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Additional Information

Master production schedule using robust optimization approaches in an automobile second-tier supplier

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Abstract

This paper considers a real-world automobile second-tier supplier that manufactures decorative surface finishings of injected parts provided by several suppliers, and which devises its master production schedule by a manual spreadsheet-based procedure. The imprecise production time in this manufacturer's production process is incorporated into a deterministic mathematical programming model to address this problem by two robust optimization approaches. The proposed model and the corresponding robust solution methodology improve production plans by optimizing the production, inventory and backlogging costs, and demonstrate the their feasibility for a realistic master production schedule problem that outperforms the heuristic decision-making procedure currently being applied in the firm under study.

Keywords: robust optimization, master production schedule, uncertainty, automotive industry

1 Introduction

Production planning consists of a process followed to not only acquire resources and raw materials, but to also plan production activities, required to transform raw materials into finished products to meet customer demand as efficiently as possible. Hence the goal of production planning is to make planning decisions by optimizing the trade-off between economic objectives and the service level or customer satisfaction (Pochet and Wolsey 2006).

According to Mula et al. (2006a) and Díaz-Madroñero et al. (2014b), five production planning areas can be considered: hierarchical production planning (HPP); aggregate production planning (APP); material requirement planning and manufacturing resources planning (MRP), especially the theory presented in Grubbstrom and Tang (2000) and its dynamics, as also presented in Grubbström et al. (2010), where robust perturbations can be evaluated; supply chain planning (SCP), as in Kovačić and Bogataj (2013), which also includes reverse principles, where the robustness of intensity and timing can be studied; master production schedule (MPS). The MPS is a mono-level lot-sizing problem used to establish an optimal production plan that provides release dates and amounts of finished products to be manufactured by minimizing production, inventory holding and overtime costs, and by meeting customer demand without backlogs and stockouts (Kimms 1998).

All the factors and parameters involved in MPS calculations can be affected by their inherent uncertainty, especially in current industrial environments with short product life-cycle products that focus on consumer preferences (Weng and Parlar 2005; Aytac and Wu 2013), and with complex manufacturing processes (Nannapaneni and Mahadevan 2014). In these situations, if the uncertainty associated with input data is not contemplated, infeasible or economically unviable production plans may be obtained. Thus according to Mula et al. (2006a), there are many research works and applications that aim to formalize uncertainty in manufacturing systems. Indeed the literature contains several surveys about modeling uncertain production problems, such as Mula et al. (2006a), Ko et al. (2010), Dolgui et al. (2013), Aloulou et al. (2014), to which readers are referred.

Different methods have been used by researchers to consider uncertainty in production planning problems, which have highlighted stochastic programming, fuzzy mathematical programming, stochastic dynamic programming and robust optimization, according to Sahinidis (2004). An MPS model can be developed to operate in an uncertain environment where statistical data are either not that reliable or not even available. In these contexts, modeling variations in uncertain input parameters with probability distributions may not be the best option and, therefore, stochastic approaches have to be avoided (Mula et al. 2008; Díaz-Madroñero et al. 2014a). When sufficient data are unavailable, robust optimization approaches can be a good alternative as opposed to stochastic programming scenario-based ones because the problem size of the counterpart robust optimization model does not increase, but provides a way to incorporate different attitudes to risk, depending on the considered robust approach (Gabrel et al. 2014; Gorissen et al. 2015).

The MPS problem tackled by this study is used in a real-world automotive second-tier supplier as a manual process that is supported by using a spreadsheet, and based on planners' personal judgment and experience. Manual procedures consider a short or myopic time perspective when planning instead of an entire view of the whole horizon planning at any time, which could generate suboptimal plans (Díaz-Madroñero et al. 2014c). In order to provide MPS problems with optimal results, a deterministic mathematical programming model, based on the previous material requirement model by Mula et al. (2006b), is presented. The uncertainty conditions related to the manufacturing process time for each finished product are incorporated into the proposed model by two different robust optimization approaches proposed by Soyster (1973) and Bertsimas and

Sim (2004). Other approaches in the robust optimization context are addressed in Werner (2008), Kuchta (2011), Kara et al. (2017), and De La Vega et al. (2017).

To illustrate the validity of this proposal, deterministic and robust optimization models are applied to the real-world automotive second-tier supplier under study and the obtained results are compared with the currently applied manual procedure. The main contributions of this proposal are summarized as so: (i) provide a new robust optimization model for MPS under process time uncertainty that contemplates the backorders, overtime and idle times of productive resources; (ii) present a real world case study from the surface finishings of injected parts for MPS under process uncertainties; (iii) demonstrate the usefulness and profitability of robust optimization models compared to the heuristic and spreadsheet-based procedures that are frequently carried out in industrial companies.

The rest of the paper is arranged as follows: Section 2 presents a literature review about MPS under uncertainty conditions. Section 3 describes the industrial problem and the current planning procedure. Section 4 proposes the mathematical programming model for the MPS problem described in the previous section. Section 5 describes the solution methodology based on robust optimization approaches. Section 6 evaluates the behavior of the proposed robust optimization model in a real-world second-tier automobile supplier. Finally, Section 7 provides conclusions and directions for further research.

2 Literature review

MPS is a production area that has attracted researchers for decades, especially for its close interaction with MRP systems (Conlon 1976). The need to obtain optimal economically competitive plans has resulted in different approaches to develop MPS optimization models (Lee and Moore 1974; González and Reeves 1983; Chu 1995). However, the uncertainty associated with the input of the parameters that are derived for internal and external industrial factors, and the growing dynamism in the economic environment, mean that they need to be incorporated into existing mathematical programming models through other analytic approaches such as: stochastic programming (Vargas and Metters 2011; Körpeolu et al. 2011; Englberger et al. 2016; Lage Junior and Godinho Filho 2017); fuzzy mathematical programming (Lehtimaki 1987; Supriyanto and Noche 2011; As'ad et al. 2015); robust optimization (Ng and Fowler 2007; Gharakhani et al. 2010; Li et al. 2011; Alem and Morabito 2012; Kawas et al. 2013; Rahmani et al. 2013; Tavakkoli-Moghaddam et al. 2014; Li and Li 2015; Sakhaii et al. 2015; Haojie et al. 2017).

