An Integer Linear Programming approach to minimize the cost of the refurbishment of a façade to improve the energy efficiency of a building

Short title: Improving the energy efficiency of a façade

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ABSTRACT

Buildings account 40% of the EU's total energy consumption. Therefore, they represent a key potential source of energy savings to fight, among others, against climate change. Furthermore, around 54% of the buildings in Spain date back before 1980, when no thermal regulation was available. The refurbishment of a façade of an old building is usually the most effective way to improve its energy efficiency, by adding layers to the external envelope in order to reduce its thermal transmittance.

This paper deals with the problem of minimizing costs for the thermal refurbishment of a façade with thickness and thermal transmittance bounds and with an intervention both on the opaque part (wall) and the transparent part (windows). Among thousands, even millions of combinations of materials and thicknesses for the different layers to be added to the opaque part, types of frame and combinations of glasses and air chambers for the transparent part, the aim is to choose the one that minimizes the cost without violating any restriction imposed to the thermal refurbishment, in particular the current energy efficiency regulations in the zone.

To optimally solve this problem, it will be modelled as an Integer Linear Programming problem with binary variables. The case study will be Building 1B of the School for Building Engineering of the Polytechnic University of Valencia, Spain. It was built in the late 1960s and has had a very inefficient energy consumption record. The optimal solution will be found among more than 6 million feasible solutions.

KEYWORDS Operations research and management sciences, mathematical programming, energy efficiency, refurbishment, thermal transmittance, façade.

1 | INTRODUCTION

Energy efficient buildings are entities whose designs, materials, constructive solutions, orientations and building systems provide a significant reduction of energy needs. Furthermore, a nZEB (nearly Zero Energy Building) shows a very high energy

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performance with a nearly zero or very low amount of required energy, covered by on-site or nearby produced renewable sources [1].

The EU building stock is quite heterogeneous and mostly composed by residential buildings. The rate goes from 60-65% in Romania, Lithuania or Czech Republic up to around 85% in southern Europe (Cyprus, Malta and Italy). Around the half of the buildings were built before 1970, before any thermal regulation as well as in some countries (Cyprus, Spain or Ireland) the stock of recent dwellings, i.e. built after 2000, represents a significant share with more efficient standards. Furthermore, the type of dwelling has an impact on the space heating energy performances since different insulation characteristics imply different specific space heating consumption (due to different wall area in contact with the outdoor) [2,3].

In the EU, buildings account for 40% of total global energy consumption and have direct and important environmental impacts, ranging from the use of raw materials for their construction and renovation to the consumption of natural resources, like water and fossil fuels, and the emission of harmful substances like greenhouse gas emissions, which represent 36% [4].

With the aims to accelerate the cost-effective renovation of existing buildings and to promote smart technologies in buildings, the Directive 2018/844/EU of the European Parliament and of the Council of 30/05/2018 amended Directive 2010/31/EU on the energy performance of buildings and Directive 2012/27/EU on energy efficiency [5]. New buildings must meet minimum standards and buildings owned and occupied by public authorities should achieve nearly zero-energy status by 31 December 2018 and other new buildings by 31 December 2020. EU countries must set optimal minimum energy performance requirements.

The International Energy Agency (IEA) and the European Commission (EC) are attempting to achieve an 80% reduction in global emissions by 2050 [6] and specific targets on 2020 and 2030.

The 2020 package is a set of binding legislation to ensure the EU meets its climate and energy targets for the year 2020 [7] and can be summarized in three key targets: 20% cut in greenhouse gas emissions (from 1990 levels), 20% of EU energy from renewables and 20% improvement in energy efficiency. The 2030 climate and energy framework [8] builds on the 2020 climate and energy package and sets other three key targets for the year 2030: at least 40% cut in greenhouse gas emissions (from 1990 levels), at least 27% share for renewable energy and finally at least 27% improvement in energy efficiency. Finally, the European Commission is looking at cost-efficient ways to make the European economy more climate-friendly and less energy-consuming [9]. Its low-carbon economy roadmap suggests that by 2050, the EU should cut greenhouse gas emissions to 80% below 1990 levels with sectors contributing. Milestones to achieve this are 40% emissions cuts by 2030 and 60% by 2040 so that the low-carbon transition seems to be feasible and affordable.

Attending this complex and changing framework, the refurbishment of existing buildings is often considered a crucial way to reduce energy use and CO_2 emissions in the building stock [10]. The lifecycle of a building can be significantly extended by effective refurbishment**.** Buildings refurbishment supports excellent opportunities to reduce energy consumption in buildings as well as encourages other sustainable refurbishment principles implementation like public healthcare, environment protection, rational use of resources, information about sustainable refurbishment dissemination and stakeholder groups' awareness [11]. Building refurbishment is the result of complex decision processes, involving many actors [12].

Energy efficiency is directly linked with bioclimatic architecture (shape and orientation of the building, solar protections, passive solar systems), a high performing building envelope (thorough insulation, high performing glazing and windows, air-sealed construction, avoidance of thermal bridges) and high performance controlled ventilation (mechanical insulation, heat recovery) as well as efficient building systems [13,14]. For instance, in the Autonomous Province of Bolzano-Bozen, South Tyrol, Italy, Casaclima-Klimahaus is an energy efficiency rating system for building introduced in 2002 [15] to improve the quality and the building performance of the local building stock.

The *U*-value, or thermal transmittance (reciprocal of *R*-value), is the rate of transfer of heat through a structure (which can be a single material or a composite), divided by the difference in temperature across that structure. The units of measurement are Watts per meter squared Kelvin ($Wm^{-2}K^{-1}$). The lower the *U*-value is, the better insulated a structure is. Workmanship and installation standards can strongly affect the thermal transmittance. If insulation is fitted poorly, with gaps and cold bridges, then the thermal transmittance can be considerably higher than desired. Thermal transmittance takes heat loss due to conduction, convection and radiation into account. Usually, the *U*-value for a specific building construction is stated in the construction drawings (BIM, Building Information Modeling). If not, the *U*-value can be calculated by finding the reciprocal of the sum of the thermal resistances of each material making up the building element in question. Note that, as well as the material resistances, the internal and external faces also have resistances, which must be added. These are fixed values according to local norms. The thermal transmittance of a wall consisting of *n* layers is given by Eq. (1) as described by McMullan [16]:

$$U = \frac{1}{\frac{1}{h_{int}} + \sum_{i=1}^{n} \frac{t_i}{\lambda_i} + \frac{1}{h_{ext}}} \qquad (1)$$

Where λ_i ($Wm^{-1}K^{-1}$) and t_i (m) represent the thermal conductivity and the thickness respectively of layer *i*, and $1/h_{ext}$ and $1/h_{int}$ (m^2KW^{-1}) represent the standard external and internal conductivity respectively for the air layers connected with the envelope.

U-values across Europe can be quite different depending on the climate conditions and the national regulation as well as the chosen standards for comfort. For instance, *U*-values for reference buildings in the north of Italy range from 0.29 up to 0.76 $Wm^{-2}K^{-1}$ [17]. *U*-value requirements or recommendations across Europe for wall, floor and roof are quite different. EURIMA [18] developed a table that provides a general description of existing requirements or the recommendation of *U*-values in 100 cities within 31 countries in Europe. The strictest requirement is given in Ljubljana with U= 0.15 $Wm^{-2}K^{-1}$. Information about climate zones and recommended *U*-values in Spain, where the presented case study is located, can be checked in [19].

This work presents a mathematical procedure to minimize costs for the refurbishment of façades of old buildings, so that these façades can meet the new regulations involving the wall's thermal transmittance, with the least possible investment, and also taking into account other construction limitations like thickness of the new façade and availability of materials. Therefore, this work can be considered a contribution in the achievement of the EU targets cited above on reduction of energy consumption.

The procedure involves the intervention both on the opaque part (wall) and on the transparent part (windows). Among thousands, even millions of combinations of materials and thicknesses for the different layers to be added to the opaque part, types of frame and combinations of glasses and air chambers for the transparent part, the aim is to choose the one that minimizes the cost, without violating any restriction imposed to the thermal refurbishment, in particular, the current energy efficiency regulations in the zone. The minimization problem will be modelled as an Integer Linear Programming (ILP) problem with binary variables. ILP is a particular case of Linear Programming (LP) [20-22] in which all variables are integer. It is worth remembering that, basically, a LP problem consists in the maximization or minimization of a function that depends linearly on a set of variables, which at the same time are interrelated through a set of linear constraints. Moreover, while a LP problem with all variables continuous has polynomial complexity, an ILP problem (as the case studied here) and the mixed case in which the LP problem has both continuous and integer variables (MILP) have exponential complexity.

The rest of the paper is organized as follows. Section 2 provides some relevant works in the literature addressing optimization problems involving energy saving in building construction, either including the composition (layers) of a façade, the use of LP or both. The definition of the general problem presented here and its ILP formulation are given in Section 3. Section 4 presents the case study: Building 1B of the School for Building Engineering of the Polytechnic University of Valencia, Spain, built in the late 1960s, with a very bad thermal transmittance of its walls. The general characteristics of the interventions are also given in this section. Section 5 shows and analyses the obtained results for different scenarios. Finally, Section 6 presents the conclusions. An appendix is provided with an exhaustive exposition of all materials taken into account and their characteristics.

2 | LITERATURE REVIEW

Literature review shows that many optimization problems involving multilayered walls and windows are aimed to find cost-optimal energy performance levels, that is, the energy performance that leads to the lowest cost during the estimated economic life cycle of a building. In this context, the structure of an external wall and the type of windows are crucial elements to take into account in the cost-optimal analysis of the envelope, while also building systems could be considered. This is the case of the work by Kurnitski et al. [23] for buildings in Estonia, whose results have been implemented in the new Estonian regulation involving requirements for new buildings, and the works by Baglivo et al. [15], Corgnati et al. [17] and Congedo et al. [24], all of them aiming or having office and mono-residential buildings in Italy as a case study.

