

The Picture Fuzzy Distance Measure in Controlling Network Power Consumption

Ngan Thi Roan^{1*}, Salvador Coll Arnau², Marina Alonso Díaz², Juan Miguel Martínez Rubio², Pedro López², Fran Andujar³, Son Hoang Le⁴, Manh Van Vu¹, Felorentin Smarandache⁵

¹VNU University of Science, Vietnam National University, Hanoi, Vietnam.

²UPV Universitat Politècnica de València, Spain.

³Universidad de Valladolid, Spain.

⁴VNU Information Technology Institute, Vietnam National University, Hanoi, Vietnam.

⁵University of New Mexico, Gallup Campus, USA.

PAPER INFO	ABSTRACT
<p>Chronicle: Received: 20 March 2020 Revised: 28 May 2020 Accepted: 30 July 2020</p>	<p>In order to solve the complex decision-making problems, there are many approaches and systems based on the fuzzy theory were proposed. In 1998, Smarandache [10] introduced the concept of single-valued neutrosophic set as a complete development of fuzzy theory. In this paper, we research on the distance measure between single-valued neutrosophic sets based on the H-max measure of Ngan et al. [8]. The proposed measure is also a distance measure between picture fuzzy sets which was introduced by Cuong in 2013 [15]. Based on the proposed measure, an Adaptive Neuro Picture Fuzzy Inference System (ANPFIS) is built and applied to the decision making for the link states in interconnection networks. In experimental evaluation on the real datasets taken from the UPV (Universitat Politècnica de València) university, the performance of the proposed model is better than that of the related fuzzy methods.</p>
<p>Keywords: Neutrosophic Set. Picture Fuzzy Set. Distance Measure. Decision Making. Interconnection Network. Power Consumption.</p>	

1. Introduction

The fuzzy theory was introduced the first time in 1965 by Zadeh [1]. A fuzzy set is determined by a membership function limited to $[0, 1]$. Until now, there is a giant research construction of fuzzy theory as well as its application. The fuzzy set is used in pattern recognition, artificial intelligent, decision making, or

* Corresponding author

E-mail address: roanngan@gmail.com

DOI: 10.22105/jfea.2020.249183.1009

data mining [2, 3], and so on. Besides that, the expansion of fuzzy theory is also an interesting topic. The interval-valued fuzzy set [4], the type-2 fuzzy set [5], and the intuitionistic fuzzy set [6] are all developed from the fuzzy set. They replaced the value type or added the other evaluation to the fuzzy set in order to overcome the inadequate simple approach of this traditional fuzzy set. Such as in 1986, the intuitionistic fuzzy set of Atanassov [6] builds up the concept of the non-membership degree. This supplement gives more accurate results in pattern recognition, medical diagnosis and decision making [7-9], and so on. In 1998, Smarandache [10] introduced neutrosophic set to generalize intuitionistic fuzzy set by three independent components. Until today, many subclasses of neutrosophic sets were studied such as complex neutrosophic sets [11, 12]. As a particular case of standard neutrosophic sets [13, 14], the picture fuzzy set introduced in 2013 by Cuong [15], considered as a complete development of the fuzzy theory, allows an element to belong to it with three corresponding degrees where all of these degrees and their sum are limited to $[0, 1]$. Concerning extended fuzzy set, some recent publications may be mentioned here as in [16-20].

As one of the important pieces of set theory, distance measure between the sets is a tool for evaluating different or similar levels between them. Some literature on the application of intuitionistic fuzzy measure from 2012 to present can be found in [7, 21-23]. In 2018, Wei introduced the generalized Dice similarity measures for picture fuzzy sets [17]. However, the definition of Wei is without considering the condition related to order relation on picture fuzzy sets. In a decision-making model, a distance measure can be used to compare the similarities between the sets of attributes of the samples and that of the input, such as in predicting dental diseases from images [24]. In this paper, we define the concept of the single-valued neutrosophic distance measure, picture fuzzy distance measure, and represent the specific measure formula. We prove the characteristics of this formula as well as the relation among it and some of the other operators of picture fuzzy sets. The proposed distance measure is inspired by the H-max distance measure of intuitionistic fuzzy sets [8]. Hence, it inherits the advantage of the cross-evaluation in the H-max and moreover it has the completeness of picture fuzzy environment.

