comfort given by a district-heating system, Privitera el at. [28] also use LP to reduce the cost of renewable energy technologies to comply with carbon emission reductions, Lindberg et al. [29] use MILP to optimize the investments and the operation of the energy technologies in the field of Zero Energy Buildings, and Ogunjuyigbe et al. [30] also use MILP with the aim of maximizing the sub-load points of photovoltaic solar energy that will be available at each period of the day, contemplating both the availability level of the solar irradiation and the user's quality of life.

This paper deals with a different and complementary approach with the embodied CO_2 emissions of the opaque part of a building's envelope, which include the life cycle of any material used in its construction: excavation, processing, construction, operation, maintenance, demolition and waste or recycling. With the aim of minimizing the embodied CO_2 emissions of this opaque part, an exact procedure based on ILP with binary variables is presented. The procedure obtains the optimal solution that contains the adequate materials and thicknesses for the different layers of the opaque part of the envelope, at the same time that other restrictions inherent to the construction of the façade are met, such as budget limitations, total thickness of the façade, current legislation about thermal transmittance, availability of materials, construction time, maintenance costs, and incompatibility between materials corresponding to different layers. The construction costs are not taken into account in this paper because only the opaque part of the envelope has been studied. Therefore, although the decision of the composition of the layers of a wall may depend on other factors (aesthetic, design, user decisions, etc.) that cannot be easily incorporated into a mathematical formulation, obtaining optimal solutions in different scenarios through the exact procedure presented here can help the designers to decide on the more appropriate solution according to selected or relevant restrictions.

In the chosen case study this exact procedure will find the best combination of materials for the different layers of the opaque part of the envelope defined by up to six layers, in order to compare the results obtained for five technologies (X-lam panels, wooden balloon-frame, MHM panels, reinforced concrete wall and brick wall) applied to the structural element of the wall (layer 3). This case study is flexible and representative for many constructive solutions. The combination of materials and thicknesses for each layer provides a total amount of 12,760,020 combinations for this 6-layer external wall. This fact shows the complexity of the presented case study, although incompatibilities between different materials corresponding to consecutive layers mean that only 1,121,760 of these combinations are feasible. Remarkably, the obtained results show that an adequate selection of materials can reduce the embodied CO_2 emissions of the opaque part of the envelope up to 78.5% for similar values of transmittance and thickness of the wall. But this reduction can reach 84.2% if very similar values of transmittance and thickness are not required.

The rest of the paper is organized as follows: Section 2 presents the definition and the ILP formulation of the problem studied here. Section 3 shows the characteristics of the case study based on a façade defined by up to six layers. Section 4 analyzes the results on different scenarios depending on thickness intervals and maximal allowed thermal transmittance. Finally, Section 5 gives some conclusions. An appendix shows all data corresponding to the used materials in the case study (conductivity, thickness, embodied CO_2 emissions, etc.).

2. Problem description and ILP formulation

This section models the problem of minimizing the embodied CO_2 emissions of the opaque part of a façade subject to certain construction restrictions as an ILP problem. Concretely, the embodied CO_2 emissions of $1m^2$ of the façade will be minimized, because if the façade has a total of S m^2 of opaque part, it is only necessary to multiply the result by *S*. For a better understanding of the formulation, some notations, the used variables and the parameters are first presented.

– Let n be the number of layers of the façade, which will be enumerated from inside to outside. Each layer $i \in \{1, \dots, n\}$ is made of one of the m_i different materials available for this layer, and given a layer $i \in \{1, \dots, n\}$, the material $j \in \{1, \dots, m_i\}$ is available in r_{j_i} different thicknesses.

- For each $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m_i\}$ and $k \in \{1, \dots, r_{j_i}\}$, the following parameters are considered:
  - $k_{i,j,k}$ number of kg that weighs $1m^2$ of material *j* with type of thickness *k* available for layer *i*.
  - $kco2_{i,j}$ number of kg of embodied CO_2 for each kg of material *j* available for layer *i*.
  - $t_{i,j,k}$ thickness corresponding to material *j* with type of thickness *k* available for layer *i* (note that *k* indicates the type of thickness, not the thickness).
  - $c_{i,j,k}$ cost of placing in layer *i* $1m^2$ of material *j* with type of thickness *k* available for layer *i*.
  - $ct_{i,j,k}$ time of placing in layer *i* $1m^2$ of material *j* with type of thickness *k* available for layer *i*.
  - $mc_{i,j,k}$ maintenance cost for 1 m^2 of material *j* located in layer *i* with type of thickness *k*, given a period of time.
- The total thickness of the external wall is comprised between bounds T_{min} and T_{max}.
- Let U_{max} be the maximum thermal transmittance allowed for the opaque part of the façade.
- Let B_{max} be the maximum budget allowed to construct $1m^2$ of the opaque part of the façade.
- Let CT_{max} be the maximum allowed construction time for $1m^2$ of the opaque part of the façade.
- Let MC_{max} be the maximum allowed maintenance cost for $1m^2$ of the opaque part of the façade, given a period of time.
- Given two consecutive layers, there may exist incompatibilities between some materials and thicknesses corresponding to these layers (see examples in [25]).
- The variables of the ILP problem are the binary variables $x_{i,j,k}$ whose value are 1 if layer *i* is made with material *j* and type of thickness *k*, and 0 otherwise, $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m_i\}$ and $k \in \{1, \dots, r_{j_i}\}$.
- As explained in [26], given a material *j*, with $j \in \{1, \dots, m_i\}$ for some $i \in \{1, \dots, n\}$, and let λ_j be its thermal conductivity, from the transmittance formula for a wall composed of several layers [31], the linear constraint to comply with the thermal transmittance upper bound for the opaque part of the façade is:

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}}\frac{t_{i,j,k}}{\lambda_j}x_{i,j,k} \geq \frac{1}{U_{max}}-\frac{1}{h_{int}}-\frac{1}{h_{ext}} \quad (1)$$

Where $1/h_{ext}$ and $1/h_{int}$ (m^2KW^{-1}) represent the standard external and internal resistances respectively for the air layers connected with the façade.