Other works focus only on the external walls or at most on the entire building envelope, but always with the aim of improving different parameters involved in energy efficiency, mainly using multi objective genetics algorithms. For instance, Baglivo and Congedo [14], which find best sequences of layers for precast external walls, Sambou et al. [25], that try to maximize both the thermal insulation and the thermal inertia, Baglivo et al. [26], that uses a parameter known as operative air temperature (TOP) to evaluate how each component of the envelope impacts the thermal behavior of the whole building, and Di Perna et al. [27], that demonstrate how walls with the same stationary and periodic thermal transmittance, but with different inertial area heat capacity, behave very differently from the point of view of indoor comfort.

Note that none of the works cited above takes into account the construction cost of the façade in the optimization problem, only construction works and components related to energy performance are considered [23], or the cost is calculated subsequently for the best solutions [14]. Like the works mentioned above, the one presented here aims at finding the best structure for a façade, but although its takes into account energy efficiency through the thermal transmittance, the objective will not be to optimize this efficiency but the cost of construction. Another substantial difference is that this work presents an exact procedure to find the optimal solution, not a heuristic one or a comparison between different options.

On the other hand, LP has proved its effectiveness to model many real optimization problems [28], and it is being increasingly applied in the field of building construction, particularly to solve optimization problems involving energy saving. For instance, Privitera et al. [29] demonstrate how LP can be applied to assist in the choice of renewable energy technologies for use in buildings to meet CO2 emissions reduction. Ashouri et al. [30] tackle the problem of the optimal selection and sizing of a smart building system (heating and cooling systems, thermal and electrical storages, and renewable energy sources) by using MILP. Lindberg et al. [31] also use a MILP problem to investigate cost-optimal solutions for Zero Energy Buildings for different energy indicators with a financial perspective. MILP is also used by Ogunjuyigbe et al. [32] to solve a problem of allocating electrical power to appliances in residential building with intermittent photovoltaic source. They try to maximize the sub-load points that will be available at each period of the day. Soler et al. [33] use ILP with binary variables to model the problem of minimizing the thermal transmittance of an external wall under certain restrictions. Finally, Salandin and Soler [34] present an ILP formulation of the problem of minimizing the cost of construction of the opaque part of a new building's external wall, including a restriction on energy efficiency.

The work presented here can be considered a generalization of the last cited paper in several aspects. Part of the refurbishment of a façade can be considered as "pasting a new wall" to the old wall, but taking into account all the characteristics of the last one. The main difference between both works is that, as stated above, here the procedure involves the intervention both on the opaque part and on the transparent part (frame and combinations of glasses and air chambers), which increases the types and number of variables and restrictions. Moreover, variables of both opaque and transparent part are related through thickness restrictions. Note that, unlike other papers cited above, in order to improve the global thermal behaviour of the façade, this work does not take into account internal loads and the internal mass, due to its limited dynamic interaction with the outdoor environment, as described in [27]. Finally, this works takes into account a very important restriction not considered in [34]: to avoid rising damp, the first lower meter of the opaque part of the façade can contain different thermal insulation material than the rest of the façade. Again, this fact increases the types and number of variables and restrictions.

3 | DEFINITION AND ILP FORMULATION OF THE PROBLEM

The problem of minimizing the cost of refurbishment of a façade to comply with current energy efficiency regulations and other restrictions is formulated in this section as an ILP problem. For a better understanding of the formulation, some notations, the used variables and parameters are presented before.

- Let $S = S_w + S_o$ be the total surface in m^2 of the façade, where S_w is the surface corresponding to the windows *and* S_o is the surface corresponding to the opaque part of the façade. Furthermore, $S_o = S_o^l + S_o^u$, where S_o^l is the surface corresponding to the first lower meter of the opaque part of the façade, and S_o^u is the upper surface to the first lower meter of the opaque part of the façade. Those parts can show different thermal insulation materials in order to avoid rising damp. In the same way, $S_w = S_w^f + S_w^g$, where S_w^f is the total surface of the windows corresponding to the frame, and S_w^g is the total surface of the windows corresponding to the glass. These four surfaces will be also taken into account on preliminary calculations such as computing the price and the transmittance of the different types of windows to cover surface S_w, or to determine the availability or the price of certain materials for the wall.
- Let μ be the number of layers of the original façade, let σ be the sum of the quotients t_i/λ_i where λ_i ($Wm^{-1}K^{-1}$) and t_i (m) represent the thermal conductivity and the thickness respectively of each layer i of the original façade, and let τ be the sum of these thicknesses t_i, $i \in \{1, \dots, \mu\}$.
- Let n be the number of layers to be added to the external side of the original façade, which will be numbered from the inside (layer attached to the existing wall) to the outside. Each layer $i \in \{1, \dots, n\}$ is made of one of the m_i different materials available for this layer, and given a layer $i \in \{1, \dots, n\}$, the material $j \in \{1, \dots, m_i\}$ is available in r_{j_i} different thicknesses. For each $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m_i\}$, $k \in \{1, \dots, r_{j_i}\}$, the following parameters are defined:
  - $t_{i,j,k}$ thickness corresponding to material j with type of thickness k available for layer i.
  - $c_{i,j,k}$ cost of placing in layer i 1m^2 of material j with type of thickness k.
- Let n_{lo} be the different options for the first lower meter of the thermal insulation (first layer of S_o). For each $j \in \{1, \dots, n_{lo}\}$ there are available r_j different thicknesses of this option. For each $j \in \{1, \dots, n_{lo}\}$ and $k \in \{1, \dots, r_j\}$ the following parameters are defined:
  - $t_{j,k}^{lo}$ thickness corresponding to option j and type of thickness k for the chosen material.
  - $c_{j,k}^{lo}$ cost of placing on the first layer of the lower opaque part 1m^2 of the chosen material with option j and type of thickness k.
- Let n_f and n_g be the number of different window's frames considered and the number of different combinations of glasses and air chambers considered for the windows respectively, all of them complying with the maximum allowed transmittance for these materials in the zone. For each $i \in \{1, \dots, n_f\}$ and $j \in \{1, \dots, n_g\}$ the following parameters are defined:
  - t_i^f thickness corresponding to window's frame of type i.

- c_i^f cost of placing 1m^2 of frame *i* in the windows.
- c_j^g cost of placing 1m^2 of a combination of glasses and air chambers type *j* in the windows.

– Let *F* be the fixed cost of removing old windows and pre-frame preparations.
– Given two consecutive layers, there may exist incompatibilities between some materials and thicknesses corresponding to these layers, as described in [34].
– Given a type of frame and a type of combination of glasses and air chambers, it may exist incompatibility between both elements, mainly due to their thicknesses.
– The total thickness of added layers is comprised between bounds t_{min} and t_{max}.
– Let U_{max} be the maximum thermal transmittance allowed for the external wall (its upper opaque part), according to the legislation for the climate zone where the building is located.
– *Let* W_{min} be the minimum difference set between the final wall thickness and the new window's frame thickness.
– Let $x_{i,j,k}$ be a binary variable which value is 1 if layer *i* is made with material *j* and type of thickness k, and 0 otherwise, $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m_i\}$, $k \in \{1, \dots, r_{j_i}\}$. Note that for *i*=1, this variable is associated only with the upper opaque part.
– Let $y_{j,k}$ be a binary variable which value is 1 if option *j* of thermal insulation with thickness type k is chosen for the first layer of the lower opaque part, and 0 otherwise, $j \in \{1, \dots, n_{lo}\}$, $k \in \{1, \dots, r_j\}$.
– Let z_i^f be a binary variable which value is 1 if type of frame *i* is chosen for the refurbishment, and 0 otherwise, $i \in \{1, \dots, n_f\}$.
– Let z_i^g be a binary variable which value is 1 if type *i* of combination of glasses and air chambers is chosen for the refurbishment, and 0 otherwise, $i \in \{1, \dots, n_g\}$.
– Given a material j, with $j \in \{1, \dots, m_i\}$ for some $i \in \{1, \dots, n\}$, and let λ_j be its thermal conductivity, following the calculations given in [34], the linear constraint to comply with the thermal transmittance upper bound for the upper opaque part of the façade is:

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} \frac{t_{i,j,k}}{\lambda_j} x_{i,j,k} \geq \frac{1}{U_{max}} - \frac{1}{h_{int}} - \frac{1}{h_{ext}} - \sigma \qquad (2)$$

Taking into account all the concepts, restrictions and suppositions given above, the problem of minimizing the refurbishment cost of a façade can be formulated mathematically as the following ILP problem, defined through Eqs. (3) to (17):

$$\begin{aligned} Minimize \quad & S_o \sum_{i=2}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} c_{i,j,k} x_{i,j,k} + S_o^u \sum_{j=1}^{m_1} \sum_{k=1}^{r_{j_1}} c_{1,j,k} x_{1,j,k} + S_o^l \sum_{j=1}^{n_{lo}} \sum_{k=1}^{r_j} c_{j,k}^{lo} y_{j,k} \\ & + S_w^f \sum_{i=1}^{n_f} c_i^f z_i^f + S_w^g \sum_{i=1}^{n_g} c_i^g z_i^g + F \end{aligned} \qquad (3)$$

s.t.:

$$\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} x_{i,j,k} = 1 \quad \forall i \in \{1, \dots, n\} \tag{4}$$

$$\sum_{j=1}^{n_{lo}}\sum_{k=1}^{r_j} y_{j,k} = 1 \tag{5}$$

$$\sum_{i=1}^{n_f} z_i^f = 1 \tag{6}$$

$$\sum_{i=1}^{n_g} z_i^g = 1 \tag{7}$$

$$\sum_{j=1}^{m_1}\sum_{k=1}^{r_{j_1}} t_{1,j,k} x_{1,j,k} = \sum_{j=1}^{n_{lo}}\sum_{k=1}^{r_j} t_{j,k}^{lo} y_{j,k} \tag{8}$$

$$t_{min} \le \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} t_{i,j,k} x_{i,j,k} \le t_{max} \tag{9}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} t_{i,j,k} x_{i,j,k} + \tau - \sum_{i=1}^{n_f} t_i^f z_i^f \ge W_{min} \tag{10}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{k=1}^{r_{j_i}} \frac{t_{i,j,k}}{\lambda_j} x_{i,j,k} \ge \frac{1}{U_{max}} - \frac{1}{h_{int}} - \frac{1}{h_{ext}} - \sigma \tag{11}$$

$$x_{i,j,k} + x_{(i+1),j',k'} \le 1 \quad \forall\, (i,j,k-(i+1),j',k') - \mathit{incompatible} \tag{12}$$

$$z_i^f + z_j^w \le 1 \quad \forall\, (i,j) - \mathit{incompatible} \tag{13}$$

$$x_{i,j,k} \in \{0,1\} \quad \forall\, i \in \{1, \dots, n\}, j \in \{1, \dots, m_i\}, k \in \{1, \dots, r_{j_i}\} \tag{14}$$

$$y_{j,k} \in \{0,1\} \qquad \forall\, j \in \{1, \dots, n_{lo}\}, k \in \{1, \dots, r_j\} \tag{15}$$

$$z_i^f \in \{0,1\} \qquad \forall\, i \in \{1, \dots, n_f\} \tag{16}$$

$$z_i^g \in \{0,1\} \qquad \forall\, i \in \{1, \dots, n_g\} \tag{17}$$

Where:

