A Choquet integral-based hesitant fuzzy gained and lost dominance score method for multi-criteria group decision making considering the risk preferences of experts: Case study of higher business education evaluation

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Abstract

With the rapid development of higher business education, higher business education evaluation has attracted considerable attention of researchers and practitioners. The higher business education evaluation is an essential part of the development of a business school, which has a direct impact on its resource distribution. The higher business education evaluation can be considered as a multiple criteria group decision making (MCGDM) problem that involves a group of experts. Due to the complexity of the decision-making problem, decision criteria are not fully independent to each other, and the assumption of complete rationality of experts is usually invalid in many situations. In this paper, we propose a Choquet integral-based hesitant fuzzy gained and lost dominance score method to address the two important issues regarding the interactions among criteria and the behavior preference characteristics of experts in MCGDM problems. Firstly, a comprehensive distance measure of hesitant fuzzy sets is introduced by considering the relative importance of two separations. Then, a Choquet integral-based hesitant fuzzy gained and lost dominance score method based on the prospect theory is proposed to address the MCGDM problems in which experts make decision with the risk preference psychology. Finally, an illustrative example of higher business education evaluation is provided to demonstrate the applicability of the proposed method, and the sensitivity and comparative analysis are also completed to verify the validity of the proposed method.

Keywords: Multiple criteria group decision making; Hesitant fuzzy set; gained and lost dominance score method; Choquet integral; Prospect theory

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1. Introduction

With the acceleration of the world economic globalization, the economic trade exchanges between China and other economies become increasingly frequent, especially since China entered the WTO (World Trade Organization) in 2001. Consequently, the demand for business talents in Chinese socioeconomic development is growing significantly. In the last two decades, Chinese higher business education has experienced a rapid development. The higher business education evaluation plays a vital role in the development of advance-business education. It can not only promote the benign competition of business institutes and colleges, but also improve the quality of higher business education (Adam, 2016). To comprehensively and objectively evaluate the performance of higher business education, multiple factors and stakeholders should be taken into consideration. Therefore, the higher business education evaluation can be regarded as a multiple criteria group decision making (MCGDM) problem including multiple evaluation criteria such as school-running resources, education process and scientific research achievements.

In actual situations, for education evaluation problems, an assessment specialist usually cannot grasp all precise information regarding all criteria. The hesitant fuzzy set (HFS) introduced by Torra (2010) is a powerful tool to deal with vague and uncertain information. As people may hesitate among several possible values when determining the membership of an element to a given set, the HFS, whose membership degree includes several possible values, can represent such a situation exactly. Since the HFS was proposed, a large number of studies have been published (Bustince et al., 2015; Liao et al., 2018, 2019b; Rodriguez et al., 2014). Thanks to these achievements, many complex and uncertain decision-making problems can be solved.

The gained and lost dominance score (GLDS) method, originally proposed by Wu and Liao (2019), is a useful outranking method to solve multiple criteria decision making (MCDM) problems. The main advantage of the GLDS method is that it produces more reasonable and robust results than other MCDM methods. It considers both the gain and loss in uncertain context. In many MCDM methods, such as the VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), the utility values of some criteria are completely compensated by those of other criteria, while in the GLDS method, the bad performances of some criteria cannot be compensated by the good performances of other criteria. Thus, using the GLDS method for MCDM can avoid selecting an alternative performing too bad under any criterion. Motivated by the originally GLDS method proposed by Wu and Liao (2019), Fu et al. (2018) studied the underground mining method selection problem in China in which the evaluation information was represented by hesitant fuzzy linguistic term sets (Rodriguez, Martinez & Herrera, 2012; Liao, Xu, Zeng & Merigó, 2015). Liao et al. (2019c) further investigated the probabilistic linguistic GLDS method with the logarithm-multiplicative analytic hierarchy process and then applied the integrated method for life satisfaction evaluation in earthquake-hit area. Although the GLDS method has been applied in various fuzzy environments, few scholars, to the best of our knowledge, pay attention to the extension of this method in the hesitant fuzzy context. As mentioned earlier, a lot of imperfect and imprecise information is involved in the process of higher business education evaluation and the hesitant fuzzy set is adequate to cope with the situation where the evaluations can only be expressed by a set of possible values. In this regard, it is necessary to investigate the hesitant fuzzy GLDS method by considering the advantages of both the HFS and GLDS method to address this issue well.

In addition, although the GLDS method has advantages over other MCDM methods, there are still two strikes against it. On one hand, the distance measure used in the dominance flow function is inadequate to tackle the hesitant fuzzy information that has the nature of uncertainty and complexity. As a useful technique to capture the difference between HFSs, various distance measures of HFSs have been investigated by scholars, including but not limited to the generalized hesitant normalized distance (Xu & Xia, 2011), generalized hesitant fuzzy synergetic weighted distance measure (Peng et al., 2013) and normalized generalized distance measure (Tang et al., 2018). However, since those distance measures of HFSs are based on distinct weighted mean operators, unreasonable results may be produced. For example, let $h_1 = \{0.1, 0.2\}$, $h_2 = \{0.3, 0.4\}$ and $h_3 = \{0.1, 0.2, 0.5, 0.6\}$ be three hesitant fuzzy elements (HFEs). Using the normalized generalized distance measure to compute the distances $d(h_1, h_2)$ and $d(h_1, h_3)$, we have $d(h_1, h_2) = 0.2$ and $d(h_1, h_3) = 0.2$, respectively. Because of the significant difference between h_2 and h_3, the above results are not reasonable. So, it is essential to develop a new distance measure to meet this research challenge. In this paper, we introduce a novel distance measure for HFSs based on the reference (utopia) point theory (Yu, 1973), which can overcome the above flaw well.

On the other hand, the original GLDS method is incapable to handle the situation in which the decision criteria are not independent to each other. In practical decision-making problems, the inter-dependency of criteria has become such a common phenomenon that could not be ignored in MCDM. How to model the inter-dependent characteristic between criteria is a hot topic in the field of MCDM. To portray the inter-dependency of arbitrary sets, Choquet (1954) introduced the concept of capacities that provides an ability to represent the importance of a set of criteria. Afterwards, for integrating the information in fuzzy environment, a few Choquet integral-based aggregation operators were proposed by Wei et al. (2013), Joshi and Kumar (2016) and Yager (2018). A hesitant fuzzy linguistic Choquet integral arithmetic (HFLCIA) operator for MCDM problems was also proposed (Liao et al., 2019a). Inspired by the HFLCIA operator, we propose a hesitant fuzzy Choquet integral arithmetic (HFCIA) operator in this study. On this basis, we introduce a Choquet integral-based GLDS method for MCGDM problems with interactive decision criteria. In this way, we can bridge the second research gap of the original GLDS method.

Moreover, experts usually make a decision under risk. In such a case, the expected utility theory may be inadequate for the MCGDM problems that involve a panel of experts with risk preference characteristics. In other words, the assumption of completely rational decision-makers is invalid in this situation. Given that the utility theory is not an appropriate model to capture the attitude characteristic, Kahneman and Tversky (1979) introduced the prospect theory for decision-making problems in which experts have incompletely rational psychological aspects. Since the prospect theory aligns closely with the psychological characteristics of human, MCDM methods based on the prospect theory have recently attracted considerable attention of scholars. For example, Qin et al. (2017) researched an extended TODIM method using the prospect theory for the green supplier selection problem. Based on the prospect theory, Wang et al. (2018) established failure modes and effect analysis (FMEA) framework that takes into account decision-makers' psychological characteristics. Liu et al. (2019) proposed a prospect cross-efficiency (PCE) model to capture the risk preference psychology of decision-makers. As a new member of the family of decision-making methods, the GLDS method failed to consider the attitude characteristics of experts. To fill this gap, we develop a prospect dominance flow function which characterizes the risk preferences of experts.

Based on the above analysis, in this study, we dedicate to proposing a Choquet integral-based hesitant fuzzy GLDS (HF-GLDS) method to handle the higher business education evaluation problem in which the experts have the risk preference psychology. The highlights of this paper can be summarized as follows:

(1) We first propose a compromise value function of HFSs, based on which, a comparison method of HFEs is developed. Then, we introduce a comprehensive distance measure to capture the difference between HFEs.

(2) Considering the attitude characteristics of experts, we introduce a novel dominance flow function based on the prospect theory.

(3) To solve the complex MCGDM problem involving a set of interactive criteria, a Choquet integral-based HF-GLDS method is proposed. We use a higher business education evaluation problem to illustrate the proposed method.

The remainder of this paper is structured as follows: Section 2 reviews relevant concepts about the HFS, Choquet integral and GLDS method. Section 3 introduces a comprehensive distance measure for HFSs based on the compromise values of HFSs and their relative importance. We propose a dominance flow function considering the loss-aversion, and then develop a Choquet integral-based HF-GLDS method considering the risk preferences of experts in Section 4. Section 5 gives an illustrative example and provides sensitivity and comparative analyses to test the applicability and validity of the proposed method. Section 6 describes an extension of the used $\lambda -$ fuzzy measure to allow the detection of the interaction between any pairs of criteria. Conclusions are provided in Section 7.

2. Preliminaries

In this section, the basic concepts of HFSs and the Choquet integral are reviewed. The idea of the original GLDS method is also presented in this section.

2.1. Hesitant fuzzy set

Since it is difficult for experts to evaluate an object with certain values, Torra (2010) introduced the concept of HFS which provides a set of possible values to express experts' preferences. In a real-life decision-making process, experts may hesitate if they think several evaluation values are all possible about the degree of an element to a given set (Wang & Xu, 2016). This situation can only be modeled by the HFS that permits experts to assign a set of possible values of the membership degree to express their preferences. The HFS is much closer to people's cognition than other extensions of fuzzy set such as the interval-valued fuzzy set, type 2 fuzzy set and intuitionistic fuzzy set.