The decision-making problems appear in most areas aiming to provide the optimal solution. Saving interconnection network power is always interested, researched and becoming more and more urgent in the current technological era. In 2010, Alonso et al. introduced the power saving mechanism in regular interconnection network [25]. This decision-making model dynamically increases or reduces the number of links based on a thresholds policy. In 2015, they continue to study power consumption control in fat-tree interconnection networks based on the static and dynamic thresholds policies [26]. In general, these threshold policies are rough and hard because they are without any fuzzy approaches, parameter learning and optimizing processes. In 2017, Phan et al. [27] proposed a new method in power consumption estimation of network-on-chip based on fuzzy logic. However, this fuzzy logic system based on Sugeno model [27] is too rudimentary and the parameters here are chosen according to the authors' quantification.

In this paper, aiming to replace the above threshold policy, an Adaptive Neuro Picture Fuzzy Inference System (ANPFIS) based on picture fuzzy distance measure is proposed to make the decisions for the link states in interconnection networks. ANPFIS is a modification and combination between Adaptive Neuro Fuzzy Inference System (ANFIS) [28-30], picture fuzzy set, and picture fuzzy distance measure. Hence, ANPFIS operates based on the picture fuzzification and defuzzification processes, the picture fuzzy operators [18] and distance measure, and the learning capability for automatic picture fuzzy rule generation and

parameter optimization. In order to evaluate performance, we tested the ANPFIS method on the real datasets of the network traffic history taken from the UPV (Universitat Politècnica de València) university with related methods. The result is that ANPFIS is the most effective algorithm.

The rest of the paper is organized as follows. Section 2 provides some fundamental concepts of the fuzzy, intuitionistic fuzzy, single-valued neutrosophic, and picture fuzzy theories. Section 3 proposes the distance measure of single-valued neutrosophic sets and points out its important properties. Section 4 shows the new decision-making method named Adaptive Neuro Picture Fuzzy Inference System (ANPFIS) and an application of ANPFIS to controlling network power consumption. Section 5 shows the experimental results of ANPFIS and the related methods on real-world datasets. Finally, conclusion is given in Section 6.

2. Preliminary

In this part, some concepts of the theories of fuzzy sets, intuitionistic fuzzy sets, single-valued neutrosophic sets, and picture fuzzy sets are showed.

Let X be a space of points.

Definition 1. [1]. A fuzzy set (FS) A in X ,

$$A = \{ (x : \mu_A(x)) \mid x \in X \}, \tag{1}$$

is characterized by a membership function, μ_A , with a range in $[0,1]$.

Definition 2. [6]. A intuitionistic fuzzy set (IFS) A in X ,

$$A = \{ (x : \mu_A(x), \nu_A(x)) \mid x \in X \}, \tag{2}$$

is characterized by a membership function μ_A and a non-membership function ν_A with a range in $[0,1]$ such that $0 \leq \mu_A + \nu_A \leq 1$.

Definition 3. [31]. A Single-Valued Neutrosophic Set (SVNS) A in X ,

$$A = \{ (x : T_A(x), I_A(x), F_A(x)) \mid x \in X \}, \tag{3}$$

is characterized by a truth-membership function T_A , an indeterminacy-membership function I_A , and a false-nonmembership function F_A with a range in $[0,1]$ such that $0 \leq T_A + I_A + F_A \leq 3$.

Definition 4. [15]. A Picture Fuzzy Set (PFS) A in X ,

$$A = \{ (x : \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X \}, \tag{4}$$

is characterized by a positive membership function μ_A , a neutral function η_A , and a negative membership function ν_A with a range in $[0,1]$ such that $0 \leq \mu_A + \eta_A + \nu_A \leq 1$.

We denote that $SVNS(X)$ is the set of all SVNSs in X and $PFS(X)$ is the set of all PFSs in X . We consider the sets N^* and P^* defined by

$$N^* = \{x = (x_1, x_2, x_3) \mid 0 \leq x_1, x_2, x_3 \leq 1\}, \quad (5)$$

$$P^* = \{x = (x_1, x_2, x_3) \mid 0 \leq x_1, x_2, x_3, x_1 + x_2 + x_3 \leq 1\}. \quad (6)$$

Definition 5. The orders on N^* and P^* are defined as follows

- $x \leq y \Leftrightarrow (x_1 < y_1, x_3 \geq y_3) \vee (x_1 = y_1, x_3 > y_3) \vee (x_1 = y_1, x_3 = y_3, x_2 \leq y_2), \forall x, y \in P^*, [19].$
- $x \sqsubseteq y \Leftrightarrow x_1 \leq y_1, x_2 \leq y_2, x_3 \geq y_3, \forall x, y \in N^*.$

Clearly, on P^* , if $x \sqsubseteq y$ then $x \leq y$.

