Behaviour study of an eccentric pulley transmission system using systemic methods

Iñaki Esnal-Angulo & Bernabé Hernandis-Ortuño

* Red Internacional de Investigación en Diseño Sistémico, Universitat Politècnica de València, Valencia, España. iaesan@doctor.upv.es

b Dpto. Ingeniería Gráfica, Universitat Politècnica de València, Valencia, España. bherm@upv.es

Received: November 10th, 2018. Received in revised form: April 30th, 2019. Accepted: May 17th, 2019

Abstract
This paper describes a case study carried out on the behaviour of an eccentric pulley transmission system housed in a manually operated abdominal-intestinal assistant device for human use. The aim is to establish a systemic framework that serves as a validation tool for mechanical systems in the initial stages of the product design processes. The proposed study system describes the device transmission process as a function of the input angle variation (dα), the height of the user’s feet (HF) and the pulley curvature function (C), resulting from the variation of its radius (dR) over time (dt). In order to explore and compare the different behaviours and identify possible solutions four different configurations of curvatures were proposed. Causal diagrams and differential equations describe the simulation scenario. The resulting application model supports the use of a systemic frame and methods as a pre-response to the validation of design proposals.

Keywords: systems; design; product; validation; transmission.

1. Introduction
Products in general, especially those for human use, are increasing demand for an increased degree of specialisation. It is advisable, therefore, to consider each aspect of the product as an opportunity for innovation, highlighting qualities in order to favour product differentiation in the market. On the other hand, this trend towards specialisation implies the existence of more information associated with the product-system, giving rise to increasingly complex and multidisciplinary products. The handling of this large amount of information has today become one of the paradigms of innovation itself.

Because of this, new study scenarios have emerged, not only on the fundamental aspects of objects, but also to deepen...
the relationships that arise between them and their context.

In this sense, the General Systems Theory (GST), developed by Bertalanffy [1] in the mid-twentieth century, offers us a holistic and integrating vision, capable of representing all the parties involved focusing on the relationships between their components. This systemic approach enables analysis of a system from the relationship between the elements that integrate it.

In 2000, B. Hernandis and E. Iribarren [2,3], based on Ashby’s ultra-stable system schemes [4], demonstrated that product design can be proposed from a systemic perspective. As a system, a product is defined as a set of elements (variables and parameters) interconnected with each other, which works as a whole by sharing energy and information with the medium in which they exist. This consideration implies the existence of an Outer System (context) from which the system under study or Inner System (product system) is nourished and is defined according to external conditions, thus adapting to a particular medium state.

This perspective was considered by Briede et al. [5] as an information modelling tool for the communication flow management in the customer-user-expert environment. They used the Hernandis’ systemic model for product design (Fig. 1) as a strategy to articulate and involve stakeholders in order to solve and foresee the future problems in Small and Medium-sized Enterprises.

Cardozo et al. [6,7], on the other hand, used it to describe and categorize the relationships between endogenous and exogenous elements into the product-system in order to explain the product families variability in terms of scalability and flexibility. They states that “the multiple products of a family are an integral and integrable system which can be handled a larger amount of information and to foresee the possible variations that system will undergo with the change of any of the initially established conditions.

The particular feature of Hernandez and Iribarren’s model (Fig. 1) is that it classifies the elements that make up the set into three subsystems of the same hierarchical order, as opposed to the hierarchical ultra-stable model proposed by Ashby. This allows information to travel from one isosystem to another and share the same degree of relevance, even though the transformation functions, a priori, do not have the commutative property [9]. The three subsystems represent the fundamental aspects of design: functional, ergonomic and formal. This categorisation and model structure oriented to product design favours reduction of the risk of omitted variables. Some of these variables might be essential for the definition and approach of the system under study, thus promoting a high degree of accuracy and fidelity in the simulation of reality.

To understand the information flows emerging from the relationships between the system elements, tools were needed that make it possible to abstract and simplify those agents and the phenomena that arise between them. This facilitates visualisation and allows the designer (decision-making agent) to foresee the possible variations that system will undergo with the change of any of the initially established conditions.