Let X be a reference set. An HFS H on X is defined in terms of a function h that returns a subset of $[0,1]$. Mathematically, it can be represented in the following form (Xia & Xu, 2011):

$$H = \{ < x, h_A(x) > | \, x \in X \} \tag{1}$$

where $h_A(x)$ is a set of values in $[0,1]$, indicating the possible membership degrees of $x \in X$ to the set A. The basic component of an HFS is called a hesitant fuzzy element (HFE) (Xia & Xu, 2011).

Bedregal et al. (2014) introduced the typical hesitant fuzzy set that considers only a set of all finite non-empty subsets of unitary interval $[0,1]$. For real-life MCDM problems, the HFS can be easily used in the cases that aggregation or score function is considered as a tool of ranking HFEs (Rodríguez et al., 2016). In this paper, we only consider the typical HFSs as the base of ranking strategy. A comparison scheme was developed to rank any pair of HFEs using the score and variance functions of HFEs (Xia & Xu, 2011). An obvious drawback of this comparison scheme is that the twice-comparison is generally required. To overcome this defect, a comprehensive score function (Liao et al., 2019a) was introduced in the process of eliciting the order of HFEs, which simultaneously considers the average value and hesitancy degree of an HFE. The proposed score function can be shown as follows (Liao et al., 2019a):

$$E(h)=\left(\frac{1}{\#h}\sum_{\gamma\in h}\gamma^{(1-HD(h))}\right)^{1/(1-HD(h))} \tag{2}$$

and

$$HD(h)=\sqrt{\frac{1}{\#h}\sum_{\gamma\in h}(\gamma-\tilde{\gamma})^2} \tag{3}$$

where γ is the possible membership grade, $\#h$ is the number of elements in h, $\tilde{\gamma}$ indicates the arithmetic mean of γ.

2.2. The Choquet integral

Let $C=\{c_1,\ldots,c_n\}$ be a non-empty finite set of criteria, and $A=\{a_1,\ldots,a_m\}$ be a set of alternatives. For an MCDM problem, x_{ij} denotes the evaluation of alternative a_i with respect to criterion c_j.

Given that the inter-dependence may exist among the decision criteria of MCDM problems, it is essential to take into account the sophisticated relations of criteria in the process of aggregating evaluation information. The Choquet integral (Choquet, 1954), as a type of aggregation function, provides an opportunity to integrate the evaluation information on dependent criteria. This method is based on the fuzzy measure that substitutes the additive property for monotonicity (Grabisch, 1996). Considering the difficulty of calculating the fuzzy measures of the subsets of criteria, Sugeno (1974) proposed the $\lambda-$fuzzy measure to tackle this issue. For the facility of calculation, in this paper, we use the $\lambda-$fuzzy measure to capture the interaction between decision criteria.

Suppose that the fuzzy measure $\mu(\cdot)$ represent the importance degree of a subset of criteria and 2^C is the power set of C. Then, the set function, $\mu:2^C\rightarrow[0,1]$, satisfies the following conditions (Sugeno, 1974):

(1) (Boundary conditions) $\mu(\varnothing)=0$ and $\mu(C)=1$,

(2) (Monotonicity condition) $\mu(A)\geq\mu(B)$ if $B\subseteq A$,

where A and B are subsets of the criterion set $C=\{c_1,\ldots,c_n\}$, and $\mu(A)$, $\mu(B)$ and $\mu(C)$ indicate the weights of corresponding subsets.

Using the fuzzy measure to capture the importance degrees of criteria provides an ability to model the interaction between criteria. In this regard, the use of fuzzy measure makes it possible to handle the situation that $\sum_{j=1}^{n}\mu(c_j)\neq 1$.

Because of the difficulty of calculating the fuzzy measure, Sugeno (1974) proposed the following $\lambda-$fuzzy measure:

$$\mu(A \cup B) = \mu(A) + \mu(B) + \lambda\mu(A)\mu(B) \tag{4}$$

where $\lambda \in [-1, \infty)$ and $A \cap B = \varnothing$.

λ represents the interaction between the sets of criteria. In particular, if $\lambda > 0$, there exists positive interaction between criteria; if $\lambda < 0$, there exists negative interaction; if $\lambda = 0$, the fuzzy measure would reduce to a simply additive measure as:

$$\mu(A) = \sum_{c_j \in A} \mu(c_j) = \sum_{j \in \{1,\ldots,|A|\}} w_j, \quad \text{for any } A \subseteq C \tag{5}$$

For a finite set C, the $\lambda-$fuzzy measure satisfies (Chiou et al., 2005):

$$\begin{aligned} \mu(C) &= \sum_{j=1}^{n} \mu(c_j) + \lambda \sum_{j_1=1}^{n-1} \sum_{j_2=j_1+1}^{n} (\mu(c_{j_1})\mu(c_{j_2}) + \ldots + \lambda^{n-1}\mu(c_1)\cdots\mu(c_n)) \\ &= \frac{1}{\lambda}\left(\prod_{j=1}^{n}(1+\lambda\mu(c_j)) - 1\right) \end{aligned} \tag{6}$$

Let $\mu(C) = 1$. Then, Eq. (6) can be rewritten as:

$$\lambda + 1 = \prod_{j=1}^{n}(1 + \lambda\mu(c_j)) \tag{7}$$

By Eq. (7), λ can be uniquely determined as an average interactive index of the given criteria. The $\lambda-$fuzzy measure is useful when the single criterion importance is obtained. In this case, it can be used to calculate the Choquet integral. If the fuzzy measure of each criterion is difficult to obtain, the $\lambda-$fuzzy measure could give a way to other forms of fuzzy measure. One of the most well-known techniques is the Möbius transform of fuzzy measure. We will discuss it in Section 6 in detail.

Based on the $\lambda-$fuzzy measure μ, the Choquet integral for MCDM problems can be defined as follows (Joshi & Kumar, 2016):

$$C_\mu(x) = \sum_{j=1}^{n} f\left(x_{\sigma(j)}\right)\left(\mu(A_j) - \mu(A_{j+1})\right) \tag{8}$$

where $f(x_j)$ is a utility function under criterion c_j. $\sigma(\cdot)$ is the permutation of criteria such that $0 \le f\left(x_{\sigma(1)}\right) \le f\left(x_{\sigma(2)}\right) \le \ldots \le f\left(x_{\sigma(n)}\right)$, and $A_j = \left\{x_{\sigma(j)}, x_{\sigma(j+1)}, \ldots, x_{\sigma(n)}\right\}$ is a subset of $n - j + 1$ biggest components in X with $A_{n+1} = \varnothing$.

Motivated by the Choquet integral (Choquet, 1954), Angilella et al. (2016) developed a Choquet integral-based Multiple Criteria Hierarchy Process (MCHP) to solve the university ranking problem. Based on a case study about the appraisal of an abandoned quarry, a multiple criteria preference aggregation model using the Choquet integral was developed by Bottero et al. (2018). Some Choquet integral operators were proposed to integrate the evaluations under fuzzy and linguistic environments for MCDM problems (Yu et al., 2011; Joshi & Kumar, 2016; Jin et al., 2018). However, these models can hardly tackle the hesitant fuzzy information. In this paper, we propose an HFCIA operator to aggregate the evaluations expressed in HFEs.

2.3. Gained and lost dominance score (GLDS) method

The GLDS method, originally proposed by Wu and Liao (2019), is a powerful outranking technique based on dominance scores. The main idea of this method is to obtain the dominance degree between alternatives using the dominance flow function. It takes into account the group utility and individual regret value of each alternative at the same time. The procedure of the GLDS method is described as follows:

Step 1. Let $w=(w_1,w_2,\ldots,w_n)^T$ indicate the weight vector of n criteria. Calculate the normalized dominance flow $df_j^N(a_i,a_v)$ of alternative a_i to alternative a_v with respect to criterion c_j by:

$$df_j^N(a_i,a_v)=\frac{df_j(a_i,a_v)}{\sqrt{\sum_{v=1}^{m}\sum_{i=1}^{m}\left(df_j(a_i,a_v)\right)^2}} \tag{9}$$

where $df_j(a_i,a_v)=x_{ij}-x_{vj}$ for benefit criterion c_j and $df_j(a_i,a_v)=x_{vj}-x_{ij}$ for cost criterion c_j.

Step 2. Calculate the gained dominance score of alternative a_i under all criteria by:

$$DS_1(a_i)=\sum_{j=1}^{n}\left(w_j\sum_{v=1}^{m}df_j^N(a_i,a_v)\right),\ \text{for}\ \ i=1,2,\ldots,m \tag{10}$$

$DS_1(a_i)$ can be regarded as the "group utility" value of alternative a_i. The first subordinate rank set $R_1=\{r_1(a_1),r_1(a_2),\ldots,r_1(a_m)\}$ can be obtained by listing $DS_1(a_i)$ $(i=1,2,\ldots,m)$ in descending order.

Step 3. Calculate the lost dominance score of alternative a_i under all criteria by:

$$DS_2(a_i)=\max_j\left(w_j\max_v df_j^N(a_v,a_i)\right),\ \text{for}\ \ i=1,2,\ldots,m \tag{11}$$

$DS_2(a_i)$ can be deemed as the "individual regret" value of alternative a_i. The second subordinate rank set, $R_2=\{r_2(a_1),r_2(a_2),\cdots,r_2(a_m)\}$, is derived by listing $DS_2(a_i)$ $(i=1,2,\ldots,m)$ in ascending order.