- Eq. (3) is the objective function, that is, the total cost of the refurbishment.
- Eqs. (4) and (5) ensure that each layer is made exactly of one material with a given thickness. Note that layer 1 may have one material for its lower part and another one for its upper part.

- Eqs. (6) and (7) guarantee that only one type of frame and one type of combination of glasses and air chambers are chosen for the whole transparent part.
- Eq. (8) ensures that both the lower part of layer 1 and its upper part have the same thickness.
- Eq. (9) restricts the total thickness of added layers within the established bounds.
- Eq. (10) guaranties that the difference between the final wall thickness and the new window’s frame thickness is at least W_{min}.
- Eq. (11) is the key restriction with respect to energy efficiency. It ensures that the upper opaque part of the wall does not exceed the maximal allowed thermal transmittance.
- Eq. (12) forbids to place a material *j’* with thickness *k’* in the next layer to the one (layer *i*) containing the material *j* with thickness *k*. This fact is denoted by (*i*,*j*,*k*-(*i*+1),*j’*,*k’*)-incompatibility. At most one of the two materials will appear in the corresponding layer.
- In the same way, Eq. (13) forbids to use at the same time a type of frame *i* and a type of combination of glasses *j* if it exists incompatibility between both elements, mainly due to their thicknesses. This fact is denoted by (*i*,*j*)-incompatibility.
- Finally, Eqs. (14) to (17) define the variables of the problem as binary.

Note that the above formulation contains the most usual constrains given in the refurbishment of a façade, but it could include other types of linear constraints to fit as much as possible the real problem.

Note also that this problem has only considered adding layers to the outside of the original wall. This solution presents many advantages: reduction of thermal bridges, elimination of internal condensation, as well as the formation of mould both superficial and interstitial, and the appearance of moisture filtrations, improved acoustic insulation and a better protection of the façade against climatic aggressions and of the structure against thermal shocks. Furthermore, there is no loss of useful space inside the houses with better final finishes and no stress for users as no demolition or interior work are required.

An internal coat may be an interesting alternative when internal rehabilitation works are being carried out to take advantage of the synergies generated by the workforce. Likewise, this insulation option is also intended for those cases in which it is not possible to proceed with a refurbishment of the exterior façade. In fact, it is the only viable system in buildings that have a degree of protection for historical heritage. This kind of intervention has a medium-high cost as well as multiple disadvantages of performance and habitability. There is a loss of useful surface inside the building, as the walls gain a thickness of 5 cm or more without solving the problem of thermal bridges. Moreover, attention must be paid to the finishing of some areas such as doors and windows.

Finally, although overestimated, a double intervention that adds both external and internal layers to the original wall is feasible.

The ILP formulation presented here considers the case of adding layers to the outside of the original wall, which, as it has been said, is the most preferable option, and it will be the subject of the case study. In any case, it can be easily adapted and extended to the other options described above.

4 | CASE STUDY AND CHARACTERISTICS OF THE INTERVENTION

The case study focuses on a building located in Valencia, a city in eastern Spain with an altitude of 16 m, latitude of 30º28’00” N and longitude of 0º22’30” W. The climate of Valencia presents warm summers and mild winters. January is the coldest month, with average maximum temperatures of 16-17 °C over the last 30 years and minimum temperatures of 7-8 °C. Snowfall and subzero temperatures are extremely rare within the urban core of the city. The warmest month is August, with average maximum temperatures of 30-31 °C and minimum temperatures of 21-23 °C. Legislation in Spain [19] divides the territory into five climate zones according to winter climate severity, from A (less severe) to E (most severe). Valencia belongs to zone B.

Building 1B of the School for Building Engineering in the Vera Campus of the Polytechnic University of Valencia will be the case study. It was built in the late 1960s as first development and its classification is *F* for energy consumptions (in a scale from *A* to *G*), with 350 $kW{\cdot}h{\cdot}m^{-2}{\cdot}\text{year}^{-1}$, and *E* for CO_2 emissions, with 64 $kgCO_2{\cdot}m^{-2}{\cdot}\text{year}^{-1}$. The opaque part of the façade is a 3-layer wall with two external layers of concrete (20 mm each) and an interior layer made by simple wood chips mixed with mortar (60 mm). The estimated transmittance of the opaque part is U_{opaque}= 1.2 $Wm^{-2}K^{-1}$, and the estimated transmittance of the transparent part (windows with metallic frame and simple glass) is $U_{frame} = U_{glass} = 5.7\ Wm^{-2}K^{-1}$. Both transmittances are over the current legal limits for the corresponding climatic zone where Valencia is located: $U_{max,opaque}$=0.82 $Wm^{-2}K^{-1}$ and $U_{max,windows}$=3.6(east)-3.8(north) $Wm^{-2}K^{-1}$.

Two façades will be studied: façade 1, which shows the highest amount of transparent part (54%), on the east (see Figure 1), and façade 2, which corresponds to the largest surface (307 m^2) on the north side. Table 1 summarizes the main data of the two façades, including total surfaces, surfaces both of the opaque and the transparent part, and the *U*-values corresponding to the different parts, where $U_{\text{transparent}} = 0.86 \cdot U_{\text{glass}} + 0.14 \cdot U_{\text{frame}}$.

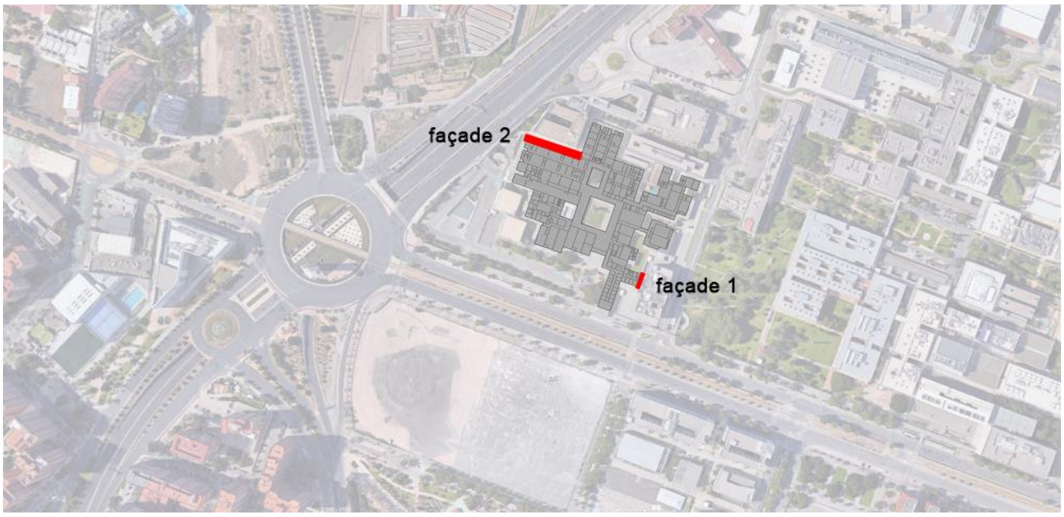

FIGURE 1 Façades 1 and 2 in the School for Building Engineering and their location on the Vera Campus of the Polytechnic University of Valencia

The suggested refurbishment solution is an ETICS (*External Thermal Insulation Composite Systems*) and includes the removal of the old windows, the preparation of the "holes", new windows with double glass (standard, low emissive and with solar control) or triple glass (low emissive and with solar control) and 5 options for the frame (PVC, aluminium, aluminium with heat break, aluminium and wood, wood) as well as an added multilayer "coat" with flexible configuration (thermal insulation, air chamber, new panel and external finish). Figure 2 shows from left to right, the existing façade, the configuration with thermal insulation, plaster and painting (Type 1), the configuration with thermal insulation, air gap and bricks (Type 2) as well as the configuration with thermal insulation, air gap and ceramic/composite panel (Type 3).