The problem of minimizing the embodied CO_2 emissions of $1m^2$ of the opaque part of a façade can be formulated mathematically as the following ILP problem, defined through Eqs. (2) to (10):

$$Minimize \quad \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} kco2_{i,j}\cdot k_{i,j,k}\cdot x_{i,j,k} \quad (2)$$

s.t.:

$$\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} x_{i,j,k} = 1 \quad \forall i \in \{1,\dots,n\} \quad (3)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} c_{i,j,k}x_{i,j,k} \leq B_{max} \quad (4)$$

$$T_{min} \leq \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} t_{i,j,k}x_{i,j,k} \leq T_{max} \quad (5)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}}\frac{t_{i,j,k}}{\lambda_j}x_{i,j,k} \geq \frac{1}{U_{max}}-\frac{1}{h_{int}}-\frac{1}{h_{ext}} \quad (6)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} ct_{i,j,k}x_{i,j,k} \leq CT_{max} \quad (7)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} mc_{i,j,k}x_{i,j,k} \leq MC_{max} \quad (8)$$

$$x_{i,j,k} + x_{(i+1),j',k'} \leq 1 \quad \forall\,(i,j,k-(i+1),j',k') - \ incompatible \quad (9)$$

$$x_{i,j,k} \in \{0,1\} \quad \forall\, i \in \{1,\dots,n\}, j \in \{1,\dots,m_i\}, k \in \{1,\dots,r_{j_i}\} \quad (10)$$

Where:

- Eq. (2) is the objective function, that is, the total embodied CO_2 emissions in *kg*.
- Eq. (3) ensures that each layer is made exactly of one material with a given thickness.
- Eq. (4) guarantees that the maximum budget is not exceeded.
- Eq. (5) restricts the total thickness of the façade within the established bounds.

- Eq. (6) ensures that the opaque part of the façade does not exceed the maximal allowed thermal transmittance.
- Eq. (7) restricts the time limit to construct 1 m^2 of the wall.
- Eq. (8) guaranties also a limit to the maintenance cost of 1 m^2 of the wall during a given period of time.
- Eq. (9) forbids to place a material *j'* with thickness *k'* in the next layer to the one (layer *i*) containing the material *j* with thickness *k* (this fact is denoted as (*i*,*j*,*k*-(*i*+1),*j'*,*k'*)-incompatibility). At most one of the two materials will appear in the corresponding layer.
- Finally, Eq. (10) defines the variables of the problem as binary.

Note that the formulation given above can be extended with new restrictions involving other parameters related to the opaque part of the façade, in order to fit as much as possible the real problem. The only condition is that these restrictions must be linear.

3. Case study

The case study is based on an envelope defined by up to six layers according to different technological and constructive solutions for generating interesting alternative scenarios. At the same time, each layer can have different thicknesses, depending on the material chosen for it. Layer 1 may be plaster, plaster clay, wood or gypsum plaster board. Layer 2 is the gap for building systems, which will not exist in case layer 1 is made of plaster gypsum and layer 3 is made of brick wall. Layer 3 constitutes the structural element of the wall with 5 different technologies and 8 options: X-lam panels, wooden balloon-frame, MHM panels (Massiv-Holz-Mauer), reinforced concrete wall or brick wall (normal, hi-performance, insulation-filled and clay).

X-lam panels, wooden balloon-frame and MHM panels (Massiv-Holz-Mauer) are different wood technologies. Each one has a different assembly system and the total mass of a wall depends on it. The balloon-frame is a system based on a light wood frame module. The MHM panels and the X-lam panels are massive systems based on multi plywood modules with different assembly system. In the X-lam panels the layers are glued, while in the MHM panels the layers are mechanically assembled with nails. Reinforced concrete wall and brick wall are traditional based on concrete or bricks and mortar.

Layer 4 provides the thermal insulation of the envelope made of natural (expanded cork, coconut fiber or sheep wool) or synthetic materials (projected polyurethane, extruded polystyrene, expanded polystyrene, mineral wool or nanoporous gel). Layer 5 is an intermediate layer connected with the final layer 6 and similar to layer 2. Finally, layer 6 is the exterior finishing, made of plaster cement, aluminum or wood board, limestone or face brick. Layer 5 will not exist in case layer 6 is made of plaster cement. Table 1 gives a summary of the different materials and thicknesses used for each one of the 6 layers (Table A in the Appendix provides full data), while Figure 1 shows the constructive detail of the envelope.

Table 1. Layers, materials and thickness used in the 6-layer facade of the case study.

LAYER	MATERIALS	OPTIONS
L1	Plaster	1.0, 1.3, 1.5, 1.8, 2.0 *cm*
	Gypsum plaster board	1.25 *cm*
	Plaster clay	1.0, 1.3, 1.5, 1.8, 2.0 *cm*
	Wood board	1.0, 1.6, 1.9 *cm*
L2	Light vent. Air gap	3.0, 5.0, 8.0, 10.0 *cm*
	Air gap without vent	3.0, 5.0, 8.0, 10.0 *cm*
	Mineral wool	3.0, 5.0, 8.0, 10.0 *cm*
L3	X-lam,	10.0, 20.0, 29.7 *cm*
	Balloon frame	20.0, 30.0 *cm*
	MHM	20.0, 25.0, 29.5, 34.0 *cm*
	Concrete wall	15.0, 30.0 *cm*
	Brick wall	25.0, 30.0 *cm*
	Brick wall hi-performance	25.0, 30.0 *cm*
	Brick wall insulation fitted	25.0, 30.0 *cm*
	Brick wall clay	25.0, 30.0 *cm*
L4	Projected Polyurethane	3.0, 4.0, 5.0, 6.0, 8.0, 9.0, 10.0, 12.0 *cm*
	Extruded polystyrene	3.0, 4.0, 5.0, 6.0, 8.0, 9.0, 10.0, 12.0 *cm*
	Mineral wool	3.0, 4.0, 5.0, 6.0, 8.0, 9.0, 10.0, 12.0 *cm*
	Expanded polystyrene	3.0, 4.0, 5.0, 6.0, 8.0, 9.0, 10.0, 12.0 *cm*
	Expanded cork	1.0, 2.0, 3.0 *cm*
	Coconut fiber	4.0 *cm*
	Sheep wool	10.0, 15.0 *cm*
	Nanoporous gel	1.0, 2.0, 3.0 *cm*
L5	Light vent. Air gap	3.0, 5.0, 8.0, 10.0 *cm*
	Air gap without vent	3.0, 5.0, 8.0, 10.0 *cm*
L6	Plaster cement	1.0,1.3,1.5, 1.8, 2.0 *cm*
	Aluminium board	0.15, 0.2 *cm*
	Limestone	1.0, 1.5 *cm*
	Wood board	1.0 *cm*
	Face brick	11.5, 12.0 *cm*

Taking into account the different thicknesses chosen for the different materials of each layer, a total amount of 12,760,020 combinations for this envelope can be obtained, but due to several incompatibilities between some materials, the number of possible combinations detected through Eq. (7) is reduced to 1,121,760. The established incompatibilities are the following:

- Plaster Gypsum and plaster clay in layer 1 are incompatible with the existence of layer 2.