1.1. System Dynamics

The use of System Dynamics makes it possible to simulate and analyse the information behaviour and the feedback loops produced over time [10]. This method has been used widely in the industrial sector for studying industrial processes [11] or control of contracting cycles, as well as for socio-economic, environmental, meteorological, biological, psychological, demographic systems, etc. [12,13] and even global systems, such as the one proposed for the 1st Club of Rome for simulation of world evolution [14].

This view of systems behaviour contemplates a series of techniques, such as causal diagrams, which represent the interaction flows between the variables and their transformation over time. With them, therefore, a complete visualisation of the general evolution of the system can be obtained. It is for this reason that this methodology has been chosen for the present study, since this makes it possible to handle a larger amount of information and to foresee the consequences arising from the decision-making in the design stage with greater rigour.

The principle followed on the representation of the transmission system of the pulley is homologous to any other transformation process flow, which basically, according to López-Moreda [15], there is an input information that undergoes a transformation process and an output information is obtained.

On the other hand, cybernetic methods, such as control tools, allow for quantitative and detailed analysis of the transformations of system variables. According to Proncheva and Markhov [16], this approach makes it possible to base the system dynamics on two basic principles: “the equations that define the variables must be able to be extrapolated to all the other variables of the system”, i.e. they satisfy the same criteria:

$$\frac{dy}{dt} = y^+y^-$$  \hspace{1cm} (1)

Where, in this case, $y^+$ is the expression of the positive variance ratios (including all the factors related to the increase of the variable $y$) and $y^-$ is the expression of the

![Figure 1. Systemic product model.](source: [2].)
negative variance ratios (including all the factors related to the decrease of the variable $y$).

In consequence, “all the ratios (positive and negative) can be presented as a function set”:

$$y^+ = g(y_1, y_2, y_3, ..., y_n) = f(F_1, F_2, F_3, ..., F_k) = f_1(F_1), f_2(F_2), f_3(F_3), ..., f_k(F_k)$$

The analytical expressions enable a detailed description of the variations, facilitating the graphical representation of the action range and providing a clear and concise visualisation of all the states that the variables will adopt over time.

2. Materials and Methods

A case study for experimentation based on the transmission drive mechanism of a domestic abdominal-intestinal assistance device was proposed (Fig. 2).

The transmission drive subsystem (Fig. 3) is physically solved by means of a lever ($L_1$) fixed to an eccentric pulley (PA) that varies its radius ($r$) over the angle rotated. This in turn, through a belt transmission, drives a wheel (WA) with a second lever ($L_2$) which has a pedal at the end where the user places their feet. In this way, the rotation of the actuating lever results in raising of the user’s feet and, thereby, the different postural states of the lower train kinematic chain, until a position close to squatting is reached. (Fig. 4).

In the initial stage (<45º), the user position adopted can generate fatigue, as the effort required is determined by the resulting moments, so it is advisable to reduce the loads (reducing action). On the contrary, in the final stage (>45º), where there is less postural risk, these loads increase (multiplier action). In this way, in addition to reducing the risk of injury, greater biomechanical efficiency of the energy used is obtained.

The eccentricity of the pulley curvature results in a non-constant radius variation, which produces significant ergonomic repercussions for the user and therefore, their experience in use.

The different curvatures proposed (Fig. 5) for the simulation have a minimum radius ($r_1$) equal to the output radius ($r_1 = r_w$) and a maximum radius ($r_2$) greater than the output radius ($r_2 > r_w$). The different trajectories proposed between $r_1$ and $r_2$ correspond to the different curvatures resulting from the specific point union states of the radius variable.

For the pulley profiles: curve 1 and 2 are based on the eccentricity variation; curve 3 is based on the curvature discontinuity (by addition of an inflection point); and curve 4 is based on the absence of curvature (or total eccentricity).

The evaluation criteria for the different curvatures was established by the optimum vertical load distance (height of the user’s feet). It was determined by the usability test carried out by the authors [17] based on Rad’s work in 2002, on the
relationship between the user position and the puborectalis muscle angle \[18\]. The optimal elevation value was established at 200–250 mm for an adult male user of 50 ± 5 percentile (≈1,740 mm, ≈1,660 mm in Spain) according to the UNE EN ISO 7250: 1998 standard \[19\] and the Instituto para la Salud e Higiene en el Trabajo, INSHT (Institute of Occupational Health and Safety) \[20\].