Step 4. Calculate the global score of each alternative using the following expression:

$$BS_i = \frac{DS_1(a_i)}{\sqrt{\sum_{i=1}^{m}\left(DS_1(a_i)\right)^2}} \cdot \frac{m - r_1(a_i) + 1}{m(m+1)/2} - \frac{DS_2(a_i)}{\sqrt{\sum_{i=1}^{m}\left(DS_2(a_i)\right)^2}} \cdot \frac{r_2(a_i)}{m(m+1)/2}, \text{ for } i = 1, 2, \ldots, m \tag{12}$$

Step 5. Rank the alternatives according to the descending order of the global scores of alternatives.

From the above procedure, it is easy to see that the global score is a comprehensive value involving two different score values and two different ranks. In other words, the GLDS method takes into account the values of "group utility" and "individual regret", simultaneously. In this regard, the GLDS method can avoid selecting an alternative performing too badly on some criteria as obtained by the simple weighted averaging operators.

2.4. Literature review on higher business education evaluation

Over the past decades, many MCDM methods have been developed by researchers and scholars for university evaluation and ranking. Kao and Lin (2011) used the data envelopment analysis (DEA) to handle the qualitative evaluation of UK universities. Wu et al. (2012) developed a hybrid MCDM method involving the analytic hierarchy process (AHP) and VIKOR method for higher education evaluation. Kabak and Dagdeviren (2014) integrated the analytic network process (ANP) and PROMETHEE (preference ranking organization method for enrichment evaluations) method for ranking universities. Angilella et al. (2016) developed a Choquet integral preference model to obtain the ranking of universities. Chen et al. (2016) proposed an outranking method based on the probability theory to evaluate university faculties. Kobina et al. (2017) investigated several power aggregation operators for undergraduate school ranking problems with the probabilistic linguistic information. Corrente et al. (2017) studied the university ranking problem based on the extended ELECTRE III method. Liu et al. (2019) proposed a prospect cross-efficiency (PCE) model to evaluate universities in China. An overview was offered based on the comparison between the GLDS method used in this paper and the other methods proposed in previous researches, shown in Table 1.

Table 1. Comparisons between different MCDM methods for university ranking used in 2011-2019

Reference	**Ranking method**	**The category of ranking method**	**Information form**	**Interaction among criteria**	**Group decision**
This paper	GLDS	Outranking	HFEs	√	√
Liu et al. (2019)	PCE	Utility value	Crisp values	×	×
Corrente et al. (2017)	ELECTRE III	Outranking	Crisp values	√	×
Kobina et al. (2017)	-	Utility value	PLEs	×	√
Chen et al. (2016)	-	Outranking	PHFLEs	×	√
Angilella et al. (2016)	MCHP	Utility value	Crisp values	√	×

Kabak and Dagdeviren (2014)	ANP and PROMETHEE	Outranking	Crisp values	×	×
Wu et al. (2012)	AHP and VIKOR	Outranking	Crisp values	×	×
Kao and Lin (2011)	DEA	Utility value	Fuzzy number	×	×

From Table 1, we can see that many ranking methods have been applied to handle the university ranking problem. However, few literature has investigated the emerging GLDS method for higher education evaluation. In addition, few MCDM methods took into account the risk preference psychology of experts and the inter-dependency between criteria, simultaneously. Therefore, it is necessary to improve the GLDS method to address the complicated MCDM problem involving the behavior preference and inter-dependency characteristics. It not only extends the boundary of applications of the GLDS method, but also addresses the increasing sophisticated higher education evaluation problems.

3. A comprehensive distance measure between hesitant fuzzy sets

For MCGDM problems, it could be difficult to determine an optimal solution if conflicting criteria are involved in the evaluation process. To derive a final solution, Yu (1973) explored the compromise solution instead of the optimal solution to address this issue. The main idea of this method is to find a compromise value between positive ideal solution (PIS) and negative ideal solution (NIS). Based on the compromise theory, in this section, we first propose a compromise value function for HFEs based on the reference point, and then develop a comprehensive distance measure for HFEs using the proposed compromise value function.

3.1. The compromise value function for hesitant fuzzy sets

In the following, we propose a compromise value function which takes into account the relative importance of the distances to PIS and NIS. Based on the score function of HFEs (Liao et al., 2019a) given as Eq. (2), we develop a score value-based distance function of an HFE to the PIS in the following form:

Definition 1. Let h be an HFE. The score value-based distance between h and $\tilde{1}$ can be defined as:

$$d(h,\tilde{1})=\left(\frac{1}{\#h}\sum_{\gamma\in h}(\tilde{1}-\gamma)^{(1-HD(h))}\right)^{1/(1-HD(h))} \tag{13}$$

where $\#h$ denotes the number of elements in h. $\tilde{1}=\{1,1,\cdots,1\}$ and the number of elements in the set $\tilde{1}$ is $\#h$.

In analogous, the distance between the HFE h and $\tilde{0}$ can be determined by the score value-based distance function $d(h,\tilde{0})$ using the following expression:

$$d(h,\tilde{0})=\left(\frac{1}{\#h}\sum_{\gamma\in h}(\gamma-\tilde{0})^{(1-HD(h))}\right)^{1/(1-HD(h))} \tag{14}$$

where $\tilde{0}=\{0,0,\ldots,0\}$ and the number of elements in the set $\tilde{0}$ is $\#h$.

Remark 1. It is noteworthy that the score values-based distance function contains a hesitancy degree function $HD(h)$, reflecting the uncertainty of the expert's evaluation. For any two HFEs h_1 and h_2, if $HD(h_1)$ is higher than $HD(h_2)$, then, the former expert's evaluation has a larger uncertainty. Specifically, $HD(h)=0$ implies that the expert evaluates an object without hesitancy.

In MCDM, balancing the separations to the PIS and NIS plays an important role in finding the compromise solution(s) (Kuo, 2017). To address this issue, we associate ω and $(1-\omega)$ to the two separation measures of an alternative, where ω is the relative importance of d_j^- and $(1-\omega)$ is the relative importance of d_j^+, $0<\omega<1$. Based on the proposed distance measure function, we develop a distance-based compromise value function under the hesitant fuzzy environment, which can be defined as follows:

Definition 2. For a criterion c_j, h_j is an HFE that indicates the evaluation of an alternative on criterion c_j. Then, the distance-based compromise value function of h_j is defined as follows:

$$CV(h_j)=\omega d_j^- -(1-\omega)d_j^+ \tag{15}$$

where $d_j^+=\{d(h_j,\tilde{1})\,|\,j\in C_1, d(h_j,\tilde{0})\,|\,j\in C_2\}$ and $d_j^-=\{d(h_j,\tilde{0})\,|\,j\in C_1, d(h_j,\tilde{1})\,|\,j\in C_2\}$. Here, C_1 indicates the set of benefit criteria, and C_2 indicates the set of cost criteria.

Obviously, $CV(h_j)\in[-1,1]$, for $j=1,2,\ldots,n$. The two distance measures, d_j^+ and d_j^-, can be deemed as the two separations of the compromise value function by which the compromise solution is determined.

Example 2. Consider $h=\{0.2,0.3,0.5\}$ as an evaluation value under a benefit criterion. Based on Eqs. (13) and (14), we obtain the score value-based distances of the HFE as: $d(h,\tilde{0})=0.331$, $d(h,\tilde{1})=0.665$. Then, if $\omega=0.5$, we have the compromise value as $CV(h)=-0.167$.

According to Definition 2, the following theorem can be derived.

Theorem 1. Let h be an HFE defined in the finite set of X. Then, the compromise value function is monotonically non-deceasing with respect to h. Namely, for any two HFEs h_1 and h_2, if $h_1 \precsim h_2$, then

$CV(h_1) \le CV(h_2)$.

Proof. Please see Appendix A. 1.

According to Eqs. (13) and (14), the distance-based compromise value considering the relative importance can be determined by the score value-based distance measures $d(h_j, \tilde{1})$ and $d(h_j, \tilde{0})$, which are both real numbers. In fact, the distance-based compromise value $CV(h_j)$ makes a trade-off between the PIS d_j^+ and NIS d_j^-. The distance-based compromise value provides an intuitive interpretation of ranking for HFSs. This means that the distance-based compromise value function can be deemed as a useful tool to explore the ranking of HFSs. In this sense, we can derive the ranking of any two HFEs h_1 and h_2 using their compromise values.

Theorem 2. Let h_1 and h_2 be any two HFEs, and $CV(h_1)$ and $CV(h_2)$ are the compromise values of them. Then, the ranking of h_1 and h_2 can be determined as follows:

(1) If $CV(h_1) < CV(h_2)$, then h_1 is inferior to h_2, denoted as $h_1 \prec h_2$;

(2) If $CV(h_1) > CV(h_2)$, then h_1 is superior to h_2, denoted as $h_1 \succ h_2$;

(3) If $CV(h_1) = CV(h_2)$, then h_1 is indifferent to h_2, denoted as $h_1 \sim h_2$.

Proof. Please refer to Appendix A.2.

Example 3. Suppose that there are two HFEs $h_1 = \{0.2, 0.3, 0.5\}$ and $h_2 = \{0.4, 0.7\}$ on a benefit criteria. Based on Eq. (15), we can calculate the compromise values as: $CV(h_1) = -0.167$ and $CV(h_2) = 0.050$. Since $CV(h_1) < CV(h_2)$, we can obtain $h_1 \prec h_2$ according to Theorem 2. In other words, h_1 is inferior than h_2.