The scope of this section is the MPS problem based on robust optimization programming approaches. Ben-Tal and Nemirovski (1998) and Bertsimas and Sim (2004) are the most popular robust optimization methods in the MPS area, but other popular robust optimization approaches, such as those proposed by Soyster (1973) or Mulvey et al. (1995), have been successfully applied to other production planning models. (Gharakhani et al. 2010; Li et al. 2011; Alem and Morabito 2012; Tavakkoli-Moghaddam et al. 2014; Sakhaii et al. 2015; Haojie et al. 2017) opt for the (Bertsimas and Sim 2004) approach, while Ng and Fowler (2007), Kawas et al. (2013), Rahmani et al. (2013) and Li and Li (2015) consider other approaches like those indicated in Table 1.

According to Childerhouse and Towill (2002), Wang and Shu (2005) and Peidro et al. (2009), sources of uncertainty can be divided into three groups: supply, process and demand. Uncertainty in supply is caused by variability in lead times or the quality of items provided by suppliers. Uncertainty in demand corresponds to variability of customer orders due to inexact forecasts or their changing preferences. Process uncertainty is the result of poorly reliable manufacturing processes because of, for example, machine breakdowns or uncontrollable setup times Gharakhani et al. (2010), Kawas et al. (2013), Tavakkoli-Moghaddam et al. (2014) and Sakhaii et al. (2015) contemplate the uncertainty inherent to the manufacturing process in isolation. However, other authors like Alem and Morabito (2012) and Rahmani et al. (2013) add demand levels to the considered uncertain parameters, while Ng and Fowler (2007) also include supply uncertainty. Finally, demand uncertainty is considered separately in the proposals by Li et al. (2011), Li and Li (2015) and Haojie et al. (2017), as shown in Table 2.

Another dimension considered in this review is the structure of the objective function, which depends on the extension of the production planning problem addressed by each contribution. In this context, the most frequent component is minimization of production costs, including setup costs, inventory holding costs and stockout costs (see Table 2). Other costs included in the objective functions of the reviewed studies are labor and hiring/firing costs, supply costs, and other costs related to capacity expansion, failing inspections or breakdowns. Finally, robust optimization MPS models are frequently validated with random generated instances to carry out computational experiments, and only Ng and Fowler (2007), Li et al. (2011), Alem and Morabito (2012) and Rahmani et al. (2013) present real applications in the electronics and semiconductors, refineries, furniture and household appliances industrial sectors.

After a review process, the following issues are highlighted for the robust optimization MPS problem: (1) although stockouts costs are considered by several authors, the possibility of delaying customer orders and considering backorders is not addressed; (2) apart from labor costs, the overtime and idle time of productive resources, and production machines, are not frequently found in the reviewed models; (3) the shortage of validated robust optimization MPS in different real-world industrial sectors. These aspects are taken into account to address the robust optimization MPS problem in this work.

Table 1. The main characteristics of the reviewed articles

	Robust optimization approach	Uncertainty source	Objective function members	Industrial sector application
Ng and Fowler (2007)	Atamturk and Zhang (2007)	Demand Process Supply	Inventory holding costs Stockout costs	Electronics and semiconductors
Gharakhani et al. (2010)	Bertsimas and Sim (2004)	Process	Capacity expansion costs	Random generated instances
Li et al. (2011)	Ben-Tal and Nemirovski (1998) Bertsimas and Sim (2004)	Demand	Total profit Production costs Non quality costs	Refinery
Alem and Morabito (2012)	Bertsimas and Sim (2004)	Demand Process	Production costs Setup costs Stockout costs	Furniture
Kawas et al. (2013)	Ben-Tal and Nemirovski (1998)	Process	Total profit Failing inspections costs	Random generated instances
Rahmani et al. (2013)	Mulvey et al. (1995) Yu and Li (2000)	Demand Process	Production costs Supply costs Labor costs Inventory holding costs Hiring/firing costs Stockout costs	Household appliances
Tavakkoli-Moghaddam et al. (2014)	Bertsimas and Sim (2004)	Process	Production costs Supply costs Inventory holding costs Stockout costs	Random generated instances
Li and Li (2015)	Li et al. (2012)	Demand	Total profit Production costs Labor costs	Random generated instances
Sakhaii et al. (2015)	Bertsimas and Sim (2004)	Process	Production costs Labor costs Hiring/firing costs Stockout costs Breakdown costs	Random generated instances
Haojie et al. (2017)	Bertsimas and Sim (2004)	Demand	Production costs Setup costs Inventory holding costs	Random generated instances

3 Problem description

The case study presented herein corresponds to a company from the automobile sector. It produces the decorative surface finishings of the injected parts provided by several suppliers. It is a capacitated process with only one available resource. The process also includes a production sequence in produces a limited number of batches. Each item has a different lot size depending on the volume of the part. Each batch performs the following cyclic process:

1. Injection parts (raw material) are placed inside the machine in specific batches per item
2. Raw material is processed inside the resource (machine)
3. Processed parts leave the machine and are checked
4. The parts that pass quality controls are packed (faulty parts are classified as scrap and are placed inside containers to be disposed of later).