2.1. Essay method

The process method carried out during the experimentation was, in the first place, determination of the simulation scenario. The device dynamics agents (parameters and variables), the information flows and the transformation functions were identified. The initial test conditions were established and the use cycle simulation for the four proposed pulley curvature configurations was performed. The data collected were graphically represented. Data analysis was carried out by comparison and quantitative evaluation of the different proposals.

2.2. Essay preparation

Based on the previous abdominal-intestinal assistant modelling carried out by the authors \[21\], the suprasystem “context” or Outer System was established. Following Hernandis’ model, the System under Study was established, along with the agents intervening in the system (parameters and variables). The physical elements (components) related with these agents were identified and the variables that underlie them were quantified for the simulation measurements. At this point, a distinction was made between

parameters – whose values are established by the modeller and remain unchanged during the simulation process – and variables – which are those elements that vary due to the action of some transformation factor or disturbance. The pulley radius (r\(_1\)) was established as an operand; and the rotation angle (θ) as operator, with the pulley curvature (C) being the agent that acts as a transformation factor and performs this variation.

The system information flow was described by Forrester’s and causal diagrams, in which the direction and sense of the information generated within the system is shown, as well as the nature of each element. The analytical expression of the behaviour of each element studied was described by differential equations from the canonical control equations, extracted from the principles provided by Proncheva and Markhov.

Subsequently, the parameter values were established to define the conditions of the simulation as follows: Maximum pulley radius (r\(_2\) = 40 mm), Minimum pulley radius (r\(_1\) = 20 mm), Auxiliary radius (r\(_3\) = 10 mm), Wheel radius (r\(_w\)) = 20 mm), Belt deformation (BD = 0.05%), Input lever length (L\(_1\) = 550 mm) and Output lever length (L\(_2\) = 250 mm). (Table 1).

Identification was performed of the variables that underlie the physical elements that intervene in the study and that were subject to assessment after the simulation. Their action ranges were established, as well as the initial values as follows: Input Lever angle (IL = 0º), Pulley angle (PA = 0º), Initial curvature (C\(_1\) = 40 mm), Continuity curvature (C\(_C\) = 1), Wheel angle (WA = 0º), Output Lever angle (OL = 0º) and Height of feet (HF = 0 mm).

The initial conditions of the simulation were completed by determining the duration of the cycle of use (t = 1s).

2.3. Simulation

The simulation was carried out with parametric modelling
CAD (Autodesk Inventor v2013) and dynamic simulation (Dynamics Module) software.

The system under study was three-dimensionally modelled, assigning values to the parameters and degrees of freedom of each component. Two-dimensional diagrams graphically represented the data obtained.

A polynomial linear interpolation method with a least squares adjustment was used for the functions for construction of the different curvatures, whose values were collected and ordered in a table (Table 3), as well as the height of feet variable values (HF).

3. Results

The identification of the agents that intervene and make up the system under study and the interdependence relationships were described by diagrams as shown in Figs. 6 and 7.

The interdependence resulting from the intervening variables in the system under study was described by means of the following expressions:

\[
\frac{dPA}{dLI} = 1; \\
\frac{dr}{dPA} = C; \\
\frac{dWA}{dr'} = \frac{dr'}{dPA} * 1 - BD \\
\frac{dLO}{dWA} = 1 \\
\frac{dLI}{dt} = \text{cte};
\]

The previous expressions were built relating to the predecessor element (emitter), rather than to the time parameter. This displays the information flow, increasing the traceability and emphasising the relationship between the elements.

Expression number 3 describes the variation of the pulley angle (dPA) depending on the variation in input lever angle (dLI or dα). It indicates that both variables have the same value, as the components they represent are solidly connected. This means that when the user operates the mechanism by turning the input lever, the pulley will also rotate with the same linear speed.

Equation number 4 reflects the discrete value of the radius at a given moment, which is defined by the variation in pulley radius (dr) as a function of the variation in pulley angle (dPA). Because it starts from a maximum initial value of r2 and a minimum final value of r1, it can be inferred that the resulting value will be included in that interval (r2 < r' < r1). It will be determined, in each case, by the curvature function.