Example 4. Using the input data in Wang and Xu (2016), we can derive the ranking of the two HFEs $h_1 = \{0.1, 0.6, 0.8\}$ and $h_2 = \{0.2, 0.4, 0.9\}$ based on Theorem 2. It is easy to find that they have the same average value and the same deviation degree using the comparison technique proposed by Liao et al. (2014). However, with the aid of the distance-based compromise value function, we have $CV(h_1) = -0.005$ and $CV(h_1) = 0.005$. We can distinguish h_1 and h_2 using the suggested ranking method.

According to Definition 2, it is easy to observe that the boundary condition $-1 \le CV(h) \le 1$ is satisfied. Based on the set theory, we can demonstrate that the proposed value function is endowed with partial orders by Theorem 3.

Theorem 3. Let H be a non-empty set of HFEs in X, and the preference relation "$\preceq$" on the set H be a binary relation. Then, the order preference relation "$\preceq$" is a partial order and $(H,[\bar{0},\bar{1}],\preceq)$ is a complete bounded lattice with the smallest element $\bar{0}=\{0,0,\ldots,0\}$ and the greatest element $\bar{1}=\{1,1,\ldots,1\}$.

Proof. Please refer to Appendix A.3.

The proposed order relationship of HFEs can be used in the Choquet integral operators, which require a permutation among HFEs. It would be an interesting topic to discuss the admissible order of HFEs given that the envelope of an HFE is an interval value and the admissible order of interval values has been discussed by many scholars (Bustince, Fernández, Kolesárová, & Mesiar, 2013). We will pay attention to this issue in the future.

3.2. A comprehensive distance measure of HFSs based on the compromise value function

Although the orders of HFEs are vital to capture the relations between HFEs, the ranking relation is unable to identify the difference level which play an important role in the GLDS method. In this regard, the distance measure plays a significant role to characterize this relation. However, existing distance measures of HFEs based on weighted mean operators may produce unreasonable results as justified in the Introduction. To improve the validity of the distance measure of HFEs, we propose a compromise distance measure for HFSs using the compromise value function.

Definition 3. Let h_{1j} and h_{2j} be two HFEs. Then, the comprehensive distance measure between h_{1j} and h_{2j} is defined as:

$$\tilde{d}(h_{1j},h_{2j})=\left|CV(h_{1j})-CV(h_{2j})\right|,\ \text{for}\ \ j=1,2,\ldots,n \tag{16}$$

where h_{1j} denotes the evaluation under criterion c_j, and $CV(h_{1j})$ is the compromise value of h_{1j}.

Remark 2. It is worth noting that the comprehensive distance measure contains a significant coefficient ω which has not been displayed in Eq. (16). If $\omega=0.5$, then $\tilde{d}(h_{1j},h_{2j})=0.5\left|(d_{1j}^{-}-d_{1j}^{+})-(d_{2j}^{-}-d_{2j}^{+})\right|=\left|d_{1j}^{+}-d_{2j}^{+}\right|$. Namely, the comprehensive distance measure will reduce to a distance measure only considering one reference point. In this special case, there is no need to consider the NIS in this situation.

According to Definition 3, it is easy to find that the proposed comprehensive distance measure satisfies the properties shown in Theorem 4.

Theorem 4. Let h_{1j}, h_{2j} and h_{3j} be three HFEs. The comprehensive distance measure $\tilde{d}$ is a real number, and the following properties are satisfied:

(1) $0 \le \tilde{d}(h_{1j}, h_{2j}) \le 1$;

(2) $\tilde{d}(h_{1j}, h_{2j}) = 0$, iff $h_{1j} = h_{2j}$;

(3) $\tilde{d}(h_{1j}, h_{2j}) = \tilde{d}(h_{2j}, h_{1j})$;

(4) $\tilde{d}(h_{1j}, h_{3j}) \le \tilde{d}(h_{1j}, h_{2j}) + \tilde{d}(h_{2j}, h_{3j})$.

4. A Choquet integral-based GLDS method considering the risk preferences of experts for MCGDM problems

In this section, we propose a Choquet integral-based HF-GLDS method for MCGDM problems by using the comprehensive distance measure introduced in Section 3. The proposed method considers not only the inter-dependent characteristic between decision criteria, but also the behavior preferences of experts.

A general MCGDM problem is mainly composed by a set of alternatives $A=\{a_1,\ldots,a_m\}$, a finite set of criteria $C=\{c_1,\ldots,c_n\}$ and a set of experts $E=\{e_1,\ldots,e_q\}$ where q denotes the number of experts. $s=(s_1,\ldots,s_q)^T$ is a weight vector associated with the experts, which satisfies $s_k \in [0,1]$ and $\sum_{k=1}^{q} s_k = 1$. $D^k = (h_{ij}^k)_{m\times n}$ is the individual hesitant fuzzy decision matrix, where h_{ij}^k is the evaluation of alternative a_i with respect to criterion c_i given by expert e_k, $k=1,2,\ldots,q$, which can be expressed as follows:

$$D^k = \begin{array}{c} \\ a_1 \\ \vdots \\ a_i \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & \cdots & c_j & \cdots & c_n \end{array} \\ \begin{pmatrix} h_{11}^k & \cdots & h_{1j}^k & \cdots & h_{1n}^k \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{i1}^k & \cdots & h_{ij}^k & \cdots & h_{in}^k \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_{m1}^k & \cdots & h_{mj}^k & \cdots & h_{mn}^k \end{pmatrix} \end{array}$$

Based on the prospect theory (Tversky & Kahneman, 1992), we propose a novel dominance degree function for HFSs by considering the risk attitude of experts, which is defined as follows:

$$dd_j(h_i, h_v) = \begin{cases} \tilde{d}(h_{ij}, h_{vj}), \text{ if } CV(h_{ij}) - CV(h_{vj}) \ge 0 \\ -\dfrac{1}{\theta}\tilde{d}(h_{vj}, h_{ij}), \text{ if } CV(h_{ij}) - CV(h_{vj}) < 0 \end{cases} \tag{17}$$

where θ denotes the loss-aversion parameter associated with experts. If $0 < \theta < 1$, then the sensitivity of losses is high; if $\theta > 1$, then the sensitivity of losses is low. $\tilde{d}(h_{ij}, h_{vj})$ is the distance between h_{ij} and h_{vj}, and $\tilde{d}(h_{vj}, h_{ij})$ is the distance between h_{vj} and h_{ij}. $CV(h_{ij}) - CV(h_{vj})$ indicates the difference of the compromise values.

For MCGDM problems, suppose that h_{ij}^k and h_{vj}^k are the evaluations given by expert e_k ($k=1,2,\ldots,q$). Then, the overall dominance flow of alternative a_i over alternative a_v on criterion c_j corresponding to the expert group can be defined as:

$$df_j(a_i,a_v)=\sum_{k=1}^{q} s_k dd_j(h_i^k,h_v^k) \tag{18}$$

As we know, normalizing evaluation values is an essential process when not all criteria are established on the same dimension. In this regard, we integrate the evaluations given by the k experts using Eq. (18). Then, to aggregate the dominance flows under different criteria, we should normalize the dominance flows by:

$$df_j^N(a_i,a_v)=\frac{df_j(a_i,a_v)}{\sqrt{\sum_{v=1}^{m}\sum_{i=1}^{m}\left(df_j(a_i,a_v)\right)^2}} \tag{19}$$

To obtain the group utility value of alternative a_i, the essential procedure is to aggregate the evaluations under all criteria. Considering that the sophisticated relationship usually exists in real MCGDM problems, it is necessary to model the inter-dependency relation between criteria. Using the HFLCIA operator proposed by Liao et al. (2019) to aggregate the alternative evaluations provides an ability to depict the inter-dependency relation between criteria. Motivated by the HFLCIA operators, we propose a hesitant fuzzy Choquet integral arithmetic (HFCIA) operator to integrate the hesitant fuzzy evaluations on inter-dependent criteria as follows:

First, based on Eq. (19), we can obtain the gained dominance score of alternative a_i over alternative a_v under criterion c_j as:

$$df_j^N(a_i)=\sum_{v=1}^{m} df_j^N\left(a_i,a_v\right) \tag{20}$$

Then, the HFCIA operator, which can aggregate the individual dominance scores into the gained dominance score of alternative a_i over alternative a_v under all decision criteria, is defined in the follow form:

$$\begin{aligned} DS_1(a_i) &= HFCIA\left(df_1^N(a_i),df_2^N(a_i),\ldots,df_n^N(a_i)\right) \\ &= \bigoplus_{j=1}^{n}\left(df_{\sigma(j)}^N(a_i)\right)\left(\mu\left(C_{\sigma(j)}\right)-\mu\left(C_{\sigma(j+1)}\right)\right) \\ &= \sum_{j=1}^{n}\left(df_{\sigma(j)}^N(a_i)\right)\left(\mu\left(C_{\sigma(j)}\right)-\mu\left(C_{\sigma(j+1)}\right)\right) \end{aligned} \tag{21}$$

where $\sigma(j)$ $\left(j=1,2,\ldots,n\right)$ is a permutation of $\left(1,2,\ldots,n\right)$ such that $CV(h_{\sigma(1)})\le CV(h_{\sigma(2)})\le\ldots\le CV(h_{\sigma(n)})$ and

$C_{\sigma(n+1)} = \varnothing$, and $C_{\sigma(j)} = \{c_{\sigma(j)}, c_{\sigma(j+1)}, \ldots, c_{\sigma(n)}\}$ is a subset of $n - j + 1$ biggest components in C.