Remark 1. The lattice (P^*, \leq) is a complete lattice [19] but (P^*, \sqsubseteq) is not. For example, let $x = (0.2, 0.3, 0.5)$ and $y = (0.3, 0, 0.7)$, then there is not any supremum value of x and y on (P^*, \sqsubseteq) .

Otherwise, we have $\sup(x, y) = (0.3, 0, 0.5)$ on the lattice (P^*, \leq) . We denote the units of (P^*, \leq) as follows $0_{P^*} = (0, 0, 1)$ and $1_{P^*} = (1, 0, 0)$ [19]. It is easy to see that 0_{P^*} and 1_{P^*} are also the units on (P^*, \sqsubseteq) . Now, some logic operators on $PFS(X)$ are presented.

Definition 6. [19]. A picture fuzzy negation N is a function satisfying

$$N : P^* \rightarrow P^*, N(0_{P^*}) = 1_{P^*}, N(1_{P^*}) = 0_{P^*}, \text{ and } N(x) \geq N(y) \Leftrightarrow x \leq y.$$

Example 1. For every $x \in P^*$, then $N_0(x) = (x_3, 0, x_1)$ and $N_s(x) = (x_3, x_4, x_1)$ are picture fuzzy negations, where $x_4 = 1 - x_1 - x_2 - x_3$.

Remark 2. The operator N_0 also satisfies $N_0(x) \sqsubseteq N_0(y) \Leftrightarrow x \sqsubseteq y, \forall x, y \in P^*.$

Now, let $x, y, z \in P^*$ and $I(x) = \{y \in P^* : y = (x_1, y_2, x_3), 0 \leq y_2 \leq x_2\}.$

Definition 7. [19] A picture fuzzy t-norm T is a function satisfying

$$T : P^* \times P^* \rightarrow P^*, T(x, y) = T(y, x), T(T(x, y), z) = T(x, T(y, z)),$$

$$T(1_{P^*}, x) \in I(x), \text{ and } T(x, y) \leq T(x, z), \forall y \leq z.$$

Definition 8. [19] A picture fuzzy t-conorm S is a function satisfying

$$S : P^* \times P^* \rightarrow P^*, S(x, y) = S(y, x), S(S(x, y), z) = S(x, S(y, z)),$$

$$S(0_{P^*}, x) \in I(x), \text{ and } S(x, y) \leq S(x, z), \forall y \leq z.$$

Example 2. For all $x, y \in P^*$, the following operators are the picture fuzzy t-norms:

- $T_0(x, y) = (\min(x_1, y_1), \min(x_2, y_2), \max(x_3, y_3)).$
- $T_1(x, y) = (x_1 y_1, x_2 y_2, x_3 + y_3 - x_3 y_3).$
- $T_2(x, y) = (\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), \min(1, x_3 + y_3)).$
- $T_3(x, y) = (\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), x_3 + y_3 - x_3 y_3).$
- $T_4(x, y) = (x_1 y_1, \max(0, x_2 + y_2 - 1), x_3 + y_3 - x_3 y_3).$
- $T_5(x, y) = (\max(0, x_1 + y_1 - 1), x_2 y_2, x_3 + y_3 - x_3 y_3).$

Example 3. For all $x, y \in P^*$, the following operators are the picture fuzzy t-conorms:

- $S_0(x, y) = (\max(x_1, y_1), \max(0, x_2 + y_2 - 1), \min(x_3, y_3)).$
- $S_1(x, y) = (x_1 + y_1 - x_1 y_1, x_2 y_2, x_3 y_3).$
- $S_2(x, y) = (\min(1, x_1 + y_1), \max(0, x_2 + y_2 - 1), \max(0, x_3 + y_3 - 1)).$
- $S_3(x, y) = (x_1 + y_1 - x_1 y_1, \max(0, x_2 + y_2 - 1), \max(0, x_3 + y_3 - 1)).$
- $S_4(x, y) = (x_1 + y_1 - x_1 y_1, \max(0, x_2 + y_2 - 1), x_3 y_3).$
- $S_5(x, y) = (x_1 + y_1 - x_1 y_1, x_2 y_2, \max(0, x_3 + y_3 - 1)).$

Remark 3. For all $x, y, z \in P^*$ and $y \sqsubseteq z$, the operators $T_{i(i=0, \dots, 5)}$ also satisfy the condition $T(x, y) \sqsubseteq T(x, z)$.