TABLE 1 Data of the chosen façades

	Façade 1 (East)	**Façade 2 (North)**
Surface [m^2]	72.80	307.00
Opaque part [m^2] (%)	33.6 (46.2)	267.8 (87)
U_{opaque} [$Wm^{-2}K^{-1}$]	1.2	1.2
MAX admissible U_{opaque} in zone *B3*	*0.82*	*0.82*
Transparent part [m^2] (%)	39.2 (53.8)	39.2 (13)
$U_{transparent}$ [$Wm^{-2}K^{-1}$]	5.7	5.7
MAX admissible $U_{transparent}$ in zone *B3*	*3.6*	*3.8*
Frame [m^2] (units)	5.6 (14)	5.6 (14)
U_{frame} [$Wm^{-2}K^{-1}$]	5.7	5.7
Glass [m^2]	33.6	33.6
U_{glass}[$Wm^{-2}K^{-1}$]	5.7	5.7

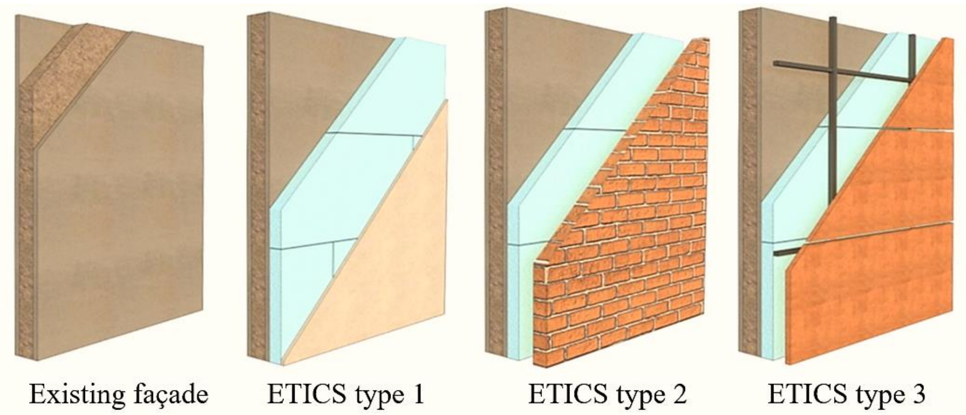

FIGURE 2 Existing façade and suggested ETICS solutions

Summarized data are included in Tables 2 and 3. Table 2 shows all the materials considered for the different layers to be added to the opaque part, as well as an interval of thickness for each one of them. This table also includes the composition of the existing façade. Concretely, 61 options have been chosen for the thermal insulation of the upper part of the façade, 12 options for the thermal insulation of the lower part, 9 options for the air chamber, 17 options for the new panel and 10 options for the external finish. On the other hand, Table 3 shows the materials considered for window's frame, as well as the different combinations of glasses and air chambers: 8 options for the windows's frame, 48 options for double glass and 12 options for triple glass.

TABLE 2 Materials for the opaque part

Layer	Function	Material (options)	Thickness [mm]
Layer Ef	Existing façade	Concrete plate with wood chips and mortar	100
Layer Ti	Thermal insulation	Projected Polyurethane (4)	30 up to 60
		Extruded polystyrene (12)	30 up to 60
		Mineral wool (12)	30 up to 60
		Expanded polystyrene (24)	30 up to 60
		Expanded cork (8)	30 up to 60
		Nanoporous gel (1)	30
Layer Ag	Air gap	Light ventilated (4)	30, 50, 80, 100
		Not ventilated (4)	30, 50, 80, 100
		Absence (1)	0
Layer Np	New panel	Face brick (3)	115
		Pressed facing brick (2)	120
		Composite panel (1)	30
		Extruded ceramic panel (1)	40
		Regular plaster (5)	10 up to 20
		Thermal plaster (5)	10 up to 20
Layer Pa	Painting	Painting (9)	1
		Absence (1)	0

TABLE 3 Materials for the transparent part

Element	Material	Thickness or Composition [mm]
Frame	PVC	max 40 mm thickness
	Aluminium	max 30 mm thickness
	Aluminium + thermal break	max 30 mm thickness
	Aluminium +Wood	from 47 up to 55 mm thickness
	Wood	from 32 up to 54 mm thickness
Double glass	Standard (15)	External glass: 4,5,6,8
		Air chamber: 6, 8, 12, 16
		Argon chamber: 12, 16
		Internal glass*: 4, 5, 6, 8*
	Low emissivity (14)	External glass: 4,6,8
		Air chamber: 6, 8, 12, 16, 20
		Argon chamber: 12, 16, 20
		Internal glass*: 6*
	Solar control (19)	External glass: 6,8
		Air chamber: 6, 8, 12, 16, 20
		Argon chamber: 12, 20
		Internal glass*: 4, 8, 12*
Triple glass	Low emissivity (6)	External glass: 4, 6, 8
		Air chamber 1: 16
		Intermediate glass: 4, 6
		Chamber 2: 16
		Argon chamber 1 and 2: 16
		internal glass: 4, 6
	Solar control (6)	External glass: 4, 6, 8
		Air chamber 1: 16
		Intermediate glass: 4, 6
		Chamber 2: 16
		Argon chamber 1 and 2: 16
		internal glass: 4, 6

Tables A.1 to A.3 in the Appendix contain the relevant data for each one of the 177 options summarized in Tables 2 and 3, to run the ILP problem: thickness, conductivity, cost and name of the correspondig variable.

Some considerations will be taken into account to obtain the best solutions to the intervention. As stated in Section 3, some of the different materials or elements are incompatibles. For instance, air chamber is incompatible with plaster in the new panel, and painting is only compatible with plaster in the new panel. In the same way, for example, the aluminium frame can admit a maximal thickness for a double glass package of 30 mm while the PVC frame admits triple glass packages up to 52 mm. Moreover, in order to avoid rising damp, the first lower meter of the thermal insulation is usually made by extruded polystyrene. On the other hand, painting in the exterior layer is not taking into account to compute the thermal transmittance of a wall because it is thermally transparent. Finally, two important constraints are set for all the considered scenarios: the added thickness in meters belongs to the interval $[t_{min}, t_{max}] = [0.05, 0.2]$, and the minimum recommended difference between the final wall thickness and the new window's frame thickness is $W_{min}=0.1m$.

The cost generator website of CYPE Ingenieros [35] was accessed on December 2018 for all costs, which always include materials, staff and site facilities, and they are described in the Appendix.

5 | ANALYSIS OF THE RESULTS

Taking into account all the chosen and available materials with their corresponding thicknesses for the different layers to be added to the opaque part, the different types of frame for the transparent part, the different combinations of glasses and air chambers (see Tables A.1 to A.3), and after considering all the incompatibilities between materials, frames and combinations of glasses and air chambers, some of them were cited in Section 4 and others can be easily shown through Tables A.2 and A.3, a total of 6,813,090 possible interventions for the refurbishment can be considered, which proves the complexity of finding the best solution under the given conditions.

Four relevant scenarios will be compared for the 2 chosen façades and the 3 ETICS solutions:

- *VLC*: lowest cost in €/m^2 in Valencia (U_{max}=0.82 $Wm^{-2}K^{-1}$).
- *Lowest U*: lowest cost in €/m^2 for the minimum possible value of U taking into account the list of available materials.
- *NATURE*: lowest cost in €/m^2 using environment friendly materials. Given the list of available materials, this implies the use of cork as thermal insulation and the use of frames made of 100% wood.
- *Thickness*: lowest cost in €/m^2 given the minimum possible thickness of the wall and U_{max}=0.82 $Wm^{-2}K^{-1}$.

Moreover, for each façade, each scenario and each ETICS solution, both the 2-glasses window option and the 3-glasses window option have been considered. This gives a total of 48 ILP problems to solve.

To solve all these ILP problems, *Mathematica* 11.2 [36] has been run on a PC Intel®CoreTMI7-6700 with 4 processors, 3.46 GHz and 8GB RAM. *Mathematica* is a widely used tool to solve mathematical, physical and engineering problems, with specific functions to solve ILP problems. Note that *Mathematica* needed only a few hundredths of CPU second to obtain each optimal solution, and for the only two cases without feasible solution, *Mathematica* needed about 20 seconds to determine the unfeasibility. A study on running times is therefore considered unnecessary.

Finally, it is worth noting that, given a façade and scenario and an option of glasses, to obtain the minimum possible value of U taking into account the list of available materials, the ILP problem has been solved many times, each time with a U value inferior in 0.01 units or even 0.005 units to the previous one, until the ILP problem becomes unfeasible. In the same way, to obtain the minimum possible thickness of the wall, the ILP problem has been solved several times, each time with a thickness inferior in 1*mm* to the previous one, until the ILP problem becomes unfeasible.

Tables 4 and 5 show the optimal solutions obtained (if possible) for the 24 ILP problems involving façade 1. Table 4 contains the results for the 12 ILP problems considering only double glass, while Table 5 contains the results for the 12 ILP problems considering only triple glass. For each feasible ILP problem, given its optimal solution, these tables contain its thermal transmittance in $Wm^{-2}K^{-1}$, its cost in euros per m^2, its thickness in *cm*. (without taking into account the original wall), as well as the chosen material with its corresponding thickness for each layer and for the frame, and the characteristics of the combination of glasses and air chambers.

Table 4 shows how for Type 1 (mortar) the thermal insulation admits 3 materials: projected polyurethane for lowest U, extruded polystyrene for *VLC* and minimum *Thickness* scenario and cork for the *NATURE* scenario. The glass configuration is quite similar with just an air chamber with double thickness for the *NATURE* scenario. On the other hand, 3 different materials have been obtained for the frame (PVC, wood and aluminium) with very similar thickness from 6.4 up to 7.1 cm.

This ETICS solution always shows the lowest thickness due to the absence of air gap. Furthermore, U-values are between 0.33 and 0.43 $Wm^{-2}K^{-1}$. Note that a minimal reduction of the final thickness (from 7.1 to 6.4 cm) produces an important increased value of U from 0.33/0.38 up to 0.43 $Wm^{-2}K^{-1}$. For Type 2 (brick) in order to achieve the lowest U=0.26 $Wm^{-2}K^{-1}$ the configuration includes a light ventilated air gap and the thermal insulation is made by nanoporous gel, which reduces also the final thickness. The solution is shared by the *VLC* scenario and the minimal *Thickness* scenario and furthermore, the *NATURE* scenario is more expensive than the *lowest U* scenario, with a double U-value (0.50 vs 0.26 $Wm^{-2}K^{-1}$). For Type 3 (panel) three kinds of frame are used, and the combination of nanoporous gel with a 10cm light ventilated air gap produces the lowest U-value (*VLC and lowest U*) of all the 48 optimal solutions. In this type there are quite big differences on U-values, ranging from 0.23 up to 0.53 $Wm^{-2}K^{-1}$. The U= 0.23 $Wm^{-2}K^{-1}$is suitable for Helsinki, Ivalo or Oulu in Finland. On the other hand, the same double glass configuration 4/6/4 is the chosen option for all types in all scenarios except the *NATURE* one that shows a 5/12/4 glass. Finally, regarding costs, the Type 1 solutions provide the cheapest ones, while the *NATURE* scenario provides the most expensive ones.