Similarly, to obtain the individual regret value of alternative a_i, we use the maximum dominance flow of alternative a_v over alternative a_i under all criteria to represent its lost dominance score. Inspired by the original GLDS method (Wu & Liao, 2019), we define the lost dominance score of alternative a_i over all other alternatives on all criteria using the following expression:

$$DS_2(a_i) = \max_j \left(\max_v df^N_{\sigma(j)}(a_v, a_i) \left(\mu(C_{\sigma(j)}) - \mu(C_{\sigma(j+1)}) \right) \right) \tag{22}$$

Finally, based on the gained and lost dominance scores and their corresponding ranks, we derive the global score of each alternative by the following formula:

$$BS_i = \frac{DS_1(a_i)}{\sqrt{\sum_{i=1}^{m} \left(DS_1(a_i)\right)^2}} \cdot \frac{m - r_1(a_i) + 1}{m(m+1)/2} - \frac{DS_2(a_i)}{\sqrt{\sum_{i=1}^{m} \left(DS_2(a_i)\right)^2}} \cdot \frac{r_2(a_i)}{m(m+1)/2} \tag{23}$$

where m is the number of alternatives. $r_1(a_i)$ is the rank of alternative a_i, which is determined in descending order of $DS_1(a_i)$, for $i = 1, 2, \ldots, m$. $r_2(a_i)$ is the rank of alternative a_i, which is determined in ascending order of $DS_2(a_i)$, for $i = 1, 2, \cdots, m$.

Based on the global scores of all alternatives, we can obtain the final ranking of alternatives.

For the facility of application, we can summarize the procedure of the Choquet integral-based HF-GLDS method for MCGDM problems in detail as follows:

Step 1. Determine the dominance flows. For an MCGDM problem, the individual decision matrix associated to different experts can be established as $D^k = (h^k_{ij})_{m \times n}$, $k = 1, 2, \ldots, q$. Then, based on Eqs. (17) and (18), the dominance flow of alternative a_i over alternative a_v corresponding to the k expert under each c_j is determined as $df^k_j(a_i, a_v)$.

Step 2. Normalize the dominance flow. Calculate the normalized dominance flow of alternative a_i over alternative a_v under criterion c_j by Eq. (19).

Step 3. Determine the fuzzy measures. Determine the weight $\mu(c_j)$ of criterion c_j, and then the fuzzy measures of criterion subset are defined by Eq. (4).

Step 4. Calculate the gained dominance scores. Calculate the gained dominance score $DS_1(a_i)$ of alternative a_i over alternative a_v under all criteria by Eqs. (20) and (21). Then, the first subordinate rank set, $R_1 = \{r_1(a_1), r_1(a_2), \ldots, r_1(a_m)\}$, can be determined according to the values of a_i in descending order.

Step 5. Calculate the lost dominance scores. Calculate the lost dominance score $DS_2(a_i)$ by Eq. (22). Then, the second subordinate rank set, $R_2 = \{r_2(a_1), r_2(a_2), \ldots, r_2(a_m)\}$, is determined by the values $DS_2(a_i)$ in ascending order.

Step 6. Aggregate the gained dominance scores and lost dominance scores. Determine the global score BS_i of alternative a_i by Eq. (23). Then, the global rank set $R = \{r(a_1), r(a_2), ..., r(a_m)\}$, of the alternatives is determined in descending order of BS_i. The higher BS_i is, the better a_i is.

Considering the complexity of MCGDM problems, the assumption that the weights given by experts satisfying additivity is invalid. Besides, the process of weight vector normalization ignores the inter-dependency among the criteria. Therefore, it is necessary to use the Choquet integral to aggregate the evaluations under the criteria that are usually not independent to each other. It is noteworthy that, in this study, the single fuzzy measure $\mu(c_j)$ is provided by a group of experts directly.

Compared with the original GLDS method, the Choquet integral-based HF-GLDS method possesses the following strength:

(1) It uses a comprehensive distance measure to capture the difference between HFEs in MCGDM problems involving multiple conflicting criteria. In the original GLDS method (Wu & Liao, 2019), an important step is to adjust the probabilistic linguistic elements into the same probability distribution before calculating the distance between two evaluation values. This extra operation on the raw data may lead to information loss. In this regard, our method overcomes not only this flaw but also the shortcoming of the original GLDS method on handling conflicting criteria.

(2) It provides an ability to depict the attitude characteristics of experts to loss. The risk preference psychology, as we know, generally exists when experts make decision under risk. It is vital to portray this psychology for MCGDM problems. While the proposed method takes into account this preference using the prospect theory, the original GLDS method ignores this psychological phenomenon.

(3) It considers the inter-dependency phenomenon among criteria. The independence of criteria is a potential

assumption of the original GLDS method that is unable to capture the inter-dependent characteristics between criteria. Using the Choquet integral to aggregate information provides an ability to handle more complicated situation in MCGDM problems in which the criteria are not independent.

5. An illustrative example: Case study of higher business education evaluation

In this section, the Choquet integral-based HF-GLDS method is implemented to derive the ranking of four business schools in Sichuan, China. Sensitivity and comparative analyses are given to demonstrate the validity and effectiveness of the proposed method.

5.1. Problem description

The university education is not only one of the most expensive consumption that most people ever make, but can exert influence on lifestyle, income and occupation of lifetime. Higher business education evaluation plays a critical role in many aspects. From the perspective of educational decision-making bodies, this evaluation process can assist universities and colleges to recognize their advantages and form their own school-running features. Furthermore, it can help management departments to make scientific educational decisions, develop educational planning and allocate educational resources rationally. From the perspective of social public, it is not only conducive to grasping the actual outputs of educational inputs of colleges and universities, but also conducive to enhancing the confidence and support of the whole society for the development of colleges and universities. From the perspective of students, it can help young people who are ready to go to college and aim to major in business make rational decisions.

In this study, we use the first perspective aforementioned to analyze. Suppose that the provincial government of Sichuan needs to assess the performances of four business schools $\{a_1, a_2, a_3, a_4\}$ and distributes education resources depending on their evaluation results. A number of studies have investigated the criteria considered in the higher education evaluation (Chen et al., 2016; Kabak & Dagdeviren, 2014; Wu et al., 2012). In current study, four criteria are considered in the evaluation process. Their detailed descriptions are shown in Table 1.

Table 1. Establishing the evaluation criteria.

Evaluation objective	**Criteria**	**Description of criteria**
Higher business education evaluation	School-running resources c_1	It contains the number of doctor and master degree authorization centers and reflects proportion of teachers with deputy senior professional titles.
	Scientific research outputs c_2	We utilize the number of national projects and average annual research funding to represent scientific research outputs.

Talent cultivation c_3	It refers to the number of students, teaching achievement Award and number of national excellent courses.
International exchanges c_4	It mainly involves two aspects, international academic conference and overseas exchanges.

Suppose that three experts $\{e_1, e_2, e_3\}$ are invited to evaluate the four business schools and give a ranking. The weight vector of the three expert is $s = (0.3, 0.3, 0.4)$. These experts use HFSs to evaluate the performances of the four business schools under the four criteria. The decision matrices can be shown in Tables 2-4.

Table 2. The decision matrix D_1 .

	c_1	c_2	c_3	c_4
a_1	$\{0.5, 0.6, 0.8\}$	$\{0.3, 0.6\}$	$\{0.2, 0.4\}$	$\{0.3, 0.4, 0.5\}$
a_2	$\{0.4, 0.6, 0.7\}$	$\{0.4, 0.7\}$	$\{0.6, 0.7, 0.8\}$	$\{0.6\}$
a_3	$\{0.6\}$	$\{0.3, 0.4\}$	$\{0.8, 0.9\}$	$\{0.6, 0.8\}$
a_4	$\{0.9, 1.0\}$	$\{0.8\}$	$\{0.6, 0.8\}$	$\{0.5, 0.7\}$

Table 3. The decision matrix D_2 .

	c_1	c_2	c_3	c_4
a_1	$\{0.6, 0.8\}$	$\{0.2, 0.5\}$	$\{0.5, 0.8\}$	$\{0.8\}$
a_2	$\{0.4, 0.6\}$	$\{0.5, 0.7\}$	$\{0.7\}$	$\{0.6, 0.8\}$
a_3	$\{0.7, 0.9\}$	$\{0.5, 0.6\}$	$\{0.9\}$	$\{0.2, 0.5\}$
a_4	$\{0.8\}$	$\{0.6, 0.7\}$	$\{0.5, 0.7, 0.8\}$	$\{0.7\}$

Table 4. The decision matrix D_3 .

	c_1	c_2	c_3	c_4
a_1	$\{0.8\}$	$\{0.2, 0.3\}$	$\{0.2, 0.3, 0.4\}$	$\{0.4, 0.5\}$
a_2	$\{0.6, 0.7\}$	$\{0.5, 0.7\}$	$\{0.6\}$	$\{0.5, 0.8\}$
a_3	$\{0.8\}$	$\{0.5, 0.6\}$	$\{0.6, 0.7\}$	$\{0.5, 0.6, 0.8\}$
a_4	$\{0.9\}$	$\{0.6, 0.7, 0.8\}$	$\{0.8\}$	$\{0.8\}$

5.2. The decision-making process using the Choquet integral-based HF-GLDS method

Based on the proposed Choquet integral-based HF-GLDS method, we solve the MCGDM problem concerning the higher business education evaluation in the following stepwise procedure:

Step 1. Determine the dominance flows. Based on Eq. (17), we set $\theta = 1$, which indicates the median value about the sensitivity to loss. We set $\omega = 0.5$ and obtain the dominance flows in Tables 5-8.