Similarly, $S_{i(i=0, \dots, 5)}$ also satisfy $S(x, y) \sqsubseteq S(x, z)$.

The logic operators N, T and S on P^* are corresponding to the basic set-theory operators on $PFS(X)$ as follows.

Definition 9. Let N, T and S be the picture fuzzy negation, t-norm and t-conorm, respectively, and $A, B \in PFS(X)$. Then, the complement of A w.r.t N is defined as follows:

$$\bar{A}^N = \left\{ \left\langle x : N\left(\left(\mu_A(x), \eta_A(x), \nu_A(x)\right)\right) \right\rangle \mid x \in X \right\}, \tag{7}$$

the intersection of A and B w.r.t T is defined as follows:

$$A \cap_T B = \left\{ \left\langle x : T\left(\left(\mu_A(x), \eta_A(x), \nu_A(x)\right), \left(\mu_B(x), \eta_B(x), \nu_B(x)\right)\right) \right\rangle \mid x \in X \right\}, \tag{8}$$

and the union of A and B w.r.t T is defined as follows:

$$A \cup_S B = \left\{ \left(x : S \left((\mu_A(x), \eta_A(x), \nu_A(x)), (\mu_B(x), \eta_B(x), \nu_B(x)) \right) \right) \mid x \in X \right\}. \quad (9)$$

3. The Single-valued Neutrosophic Distance Measure and the Picture Fuzzy Distance Measure

Recently, Wei has introduced the generalized Dice similarity measures for picture fuzzy sets [17]. However, the definition of Wei is without considering the condition related to order relation on picture fuzzy sets. The new distance measure on picture fuzzy sets is proposed in this section. It is developed from intuitionistic distance measure of Wang et al. [32] and Ngan et al. [8].

Definition 10. A single-valued neutrosophic distance measure d is a function satisfying

- $d : N^* \times N^* \rightarrow [0, +\infty)$,
- $d(x, y) = d(y, x)$,
- $d(x, y) = 0 \Leftrightarrow x = y$,
- If $x \sqsubseteq y \sqsubseteq z$ then $d(x, y) \leq d(x, z)$ and $d(y, z) \leq d(x, z)$.

Definition 11. A picture fuzzy distance measure d is a single-valued neutrosophic distance measure and $d(x, y) \in [0, 1], \forall x, y \in P^*$.

Definition 12. The measure D_0 is defined as follows

$$D_0(x, y) = \frac{1}{3} \left(|x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3| + \left| \max\{x_1, y_3\} - \max\{x_3, y_1\} \right| \right), \forall x, y \in P^*. \quad (10)$$

Proposition 1. The measure D_0 is a picture fuzzy distance measure.

Proof. Firstly, we have $\left| \max\{x_1, y_3\} - \max\{x_3, y_1\} \right| \in [0, 1]$ and

$$|x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3| \leq (|x_1| + |y_1|) + (|x_2| + |y_2|) + (|x_3| + |y_3|) \leq (x_1 + x_2 + x_3) + (y_1 + y_2 + y_3) \leq 2.$$

Therefore, $0 \leq \frac{1}{3} \left(|x_1 - y_1| + |x_2 - y_2| + |x_3 - y_3| + \left| \max\{x_1, y_3\} - \max\{x_3, y_1\} \right| \right) \leq 1$.

Secondly, we obtain that $D_0(x, y) = D_0(y, x)$ since D_0 has the symmetry property between the arguments.

Thirdly, $D_0(x, y) = 0 \Leftrightarrow |x_1 - y_1| = |x_2 - y_2| = |x_3 - y_3| = \left| \max\{x_1, y_3\} - \max\{x_3, y_1\} \right| = 0 \Leftrightarrow x = y$.

Finally, let $x \sqsubseteq y \sqsubseteq z$, then $x_1 \leq y_1 \leq z_1, x_2 \leq y_2 \leq z_2, x_3 \geq y_3 \geq z_3$. We obtain that

$$|x_1 - y_1| \leq |x_1 - z_1|, |x_2 - y_2| \leq |x_2 - z_2|, |x_3 - y_3| \leq |x_3 - z_3|.$$

Moreover, $\max\{z_1, x_3\} \geq \max\{y_1, x_3\} \geq \max\{x_1, y_3\} \geq \max\{x_1, z_3\}$. Hence,

$|\max\{x_1, y_3\} - \max\{x_3, y_1\}| \leq |\max\{x_1, z_3\} - \max\{x_3, z_1\}| \cdot \lim_{x \rightarrow \infty}$. Thus, $D_0(x, y) \leq D_0(x, z)$. Similarly, we also have $D_0(y, z) \leq D_0(x, z)$.