It must be taken into account that as this process is capacitated and cyclical, the machine must not stop at any time during the production process; therefore, batches must be periodically placed inside. Those batch numbers that can be produced depend on the production time, as determined by installation (machine), start and shutdown.

This manufacturing process can become unstable as production times for certain references prolong. Specifically, up to five references of a particular surface finishing type usually show increases in their manufacturing time of up to 10%. Concerning this finish type, the main problem is achieving a given color within accepted tolerances (measured by a colorimeter). Given the novelty and technical complexity of this technology, it is normal to deviate the required color in the different chemical process stages. To this end, several measurements are made during the process to ensure that the parameters established in each stage are achieved, and different chemical solutions are added to carry out the process until its nominal values. However, these addictions can increase the required manufacturing time.

In this complex environment, production planning is calculated according to customer demand and to the available stock of raw materials. Production planning also depends on several constraints, such as the production capacity of the available productive resources and the size of the production batches for each reference.

Thus we state the robust MPS problem in the second-tier automobile supplier to be as follows:

With:

- Product data, such as production time, production scrap percentage, production lot size, etc.
- Production capacity of available resources

- Initial inventory levels
- Demand over the entire planning horizon

To determine:

- The amount of each product to be produce per period
- The inventory level of each product per period
- The backorders of each product per period
- The utilization of productive resources, including extra and idle times

The main goal to meet is to:

- Minimize the total costs, including production costs, extra and idle time costs and inventory costs, while meeting customer demand by minimizing backorder costs.

Moreover, the following assumptions are made:

- Demand levels are considered firm over the whole planning horizon because short-term demand data do not usually vary
- Uncertainty is present in the manufacturing process as it can present instability, which can affect the production time of the finished products.

3.1 Heuristic procedure for production planning

The current decision-making process is a heuristic procedure based on the use of a spreadsheet (Figure 2).

Item	Pcs / Batch	Batches in sequence	Consumption (pcs)	Balance	Raw Material Stocks
Item 1	100	1	0	780	780
Item 2	72	3	1196	247	1.443
Item 3	72	2	798	430	1.228
Item 4	72	4	1595	1262	2.857
Item 5	196	1	1086	-518	568
Item 6	60	5	1662	-323	1.339
Item 7	60	5	1662	-177	1.485
Item 8	430	1	2382	-1076	1.306
Item 9	72	2	798	270	1.068
Item 10	96	1	532	-528	4
Item 11	96	1	532	-340	192
Item 12	144	1	798	-438	360

Figure 1. Spreadsheet for the heuristic production planning procedure

The spreadsheet fields are defined as follows:

- The first column corresponds to *Item ID*, which is the unique identifier of each manufactured product.
- The second column indicates the number of parts per batch (lot size).
- The *Batches in sequence* column shows the number of batches for each item to be produced in the manufacturing sequence.
- The *Consumption (pcs)* column is the number of raw material parts to be consumed during a certain production time period. This production time is usually set for 1 production day.
- The *Balance* column shows the raw material stock estimated for each item at the end of the production period.
- The *Raw Material Stocks* column points out the current feedstock level (in parts) for each item.

Depending on the information on the demand levels, the amounts of available stock and a production lot size of each reference, the production plan is determined according to the mechanism indicated in Figure 2.

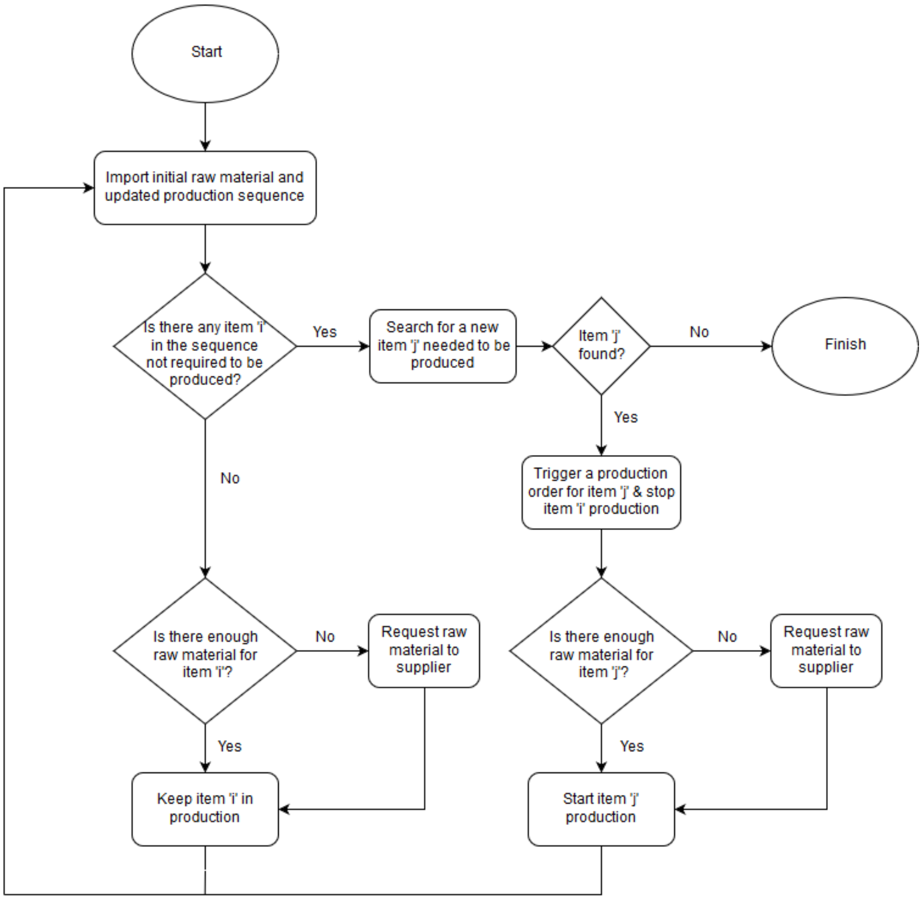

Figure 2. Current heuristic procedure for production planning

The current planning process is based on the following heuristics:

The first step is to import the current production sequence (*Item ID* & *Batches in sequence*) from the process machine, as well as the raw material stocks (*Raw Material Stocks*) of each item. All this information is obtained from the database of the implemented ERP and is loaded to the spreadsheet by a database query. After updating these data, the *Consumption* Column is automatically calculated, depending on *Item ID* and the number of *Batches in the sequence*.