Table 5. The dominance flows of one alternative over another under criterion c_1 .

	Expert e_1				Expert e_2				Expert e_3			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4

a_1	0.000	0.067	0.034	-0.317	0.000	0.201	-0.101	-0.100	0.000	0.150	0.000	-0.100
a_2	-0.067	0.000	-0.033	-0.384	0.201	0.000	-0.301	-0.300	-0.150	0.000	-0.150	-0.250
a_3	-0.034	0.033	0.000	-0.351	0.101	0.301	0.000	0.001	0.000	0.150	0.000	-0.100
a_4	0.317	0.384	0.351	0.000	0.100	0.300	-0.001	0.000	0.100	0.250	0.100	0.000

Table 6. The dominance flows of one alternative over another under criterion c_2.

	Expert e_1				Expert e_2				Expert e_3			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
a_1	0.000	-0.101	0.100	-0.350	0.000	-0.251	-0.201	-0.301	0.000	-0.350	-0.300	-0.450
a_2	0.101	0.000	0.200	-0.250	0.251	0.000	0.050	-0.050	0.350	0.000	0.050	-0.100
a_3	-0.100	-0.200	0.000	-0.450	0.201	-0.050	0.000	-0.100	0.300	-0.050	0.000	-0.150
a_4	0.350	0.250	0.450	0.000	0.301	0.050	0.100	0.000	0.450	0.100	0.150	0.000

Table 7. The dominance flows of one alternative over another under criterion c_3.

	Expert e_1				Expert e_2				Expert e_3			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
a_1	0.000	-0.401	-0.551	-0.401	0.000	-0.049	-0.249	-0.016	0.000	-0.300	-0.350	-0.500
a_2	0.401	0.000	-0.150	0.000	0.049	0.000	-0.200	0.033	0.300	0.000	-0.050	-0.200
a_3	0.551	0.150	0.000	0.150	0.249	0.200	0.000	0.233	0.350	0.050	0.000	-0.150
a_4	0.401	0.000	-0.150	0.000	0.016	-0.033	-0.233	0.000	0.500	0.200	0.150	0.000

Table 8. The dominance flows of one alternative over another under criterion c_4.

	Expert e_1				Expert e_2				Expert e_3			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
a_1	0.000	-0.200	-0.301	-0.200	0.000	0.100	0.451	0.100	0.000	-0.201	-0.184	-0.350
a_2	0.200	0.000	-0.101	0.000	-0.100	0.000	0.352	0.000	0.201	0.000	0.017	-0.149
a_3	0.301	0.101	0.000	0.100	-0.451	-0.352	0.000	-0.351	0.184	-0.017	0.000	-0.166
a_4	0.200	0.000	-0.100	0.000	-0.100	0.000	0.351	0.000	0.350	0.149	0.166	0.000

Step 2. Normalize the dominance flows. According to Eq. (19), we calculate the normalized dominance flows of alternative a_i over another a_v. The obtained results are shown in Tables 9-12.

Table 9. The normalized dominance flows of one alternative over another under criterion c_1.

	a_1	a_2	a_3	a_4
a_1	0.000	0.229	-0.033	-0.270
a_2	-0.229	0.000	-0.262	-0.499
a_3	0.033	0.262	0.000	-0.237
a_4	0.270	0.499	0.237	0.000

Table 10. The normalized dominance flows of one alternative over another under criterion c_2.

		a_2	a_3	a_4
a_1	0.000	-0.317	-0.194	-0.484
a_2	0.317	0.000	0.123	-0.168
a_3	0.194	-0.123	0.000	-0.290
a_4	0.484	0.168	0.290	0.000

a_1

Table 11. The normalized dominance flows of one alternative over another under criterion c_3.

	a_1	a_2	a_3	a_4
a_1	0.000	-0.310	-0.462	-0.395
a_2	0.310	0.000	-0.152	-0.085
a_3	0.462	0.152	0.000	0.067
a_4	0.395	0.085	-0.067	0.000

Table 12. The normalized dominance flows of one alternative over another under criterion c_4.

	a_1	a_2	a_3	a_4
a_1	0.000	-0.290	-0.075	-0.447
a_2	0.290	0.000	0.216	-0.157
a_3	0.075	-0.216	0.000	-0.373
a_4	0.447	0.157	0.373	0.000

Step 3. Determine the fuzzy measures. Suppose that the fuzzy measures of criteria c_j ($j=1,2,3,4$) are given as: $\mu(c_1)=0.4$, $\mu(c_2)=0.2$, $\mu(c_3)=0.3$, $\mu(c_4)=0.4$. Based on Eq. (4), we obtain the parameter $\lambda=-0.542$. Then we have $\mu(c_1,c_2)=0.557$, $\mu(c_1,c_3)=0.635$, $\mu(c_1,c_4)=0.713$, $\mu(c_2,c_3)=0.468$, $\mu(c_2,c_4)=0.557$, $\mu(c_3,c_4)=0.635$, $\mu(c_1,c_2,c_3)=0.766$, $\mu(c_1,c_2,c_4)=0.836$, $\mu(c_2,c_3,c_4)=0.767$, $\mu(c_1,c_3,c_4)=0.897$, $\mu(c_1,c_2,c_3,c_4)=1$.

Step 4. Calculate the gained dominance scores. By Eqs. (20) and (21), we calculate the gained dominance scores of the alternatives under different criteria. The results are listed in Table 13.

Table 13. The gained dominance scores of the alternatives.

Alternative	The gained dominance score				The overall gained dominance score
	c_1	c_2	c_3	c_4	
a_1	-0.073	-0.995	-1.167	-0.813	-0.597
a_2	-0.990	0.272	0.073	0.350	-0.033

a_3	0.058	-0.219	0.680	-0.513	0.075
a_4	1.006	0.942	0.413	0.977	0.892

Step 5. Calculate the lost dominance score. By Eq. (22), we derive the lost dominance scores listed in Table 14.

Table 14. The lost dominance scores of the alternatives.

Alternatives	The lost dominance scores				The aggregated lost dominance score
	c_1	c_2	c_3	c_4	
a_1	0.270	0.484	0.462	0.447	0.134
a_2	0.499	0.168	0.152	0.157	0.200
a_3	0.237	0.290	0.000	0.373	0.149
a_4	0.000	0.000	0.067	0.000	0.020

Step 6. Aggregate the gained dominance scores and lost dominance scores. By Eq. (23), we obtain the global dominance scores of the alternatives as $BS_1 = -0.150$, $BS_2 = -0.288$, $BS_3 = -0.137$, $BS_4 = 0.324$. According to these global dominance scores, we can rank the alternatives and select the optimal one. Since $BS_4 > BS_3 > BS_1 > BS_2$, then $a_4 \succ a_3 \succ a_1 \succ a_2$. Thus, a_4 is the best business school in Sichuan, China.

5.3. Sensitivity analysis

Compared with the original GLDS method (Wu & Liao, 2019), the proposed Choquet integral-based HF-GLDS method takes into account the attitude characteristics of experts based on the prospect theory. To find out what influence the risk attitude has on final results, we investigate the change of global dominance scores with different parameter θ. For the sake of simplification, we use the parameter θ to represent the overall loss-aversion degree of the expert group rather than individual experts. Figs. 1 and 2 present the corresponding results.

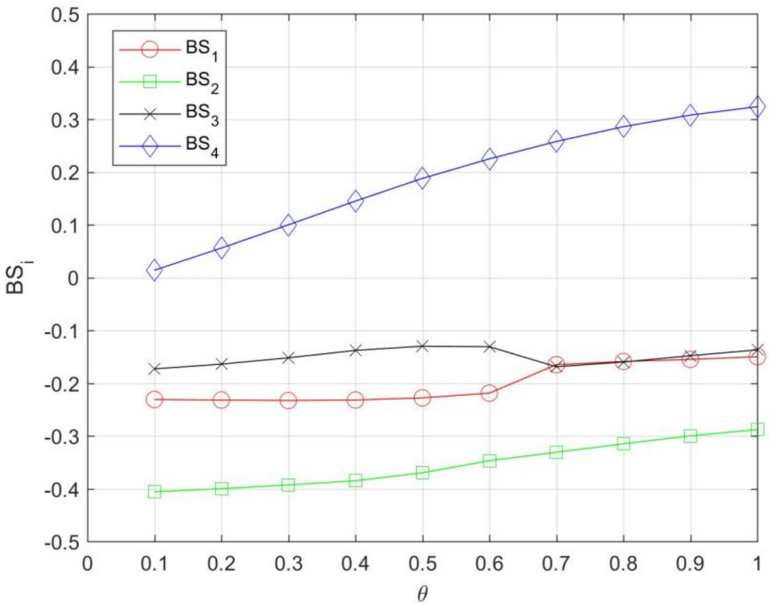

Fig. 1. Global dominance scores of the alternatives under different values of θ $(0<\theta\leq 1)$

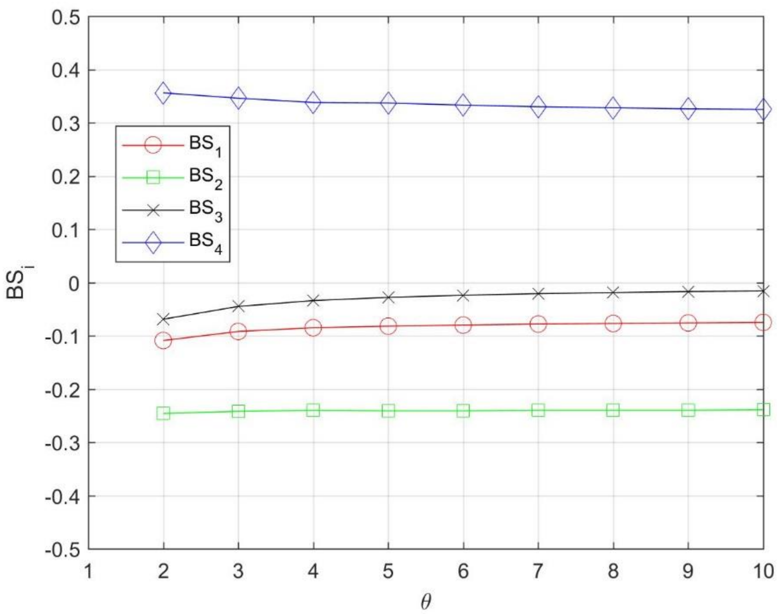

Fig. 2. Global dominance scores of the alternatives under different values of θ $(\theta>1)$

According to Figs. 1 and 2, we can derive that the final ranking keeps unchanged no matter what the value of θ is between 0 and 10. It implies that the global ranking is not sensitive to the attitude characteristics of the expert group. In other words, the results are consistent no matter the parameter θ is considered or not in this illustrative example.