- Gypsum plaster board and wood board in layer 1 are incompatible with the absence of layer 2.
- X-lam, balloon-frame, MHM and concrete wall in layer 3 are incompatible with the absence of layer 2, and also incompatible with plaster Gypsum or plaster clay in layer 1.
- Any kind of brick in layer 3 is incompatible with gypsum plaster board or wood board in layer 1.
- Air gap in layer 5 is incompatible with plaster cement and withewash in layer 6.
- The absence of layer 5 is incompatible with any kind of material in layer 6 except for plaster cement and whitewash.

The generic data of CO_2 emissions for this case study have been obtained from [9,11]. The values for vertical air flow close to the external and internal surfaces are $1/h_{ext} = 0.04\ m^2\ KW^{-1}$ and $1/h_{int} = 0.13\ m^2\ KW^{-1}$ respectively [32].

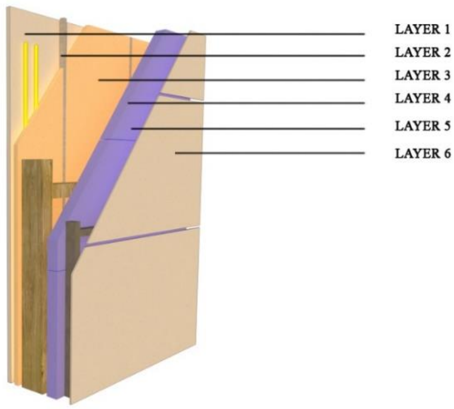

Figure 1. Constructive detail of the case study.

4. Results and discussion

A total of 70 scenarios have been considered, depending on reference maximum *U*-values (from 0.15 up to 0.94 according to European and Spanish norms) and thicknesses in intervals of 5 cm (from 15 up to 50 cm). Table 2 shows the amount of embodied CO_2 emissions produced by the optimal solution given by the ILP problem in each one of the 70 scenarios for the 5 different technologies used in the structural layer of the façade (layer 3): X-lam panels, wooden balloon-frame, MHM panels, reinforced concrete wall and brick wall. Data included in this table show the kilograms of embodied CO_2 emissions corresponding to the life cycle of all the materials used in $1m^2$ of wall for each optimal solution, depending on each structural element chosen, each allowed maximal thermal transmittance and each allowed interval of thickness. Existing requirement or recommendation for *U*-values within 31 European countries [33] are included in the first five columns of Table 2, and range from 0.15 $Wm^{-2}K^{-1}$ in Ljubliana (Slovenia) up to 0.20 $Wm^{-2}K^{-1}$ in Zürich (Switzerland), 0.30 $Wm^{-2}K^{-1}$ in

Prag (Czech Republic), 0.40 $Wm^{-2}K^{-1}$ in Marseille (France) and 0.50 $Wm^{-2}K^{-1}$ in Sofia (Bulgaria). The last five columns are related to the maximum allowed *U*-values in each one of the 5 winter climate zones in Spain (from less, A, to more sever, E) as indicated in [34], which are similar or matching to *U*-values of Mediterranean countries like Italy and Greece.

Therefore, 350 ILP problems have been solved, but not in all of them a feasible solution was obtained (those corresponding to low allowed thermal transmittances and small wall thicknesses simultaneously). A blank in a cell in Table 2 means that the corresponding ILP problem is infeasible. *Mathematica* 12 [35] has been used as ILP solver in these computational experiments and it has been run on a PC Intel® Core™ I7-9700 with 8 cores, 3GHz and 8GB RAM. The CPU times to obtain the optimal solutions vary from only 1.5 hundredths of a second in the best case to 20.3 hundredths of a second in the worst case. On the other hand, *Mathematica* needed between 13 and 14 seconds to conclude that the problem was infeasible in all cases of infeasibility. Taking into account that there are 12,760,020 solutions (feasible and unfeasible), and that an ILP problem has exponential complexity, the running times can be considered excellent and stable both for feasible and unfeasible problems. This gives more credit to the ILP procedure chosen to optimally solve the complex problem addressed here.

From the 70 studied scenarios for each material in layer 3, the three more interesting and representative scenarios considered are those corresponding to:

- The lowest embodied CO_2 among all the optimal solutions. In case of a tie, select the one corresponding to the lowest thermal transmittance allowed, and in case of a new tie, select the one corresponding to the interval of smaller thickness.
- The lowest embodied CO_2 among the optimal solutions in the minimum thickness interval. In case of a tie, select the one corresponding to the lowest thermal transmittance allowed.
- The lowest embodied CO_2 among the optimal solutions with the minimum maximal allowed *U*-value. In case of a tie, select the one corresponding to the interval of smallest thickness.

The cells corresponding to these three relevant scenarios (after selection in case of a tie) are emphasized in Tables 2 with gray shading, from darker to lighter respectively. Table 3 shows the best solution obtained by the ILP procedure for each one of these 15 cases. For each solution, the following values are given: the chosen material and thickness of each layer, the embodied CO2 emissions per m2, the exact thermal transmittance of the wall, the exact thickness of the wall and the CPU running time to solve the corresponding ILP problem. The best solution corresponding to each scenario is also emphasized with gray shading in Table 3.

Table 2
Minimum embodied *kg* of CO_2 in $1m^2$ of the external wall given an interval of thickness in *cm* (row) and a maximal allowed thermal transmittance in $Wm^{-2}K^{-1}$ (column), for 5 different technologies used in the structural element of the wall. A blank in a cell means that the corresponding ILP problem is infeasible.