From this point, two possibilities exist:

1. In the sequence, it is not necessary to manufacture item *i* as its demand is covered. If this condition is fulfilled, a new required item *j* is searched for. At this point, once again two options are available:
    a. An item *j* needs to be produced. The production of item *i* is stopped and a production order of item *j* is trigged. If there is not enough raw material to produce item *j* (*Consumption* > *Raw Material Stocks*, so *Balance* < 0), the raw material is requested from the supplier.
    b. No item *j* needs to be produced. This would end the production process.
2. In the sequence, it is necessary to keep producing item *i* in as its demand is not yet covered. In this case, once again there are two options:
    a. There is enough raw material for item *i* to continue being produced (*Consumption* < *Raw Material Stocks*, so *Balance* > 0). So item *i* remains in the production sequence.
    b. There is not enough raw material stock to keep producing item *i* (*Consumption* > *Raw Material Stocks*, so *Balance* < 0). So a purchase order to request raw material is sent to the supplier to continue producing that item.

This kind of spreadsheet-based procedure can often lead to suboptimal results and serious errors, which may involve substantial costs (Caulkins et al. 2007; Powell et al. 2008; Dzuranin and Slater 2014).

4 Model formulation

In this section, a mathematical programming model for the MPS is proposed to improve the results obtained by the manual procedure described in the previous section. First, a deterministic model is presented, based on Mula et al. (2006b), which does not consider any uncertain parameter, but contemplates relative production performance. Then this model is reformulated with two robust optimization approaches to protect against manufacturing process fluctuations. The notation defines the sets of indices, parameters and decision variables for the proposed model (Table 2).

Table 2. Notation

Sets of indices	
I:	Set of products (i =1, 2,…,I).
R:	Set of productive resources (r =1, 2,…,R).
T:	Set of planning periods (t =1, 2…T).
Parameters	
d_{it}:	Demand of product i during time period t (units).
$INVTO_i$:	Initial inventory amount of product i (units).
SR_i:	Scheduled receptions of product i during time period t (units)
ar_{ir}:	Required time of resource r to produce one unit of product i (hours)
β_i:	Manufacturing performance of product i in the available production resources (%)
lot_i:	Production lot size for product i (units)
cap_r:	Production capacity of resource r (hours)
$B0_i$:	Initial backorders of product i (units)
cp_i:	Variable cost of the normal production of one unit of product i (euros)
ci_i:	Inventory cost of one unit of product i (euros)
cb_i:	Backorder cost of one unit of product i (euros)
$cidt_r$:	Idle time hour cost of resource r (euros)
$covt_r$:	Overtime hour cost of resource r (euros)
Decision variables	
P_{it}:	Quantity of product i to be produced during time period t (units)
$INVT_{it}$:	Amount of the inventory of i at the end of time period t (units)
K_{it}:	Number of lots to produce of products i during time period t.
B_{it}:	Backorder amount of product i at the end of time period t (units)
TID_{rt}:	Idle time of resource r during time period t (hours)
TOV_{rt}:	Overtime of resource r during time period t (hours)
Objective function	
z:	Total costs (euros).

The formulation of the deterministic model for the MPS is as follows:

Minimize total costs:

$$Min\ z = \sum_{i=1}^{I}\sum_{t=1}^{T}(cp_i\, P_{it} + ci_i\, INVT_{it} + cb_i B_{it}) + \sum_{r=1}^{R}\sum_{t=1}^{T}(cidt_r\, TID_{rt} + covt_r TOV_{rt})$$

(1)

Subject to:

$$INVT_{i,t-1} + \beta_i P_{it} + SR_{it} - INVT_{it} - B_{i,t-1} + B_{it} = d_{it} \qquad \forall i, \forall t \qquad (2)$$

$$\sum_{i=1}^{I} ar_{ir} P_{it} + TID_{rt} - TOV_{rt} = cap_{rt} \qquad \forall r, \forall t \qquad (3)$$

$$P_{it} = K_{it} \cdot lot_i \qquad \forall i, \forall t \qquad (4)$$

$$B_{it} = 0 \qquad \forall i, \forall t = T \qquad (5)$$

$$P_{it}, K_{it}, INVT_{it}, B_{it}, TID_{rt}, TOV_{rt} \geq 0 \qquad \forall i, \forall t, \forall r \qquad (6)$$

$$K_{it} \in Z \qquad \forall i, \forall t \qquad (7)$$

Constraint (1) corresponds to the objective function, which aims to minimize the total costs computed over the planning horizon, including the production, inventory, backorder, idle time and overtime costs. Constraint (2) determines the inventory balance equation. Constraint (3) establishes the available capacity for normal and overtime production. Constraint (4) ensures that the amount of each item to be produced is an integer multiple of each product lot size. Constraint (5) ensures that the backorders at the end of the planning horizon are null. Constraint (6) and Constraint (7) determine the non negativity and integer conditions for the decision variables, respectively. In this MPS problem, the ar_{ir} values can vary depending on several factors, such as the operating conditions of the production resources, workers' training, etc. We consider that the other parameters are crisp because the related information over the planning horizon is well-known. Indeed the production system can be affected by the variability of this processing time and, therefore, infeasible production plans can be obtained. In this production environment, the proposed deterministic model has to be transformed with robust optimization approaches to obtain production plans that are protected against uncertain processing times.