Table 5 (triple glass) shows that the only unfeasible ILP problem occurs for the *NATURE* scenario due to the dimensional constrains for Type 1 (it is impossible to comply with Eq. (10) using mortar, wood frame and triple glass). The transparent part is solved with a composite aluminium/wood frame and a low emissivity glass 4/16/4/16/4 with argon in the chambers in the other 3 scenarios and for all types of ETICS. U-values are quite similar. For Type 2 the wood frame is the option for all scenarios and a quite wide range of insulation materials is given: extruded polystyrene for *VLC* and minimal *Thickness*, nanoporous gel for the *lowest U* and cork for the *NATURE* scenario. For Type 3 a wooden frame appears in all scenarios except for the minimum *Thickness* one, which contains a composite aluminium/wood frame. Note that unlike Table 4, in Table 5 the cost differences are not very large. This is due to the fact that triple glass is very expensive comparing with double glass, and the transparent part represents 54% of the façade's surface.

TABLE 4 Best solutions for façade 1 with double glass

Type 1_2G	***VLC***	***Lowest U***	***NATURE***	***Thickness***
U solution	0.3836	0.335	0.3986	0.4318
Cost/m2	129.37	130.82	294.96	131.15
Thickness	7.1	7.1	7.1	6.4
Ti down	Ext. polyst. dots 6cm.	Ext. polyst. dots 6cm.	Ext. polyst. dots 6cm.	Ext. polyst. dots 5cm.
Ti up	Ext. polyst. dots 6cm.	Proj. polyu. mech. 6cm.	Aglo. exp. cork dots 6cm.	Ext. polyst. dots 5cm.
Ag	No air gap	No air gap	No air gap	No air gap
Ef	Stand. Mortar 1cm.	Stand. Mortar 1cm.	Stand. Mortar 1cm.	Stand. Mortar 1.3cm.
Pa	Trad. Whitewash	Trad. Whitewash	Trad. Whitewash	Trad. Whitewash
Frame	PVC	PVC	Wood 6.8cm.	Aluminium
Glasses	S.d.g. 4/6/4	S.d.g. 4/6/4	S.d.g. 5/12/4	S.d.g. 4/6/4
Type 2_2G	***VLC***	***Lowest U***	***NATURE***	***Thickness***
U solution	0.4899	0.2595	0.5019	0.4899
Cost/m2	156.71	212.1	320.35	156.71
Thickness	17.5	20	17.5	17.5
Ti down	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.
Ti up	Ext. polyst. dots 3cm.	Nanoporous aerogel	Aglo. exp. cork dots 3cm.	Ext. polyst. dots 3cm.
Ag	Not vent. air gap 3cm.	Light vent. air gap 5cm.	Not vent. air gap 3cm.	Not vent. air gap 3cm.
Ef	Face brick 24x11.5x5	Pres. face brick 24x12x5	Face brick 24x11.5x5	Face brick 24x11.5x5
Pa	No painting	No painting	No painting	No painting
Frame	PVC	PVC	Wood 6.8cm.	PVC
Glasses	S.d.g. 4/6/4	S.d.g. 4/6/4	S.d.g. 5/12/4	S.d.g. 4/6/4
Type 3_2G	***VLC***	***Lowest U***	***NATURE***	***Thickness***
U solution	0.4631	0.2352	0.4738	0.5288
Cost/m2	169.9	210.71	333.6	176.58
Thickness	11.3	13.3	11.3	6.4
Ti down	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.
Ti up	Ext. polyst. dots 3cm.	Nanoporous aerogel	Aglo. exp. cork dots 3cm.	Ext. polyst. dots 3cm.
Ag	Not vent. air gap 8cm.	Light vent. air gap 10cm.	Not vent. air gap 8cm.	Not vent. air gap 3cm.
Ef	Composite panel	Composite panel	Composite panel	Extruded ceramic panel
Pa	No painting	No painting	No painting	No painting
Frame	S.d.g. 4/6/4	PVC	Wood 6.8cm.	Aluminium
Glasses	S.d.g. 4/6/4	S.d.g. 4/6/4	S.d.g. 5/12/4	S.d.g. 4/6/4

As a quick visual way to compare costs, *U*-values and thickness of all the ILP optimal solutions involving façade 1 and shown in Tables 4 and 5, a radar diagram is very useful. This type of diagrams can display jointly three variables in a two-dimensional chart.

The radar diagram given in Figure 3 represents the variability of *U*-values. The *lowest U* scenario shows a smaller and quite regular surface somehow elongated vertically due to the higher values for Type1_2G and Type1_3G depending on the smaller mass available in this constructive solution with mortar. The other 3 scenarios show very similar *U*-value for all solutions and consequently similar and quite regular hexagons.

Differences are quite remarkable in Type3_2G from 0.23 up to 0.53 $Wm^{-2}K^{-1}$. For the *NATURE* scenario and Type 1_3G there is no feasible solution.

TABLE 5 Best solutions for façade 1 with triple glass

Type 1_3G	***VLC***	***Lowest U***	***NATURE***	***Thickness***
U solution	0.3836	0.335		0.4308
Cost/m2	457.1	458.6	IMPOSSIBLE	458.47
Thickness	7.1	8.1		6.9
Ti down	Ext. polyst. dots 6cm.	Ext. polyst. dots 6cm.		Ext. polyst. dots 5cm.
Ti up	Ext. polyst. dots 6cm.	Proj. polyu. mech. 6cm.		Ext. polyst. dots 5cm.
Ag	No air gap	No air gap		No air gap
Ef	Stand. Mortar 1cm.	Stand. Mortar 1cm.		Stand. Mortar 1.8cm.
Pa	Trad. Whitewash	Trad. Whitewash		Trad. Whitewash
Frame	Alumin./wood 6.8cm.	Alumin./wood 6.8cm.		Alumin./wood 6.8cm.
Glasses	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4		T.g. low int. arg. 4/16/4/16/4
Type 2_3G	***VLC***	***Lowest U***	***NATURE***	***Thickness***
U solution	0.4899	0.2595	0.5019	0.4899
Cost/m2	390.6	445.99	392.44	390.6
Thickness	17.5	20,00	17.5	17.5
Ti down	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.
Ti up	Ext. polyst. dots 3cm.	Nanoporous aerogel	Aglo. exp. cork dots 3cm.	ext. polyst. dots 3cm.
Ag	Not vent. air gap 3cm.	Light vent. air gap 5cm.	Not vent. air gap 3cm.	Not vent. air gap 3cm.
Ef	Face brick 24x11.5x5	Pres. face brick 24x12x5	Face brick 24x11.5x5	Face brick 24x11.5x5
Pa	No painting	No painting	No painting	No painting
Frame	Wood 9.8cm.	Wood 9.8cm.	Wood 9.8cm.	Wood 9.8cm.
Glasses	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4
Type 3_3G	***VLC***	***Lowest U***	***NATURE***	***Thickness***
U solution	0.4631	0.2352	0.4738	0.4575
Cost/m2	403.79	444.6	405.63	497.93
Thickness	11.3	13.3	11.3	7.3
Ti down	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 3cm.	Ext. polyst. dots 4cm.
Ti up	Ext. polyst. dots 3cm.	Nanoporous aerogel	Aglo. exp. cork dots 3cm.	Ext. polyst. dots 4cm.
Ag	Not vent. air gap 8cm.	Light vent. air gap 10cm.	Not vent. air gap 8cm.	Not vent. air gap 3cm.
Ef	Composite panel	Composite panel	Composite panel	Composite panel
Pa	No painting	No painting	No painting	No painting
Frame	Wood 9.8cm.	Wood 9.8cm.	Wood 9.8cm.	Alumin./wood 6.8cm.
Glasses	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4	T.g. low int. arg. 4/16/4/16/4

Figure 4 represents the variability of the thicknesses. As expected, the lowest thickness of 6.4 cm occurs in the minimal *Thickness* scenario for Type1_2G. In this type also occurs the lowest difference between scenarios, just 7 mm. A thickness of 7.1 *cm* is shared by the other 3 scenarios. Also expected, the largest thicknesses correspond to the 8 optimal solutions problems involving Type 2, where a face brick is required with at least 11.5 *cm* width, and an air chamber. The worst case corresponds to the *Lowest U* scenario with 20 *cm* width (plus 10 *cm* of the original wall). The largest difference between the 4 scenarios occurs for Type3_2G: 6.4 *cm* for the *minimum Thickness* scenario vs 13.3 for the *VLC* and *Lowest U* scenarios. Finally note that the polygonal corresponding to the *NATURE* scenario is difficult to detect since it coincides practically in all types with *VLC* scenario and does not exist for Type_3G.