Remark 4. If d is a picture fuzzy distance measure, then d is a single-valued neutrosophic distance measure. The opposite is not necessarily true. Some picture fuzzy operations were introduced by the group of authors of this paper [18, 19]. Hence, this research is seen as a complete link to the authors' previous work on picture fuzzy inference systems. An inference system of neutrosophic theory will be developed in another paper as a future work.

Proposition 2. Let $x, y \in P^*$. The measure D_0 satisfies the following properties:

- $D_0(N_0(x), N_S(x)) = \frac{1}{3}x_4$.
- If $x_2 \geq x_4$, then $|D_0(x, N_0(x)) - D_0(x, N_S(x))| = \frac{1}{3}x_4$.
- $|D_0(x, N_0(y)) - D_0(N_0(x), y)| = \frac{1}{3}|x_2 - y_2|$.
- $|D_0(x, y) - D_0(N_0(x), N_0(y))| = \frac{1}{3}|x_2 - y_2|$.
- If $x_1 + x_3 = y_1 + y_3$, then $D_0(x, y) = D_0(N_S(x), N_S(y))$.
- If $x_1 + x_3 = y_1 + y_3$, then $D_0(x, N_S(y)) = D_0(N_S(x), y)$.
- $D_0(x, N_0(x)) = |x_1 - x_3| + \frac{1}{3}x_2$.
- $D_0(x, N_S(x)) = |x_1 - x_3| + \frac{1}{3}|x_2 - x_4|$.
- $D_0(x, I_{P^*}) = \frac{1}{3}(2 - 2x_1 + x_2 + x_3)$.
- $D_0(x, \theta_{P^*}) = \frac{1}{3}(2 - 2x_3 + x_1 + x_2)$.
- $D_0(\theta_{P^*}, I_{P^*}) = 1$.
- $D_0(x, N_0(x)) = 1$ if and only if $x \in \{\theta_{P^*}, I_{P^*}\}$.
- $D_0(x, N_S(x)) = 1$ if and only if $x \in \{\theta_{P^*}, I_{P^*}\}$.
- $D_0(x, N_0(x)) = 0$ if and only if $x_1 = x_3, x_2 = 0$.
- $D_0(x, N_S(x)) = 0$ if and only if $x_1 = x_3, x_2 = x_4$.
- $D_0((0, 0, 0), (0, a, 0)) < D_0((0, 0, 0), (a, 0, 0)) = D_0((0, 0, 0), (0, 0, a))$
 $< D_0((a, 0, 0), (0, 0, a)) = D_0((a, 0, 0), (0, a, 0)) = D_0((0, 0, a), (0, a, 0)), \forall a \in (0, 1]$.

Proof. These properties are proved as follows:

– We have $D_0(N_0(x), N_S(x)) = D_0((x_3, 0, x_l), (x_3, x_4, x_l))$

$$= \frac{1}{3}(|x_3 - x_3| + |0 - x_4| + |x_l - x_l| + |\max\{x_3, x_l\} - \max\{x_3, x_l\}|) = \frac{1}{3}x_4.$$

– We have

$$|D_0(x, N_0(x)) - D_0(x, N_S(x))| = |D_0((x_1, x_2, x_3), (x_3, 0, x_l)) - D_0((x_1, x_2, x_3), (x_3, x_4, x_l))|$$

$$= \left| \frac{1}{3}(|x_1 - x_3| + |x_2 - 0| + |x_3 - x_l| + |\max\{x_1, x_l\} - \max\{x_3, x_3\}|) \right.$$

$$\left. - \frac{1}{3}(|x_1 - x_3| + |x_2 - x_4| + |x_3 - x_l| + |\max\{x_1, x_l\} - \max\{x_3, x_3\}|) \right|$$

$$= \frac{1}{3}|x_2 - |x_2 - x_4|| = \frac{1}{3}x_4.$$

– We have

$$|D_0(x, N_0(y)) - D_0(N_0(x), y)| = |D_0((x_1, x_2, x_3), (y_3, 0, y_l)) - D_0((x_3, 0, x_l), (y_1, y_2, y_3))|$$

$$= \left| \frac{1}{3}(|x_1 - y_3| + |x_2 - 0| + |x_3 - y_l| + |\max\{x_1, y_l\} - \max\{y_3, x_3\}|) \right.$$

$$\left. - \frac{1}{3}(|x_3 - y_l| + |0 - y_2| + |x_l - y_3| + |\max\{x_3, y_3\} - \max\{y_l, x_l\}|) \right| = \frac{1}{3}|x_2 - y_2|.$$