5 Solution methodology

In order to reach a robust solution for the considered MPS problem, the deterministic model presented in Constraints (1) to (7) is transformed by adopting the robust optimization approaches by Soyster (1973) and by Bertsimas and Sim (2004).

For illustration purposes, a nominal linear programming model that includes the uncertain parameters that belong to matrix A is considered as follows:

Maximize cx (8)

subject to:

$Ax \leq b$ (9)

$l \leq x \leq u$ (10)

According to Bertsimas and Sim (2004) and to Ben-Tal and Nemirovski (2000), if a particular row i of matrix A is considered, J_i represents the set of coefficients in row i subject to uncertainty. Each entry $a_{ij}, j \in J_i$ is modeled as a symmetric and bounded random variable $\tilde{a}_{ij}, j \in J_i$ that takes the values in $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$.

5.1 The Soyster (1973) robust optimization approach

One of the seminal contributions in robust optimization was presented by Soyster (1973), who proposed a linear optimization model to construct a feasible solution for all the data that belong to a convex set. According to the Soyster (1973) approach, the robust formulation of model (8)-(10) corresponds to:

Maximize cx (11)

subject to:

$\sum_j a_{ij}x_j + \sum_{j \in J_i} \hat{a}_{ij} y_j \leq b_i \quad \forall i$ (12)

$x \geq 0$ (13)

where $\hat{a}_{i,j} = [A_{i,j}], A_j \in K_j$. Therefore, matrix $\tilde{A}$ is composed of the most extreme values of each parameter subject to uncertainty. This approach ensures that model solutions are always feasible for any uncertain parameter variation within the set limits.

To apply the Soyster (1973) robust approach in the deterministic model, the following changes in production capacity restriction are made:

$\sum_{i=1}^{I} ar_{ir} P_{it} + \sum_{i=1}^{I} arv_{ir}\, Py_{it} + TID_{rt} - TOV_{rt} = cap_{rt} \quad \forall r, \forall t$ (14)

since arv_{ir} is the maximum variation of the time required by the resource for each product, and Py is the auxiliary integer variable proposed and implemented by Soyster (1973). Finally, the non negativity constraint is added to the newly defined variables. As we can see, this is the deterministic model application for the worst case.

5.2 The Bertsimas and Sim (2004) robust optimization approach

The Bertsimas and Sim (2004) approach allows the solution to remain close to the optimum, with no excess of conservatism presented by the first robust programming approaches being reaching. First, data model *U* is defined, which contains the elements subject to variability. Since J_i is the set of coefficients of $a_{i,j}$, with $j \in J_i$, parameters that are subject to uncertainty, which implies that $\tilde{a}_{i,j}$ takes values according to a symmetric distribution with a mean equal to the nominal value of $a_{i,j}$ within the interval $[a_{i,j} - \hat{a}_{i,j}, a_{i,j} + \hat{a}_{i,j}]$. For each constraint *I,* a parameter Γ_i, which is not necessarily integer, is introduced that can take the values within the interval [0, $|J_i|$]. Parameter Γ_i adjusts the level of protection to be assumed for the model's *i-th* restriction, where the highest parameter Γ_i value is the level of protection that is taken in relation to this restriction (Bertsimas and Sim, 2004). It should be noted that it is unlikely that all coefficients $a_{i,j}$ with $j \in J_i$ vary, so only one number $\lfloor \Gamma_i \rfloor$ (the largest integer less than or equal to Γ_i) of the parameters is allowed to vary at the same time. Therefore, the proposed model is as follows:

$$\text{Maximize } c^T x \tag{15}$$

subject to:

$$\sum_j a_{i,j}x_j + \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \backslash S_i\}} \left(\sum_{j \in S_i} \hat{a}_{i,j} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{i,t_i} y_t \right) \leq b_i \quad \forall i \tag{16}$$

$$-y_i \leq x_j \leq y_i \quad \forall j \tag{17}$$

$$l \leq x \leq u \tag{18}$$

$$y \geq 0 \tag{19}$$

Here we observe that if Γ_i is the chosen integer, the ith restriction is protected by $\beta_i(x, \Gamma_i) = max_{\{S_i | S_i \subseteq J_i|, |S_i| = \Gamma_i\}} \left(\sum_{j \in S_i} \hat{a}_{i,j} |x_j| \right)$. For the value of parameter $\Gamma_i = 0$, $\beta_i(x, 0) = 0$ is obtained, which implies that the constraints coincide with the constraints of the nominal problem. When the value of parameter $\Gamma_i = |J_i|$, the same result is obtained as in the Soyster (1973) approach (the worst case). Therefore by varying $\Gamma_i \in [0, |J_i|]$, the level of model robustness can be adjusted against the level of the solution's conservatism. The previous model is non linear, but one advantage of this modeling approach is the possibility of transforming this model into a linear model with the following definition: given vector x *, the protection function of the *i-th* restriction is:

$$\beta_i(x^*, \Gamma_i) = \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| + \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left(\sum_{j \in S_i} \hat{a}_{i,j} |x_j^*| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{i,t_i} |x_j^*| \right) \quad (20)$$

whose value equals the objective function of the following linear programming problem:

$$\beta_i(x^*, \Gamma_i) = max \sum_{j \in J_i} \hat{a}_{i,j} |x_j^*| z_{i,j} \quad (21)$$

subject to:

$$\sum_{j \in J_i} z_{i,j} \leq \Gamma_i \quad (22)$$

$$0 \leq z_{i,j} \leq 1 \quad \forall j \in J_i \quad (23)$$

and it has as a dual formulation:

$$min \sum_{j \in J_i} p_{i,j} + z_i \Gamma_i \quad (24)$$

subject to:

$$z_i + \mathrm{p}_{ij} \geq \hat{a}_{i,j} |x_j^*| \quad \forall i, j \in J_i \quad (25)$$

$$\mathrm{p}_{ij} \geq 0 \quad \forall i, j \in J_i \quad (26)$$

$$z_i \geq 0 \quad \forall i, j \in J_i \quad (27)$$

If the dual problem is replaced in the initially expressed expression, an equivalent linear-type problem a is obtained:

$$\text{Maximize } cx \quad (28)$$

subject to:

$$\sum_j a_{i,j} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{i,j} \leq b_i \quad \forall i \quad (29)$$

$$z_i + p_{i,j} \geq \hat{a}_{i,j} y_j \quad \forall i, j \in J_i \quad (30)$$

$$-y_j \leq x_j \leq y_j \quad \forall j \quad (31)$$

$$l_j \leq x_j \leq u_j \quad \forall j \quad (32)$$

$$p_{i,j} \geq 0 \quad \forall i, j \in J_i \quad (33)$$

$$y_j \geq 0 \quad \forall j \quad (34)$$

$$z_i \geq 0 \quad \forall i \quad (35)$$

To apply the Bertsimas and Sim (2004) robust approach in the deterministic model, the following changes in the production capacity restrictions are made:

$$\sum_{i=1}^{I} ar_{ir}P_{it} + TID_{rt} - TOV_{rt} + Z_{rt}\omega_{rt} + \sum_{i=1}^{I} Pb_{irt} = cap_{rt} \qquad \forall r, \forall t \qquad (36)$$

since Z_{rt} and Pb_{irt} are the integral auxiliary variables implemented by Bertsimas and Sim (2004). The control parameter is also added, which allows model robustness to be regulated. This parameter determines the number of products whose processing time is affected by uncertainty. Moreover, the following restrictions are added:

$$Z_{rt} + Pb_{irt} \geq arv_i \cdot Py_{it} \qquad \forall i, \forall r, \forall t \qquad (37)$$

by adding the non negativity restrictions to the new variables introduced into the previous expressions.

After implementing these restrictions into the deterministic model, the robust model is obtained by the approach of Bertsimas and Sim (2004), with protection against uncertainty in the process regulated with the parameter; i.e., the degree of protection against uncertainty can be chosen by selecting the number of products that can change the time of their process. This is an advantage over the Soyster (1973) model because it is not that conservative, and all the references presenting variations in the manufacturing process time at the same time is not likely, which allows the model to be more flexible and cuts the robustness cost.

6 Application to an automobile second-tier supplier

The proposed models are implemented with the Maximal Software's MPL (mathematical programming language) modeling tool, installed in a PC with an Intel Core i5 2.80 GHz processor and 6 GB of RAM. The employed resolution algorithm (solver) is Gurobi (6.5.1). The input parameters are included in Microsoft Access data tables, along with the output data, which are also exported to these data tables.

The time to run the studied models was 6 periods per week, with 84 different references that correspond to approximately 80% of the volume of production in the case study.

In Annex 1, Table A.1 presents the basic data about each item, such as manufacturing performance (β_i), production time (ar_{ir}), production lot size (lot_i), unitary production costs (cp_i), inventory holding costs (ci_i) and backorder costs (cb_i). Table A.2 lists the demand levels for each product per time period (d_{it}).

To solve the model with the robust approach of Bertsimas and Sim (2004), the conservatism level is adjusted by control parameter ω. Thus it is possible to confirm that if uncertainty affects the 84 references (ω = 84), the results agree with those of the Soyster (1973) model. However, if we define ω = 0, the obtained results are the same as those obtained by the deterministic model. To compare the results between the different approaches, robustness control parameter (ω) is selected from 5 to equal 25, although the probability of more than five references being affected by the uncertainty of the process is very low.

As shown in Table 3, all the proposed models improve the heuristic procedure currently used for production planning. On the one hand, production costs are higher than in the proposed models, and the backorder costs are also much higher. This occurs because some products are manufactured above the demands levels, while the service level in other items is not completely satisfactory due to product backorders. It is important to highlight that production costs are equal for all the ω variations because the maximum quantity is produced to satisfy demand. On the other hand, the inventory costs in the heuristic procedure are higher than in the proposed models, which is mainly due to the overproduction of references in relation to demand.

The overtime cost is significantly higher in the most pessimistic models, i.e., Soyster (1973) and Bertsimas and Sim (2004) with (ω =84), but they are the approaches that can only ensure a 100% service level when the manufacturing times prolong for all the references due fluctuations in the production process. In this context, the deterministic model and the equivalent Bertsimas and Sim (2004) (for ω =84) approach present the best results, without taking into account the uncertainty in the process. However, when fluctuations in manufacturing times take place, if planning is performed with these models, backordered demand appears because of the shortage of available production time to cover the prolonged processing times. In these cases, the associated backorder costs are considerably higher than the cost of producing in extra time. Therefore, backorder costs are similar for all the models, regardless of them being deterministic or robust.

Regarding the calculation times to devise production planning, we point out the significant savings in the CPU time used by the mathematical programming models compared to the amount of time employed for the heuristic procedure. In this case, the spreadsheet is only a help tool for planners, who determine which references must be produced and their corresponding amounts by doing the necessary calculations themselves.

Therefore, even though the robustness cost seems high, it is worth assuming it by determining an appropriate robustness control parameter. This case study considers that the probability of more than five references and their manufacturing times being affected by process instability or machine breakdowns is very low. Thus adjusting a robustness level (ω) to between 1 and 5 is considered appropriate as the total obtained costs are not excessively high.