Following the kinematic chain, equation number 5 shows the relationship between the pulley radius (r') and the wheel angle (WA) and this is defined as the value of that radius multiplied by the deformation factor of the used belt material.

The relationship between the variation in the wheel angle (dWA) and the variation in the output lever angle (dLO) is expressed in equation number 6. As in equation 3, the solidly connected nature of the elements generates a result equal to 1. Finally, the relationship of variation of the input lever angle (dLI) over time (dt) is shown.

The polynomial expressions for the curvature variable (C) as a function of the rotation angle, and the graphical representations corresponding to the different cases proposed are shown below:

\[
C_1 = -4x10^{-4} + 1.33x10^{-2} - 9.78x10^{-2} + 0.780 + 19.165; \\
C_2 = -6x10^{-4} + 2.43x10^{-2} - 0.21940 + 0.963 + 18.949; \\
C_3 = -3x10^{-4} + 8.1x10^{-3} + 4.5x10^{-2} + 7.97x10^{-2} + 19.917; \\
C_4 = -2x10^{-4} + 7.5x10^{-3} + 0.1105 + 0.7170 - 1.9470 + 21.544; \text{ Correlation coefficient } R^2 = 0.99
\]

Deriving expressions for the rate of change of the radius as a function of the rotation angle, we obtained:

\[
C_1' = r'(\theta) = -1.6x10^{-3} + 0.3990 - 0.1950 + 0.78; \\
C_2' = r'(\theta) = -2.4x10^{-3} + 7.29x10^{-2} - 0.4380 + 0.963; \\
C_3' = r'(\theta) = -1.2x10^{-3} + 2.43x10^{-2} + 9x10^{-2} + 0.08; \\
C_4' = r'(\theta) = -10^{-3} + 2.8x10^{-2} - 0.3290 + 1.4340 - 1.946;
\]
With the second derivative, the expressions for radius acceleration as a function of the rotation angle were obtained:

\begin{align*}
C''_1 &= r''(\theta) = -4.8 \times 10^{-3} \theta^2 + 0.798 \theta - 0.195; \\
C''_2 &= r''(\theta) = -7.2 \times 10^{-3} \theta^2 + 0.145 \theta - 0.438; \\
C''_3 &= r''(\theta) = -3.6 \times 10^{-3} \theta^2 + 4.86 \times 10^{-2} \theta + 9 \times 10^{-3}; \\
C''_4 &= r''(\theta) = -10^{-3} \theta^4 + 2.810^{-2} \theta^3 - 0.3290^2 + 1.4340 - 1.946; 
\end{align*}

Figure 8. Position function described by the radius variable with the different proposed curvatures.
Source: The Authors.

In Fig. 8, the graphic representation of the different states of the radius variable \(r\) with respect to the input angle variable \((IL = CA = \theta)\) is shown. Only the values within the stipulated range of 90º corresponding to the drive of the device are shown. In Figs. 9 and 10, the velocity and acceleration functions of the radius with respect to the angle travelled are shown.

The graphic representation of the behaviour of the other variables (Eqs. 3, 5, 6 and 7) was omitted due to the simplicity of their interpretation.

The simulation results obtained showing the corresponding radius variable values \(r\) that meet the criterion considered for the optimal values range stipulated for the height of feet variable \((HF = 200-250 \text{ mm})\) are presented.

The initial drive stage corresponds to the input pulley movement to prepare and accommodate the device before use \((IL = \theta_1 \leq 45^\circ)\). At this stage, the user’s body is not yet in the optimum position for the application of forces, therefore a decrease in the loads in this state is considered to be of special relevance.

At this first stage, curvature number 4 is the one that showed the radius of lowest magnitude \(22.16\) compared to curve 1 \(26.77\), curve 2 \(24.63\) and curve 3 \(26.50\), which leads to a lower height \(79.53 \text{ mm}\). The user, in this case, will operate through a greater range than with the rest of the alternatives, but at a lower intensity (reducing transmission). This increases comfort during use and reduces the risk of injury. The optimal range of elevation of the user’s feet is reached when the input lever reaches 70 degrees.