In addition, it is worth noting that the influence of the parameter θ to BS_i is more significant when $0<\theta\leq 1$ than when $\theta>1$. In other words, the global score function for high loss is steeper than for low loss. This characteristic meets the basic property of the prospect theory that there is a higher sensitivity to losses than gains for experts.

5.4. Comparative analysis

To demonstrate the validity of the proposed method, we make a comparison with the hesitant fuzzy TOPSIS (HF-TOPSIS) method proposed by Xu and Zhang (2013).

Based on the above decision matrices, we determine the hesitant fuzzy PIS h_{ij}^{+} and hesitant NIS h_{ij}^{-}, where

$$h_{ij}^{+} = \{\max_{\forall i} d(h_{ij}, \tilde{0}) \mid j \in C_1, \min_{\forall i} d(h_{ij}, \tilde{0}) \mid j \in C_2\} \tag{24}$$

and

$$h_{ij}^{-} = \{\min_{\forall i} d(h_{ij}, \tilde{0}) \mid j \in C_1, \max_{\forall i} d(h_{ij}, \tilde{0}) \mid j \in C_2\} \tag{25}$$

Calculate the Euclidean distance $\tilde{d}_i^{+}$ and $\tilde{d}_i^{-}$ of alternative a_i from the PIS h_{ij}^{+} and NIS h_{ij}^{-}, respectively, by the following expressions:

$$\tilde{d}_i^{+} = \sqrt{\sum_{j=1}^{n} (d(h_{ij}, \tilde{0}) - h_{ij}^{+})^2}, \text{ for } i = 1, 2, \ldots, n \tag{26}$$

$$\tilde{d}_i^{-} = \sqrt{\sum_{j=1}^{n} (d(h_{ij}, \tilde{0}) - h_{ij}^{-})^2}, \text{ for } i = 1, 2, \ldots, n \tag{27}$$

Use Eq. (28) to derive the closeness coefficient of alternative a_i:

$$Cc_i = \frac{\tilde{d}_i^{-}}{\tilde{d}_i^{-} + \tilde{d}_i^{+}}, \text{ for } i = 1, 2, \ldots, n \tag{28}$$

The closeness coefficient values of the alternatives are calculated as: $Cc_1 = 0.193$, $Cc_2 = 0.501$, $Cc_3 = 0.593$, $Cc_4 = 0.916$. Then, the ranking of these alternatives is $a_4 \succ a_3 \succ a_2 \succ a_1$, according to the values of Cc_i in descending order.

Although the both two methods produce the same optimal alternative a_4, there are some differences of the ranking orders derived by the HF-TOPSIS and our method. While the worst alternative is a_2 using the proposed Choquet integral-based HF-GLDS method, the worst alternative is a_1 using the HF-TOPSIS. The main reason for this is that the HF-TOPSIS method ignores the interdependent characteristic among criteria and the risk preference psychology of experts in the decision-making process, while the proposed method assumes that the criteria are not independent and the experts are bounded rational. Given that these characteristics are generally involved in MCGDM problems, the proposed method is a more reasonable ranking method in the real life that provides the decision results considering the inter-dependent characteristic and behavior preference.

6. Extension

In this section, we discuss an extension of the Choquet integral. The illustrative example shows that the $\lambda-$ fuzzy measure is convenient for the computation of Choquet integral. In some cases, however, a decision-maker might provide interaction information between criteria instead of giving the values of the fuzzy measure directly. In other words, the criteria interactions might be obtained, but the $\lambda-$ fuzzy measure could not capture this feature by effective tools. To make it possible, a one-to-one representation of fuzzy measure was defined in Grabisch (1997). The idea is to decompose a fuzzy measure into unanimity games, such that the fuzzy measure is transformed into an additive form. Therefore, the $\lambda-$ fuzzy measure in Eq. (4) can be replaced by a transformed representation of fuzzy measure, called *Möbius transform* (Rota, 1964), which refers to a linear transform on the fuzzy measure:

$$\mu(B)=\sum_{A\subseteq B} m(A), \text{ for any } B\subseteq 2^C \tag{29}$$

An invertible transform of the Möbius representation can be expressed as:

$$m(A)=\sum_{B\subseteq A}(-1)^{|A|-|B|}\mu(B) \tag{30}$$

where $|A|$ indicates the cardinality of the set A. In this way, the boundary and monotonicity conditions of a fuzzy measure must be reconstructed as:

(1) (Boundary conditions) $m(\varnothing)=0$, $\sum_{T\subseteq C} m(T)=1$,

(2) (Monotonicity condition) $\sum_{T\subseteq S} m(T\cup\{c_j\})\geq 0,\ \forall c_j\in C$ and $\forall S\subseteq C\setminus\{c_j\}$.

Grabisch (1997) introduced the concept of *Shapley value* to represent criteria importance by considering the average contribution of a criterion to the whole set of criteria, which can be conveniently expressed by $\mu(\cdot)$. Given that μ is a fuzzy measure on *G*, the importance index of criterion g_j can be represented by the Shapley value (Shapley, 1953):

$$\phi(\{g_j\})=\sum_{A\subseteq G/\{g_j\}}\frac{(n-|A|-1)!|A|!}{n!}(\mu(A\cup\{g_j\})-\mu(A)), \text{ for } j=1,2,\ldots,n. \tag{31}$$

The Shapley value satisfies $0\leq\phi(\{g_j\})\leq 1$ and $\sum_{j=1}^{n}\phi(\{g_j\})=1$, which are similar to the weight in the conventional additive operators. Another *interaction index* $I(\{g_j, g_k\})$, taking into account the interactions between criteria g_j and g_k, was proposed by Murofushi and Soneda (1993), shown as:

$$I(\{g_j, g_k\}) = \sum_{A \subseteq G \setminus \{g_j, g_k\}} \frac{(n-|A|-2)!|A|!}{(n-1)!} (\mu(A \cup \{g_j, g_k\} - \mu(A \cup \{g_j\} - \mu(A \cup \{g_k\} + \mu(A)) \tag{32}$$

where $I(\{g_j, g_k\}) \in [-1,1]$, indicating the positive (resp. negative) interaction if $I(\{g_j, g_k\}) \geq 0$ (resp. $I(\{g_j, g_k\}) \leq 0$). Furthermore, the interaction index can be rewritten with the aid of the Möbius transform:

$$I(\{g_j, g_k\}) = \sum_{T \subseteq G \setminus \{g_j, g_k\}}, \frac{1}{|T|+1} m(\{g_j, g_k\} \cup T) \tag{33}$$

Using the Möbius representation of a fuzzy measure, we present an extended Choquet integral as:

$$C_\mu(f(x_1), \ldots, f(x_n)) = \sum_{T \subseteq C} m(T) \min_{c_j \in T} f(x_j) \tag{34}$$

where $f(x_j)$ is a utility function under criterion c_j. The new representation of Choquet integral is based on the Möbius transform presented in Eq. (30). Although it is more difficult to solve the Choquet integral by Eq. (34) than by Eq. (8), the reconstructed aggregation operator provides an opportunity to learn the parameter $m(\cdot)$ according to the preference information of decision-makers. This preference learning problem has been discussed in Aggarwal and Fallah Tehrani (2019). The following example illustrates the use of the transformed Choquet integral.

For a fair comparison, we use the example presented in Section 5.1. In previous example, we used the $\lambda-$fuzzy measure with $\lambda = -0.542$ given that there exists negative interaction between criteria. The extended Choquet integral can make use of the specific interaction index $I(\cdot)$ instead of using an average interaction index λ to compute the aggregated score. The parameter $m(\cdot)$ is suitable for this purpose. The gained dominance scores obtained by the extended Choquet integral for three different interaction indices of $I(\cdot)$ are respectively illustrated in panels B-D of Figure 3 in which the x axis denotes the four alternatives used in Section 5 and the y axis denotes gained dominance scores DS_1. It is clear that the overall scores or ranks of the four alternatives roughly maintain unchanged though the transformed fuzzy measures using different interaction indices. This is because, giving a set of fuzzy measure values of each criterion ($\mu = \{0.4, 0.2, 0.3, 0.4\}$), the criteria interactions are certainly negative among some criteria and then the aggregated values would not change a lot using the Choquet integral based on the negative interactions. In this sense, we conclude that the $\lambda-$fuzzy measure provides a good generality to compute the Choquet integral, and the $\lambda-$fuzzy measure-based Choquet integral is convenient to aggregate the utility values if each fuzzy measure value is known.