7 Conclusions

This paper proposes a robust optimization model to deal with the MPS problem for an automotive second-tier supplier. This model considers uncertainty related to manufacturing times. In order to solve the proposed model with a robust paradigm, the Soyster (1973) and Bertsimas and Sim (2004) approaches were considered. These robust solution methods were tested in a real automotive supply chain and were compared to a deterministic approach. All these mathematical programming approaches proved efficient as they obtained better results for the total generated costs and calculation times than those that the heuristic procedure obtained, which is currently applied in the firm under study.

The robust modeling approach by Bertsimas and Sim (2004) incorporates the possibility of dealing with process uncertainty which, by means of a control parameter, allows the protection against uncertainty to be adjusted by controlling the costs associated with the price of robustness, without reaching the excess conservatism of the robust Soyster approach (1973). Thus further research proposals include: (i) comparing the performance of alternative approaches to model uncertainty, such as fuzzy mathematical programming, fuzzy stochastic programming or fuzzy robust programming in the addressed industrial problem; (ii) including the modeling of uncertain parameters related to other external sources, such as demand and supply; (iii) extending the present MPS to other planning areas, such as MRP or inbound and outbound logistics.

Table 3. Results for the different solution procedures

	Current heuristic procedure	**Deterministic model**	**Bertsimas & Sim (2004) approach (ω=0)**	**Bertsimas & Sim (2004) approach (ω=5)**	**Bertsimas & Sim (2004) approach (ω=10)**	**Bertsimas & Sim (2004) approach (ω=25)**	**Bertsimas & Sim (2004) approach (ω=84)**	**Soyster (1973) approach**
Total costs (euros)	8009131.14	1151130.38	1151130.38	1183832.50	1208101.97	1249811.09	1285020.38	1285020.38
Production costs (euros)	1077362.35	1070643.74	1070643.74	1070643.74	1070643.74	1070643.74	1070643.74	1070643.74
Inventory costs (euros)	2368.79	846.64	846.64	888.76	1018.23	927.35	846.64	846,64
Backorder costs (euros)	6877800.00	740.00	740.00	740.00	740.00	740.00	740.00	740,00
Idle hour costs (euros)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0,00
Overtime hour costs (euros)	51600.00	78900.00	78900.00	111560.00	135700.00	177500.00	212790.00	212790.00
Inventory (units)	13809	0	0	0	0	0	0	0
Backlog (units)	0	0	0	0	0	0	0	0
Processing time (seconds)	162000.00	18.47	12.34	18.83	21.40	24.12	23.14	18.20

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Annex 1

Table A.1. Basic item data

i	β_i (%)	αr_{ir} (hours)	lot_i (units)	cp_i (euros)	ci_i (euros)	cb_i (euros)
1	90	0.0100	100	0.80	0.04	50
2	90	0.0100	100	0.80	0.04	50
3	90	0.0139	72	1.11	0.06	50
4	90	0.0139	72	1.11	0.06	50
5	90	0.0139	72	1.11	0.06	50
6	90	0.0052	192	0.42	0.02	50
7	90	0.0167	60	1.33	0.07	50
8	90	0.0167	60	1.33	0.07	50
9	90	0.0023	430	0.19	0.01	50
10	90	0.0023	430	0.19	0.01	50
11	90	0.0023	430	0.19	0.01	50
12	90	0.0139	72	1.11	0.06	50
13	90	0.0104	96	0.83	0.04	50
14	90	0.0104	96	0.83	0.04	50
15	88	0.0069	144	0.56	0.03	50
16	88	0.0069	144	0.56	0.03	50
17	90	0.0179	56	1.43	0.07	50
18	90	0.0179	56	1.43	0.07	50
19	90	0.0096	104	0.77	0.04	50
20	88	0.0208	48	1.67	0.08	50
21	88	0.0208	48	1.67	0.08	50
22	84	0.0079	126	0.63	0.03	50
23	90	0.0208	48	1.67	0.08	50
24	90	0.0208	48	1.67	0.08	50
25	88	0.0278	36	2.22	0.11	50
26	88	0.0278	36	2.22	0.11	50
27	90	0.0833	12	6.67	0.17	50
28	90	0.0139	72	1.11	0.06	50
29	90	0.0139	72	1.11	0.06	50
30	90	0.0104	96	0.83	0.04	50
31	90	0.0079	126	0.63	0.03	50
32	90	0.0208	48	1.67	0.08	50
33	90	0.0208	48	1.67	0.08	50
34	90	0.0089	112	0.71	0.04	50
35	90	0.0238	42	1.90	0.10	50
36	90	0.0238	42	1.90	0.10	50
37	90	0.0056	180	0.44	0.02	50
38	90	0.0056	180	0.44	0.02	50
39	90	0.0139	72	1.11	0.06	50
40	90	0.0333	30	2.67	0.13	50
41	90	0.0139	72	1.11	0.06	50