X-lam	**0.15** Ljubliana	**0.20** Zürich	**0.30** Prag	**0.40** Marseille	**0.50** Sofia	**0.57(E)** Burgos	**0.66(D)** Madrid	**0.73(C)** Barcelona	**0.82(B)** Seville	**0.94(A)** Malaga
[15,20[			45.495	31.159	25.579	25.579	25.579	25.579	25.579	25.579
[20,25[		61.741	32.365	25.579	25.579	25.579	25.579	25.579	25.579	25.579
[25,30[		42.059	28.612	25.579	25.579	25.579	25.579	25.579	25.579	25.579
[30,35[	58.044	34.534	26.299	25.579	25.579	25.579	25.579	25.579	25.579	25.579
[35,40[	52.416	31.309	27.019	27.019	27.019	27.019	27.019	27.019	27.019	27.019
[40,45[	45.499	31.309	31.309	31.309	31.309	31.309	31.309	31.309	31.309	31.309
[45,50[	34.534	34.534	34.534	34.534	34.534	34.534	34.534	34.534	34.534	34.534
Balloon F.										
[15,20[										
[20,25[										
[25,30[			18.685	13.105	13.105	13.105	13.105	13.105	13.105	13.105
[30,35[		29.601	13.105	13.105	13.105	13.105	13.105	13.105	13.105	13.105
[35,40[	47.306	18.835	13.105	13.105	13.105	13.105	13.105	13.105	13.105	13.105
[40,45[	29.587	18.64	13.105	13.105	13.105	13.105	13.105	13.105	13.105	13.105
[45,50[	22.06	18.64	14.545	14.545	14.545	14.545	14.545	14.545	14.545	14.545
MHM										
[15,20[										
[20,25[										
[25,30[			66.529	60.949	60.949	60.949	60.949	60.949	60.949	60.949
[30,35[		77.445	60.949	60.949	60.949	60.949	60.949	60.949	60.949	60.949
[35,40[	95.15	66.679	60.949	60.949	60.949	60.949	60.949	60.949	60.949	60.949
[40,45[	77.439	66.648	60.949	60.949	60.949	60.949	60.949	60.949	60.949	60.949
[45,50[	69.904	66.484	62.389	62.389	62.389	62.389	62.389	62.389	62.389	62.389
Concrete										
[15,20[										
[20,25[			67.932	60.405	48.385	46.069	45.258	43.665	40.489	40.489
[25,30[			55.265	46.024	41.929	40.489	40.489	40.489	40.489	40.489
[30,35[		69.672	46.219	41.929	40.489	40.489	40.489	40.489	40.489	40.489
[35,40[	87.235	56.971	44.773	40.489	40.489	40.489	40.489	40.489	40.489	40.489
[40,45[	79.871	49.444	44.773	41.929	41.929	41.929	41.929	41.929	41.929	41.929
[45,50[	65.516	49.444	46.219	46.219	46.219	46.219	46.219	46.219	46.219	46.219
Brick										
[15,20[										
[20,25[										
[25,30[			48.59	37.43	23.24	23.24	23.24	16.94	16.94	12.08
[30,35[			27.804	20.96	13.908	10.344	9.624	9.624	9.624	9.624
[35,40[		45.513	18.912	12.787	9.702	9.624	9.624	9.624	9.624	9.624
[40,45[	54.894	29.544	15.354	11.142	10.604	10.604	10.604	10.604	10.604	10.604
[45,50[	54.894	18.579	15.354	12.564	12.104	12.104	12.104	12.104	12.104	12.104

Finally, Figure 2 shows, in the same order as Table 3, the constructive detail corresponding to each one of the 15 optimal solutions given in Table 3.

Table 3. Best solution for each constructive technology in layer 3 and for each one of the three selected scenarios.

Lowest CO_2	**X-lam**	**Balloon-frame**	**MHM**	**Concrete wall**	**Brick wall**
CO_2 *Kg/m*^{2}	25.579	13.105	60.949	40.489	9.624
U *Wm*$^{-2}$*K*$^{-1}$	0.387	0.298	0.298	0.382	0.500
Thickness *cm*	24.25	34.25	34.25	38.25	38
Time *s*	0.0312	0.0468	0.0468	0.0312	0.0312
L1	G.p board 1.25*cm*	G.p board 1.25*cm*	G.p board 1.25*cm*	G.p board 1.25*cm*	Plaster clay 1*cm*
L2	L. v. air gap. 3*cm*	L. v. air gap. 8*cm*	L. v. air gap. 8*cm*	L. v. air gap. 10*cm*	Absence
L3	XLAM 10*cm*	Balloon f. 20*cm*	MHM 20*cm*	Concrete w. 15cm	B.w. clay 25*cm*
L4	Cork 1*cm*	Cork 1*cm*	Cork 1*cm*	Cork 1*cm*	Cork 1*cm*
L5	L. v. air gap 8*cm*	No v. air gap. 3*cm*	L. v. air gap 3*cm*	L. v. air gap. 10*cm*	L. v. air gap 10*cm*
L6	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*
Lowest CO_2 for lowest thickness	**X-lam**	**Balloon-frame**	**MHM**	**Concrete wall**	**Brick wall**
CO_2 *Kg/m*^{2}	25.579	13.105	60.949	40.489	12.08
U *Wm*$^{-2}$*K*$^{-1}$	0.484	0.379	0.379	0.731	0.836
Thickness *cm*	19.25	29.25	29.25	24.24	29
Time *s*	0.0156	0.0156	0.0312	0.0156	0.312
L1	G.p board 1.25*cm*	G.p board 1.25*cm*	G.p board 1.25*cm*	G.p board 1.25*cm*	Plaster clay 1*cm*
L2	L. v. air gap. 3*cm*	L. v. air gap. 3*cm*	L. v. air gap. 3*cm*	L. v. air gap. 3*cm*	Absence
L3	XLAM 10*cm*	Balloon f. 20*cm*	MHM 20*cm*	Concrete w. 15cm	B.w. clay 25*cm*
L4	Cork 1*cm*	Cork 1*cm*	Cork 1*cm*	Cork 1*cm*	Cork 2*cm*
L5	L. v. air gap 3*cm*	No v. air gap. 3*cm*	No v. air gap. 3*cm*	L. v. air gap. 3*cm*	Absence
L6	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*	Plaster cement 1*cm*
Lowest CO_2 for lowest *U*	**X-lam**	**Balloon-frame**	**MHM**	**Concrete wall**	**Brick wall**
CO_2 *Kg/m*^{2}	34.534	22.06	69.904	65.516	54.894
U *Wm*$^{-2}$*K*$^{-1}$	0.146	0.146	0.146	0.142	0.149
Thickness *cm*	47.25	48.2	48.25	48.25	44
Time *s*	0.0468	0.0312	0.0468	0.0468	0.0468
L1	G.p board 1.25*cm*	G.p. board 1.25*cm*	G.p board 1.25*cm*	G.p board 1.25*cm*	Plaster clay 1*cm*
L2	L. v. air gap. 10*cm*	L. v. air gap 3*cm*	L. v. air gap. 8*cm*	Mineral wool 8*cm*	Absence
L3	XLAM 10*cm*	Balloon f. 20*cm*	MHM 20*cm*	Concrete w. 15cm	B.w. hi-p. 25*cm*
L4	Sheep wool 15*cm*	Sheep wool 15*cm*	Sheep wool 15*cm*	Sheep wool 15*cm*	P. polyureth. 12*cm*
L5	L. v. air gap 10*cm*	L. v. air gap 8*cm*	L. v. air gap. 3*cm*	L. v. air gap. 8*cm*	L. v. air gap. 5*cm*
L6	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*	Wood board 1*cm*