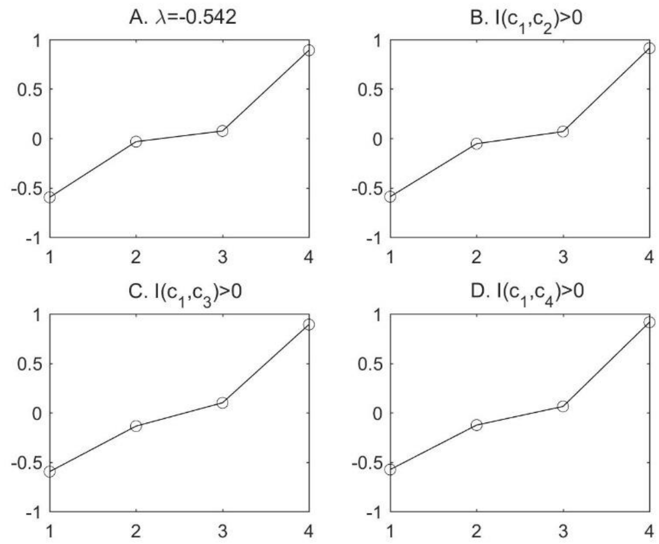

Fig. 3. Gained dominance scores using the Choquet integral based on different fuzzy measures

7. Conclusions and future research directions

The higher business education evaluation is an important issue for the development of business schools. In this regard, developing a decision-making method to conduct a higher business education evaluation is vital to promote the competitiveness of it. In general, this issue can be deemed as an MCGDM problem, which has been investigated by many scholars. But most of the MCGDM methods did not consider the inter-dependency among decision criteria. Furthermore, experts may be more sensitive to losses than gains in real world. Given that the GLDS method (Wu & Liao, 2019) is a robust and efficient group decision-making technique with an advantage of avoiding selecting an alternative performing too bad under some criteria, this study established a group decision-making model named the Choquet integral-based HF-GLDS method to solve the MCGDM problems in which the evaluation information is given as HFSs, without involving the aforementioned flaws. For this aim, we first defined a compromise value function and a comprehensive distance measure between HFSs considering the relative importance between PIS and NIS. Next, we developed a Choquet integral-based HF-GLDS method to derive the final ranking of alternatives. Finally, the proposed method was applied to a higher business education evaluation example to demonstrate the applicability of the proposed method, and the sensitivity and comparative analysis were completed to verify the validity of the proposed method. According to the results, we can come to a conclusion that the proposed method is more efficient and flexible than the HF-TOPSIS method.

In the future, it would be interesting to investigate the MCGDM problems in social network contexts by the proposed method, and further take into account the interactive characteristic between experts. It is worth developing a consensus reaching method within the framework of the proposed Choquet integral-based HF-GLDS method.

Acknowledgments

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Appendix A.1 The proof of Theorem 1.

Proof. Suppose that $h_1 = \{\gamma_1, \gamma_2, \cdots, \gamma_{l-1}, \gamma_l\}$ is an HFE involved l possible values and $h_2 = \{\gamma_1, \gamma_2, \cdots, \gamma_{l-1,} \gamma_l'\}$ is another HFE involved l possible values where γ_l' is a possible value greater than γ_l. Based on the definition of

HFS, we have $h_1 \le h_2$. Since $h_1 \le h_2$, according to the property of deviation, we have $HD(h_1) < HD(h_2)$. Then, substituting h_1 and h_2 into Eq. (13), we have

$$d(h_1,\tilde{1}) = \left(\frac{1}{l}\sum_{t=1}^{l}(1-\gamma_t)^{(1-HD(h_1))}\right)^{1/(1-HD(h_1))} = \left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))} + (1-\gamma_l)^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))},$$

$$d(h_2,\tilde{1}) = \left(\frac{1}{l}\sum_{t=1}^{l}(1-\gamma_t)^{(1-HD(h_2))}\right)^{1/(1-HD(h_2))} = \left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_2))} + (1-\gamma_l')^{(1-HD(h_2))}\right)\right)^{1/(1-HD(h_2))}.$$

Since $\gamma_l < \gamma_l'$, we have $1-\gamma_l > 1-\gamma_l'$. That is,

$$(1-\gamma_l)^{(1-HD(h_1))} > (1-\gamma_l')^{(1-HD(h_2))},$$

$$\left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))} + (1-\gamma_l)^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))} > \left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))} + (1-\gamma_l')^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))}.$$

Since $d(h,\tilde{1})$ is a monotonically non-increasing function with respect to $HD(h)$, we have

$$\left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))} + (1-\gamma_l)^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))} > \left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))} + (1-\gamma_l')^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))}$$
$$> \left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_2))} + (1-\gamma_l')^{(1-HD(h_2))}\right)\right)^{1/(1-HD(h_2))}$$

That is, $d(h_1,\tilde{1}) > d(h_2,\tilde{1})$.

It is not difficult to prove that $d(h_1,\tilde{1}) = d(h_2,\tilde{1})$ iff $h_1 = h_2$, where $d(h,\tilde{1})$ is a non-increasing function and $d(h,\tilde{0})$ is a non-decreasing function. It is clear that $CV(h)$ is a non-decreasing function. This completes the proof of Theorem 1.

Appendix A.2 The proof of Theorem 2.

Proof. First, we prove the first conclusion. Suppose that $h_1 = \{\gamma_1, \gamma_2, \cdots, \gamma_{l-1}, \gamma_l\}$ is an HFE involving l possible values and $h_2 = \{\gamma_1, \gamma_2, \cdots, \gamma_{l-1,}\gamma_l'\}$ is another HFE involving l possible values, and they have $l-1$ same possible values. Since $CV(h_1) < CV(h_2)$, we have $\omega d(h_1,\tilde{0}) - (1-\omega)d(h_1,\tilde{1}) < \omega d(h_2,\tilde{0}) - (1-\omega)d(h_2,\tilde{1})$, $d(h_1,\tilde{0}) - d(h_2,\tilde{0}) < \frac{(1-\omega)}{\omega}\left(d(h_1,\tilde{1}) - d(h_2,\tilde{1})\right)$. According to the presupposition, the two HFEs have only one different possible value. Then, we have $d(h_1,\tilde{1}) - d(h_2,\tilde{1}) > 0$. That is

$$\left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))}+(1-\gamma_l)^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))}>\left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_2))}+\left(1-\gamma_l'\right)^{(1-HD(h_2))}\right)\right)^{1/(1-HD(h_2))}$$

$$\geq\left(\frac{1}{l}\left(\sum_{t=1}^{l-1}(1-\gamma_t)^{(1-HD(h_1))}+\left(1-\gamma_l'\right)^{(1-HD(h_1))}\right)\right)^{1/(1-HD(h_1))}$$

It follows $(1-\gamma_l)^{(1-HD(h_1))}>\left(1-\gamma_l'\right)^{(1-HD(h_1))}$. Thus, we have $\gamma_l<\gamma_l'$ and then $h_1\prec h_2$. It is not difficult to prove that the second conclusion is also satisfied. Then, we only need to prove the third conclusion. We shall prove that if $CV(h_1)=CV(h_2)$, then $h_1\sim h_2$. It is clear that $\frac{d_1^- - d_2^-}{d_1^+ - d_2^+}=\frac{1-\omega}{\omega}\geq 0$ if $CV(h_1)=CV(h_2)$. We continue to prove this theorem by contradiction.

Suppose that there exists two different HFEs h_1 and h_2 such that $CV(h_1)=CV(h_2)$. This implies that for $h_1\prec h_2$ or $h_1\succ h_2$, there exists at least one pair of different HFEs h_1 and h_2 such that $CV(h_1)=CV(h_2)$. Let $h_1\prec h_2$. By Definition 1, the distance function values satisfy $d_1^->d_2^-$ and $d_1^+<d_2^+$. Then, we have $\frac{d_1^- - d_2^-}{d_1^+ - d_2^+}<0$. This inequality contradicts the assumption that $(d_1^- - d_2^-)/(d_1^+ - d_2^+)\geq 0$. Moreover, it is not difficult to prove that $(d_1^- - d_2^-)/(d_1^+ - d_2^+)<0$ for $h_1\succ h_2$. This completes the proof of Theorem 2.

Appendix A.3 The proof of Theorem 3.

Proof. To prove the order preference relation "$\prec$" is a partial order, we prove the three properties in set theory including reflexivity, anti-symmetricity and transitivity.

(1) **Reflexivity.** For an HFE $h\in H$, we have $CV(h)\leq CV(h)$. Based on Eq. (15), it is easy to find that the compromise value function is monotonically increasing with h. Thus, we have $h\preceq h$.

(2) **Anti-symmetricity.** For any two HFEs $h_1,h_2\in H$, if $h_1\preceq h_2$ and $h_2\preceq h_1$, then we have $CV(h_1)\leq CV(h_2)$ and $CV(h_2)\leq CV(h_1)$ according to Theorem 1. Thus, $CV(h_1)=CV(h_2)$. According to the monotonicity of the compromise value function, we can obtain that $h_1=h_2$.

(3) **Transitivity.** For all HFEs $h_1,h_2,h_3\in H$, if $h_1\preceq h_2$ and $h_2\preceq h_3$, then, according to Definition 1, we have $CV(h_1)\leq CV(h_2)$ and $CV(h_2)\leq CV(h_3)$. Because the compromise value $CV(h)$ is a set of real numbers, we obtain $CV(h_1)\leq CV(h_3)$. Based on the monotonic function in Eq. (15), we can derive that $h_1\preceq h_3$.