### Table 3

<table>
<thead>
<tr>
<th>(\theta_1)</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
</tr>
</thead>
<tbody>
<tr>
<td>35°</td>
<td>24.53</td>
<td>213.78</td>
<td>22.67</td>
<td>234.14</td>
</tr>
<tr>
<td>40°</td>
<td>25.59</td>
<td>237.46</td>
<td>23.57</td>
<td>260.52</td>
</tr>
<tr>
<td>45°</td>
<td>26.77</td>
<td>257.77</td>
<td>24.63</td>
<td>282.36</td>
</tr>
<tr>
<td>70°</td>
<td>34.36</td>
<td>316.24</td>
<td>33.08</td>
<td>323.37</td>
</tr>
</tbody>
</table>

In Fig. 8, the graphic representation of the different states of the radius variable \(r\) with respect to the input angle variable \((IL = CA = \theta)\) is shown. Only the values within the stipulated range of 90º corresponding to the drive of the device are shown. In Figs. 9 and 10, the velocity and acceleration functions of the radius with respect to the angle travelled are shown.

The initial drive stage corresponds to the input pulley movement to prepare and accommodate the device before use \((IL = \theta_1 \leq 45^\circ)\). At this stage, the user’s body is not yet in the optimum position for the application of forces, therefore a decrease in the loads in this state is considered to be of special relevance.

At this first stage, curvature number 4 is the one that showed the radius of lowest magnitude \(22.16\) compared to curve 1 \(26.77\), curve 2 \(24.63\) and curve 3 \(26.50\), which leads to a lower height \(79.53 \text{ mm}\). The user, in this case, will operate through a greater range than with the rest of the alternatives, but at a lower intensity (reducing transmission). This increases comfort during use and reduces the risk of injury. The optimal range of elevation of the user’s feet is reached when the input lever reaches 70 degrees.

### 4. Discussion

Based on the results, the number 4 curvature more adequately meets the objective of diminishing the loads in the initial states of use so the risk of long-term injury is lower. However, it is noticed that the results show a more detailed and clearer understanding of the process than expected and, after analysing the data of each alternative, it can be concluded that a greater optimisation level of the pulley curvature can be made in order to better adjust to the user’s needs. That is why these data will be used again to feedback to and optimise the system in future simulations focused on ergonomics to determine the magnitude of the repercussions that these changes entail in the user and subsequently implement them in the physical product model.

### 5. Conclusions

Regarding the tools used in this study, both the systemic model and the hydrodynamic diagrams are very useful and
relevant for resolving a specific design problem in which a forecast of future behaviour is required. In contrast to conventional methodology, these tools allow clearer and more complete visualisation, as well as analysis and simulation of the established conditions, thus favouring understanding and consequent decision-making, even though the most important aspects are not yet defined and it is difficult to predict the possible implications beforehand.

The incorporation of methods based on System Dynamics in the design process, on the other hand, enables us to understand system behaviour without the need to carry out physical tests, thereby favouring optimisation of both conceptualisation and development processes. This leads to a significant reduction in development time and costs. Due to this reduction in measurement and testing, design group efficiency and, consequently, its competitiveness increases.

References


I. Esnal-Angulo, is PhD in Design, Manufacture and Management of Industrial Projects from the UPV, Spain, he holds a Design Engineering MSc. and an BSc. in Industrial Design Engineering. Coordinator and research member of the rDis® (International Network for Research on Systemic Design), Deputy Editor of the rDis® academic Journal and secretary of the “Systems & Design” International Conferences. Expert on R+D and systems modelling for innovation and new products development. ORCID: 0000-0002-8351-5143

B. Hernandis-Ortuño, is PhD in Industrial Engineering, he holds a BSc. in Mechanical Engineer and an Operations Research and Systems. His is tenure professor and researcher at the Universitat Politècnica de València, Spain, director of the Master’s degree on Design, Management and Development of New Products at the UPV and President of the rDis® (International Network for Research on Systemic Design). Expert on methodologies for research, knowledge management, design and innovation. Consultant in different industrial sectors, with over 30 years of experience in the study of systems theory, complex systems, design methodology and systemic models for the innovation of products and services. ORCID: 0000-0002-5697-6382