There are two clear and predictable trends: the higher the maximum allowed thermal transmittance, the lower the emission of CO_2, while very thin or very thick thicknesses imply high values of CO_2 emissions. Another trend is related with the combination of these factors: low allowed transmittance combined with thin thickness leads usually to a lack of solution (or to a very high value of CO_2 emissions). For X-lam, the value of 25.579 CO_2/m^2 is representative for the minimum thickness and minimum embodied CO_2 scenarios while the lowest U=0.15 $Wm^{-2}K^{-1}$ shows the highest value for embodied CO_2 (58.044 CO_2/m^2 for a 30-35 cm thickness and 34.534 CO_2/m^2 for a 45-50 *cm* thickness). A low value of just 13.105

CO_2/m^2 is reached with the balloon frame constructive solution for many *U*-values and thicknesses. This seems to be the best constructive solution, because in addition to the low emission value cited above, the dispersion of values is little, although there are no feasible solutions for thickness little than 25 *cm*. On the contrary, the solution with MHM seems to be the worst, with quite high values of embodied CO_2 emissions, from 60.949 *kg* CO_2/m^2 up to 95.15 *kg* CO_2/m^2, and also with no feasible solutions for thickness little than 25 *cm*. With a concrete wall, there are no solutions for the minimum thickness interval of 15-20 *cm* and values range from 40.489 *kg* CO_2/m^2 for the minimum thickness and minimum CO_2 scenarios (and for many other scenarios) up to 87.235 *kg* CO_2/m^2. An intermediate value of 65.516 *kg* CO_2/m^2 is reached for the minimum maximal allowed *U*-value. Finally, a brick wall constructive solution with 35-40 *cm* thickness and *U* little than 0.57 $Wm^{-2}K^{-1}$ is the best solution, with the lowest embodied CO_2 emissions, only 9.624 *kg* CO_2/m^2. But his constructive solution shows higher values, compared with the balloon frame, for other *U*-values and thicknesses.

A more detailed study of these 350 results has been carried out, comparing the behavior for each allowed wall thickness and for each allowed maximum *U*-value. The main results show that for similar good values of transmittance and similar thickness of the wall, an adequate selection of materials for the layers can save up to 78.5% embodied CO_2 emissions. For instance, the global optimal solution (see first rows of Table 3) corresponding to the use of balloon frame in layer 3 produces 13.105 kg/m^2 of embodied CO_2 emissions, with a very good thermal transmittance of 0.298 $Wm^{-2}K^{-1}$ and a thickness of 34.25 cm, while the optimal solution corresponding to the use of MHM in layer 3, produces 60.949 kg/m^2 of embodied CO_2 emissions, with exactly the same thermal transmittance and thickness. The choice of the first option implies 78.5% savings in embodied CO_2 emissions compared to the second option. The same difference in emissions is obtained for the optimal solutions with the same materials, under the condition of the lowest possible thickness (see the center of Table 3). In this case, the thickness is reduced to 29.25 *cm* and the transmittance is increased to 0.379 $Wm^{-2}K^{-1}$. But this difference in CO_2 emissions can even be increased if very similar values of transmittance and thickness are not required. Thus, the global optimal solution (see first rows of Table 3) corresponding to the use of brick wall in layer 3 (9.624 kg/m^2 of embodied CO_2 emissions) represents a saving of 84.2% compared to the use of MHM (60.949 kg/m^2 of embodied CO_2 emissions).

With respect to the best solution for each constructive option in layer 3 and for each one of the three main scenarios considered (see Table 3), the most relevant results are: 9.624 *kg*

CO_2/m^2 as lowest value ever, with a brick wall solution (U= 0.50 $Wm^{-2}K^{-1}$ and 38 *cm* thickness); 25.579 *kg* CO_2/m^2 as lowest value for lowest thickness, with X-lam solution (U= 0.484 $Wm^{-2}K^{-1}$ and 19.25 *cm* thickness), and 22.06 *kg* CO_2/m^2 as lowest value for lowest U, with balloon frame solution (U= 0.146 $Wm^{-2}K^{-1}$ and 48.2 *cm* thickness). Note that the minimum embodied CO_2 emission (9.624 kg CO_2/m^2) can be reached for U-values allowed in all Spanish climate zones (A to E), and that the minimum maximal allowed U-value, 0.15 $Wm^{-2}K^{-1}$, can be reached with any foreseen technology for layer 3 as structural element of the wall.

Considering the insulation materials (layer 4) for the best solutions, cork is the most usual insulating material, except for the scenarios with lowest CO_2 for lowest U, where sheep wool or projected polyurethane are foreseen, in order to get a much thicker wall. A light ventilated air gap with different thicknesses shows no embodied CO_2 emissions and is also frequently chosen for layer 2, except for the brick wall constructive solution, due to the incompatibility of both materials. Finally, the procedure chooses wood board for layer 6 in all solution given in Table 3, except for one with absence of both layers 2 and 5 in order to minimize thickness with brick wall in layer 3. Note that the absence of layer 5 is incompatible with wood board in layer 6.