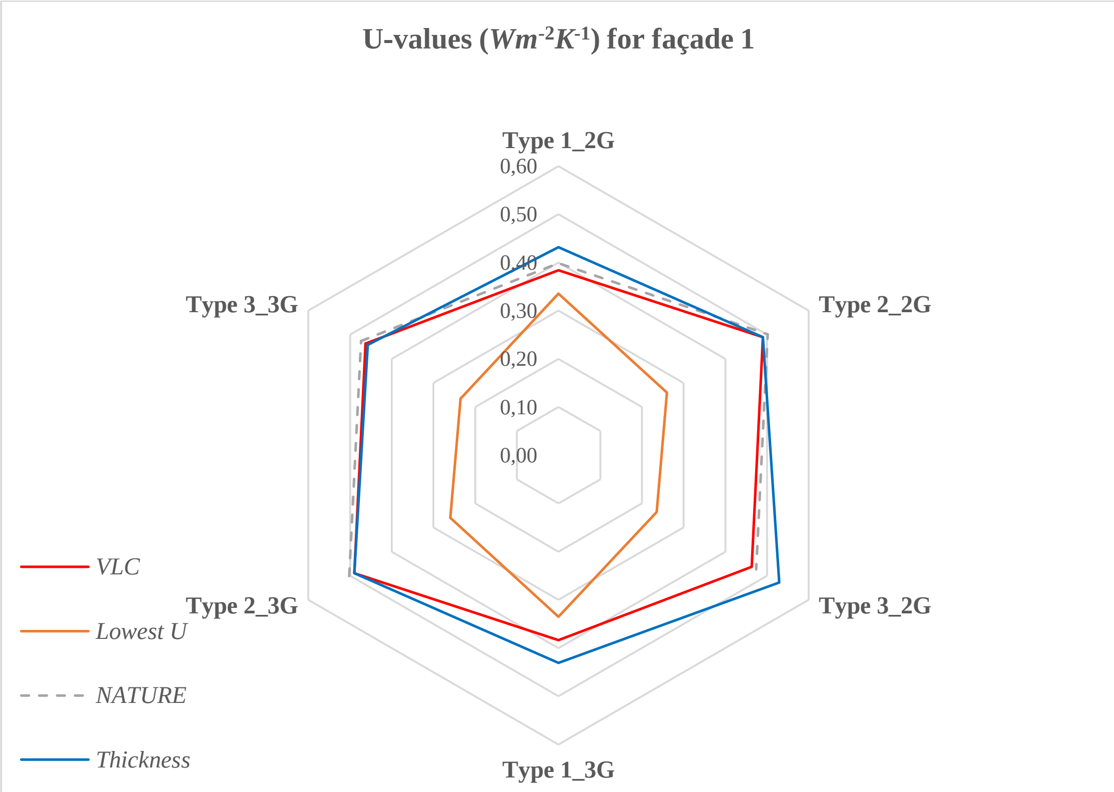

FIGURE 3 Comparing *U*-values for ETICS solutions on façade 1

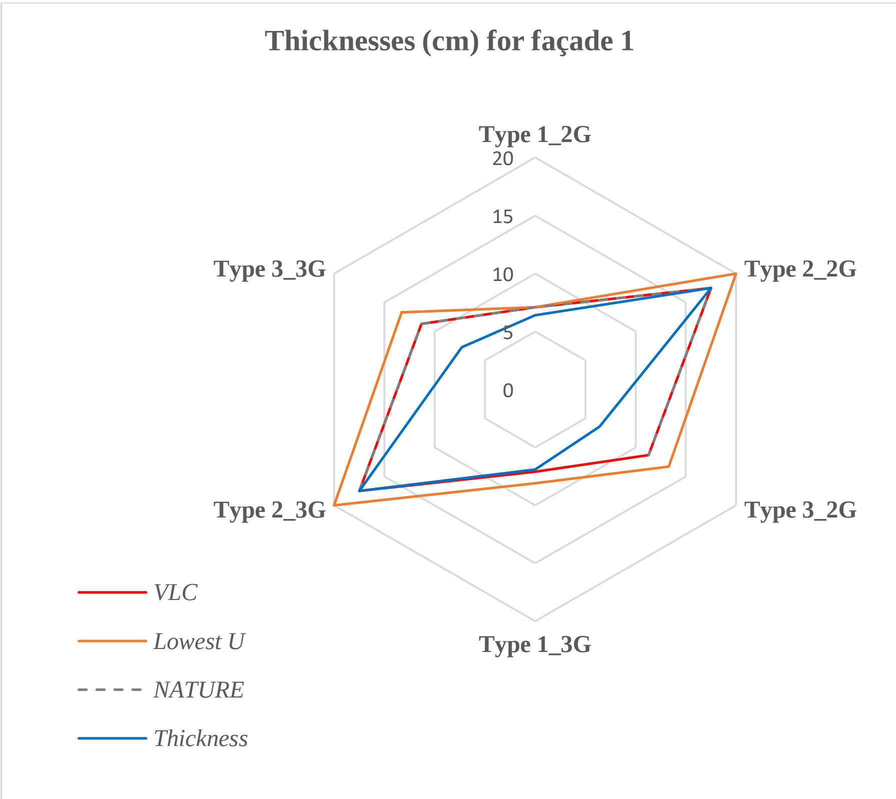

FIGURE 4 Comparing thicknesses for ETICS solutions in façade 1

Finally, Figure 5 shows the radar diagram corresponding to the optimal costs for the 24 ILP problems involving façade 1. Highest costs correspond to the triple glass windows, without an important better thermal behavior or lower thickness, as shown in Figures 3 and 4. On the contrary, Type1_2G shows the lowest costs, which is expected due to thee used materials. Finally, the *NATURE* scenario shows almost a double cost for two glasses solutions and an unfeasible solution for Type 1_3G.

Type 1_2G with *Lowest U* seems to be the best option for the required refurbishment of façade 1. With only 1.45€ per m^2 more than the cheapest solution, it improves the thermal transmittance from 0.384 to 0.335 $Wm^{-2}K^{-1}$, which is a very good value for the city of Valencia, and it is almost a quarter of the value of the original wall (1.2 $Wm^{-2}K^{-1}$).

With respect to the results obtained for the ILP problems corresponding to the 24 refurbishment solutions for the façade 2 (4 scenarios, 3 ETICS solutions, and 2 or 3 glasses), as expected, in all 24 ILP problems, the optimal solutions were the same as the corresponding problems for façade 1 (even the unfeasible solution for Type 1_3G in *NATURE* scenario), except for the costs per m^2, as also expected. As a general rule, they are much lower than for façade 1, especially in solutions with triple glass, since the proportion of opaque part in the façade 2 is much greater than that of transparent part, contrary to what happened in façade 1 (see Table 1), and prices per m^2 of the opaque part are in general much cheaper than prices per m^2 of windows. Therefore, all data corresponding to the optimal solution of the feasible ILP problems can also be seen and compared through Tables 4 and 5 and Figures 3 and 4, except for their new costs. These new costs, in euros, are given in Table 6, both for the double glass solutions and the triple glass solutions

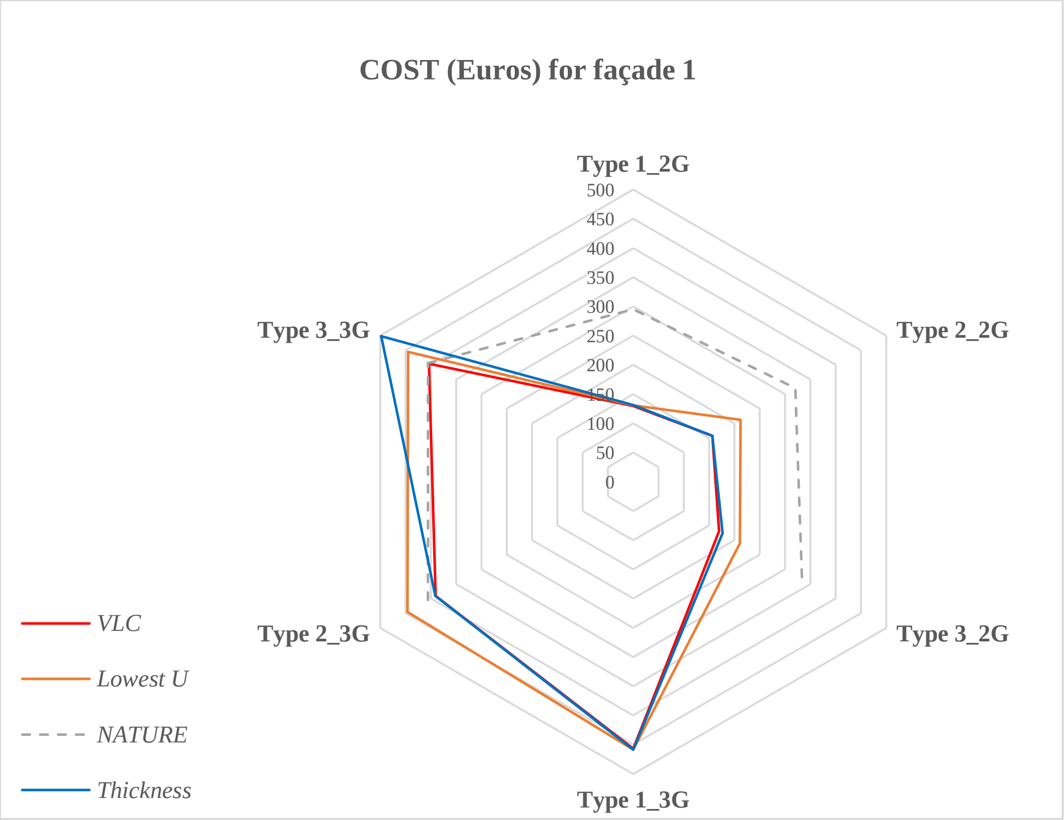

FIGURE 5 Comparing costs for ETICS solutions in façade 1

Table 6 allows an easy comparative study of the different costs, without a radar diagram. As mentioned before, costs per m^2 are much cheaper than for façade 1, but the trends are very similar: the highest costs correspond in all cases for the triple glass option, Type1_2G shows the lowest costs, but now the *NATURE* scenario is more competitive, with similar o cheaper costs than the other scenarios, except for Type 1_2G, and for Type 1_3G, where the ILP problem is unfeasible due to Eq. (10).

TABLE 6 Costs of the optimal solutions for the 24 ILP problems corresponding to façade 2

Scenario	Type 1_2G	Type 2_2G	Type 3_2G	Type 1_3G	Type 2_3G	Type 3_3G
VLC	37.67	89.35	114.29	70.21	113.94	138.87
Lowest U	41.95	236.94	234.34	74.49	261.55	258.93
NATURE	63.06	109.16	134.08	-	119.367	144.29
Thickness	38.45	89.35	124.32	72.8	113.94	147.36

6 | CONCLUSIONS

One of the most challenging targets of the EU for the coming decades is to drastically reduce the energy consumption levels. Buildings account for 40% of the EU's total energy consumption, therefore, the refurbishment of existing buildings, and particularly old ones, is one of the most effective way to achieve this objective, especially with an intervention on their façades, to decrease the thermal transmittance (rate of transfer of heat through the façade).