i	β_i (%)	αr_{ir} (hours)	lot_i (units)	cp_i (euros)	ci_i (euros)	cb_i (euros)
42	90	0.0714	14	5.71	0.29	50
43	90	0.0417	24	3.33	0.17	50
44	90	0.0083	120	0.67	0.03	50
45	90	0.0035	288	0.28	0.01	50
46	90	0.0100	100	0.80	0.04	50
47	90	0.0417	24	3.33	0.17	50
48	90	0.0091	110	0.73	0.04	50
49	90	0.0208	48	1.67	0.08	50
50	90	0.0028	360	0.22	0.01	50
51	90	0.0028	360	0.22	0.01	50
52	90	0.0125	80	1.00	0.05	50
53	90	0.0333	30	2.67	0.13	50
54	90	0.0333	30	2.67	0.13	50
55	90	0.0333	30	2.67	0.13	50
56	88	0.0556	18	4.44	0.22	50
57	88	0.0556	18	4.44	0.22	50
58	90	0.0076	132	0.61	0.03	50
59	90	0.0076	132	0.61	0.03	50
60	90	0.0278	36	2.22	0.11	50
61	90	0.0062	162	0.49	0.02	50
62	90	0.0062	162	0.49	0.02	50
63	90	0.0139	72	1.11	0.06	50
64	90	0.0040	252	0.32	0.02	50
65	90	0.0040	252	0.32	0.02	50
66	90	0.0125	80	1.00	0.05	50
67	90	0.0250	40	2.00	0.10	50
68	90	0.0833	12	6.67	0.33	50
69	90	0.0125	80	1.00	0.05	50
70	90	0.0833	12	6.67	0.33	50
71	90	0.0417	24	3.33	0.17	50
72	90	0.0139	72	1.11	0.06	50
73	90	0.0278	36	2.22	0.11	50
74	90	0.0500	20	4.00	0.20	50
75	90	0.0139	72	1.11	0.06	50
76	90	0.0093	108	0.74	0.04	50
77	90	0.0093	108	0.74	0.04	50
78	90	0.0278	36	2.22	0.11	50
79	90	0.0250	40	2.00	0.10	50
80	90	0.0250	40	2.00	0.10	50
81	90	0.0156	64	1.25	0.06	50
82	90	0.0156	64	1.25	0.06	50
83	90	0.0104	96	0.83	0.04	50
84	90	0.0104	96	0.83	0.04	50

Table A.2. Item demand per period

	d_{it}					
i	***t*=1**	***t*=2**	***t*=3**	***t*=4**	***t*=5**	***t*=6**
1	1300	1300	1300	1300	1300	1300
2	1600	1600	1600	1600	1600	1600
3	3600	3600	3600	3600	3600	3600
4	6000	6000	6000	6000	6000	7000
5	6000	6000	6000	6000	6000	7000
6	300	0	300	0	300	300
7	4500	4500	4500	4500	4500	4500
8	4500	4500	4500	4500	4500	4500
9	4500	4500	4500	4500	4500	4500
10	4500	4500	4500	4500	4500	4500
11	772	772	0	772	0	0
12	1417	1215	1215	1215	1215	2154
13	1013	1215	810	1013	1013	1724
14	1013	810	810	1013	1215	1939
15	1620	1417	1215	1822	1100	1508
16	1417	1620	1013	1822	1215	1293
17	622	420	203	608	1085	0
18	507	405	405	608	405	0
19	0	590	398	398	398	0
20	1898	1544	1367	1735	1876	2972
21	1594	1544	1367	1735	1876	2972
22	1234	1234	1080	1234	1388	2134
23	103	0	102	102	155	108
24	0	50	102	102	155	108
25	1700	1700	1700	1700	1700	1700
26	1700	1700	1700	1700	1700	1700
27	1700	1700	1700	1700	1700	1700
28	400	400	400	400	400	400
29	400	400	400	400	400	400
30	200	200	200	200	200	200
31	300	300	300	300	300	300
32	500	500	500	500	500	500
33	500	500	500	500	500	500
34	120	0	0	100	0	0
35	1200	1200	1200	1200	1200	1200
36	1200	1200	1200	1200	1200	1200

	d_{it}					
i	***t*=1**	***t*=2**	***t*=3**	***t*=4**	***t*=5**	***t*=6**
37	1200	1200	1200	1200	1200	1200
38	1200	1200	1200	1200	1200	1200
39	1500	1500	1500	1500	1500	1500
40	0	0	0	400	0	0
41	300	0	300	300	300	0
42	400	400	400	400	500	500
43	1500	1500	1500	1500	1500	1500
44	1500	1500	1500	1500	1500	1500
45	1000	1000	1000	1000	1000	1000
46	200	0	200	200	0	200
47	100	100	150	150	150	150
48	0	100	150	150	150	150
49	100	100	150	150	150	150
50	4500	4500	4500	4500	4500	4500
51	4500	4500	4500	4500	4500	4500
52	600	600	600	600	600	600
53	593	608	528	579	564	553
54	427	499	514	499	369	431
55	1605	1485	2025	2025	2155	2539
56	1350	1350	1350	1350	1350	1350
57	1350	1350	1350	1350	1350	1350
58	1200	1200	1200	1200	1200	1200
59	1200	1200	1200	1200	1200	1200
60	1300	1300	1300	1300	1300	1300
61	1200	1200	1200	1200	1200	1200
62	1200	1200	1200	1200	1200	1200
63	600	600	600	600	600	600
64	250	250	250	250	350	300
65	250	250	250	250	350	300
66	800	800	800	800	800	800
67	2500	2500	2800	2800	2800	2800
68	0	400	400	0	0	0
69	1500	1500	1500	1500	1500	1500
70	1200	1200	1200	1200	1200	1200
71	0	0	0	0	0	180
72	1300	1300	1300	1300	1300	1300
73	600	600	600	600	600	600
74	500	500	500	500	500	500

	d_{it}					
i	***t*=1**	***t*=2**	***t*=3**	***t*=4**	***t*=5**	***t*=6**
75	400	400	400	400	400	400
76	420	420	420	420	420	420
77	420	420	420	420	420	420
78	1500	1200	1200	800	800	500
79	1500	1500	1500	1500	1000	750
80	1500	1500	1500	1500	1000	750
81	3300	3300	3300	3300	3300	3300
82	3300	3300	3300	3300	3300	3300
83	3300	3300	3300	3300	3300	3300
84	3300	3300	3300	3300	3300	3300