Figure 2. Best solution for each constructive technology in layer 3 and for each one of the three selected scenarios.

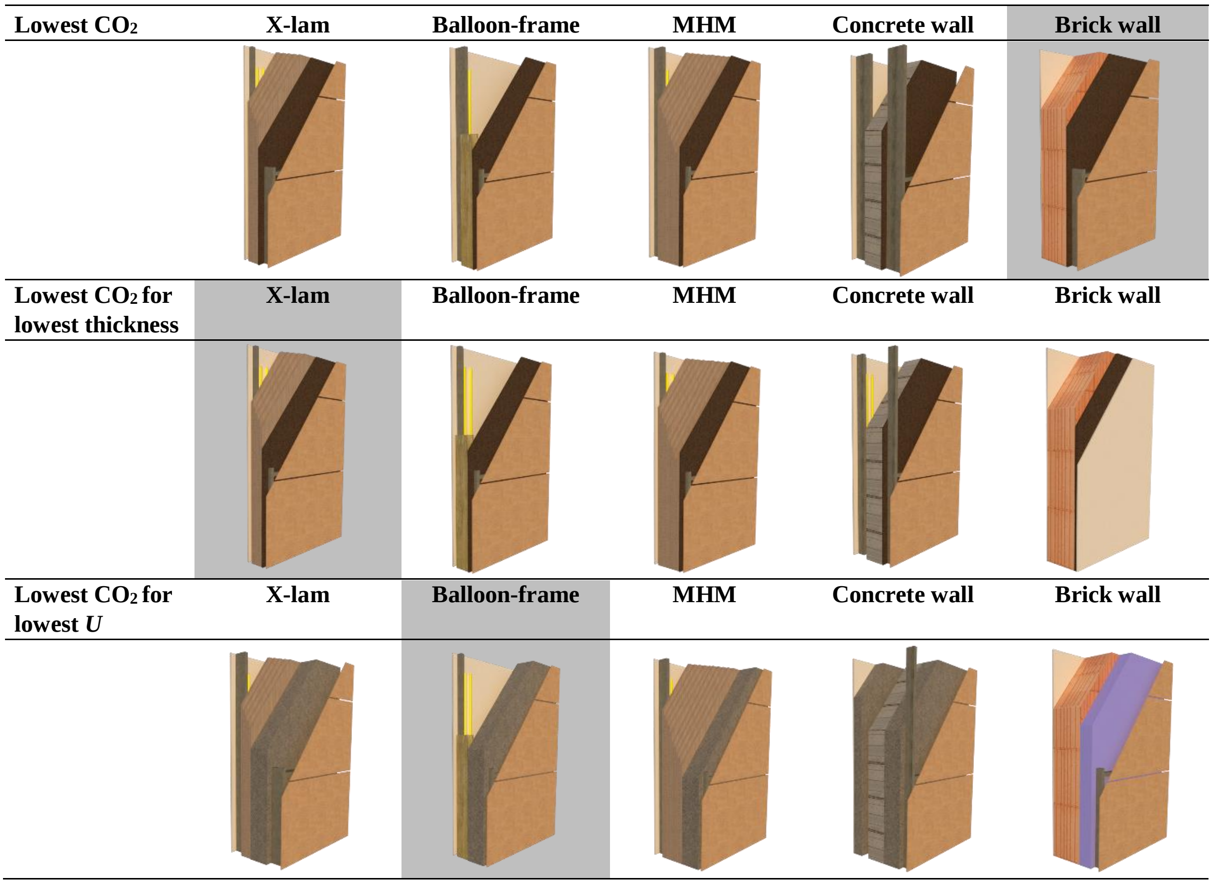

Figure 3 shows on a radar diagram the minimum embodied CO_2 emissions produced in each one of the 70 scenarios considered, independently of the constructive solution used in layer 3. That is, for each scenario of thickness interval and maximal allowed thermal transmittance, Figure 3 considers the best solution obtained by the ILP procedure among the 5 constructive technologies applied to the structural element of the wall. This kind of summarizing radar diagram can be very effective to jointly present complex multi-criteria scenarios. Values for the embodied CO_2 range from 0 up to 80 kg CO_2/m^2, and they can be read in the concentric circles. The *U*-values are represented as sector values (0.15 up to 0.94 $Wm^{-2}K^{-1}$) with the indication (A to E) of the Spanish climate zones. Each polygonal line, which can be open or closed, represents a specific thickness interval. The absence of a feasible solution, usually for low maximal allowed *U*-values, is represented by an open polygon like for lines/thicknesses 15-20, 20-25 and 25-30 *cm*. As it is easily observable in Figure 3, the best and almost identical behaviors (between 9.624 and 13.105 kg CO_2/m^2) are given for the lines/thicknesses 30-35, 35-40, 40-45 and 45-50 *cm* except for small values of *U* (less than 0.3 $Wm^{-2}K^{-1}$).