With the aim of making the least expensive possible the refurbishment of old building façades, this paper has presented an ILP procedure that models the problem of minimizing the cost of refurbishment of a façade, with interventions both on the opaque part and the transparent part, that takes into account thermal transmittance bounds and other restrictions. In this way, among even millions of solutions to the problem, the cheapest one can be quickly obtained with the use of an appropriate computer tool. This will facilitate investment in this kind of interventions to improve the energy efficiency of the buildings, as well as consider other aspects as the use of environmental friendly materials.

This ILP procedure has been applied to two façades of the oldest building of the Polytechnic University of Valencia, under different scenarios and ETICS suggested solutions, showing that the current thermal transmittance of their opaque part, 1.2 $Wm^{-2}K^{-1}$ can be reduced up to 0.23 $Wm^{-2}K^{-1}$ by choosing the adequate materials for the added layers of the wall.

For a greater improvement of the energy efficiency of old buildings, with minimum cost, the present work could be extended in future research to the whole envelope of the buildings, including the fifth façade (roof), as well as trying to consider other factors of energy efficiency, in addition to thermal transmittance. That would minimize the cost of the refurbishment of the whole envelope considering its final energy efficiency.

NOMENCLATURE

Notations	**Explanations**
Sizes of sets	
μ	Number of layers of the original façade
n	Number of layers to be added to the external side of the original façade
m_i	Number of materials available for layer *i*
r_{j_i}	Number of thicknesses available for material *j* of layer *i*
n_{lo}	Number of options for the first lower meter of the thermal insulation
r_j	Number of thicknesses available for option *j* of the first lower meter of the thermal insulation
n_f	Number of options for the window's frame
n_g	Number of options for combinations of glasses and air chambers
Parameters	
S	Total surface of the façade (all surfaces in m^2)
S_o	Surface of the opaque part of the façade
S_o^l	Surface of the first lower meter of the opaque part of the façade
S_o^u	Upper surface to the first lower meter of the opaque part of the façade
S_w	Surface of the transparent part of the façade
S_w^f	Surface of the transparent part corresponding to the frame
S_w^g	Surface of the transparent part corresponding to the glass
λ_i	Thermal conductivity of layer/material *i* ($Wm^{-1}K^{-1}$)
t_i	Thickness of layer *i* of the original façade (all thicknesses in *m*)
σ	Sum of the quotients t_i/λ_i of each layer *i* of the original façade
τ	Sum of the thicknesses t_i of each layer *i* of the original façade
$t_{i,j,k}$	Thickness of material *j* with type of thickness *k* available for layer *i*
$c_{i,j,k}$	Cost of placing in layer *i* 1m^2 of material *j* with type of thickness *k* (all costs in €)
$t_{j,k}^{lo}$	Thickness of option *j* and type of thickness *k* for the first lower meter of the thermal insulation
$c_{j,k}^{lo}$	Cost of placing on the first layer of the lower opaque part 1m^2 of option *j* and type of thickness *k* of the thermal insulation.
t_i^f	Thickness of window's frame of option *i*
c_i^f	Cost of placing 1m^2 of frame *i*
c_j^g	Cost of placing 1m^2 of a combination of glasses and air chambers type *j*
F	Cost of removing all old windows and pre-frame preparations
t_{min} and t_{max}	Bounds for the total thickness of the added layers
U	Thermal transmittance ($Wm^{-2}K^{-1}$)
U_{max}	Maximum thermal transmittance allowed for the opaque part of the wall.
W_{min}	Minimum difference set between the final wall thickness and the new window's frame thickness
$1/h_{ext}$ and $1/h_{int}$	Standard external and internal conductivity respectively for the air layers connected with the envelope (m^2KW^{-1})
Variables	

$x_{i,j,k}$	Value 1 if layer *i* is made with material *j* and type of thickness *k*, and 0 otherwise
$y_{j,k}$	Value 1 if option *j* of thermal insulation with thickness type *k* is chosen for the first layer of the lower opaque part, and 0 otherwise
z_i^f	Value 1 if type of frame *i* is chosen for the refurbishment, and 0 otherwise
z_i^g	Value 1 if type *i* of combination of glasses and air chambers is chosen for the refurbishment, and 0 otherwise

CONFLICTS OF INTEREST

This work does not have any conflicts of interest.

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APPENDIX

TABLE A.1 Data of the materials for the four layers of the opaque part

Building material	Thickness *cm*	Conductivity $Wm^{-1}K^{-1}$	Cost *€/m2*	Variable
Exisitng façade	10	1.2	0	
Layer 1 (thermal insulance)				
Projected Polyurethane mech. proyected	3	0.028	8.55	$x_{1,1,1}$
Projected Polyurethane mech. proyected	4	0.028	10.87	$x_{1,1,2}$
Projected Polyurethane mech. proyected	5	0.028	13.12	$x_{1,1,3}$
Projected Polyurethane mech. proyected	6	0.028	14.85	$x_{1,1,4}$
Extruded polystyrene dots	3	0.034	6.11	$x_{1,2,1}$/ $y_{1,1}$
Extruded polystyrene adh. mortar	3	0.034	8.19	$x_{1,3,1}$/ $y_{2,1}$
Extruded polystyrene mech. fix	3	0.034	7.78	$x_{1,4,1}$/ $y_{3,1}$
Extruded polystyrene dots	4	0.034	6.73	$x_{1,2,2}$/ $y_{1,2}$
Extruded polystyrene adh. mortar	4	0.034	8.81	$x_{1,3,2}$/ $y_{2,2}$
Extruded polystyrene mech. fix	4	0.034	8.4	$x_{1,4,2}$/ $y_{3,2}$
Extruded polystyrene dots	5	0.034	7.35	$x_{1,2,3}$/ $y_{1,3}$
Extruded polystyrene adh. mortar	5	0.034	9.44	$x_{1,3,3}$/ $y_{2,3}$
Extruded polystyrene mech. fix	5	0.034	9.03	$x_{1,4,3}$/ $y_{3,3}$
Extruded polystyrene dots	6	0.034	7.97	$x_{1,2,4}$/ $y_{1,4}$
Extruded polystyrene adh. mortar	6	0.034	10.05	$x_{1,3,4}$/ $y_{2,4}$
Extruded polystyrene mech. fix	6	0.034	9.64	$x_{1,4,4}$/ $y_{3,4}$
Expanded polystyrene straight cut dots	3	0.029	8.85	$x_{1,5,1}$
Expanded polystyrene straight cut proy adh. mortar	3	0.029	10.93	$x_{1,6,1}$
Expanded polystyrene straight cut mech. fix	3	0.029	10.39	$x_{1,7,1}$
Expanded polystyrene half cut dots	3	0.029	8.93	$x_{1,8,1}$
Expanded polystyrene half cut proy adh. mortar	3	0.029	11.01	$x_{1,9,1}$
Expanded polystyrene half cut mech. fix	3	0.029	10.47	$x_{1,10,1}$
Expanded polystyrene straight cut dots	4	0.029	10.37	$x_{1,5,2}$
Expanded polystyrene straight cut proy adh. mortar	4	0.029	12.45	$x_{1,6,2}$
Expanded polystyrene straight cut mech. fix	4	0.029	11.91	$x_{1,7,2}$
Expanded polystyrene half cut dots	4	0.029	10.48	$x_{1,8,2}$
Expanded polystyrene half cut proy adh. mortar	4	0.029	12.56	$x_{1,9,2}$
Expanded polystyrene half cut mech. fix	4	0.029	12.02	$x_{1,10,2}$
Expanded polystyrene straight cut dots	5	0.029	11.91	$x_{1,5,3}$
Expanded polystyrene straight cut proy adh. mortar	5	0.029	13.99	$x_{1,6,3}$
Expanded polystyrene straight cut mech. fix	5	0.029	13.45	$x_{1,7,3}$
Expanded polystyrene half cut dots	5	0.029	12.04	$x_{1,8,3}$
Expanded polystyrene half cut proy adh. mortar	5	0.029	14.12	$x_{1,9,3}$
Expanded polystyrene half cut mech. fix	5	0.029	13.58	$x_{1,10,3}$
Expanded polystyrene straight cut dots	6	0.029	13.43	$x_{1,5,4}$
Expanded polystyrene straight cut proy adh. mortar	6	0.029	15.51	$x_{1,6,4}$
Expanded polystyrene straight cut mech. fix	6	0.029	14.97	$x_{1,7,4}$
Expanded polystyrene half cut dots	6	0.029	13.6	$x_{1,8,4}$
Expanded polystyrene half cut proy adh. mortar	6	0.029	15.68	$x_{1,9,4}$
Expanded polystyrene half cut mech. fix	6	0.029	15.14	$x_{1,10,4}$
Mineral wool dots	3	0.035	9.87	$x_{1,11,1}$
Mineral wool mech. fix	3	0.035	10.74	$x_{1,12,1}$
Mineral wool proy adh. mortar	3	0.035	12.73	$x_{1,13,1}$
Mineral wool dots	4	0.035	11.48	$x_{1,11,2}$
Mineral wool mech. fix	4	0.035	12.34	$x_{1,12,2}$
Mineral wool proy adh. mortar	4	0.035	14.33	$x_{1,13,2}$
Mineral wool dots	5	0.035	13.22	$x_{1,11,3}$
Mineral wool mech. fix	5	0.035	14.09	$x_{1,12,3}$
Mineral wool proy adh. mortar	5	0.035	16.08	$x_{1,13,3}$

TABLE A.1 (Continuation)