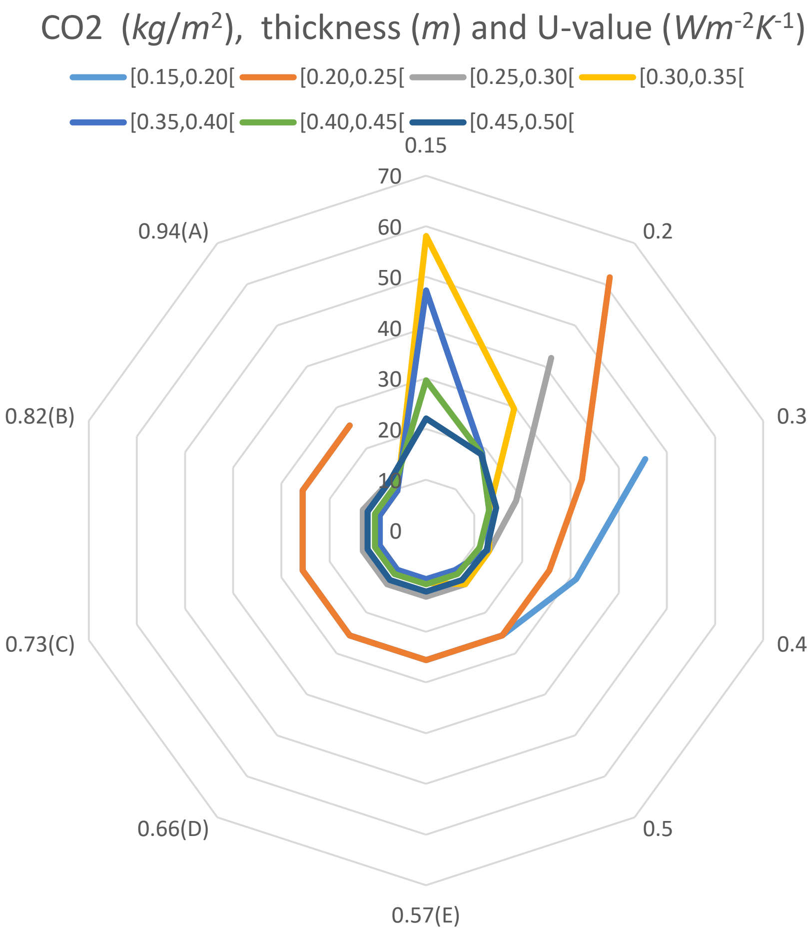

Figure 3. Comparing the embodied CO_2 (concentric circles) for thickness interval (lines) vs maximal allowed *U*-value (sectors).

All main trends are confirmed here: the higher the maximum allowed thermal transmittance, the lower the emission of CO_2, while very thin thicknesses imply high values of CO_2 emissions. The lowest U, 0.15 $Wm^{-2}K^{-1}$, suitable for the worst Alps climate conditions, is not feasible under a 30 *cm* thickness and always for quite high embodied CO_2 values.

Note that this case study does not take into account Eq. (4). That is, its aim has been to obtain the best combination of materials for the different layers of a wall, in order to minimize embodied CO_2 emissions depending on thermal transmittance bounds, regardless of the cost of these materials. It is worth noting that the case study considers thermal transmittance values corresponding to 6 European countries. The material and labor costs of these countries may differ greatly from each other. The limitation to a certain budget in one country may not really be a restriction in another country. On the other hand, meeting, for instance, a maximum thermal transmittance of 0.15 (Ljubljana) it would not be very reasonable to apply it in Spain which is a much hotter climatic zone. Therefore, the price for a solution in Ljubljana may not make sense in Spain. Thus, a comparison taking into account price limitations is not easy in this case study.

5. Conclusions

This paper presents an exact procedure based on ILP that minimizes the embodied CO_2 emissions of the opaque part of a building envelope. The application of this procedure to a representative and flexible case study shows that an adequate selection of constructive solutions and thermal insulation materials can reduce the embodied CO_2 emissions of the opaque part of the envelope up to 78.5% for similar values of transmittance and thickness of the wall. This reduction can even reach 84.2% if very similar values of transmittance and thickness are not required. In this way, the concept of embodied CO_2 emissions of a material, which include the emissions along its whole life cycle, is an appropriate parameter to measure the impact of the construction of a building for the sustainability of the planet.

Life cycle energy/carbon cost of building materials must be a focus in a global market analysis, considering the environmental cost like an economic cost, and therefore, trying to replace materials with high embodied energy and CO_2 emissions with others (new or with reuse of construction waste) that guarantee a more sustainable future.

As future research, this ILP procedure could be extended to study both embodied energy and CO_2 emissions of the whole building envelope, including the transparent part and the roof, although the number of variables and restrictions would grow considerably. The combined study of costs and embodied energy/CO_2 can rise to future standards for low embodied energy/carbon buildings, and support the adoption of LCAs as decision-making tools in order to attend the crucial debate of environmental optimal requirements of buildings.

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Appendix

Table A. Data of the materials for the six layers of the opaque part. Data obtained from databases [9,11].

Material	Conductivity $Wm^{-1}K^{-1}$	Thickness m	Density kg/m^2	$Kg\ CO_2/kg$	Variable
Layer 1 (interior)					
Plaster gypsum	0.6	0.01	13	0.17	$x_{1,1,1}$
	0.6	0.013	16.9	0.17	$x_{1,1,2}$
	0.6	0.015	19.5	0.17	$x_{1,1,3}$
	0.6	0.018	23.4	0.17	$x_{1,1,4}$
	0.6	0.02	26	0.17	$x_{1,1,5}$
Gypsum plaster board	0.21	0.0125	11.25	0.26	$x_{1,2,1}$
Plaster clay	0.81	0.01	13	0.02	$x_{1,3,1}$
	0.81	0.013	16.9	0.02	$x_{1,3,2}$
	0.81	0.015	19.5	0.02	$x_{1,3,3}$
	0.81	0.018	23.4	0.02	$x_{1,3,4}$
	0.81	0.02	26	0.02	$x_{1,3,5}$
Wood board	0.2	0.01	7.8	0.67	$x_{1,4,1}$
	0.2	0.016	12.48	0.67	$x_{1,4,2}$
	0.2	0.019	14.82	0.67	$x_{1,4,3}$
Layer 2 (air gap or wool)					
Light ventilated air gap	0.08	0.03	0	0	$x_{2,1,1}$
	0.09	0.05	0	0	$x_{2,1,2}$
	0.09	0.08	0	0	$x_{2,1,3}$
	0.1	0.1	0	0	$x_{2,1,4}$
No ventilated air gap	0.17	0.03	0	0	$x_{2,2,1}$
	0.18	0.05	0	0	$x_{2,2,2}$
	0.18	0.08	0	0	$x_{2,2,3}$
	0.18	0.1	0	0	$x_{2,2,4}$
Mineral wool	0.04	0.03	3.9	1.93	$x_{2,3,1}$
	0.04	0.05	6.5	1.93	$x_{2,3,2}$
	0.04	0.08	10.4	1.93	$x_{2,3,3}$
	0.04	0.1	13	1.93	$x_{2,3,4}$
Absence of this layer	-	0	0	0	$x_{2,4,1}$
Layer 3 (structural element)					
X-lam	0.13	0.1	49.5	0.42	$x_{3,1,1}$
	0.13	0.2	99	0.42	$x_{3,1,2}$
	0.13	0.297	147.015	0.42	$x_{3,1,3}$
Balloon-frame	0.13	0.2	19.8	0.42	$x_{3,2,1}$
	0.13	0.3	29.7	0.42	$x_{3,2,2}$
MHM	0.13	0.2	108	0.52	$x_{3,3,1}$
	0.13	0.25	135	0.52	$x_{3,3,2}$
	0.13	0.295	159.3	0.52	$x_{3,3,3}$
	0.13	0.34	183.6	0.52	$x_{3,3,4}$
Concrete wall	2.1	0.15	357	0.1	$x_{3,4,1}$
	2.1	0.3	714	0.1	$x_{3,4,2}$
Brick wall	0.5	0.25	300	0.18	$x_{3,5,1}$
	0.5	0.3	360	0.18	$x_{3,5,2}$
Brick wall hi-performance	0.133	0.25	182.5	0.18	$x_{3,6,1}$
	0.133	0.3	219	0.18	$x_{3,6,2}$
Brick wall insulation filled	0.14	0.25	200	0.18	$x_{3,7,1}$
	0.14	0.3	240	0.18	$x_{3,7,2}$
Brick wall clay	0.5	0.25	375	0.02	$x_{3,8,1}$
	0.5	0.3	450	0.02	$x_{3,8,2}$
Layer 4 (thermal insulance)					
Projected Polyurethane	0.03	0.03	1.2	4.3	$x_{4,1,1}$
	0.03	0.04	1.6	4.3	$x_{4,1,2}$
	0.03	0.05	2	4.3	$x_{4,1,3}$
	0.03	0.06	2.4	4.3	$x_{4,1,4}$
	0.03	0.08	3.2	4.3	$x_{4,1,5}$
	0.03	0.09	3.6	4.3	$x_{4,1,6}$
	0.03	0.1	4	4.3	$x_{4,1,7}$
	0.03	0.12	4.8	4.3	$x_{4,1,8}$