Building material	**Thickness** *cm*	**Conductivity** $Wm^{-1}K^{-1}$	**Cost** *€/m2*	**Variable**
Wood wool mech. fix	3	0.09	17.29	$x_{1,14,1}$
Wood wool mech. fix	4	0.09	18.72	$x_{1,14,2}$
Wood wool mech. fix	5	0.09	21.46	$x_{1,14,3}$
Agglomerate of expanded cork dots	3	0.036	14.82	$x_{1,15,1}$
Agglomerate of expanded cork mech. fix	3	0.036	15.63	$x_{1,16,1}$
Agglomerate of expanded cork dots	4	0.036	18.43	$x_{1,15,2}$
Agglomerate of expanded cork mech. fix	4	0.036	19.24	$x_{1,16,2}$
Agglomerate of expanded cork dots	5	0.036	22.05	$x_{1,15,3}$
Agglomerate of expanded cork mech. fix	5	0.036	22.86	$x_{1,16,3}$
Agglomerate of expanded cork dots	6	0.036	25.66	$x_{1,15,4}$
Agglomerate of expanded cork mech. fix	6	0.036	26.47	$x_{1,16,4}$
Nanoporous aerogel	3	0.013	199.03	$x_{1,17,1}$
Layer 2 (air gap)				
Light ventilated air gap	3	0.08	0	$x_{2,1,1}$
Light ventilated air gap	5	0.09	0	$x_{2,1,2}$
Light ventilated air gap	8	0.09	0	$x_{2,1,3}$
Light ventilated air gap	10	0.09	0	$x_{2,1,4}$
Not ventilated air gap	3	0.17	0	$x_{2,2,1}$
Not ventilated air gap	5	0.18	0	$x_{2,2,2}$
Not ventilated air gap	8	0.18	0	$x_{2,2,3}$
Not ventilated air gap	10	0.18	0	$x_{2,2,4}$
No air gap	0	-	0	$x_{2,3,1}$
Layer 3 (external coating)				
Face brick 24x11.5x5 waterproof	11.5	0.76	81.25	$x_{3,1,1}$
Face brick 24x11.3x5.2 clinker	11.5	0.76	89.93	$x_{3,2,1}$
Face brick 29x11.5x5 waterproof	11.5	0.76	83.14	$x_{3,3,1}$
Pressed face brick 24x12x4	12	0.76	119.3	$x_{3,4,1}$
Pressed face brick 24x12x5	12	0.76	112.83	$x_{3,5,1}$
Composite panel	0.3	2.09	109.83	$x_{3,6,1}$
Extruded ceramic panel	0.4	3.38	121.19	$x_{3,7,1}$
Standard mortar	1	0.93	17.28	$x_{3,8,1}$
Standard mortar	1.3	0.93	18.66	$x_{3,8,2}$
Standard mortar	1.5	0.93	19.49	$x_{3,8,3}$
Standard mortar	1.8	0.93	20.87	$x_{3,8,4}$
Standard mortar	2	0.93	21.79	$x_{3,8,5}$
Thermal mortar	1	0.67	27.45	$x_{3,9,1}$
Thermal mortar	1.3	0.67	29.01	$x_{3,9,2}$
Thermal mortar	1.5	0.67	30.31	$x_{3,9,3}$
Thermal mortar	1.8	0.67	31.78	$x_{3,9,4}$
Thermal mortar	2	0.67	34.03	$x_{3,9,5}$
Layer 4 (painting)				
Lime paint	0.1	-	8.40	$x_{4,1,1}$
Traditional whitewashing	0.1	-	2.86	$x_{4,2,1}$
Whitewashed on exterior facing	0.1	-	10.72	$x_{4,3,1}$
Plastic paint	0.1	-	7.66	$x_{4,4,1}$
Thermal plastic paint	0.1	-	16.07	$x_{4,5,1}$
Silicate paint	0.1	-	10.21	$x_{4,6,1}$
Glazing silicate paint	0.1	-	9.50	$x_{4,7,1}$
Pliolite paint	0.1	-	9.49	$x_{4,8,1}$
Silicone resin paint	0.1	-	10.3	$x_{4,9,1}$
No painting	0	-	0	$x_{4,10,1}$

TABLE A.2 Data of the substitution of the old windows and of the new frames

Material	**Extra data**	**Thickness** *cm*	**Conductivity** $Wm^{-1}K^{-1}$	**Cost** *€/(m2 of window)*	**Variable**
Removing old windows	No glass recuperation			10.28 per unit	
	No frame recuperation			5.11 per unit	
Pre frame				Always incluided	
New frame aluminium	Glass max 3 cm	6.4	1.8	178.67	z^f_1
With thermal break	Glass max 3 cm	6.4	0.8	271.67	z^f_2
New frame wood	Glass max 3.2/ min 2.1 cm	6.8	1.43	468.35	z^f_3
	Glass max 4.2/ min 3.2 cm	7.8	1.3	540.71	z^f_4
	Glass max 5.4/min 4.3 cm	9.8	1.18	552.14	z^f_5
New frame aluminium/wood	Glass max 4.7/ min 1.7 cm	6.8	1.33	726.43	z^f_6
	Glass max 5.5/ min 2.5 cm	7.8	1.13	776.43	z^f_7
New frame PVC	Glass max 4 cm	7	1.3	176	z^f_8

TABLE A.3 Data of the combinations of the new glasses and air chambers

Type	Combination data *mm*	Thickness *cm*	Conductivity $Wm^{-1}K^{-1}$	Cost *€/m2*	Variable
Standard double glass	4/6/4	1.4	3.3	39.98	$z^g{}_1$
	4/6/6	1.6	3.3	40.09	$z^g{}_2$
	4/8/4	1.6	3.1	40.39	$z^g{}_3$
	4/12/4	2	3.1	40.39	$z^g{}_4$
	4/12arg/4	2	2.7	48.2	$z^g{}_5$
	5/8/4	1.7	3.1	48.97	$z^g{}_6$
	5/12/4	2.1	2.9	49.58	$z^g{}_7$
	5/12arg/4	2.1	2.7	56.77	$z^g{}_8$
	6/8/5	1.9	3.3	62.85	$z^g{}_9$
	8/12/6	2.6	2.6	81.19	$z^g{}_{10}$
	8/12arg/6	2.6	2.5	88.37	$z^g{}_{11}$
	8/16/6	3	2.8	82.32	$z^g{}_{12}$
	8/16arg/6	3	2.7	89.51	$z^g{}_{13}$
	8/16/8	3.2	2.7	94.74	$z^g{}_{14}$
	8/16arg/8	3.2	2.6	103.36	$z^g{}_{15}$
Low emissivity double glass	4/6/6	1.4	2.5	129.8	$z^g{}_{16}$
	4/8/6	1.8	2.1	130.21	$z^g{}_{17}$
	6/8/6	2	2.1	136.51	$z^g{}_{18}$
	6/12/6	2.4	1.6	137.16	$z^g{}_{19}$
	6/12arg/6	2.4	1.3	144.31	$z^g{}_{20}$
	6/16/6	2.8	1.4	138.25	$z^g{}_{21}$
	6/16arg/6	2.8	1.1	145.43	$z^g{}_{22}$
	8/8/6	2.2	2.1	159.26	$z^g{}_{23}$
	8/12/6	2.6	1.6	159.87	$z^g{}_{24}$
	8/12arg/6	2.6	1.3	167.06	$z^g{}_{25}$
	8/16/6	3	1.4	161.31	$z^g{}_{26}$
	8/16arg/6	3	1.1	168.19	$z^g{}_{27}$
	8/20/6	3.6	1.4	162.64	$z^g{}_{28}$
	8/20arg/6	3.6	1.1	169.63	$z^g{}_{29}$
Double glass with solar control	6/6/4	1.6	3.3	134.22	$z^g{}_{30}$
	6/8/4	1.8	3.1	134.33	$z^g{}_{31}$
	6/12/4	2.2	2.8	135.25	$z^g{}_{32}$
	6/16/4	2.6	2.7	136.37	$z^g{}_{33}$
	6/20/4	3	2.7	137.81	$z^g{}_{34}$
	6/20arg/4	3	2.6	144.99	$z^g{}_{35}$
	6/16/8	3	3.2	162.88	$z^g{}_{36}$
	6/16/12	3.4	2.7	183.83	$z^g{}_{37}$
	6/20/12	3.8	2.7	185.27	$z^g{}_{38}$
	6/20arg/12	3.8	2.6	192.45	$z^g{}_{39}$
	8/6/4	1.8	3.2	134.22	$z^g{}_{40}$
	8/6/12	2.6	3.2	134.22	$z^g{}_{41}$
	8/12/4	2.4	2.8	135.24	$z^g{}_{42}$
	8/12/12	3.2	2.8	182.7	$z^g{}_{43}$
	8/12arg/12	3.2	2.6	189.88	$z^g{}_{44}$
	8/20/4	3.2	2.7	137.81	$z^g{}_{45}$
	8/20/8	3.6	2.7	164.11	$z^g{}_{46}$
	8/20/12	4	2.7	185.27	$z^g{}_{47}$
	8/20arg/12	4	2.5	192.45	$z^g{}_{48}$
Triple glass low int/ext emissivity	4/16arg/4/16arg/4	4.4	0.6	107.91	$z^g{}_{49}$
	6/16arg/4/16arg/6	4.8	0.6	129.32	$z^g{}_{50}$
	6/16arg/6/16arg/6	5	0.6	138.22	$z^g{}_{51}$
	8/16arg/6/16arg/6	5.2	0.6	158.39	$z^g{}_{52}$
	8/16arg/4/16arg/6	5	0.6	149.48	$z^g{}_{53}$
	8/16arg/4/16arg/4	4.8	0.6	138.78	$z^g{}_{54}$
With added solar control	4/16arg/4/16arg/4	4.4	0.6	136.34	$z^g{}_{55}$
	6/16arg/4/16arg/6	4.8	0.6	168.38	$z^g{}_{56}$
	6/16arg/6/16arg/6	5	0.6	177.29	$z^g{}_{57}$
	8/16arg/6/16arg/6	5.2	0.6	206.5	$z^g{}_{58}$
	8/16arg/4/16arg/6	5	0.6	197.59	$z^g{}_{59}$
	8/16arg/4/16arg/4	4.8	0.6	186.89	$z^g{}_{60}$