Table A. Continuation.

Material	Conductivity $Wm^{-1}K^{-1}$	Thickness *m*	Density kg/m^2	*Kg* CO_2/*kg*	Variable
Layer 4 (cont.)					
Extruded polystyrene	0.04	0.03	1.14	4.2	$x_{4,2,1}$
	0.04	0.04	1.52	4.2	$x_{4,2,2}$
	0.04	0.05	1.9	4.2	$x_{4,2,3}$
	0.04	0.06	2.28	4.2	$x_{4,2,4}$
	0.04	0.08	3.04	4.2	$x_{4,2,5}$
	0.04	0.09	3.42	4.2	$x_{4,2,6}$
	0.04	0.1	3.8	4.2	$x_{4,2,7}$
	0.04	0.12	4.56	4.2	$x_{4,2,8}$
Mineral wool	0.04	0.03	3.9	1.93	$x_{4,3,1}$
	0.04	0.04	5.2	1.93	$x_{4,3,2}$
	0.04	0.05	6.5	1.93	$x_{4,3,3}$
	0.04	0.06	7.8	1.93	$x_{4,3,4}$
	0.04	0.08	10.4	1.93	$x_{4,3,5}$
	0.04	0.09	11.7	1.93	$x_{4,3,6}$
	0.04	0.1	13	1.93	$x_{4,3,7}$
	0.04	0.12	15.6	1.93	$x_{4,3,8}$
Expanded polystyrene	0.035	0.03	0.9	4.17	$x_{4,4,1}$
	0.035	0.04	1.2	4.17	$x_{4,4,2}$
	0.035	0.05	1.5	4.17	$x_{4,4,3}$
	0.035	0.06	1.8	4.17	$x_{4,4,4}$
	0.035	0.08	2.4	4.17	$x_{4,4,5}$
	0.035	0.09	2.7	4.17	$x_{4,4,6}$
	0.035	0.1	3	4.17	$x_{4,4,7}$
	0.035	0.12	3.6	4.17	$x_{4,4,8}$
Cork	0.04	0.01	1.2	0.6	$x_{4,5,1}$
	0.04	0.02	2.4	0.6	$x_{4,5,2}$
	0.04	0.03	3.6	0.6	$x_{4,5,3}$
Coconut fiber	0.05	0.04	3.6	2.13	$x_{4,6,1}$
Sheep wool	0.04	0.1	3	2.15	$x_{4,7,1}$
	0.04	0.15	4.5	2.15	$x_{4,7,2}$
Nanoporous aerogel	0.013	0.01	1.5	4.2	$x_{4,8,1}$
	0.013	0.02	3	4.2	$x_{4,8,2}$
	0.013	0.03	4.5	4.2	$x_{4,8,3}$
Layer 5 (air gap)					
Light ventilated air gap	0.08	0.03	0	0	$x_{5,1,1}$
	0.09	0.05	0	0	$x_{5,1,2}$
	0.09	0.08	0	0	$x_{5,1,3}$
	0.1	0.1	0	0	$x_{5,1,4}$
No ventilated air gap	0.17	0.03	0	0	$x_{5,2,1}$
	0.18	0.05	0	0	$x_{5,2,2}$
	0.18	0.08	0	0	$x_{5,2,3}$
	0.18	0.1	0	0	$x_{5,2,4}$
Absence of this layer	-	0	0	0	$x_{5,3,1}$
Layer 6 (exterior)					
Plaster cement and	0.8	0.01	18	0.16	$x_{6,1,1}$
whitewash	0.8	0.013	23.4	0.16	$x_{6,1,2}$
	0.8	0.015	27	0.16	$x_{6,1,3}$
	0.8	0.018	32.4	0.16	$x_{6,1,4}$
	0.8	0.02	36	0.16	$x_{6,1,5}$
Aluminum sheet	200	0.0015	4.2	6.55	$x_{6,2,1}$
	200	0.002	5.6	6.55	$x_{6,2,2}$
Limestone	1.3	0.01	17	0.18	$x_{6,3,1}$
	1.3	0.015	25.5	0.18	$x_{6,3,2}$
Wood board	0.15	0.01	6.3	0.23	$x_{6,4,1}$
Face Brick	0.8	0.115	207	0.23	$x_{6,5,1}$
	0.8	0.12	216	0.23	$x_{6,5,2}$