– We have

$$|D_0(x, y) - D_0(N_0(x), N_0(y))| = |D_0((x_1, x_2, x_3), (y_1, y_2, y_3)) - D_0((x_3, 0, x_l), (y_3, 0, y_l))|$$

$$= \left| \frac{1}{3}(|x_1 - y_l| + |x_2 - y_2| + |x_3 - y_3| + |\max\{x_1, y_3\} - \max\{x_3, y_l\}|) \right.$$

$$\left. - \frac{1}{3}(|x_3 - y_3| + |0 - 0| + |x_l - y_l| + |\max\{x_3, y_l\} - \max\{y_3, x_l\}|) \right| = \frac{1}{3}|x_2 - y_2|.$$

– We have $D_0(N_S(x), N_S(y)) = D_0((x_3, x_4, x_l), (y_3, y_4, y_l))$

$$= \frac{1}{3}(|x_3 - y_3| + |x_4 - y_4| + |x_l - y_l| + |\max\{x_3, y_l\} - \max\{y_3, x_l\}|). \text{ Further,}$$

$$|x_4 - y_4| = |(I - x_1 - x_2 - x_3) - (I - y_1 - y_2 - y_3)| = |x_2 - y_2|. \text{ Thus, } D_0(N_S(x), N_S(y)) = D_0(x, y).$$

- We have $D_0(x, N_S(y)) = D_0((x_1, x_2, x_3), (y_3, y_4, y_1))$

$$= \frac{I}{3} (|x_1 - y_3| + |x_2 - y_4| + |x_3 - y_1| + |\max\{x_1, y_1\} - \max\{y_3, x_3\}|)$$

In other hand,

$$D_0(N_S(x), y) = D_0((x_3, x_4, x_1), (y_1, y_2, y_3))$$

$$= \frac{I}{3} (|x_3 - y_1| + |x_4 - y_2| + |x_1 - y_3| + |\max\{x_3, y_3\} - \max\{y_1, x_1\}|)$$

Further,

$$|x_2 - y_4| = |x_2 - I + y_1 + y_2 + y_3| = |x_2 - I + x_1 + y_2 + x_3| = |y_2 - x_4|$$

Thus,

$$D_0(x, N_S(y)) = D_0(N_S(x), y) \lim_{x \rightarrow \infty}$$
- We have $D_0(x, N_0(x)) = D_0((x_1, x_2, x_3), (x_3, 0, x_1))$

$$= \frac{I}{3} (|x_1 - x_3| + |x_2 - 0| + |x_3 - x_1| + |\max\{x_1, x_1\} - \max\{x_3, x_3\}|) = |x_1 - x_3| + \frac{I}{3} x_2$$
- We have
$$D_0(x, N_S(x)) = D_0((x_1, x_2, x_3), (x_3, x_4, x_1))$$

$$= \frac{I}{3} (|x_1 - x_3| + |x_2 - x_4| + |x_3 - x_1| + |\max\{x_1, x_1\} - \max\{x_3, x_3\}|) = |x_1 - x_3| + \frac{I}{3} |x_2 - x_4|$$
- We have $D_0(x, I_{p'}) = D_0((x_1, x_2, x_3), (1, 0, 0))$

$$= \frac{I}{3} (|x_1 - 1| + |x_2 - 0| + |x_3 - 0| + |\max\{x_1, 0\} - \max\{x_3, 1\}|) = \frac{I}{3} (2 - 2x_1 + x_2 + x_3)$$
- We have $D_0(x, 0_{p'}) = D_0((x_1, x_2, x_3), (0, 0, 1))$

$$= \frac{I}{3} (|x_1 - 0| + |x_2 - 0| + |x_3 - 1| + |\max\{x_1, 1\} - \max\{x_3, 0\}|) = \frac{I}{3} (2 - 2x_3 + x_1 + x_2)$$
- We have $D_0(0_{p'}, I_{p'}) = D_0((1, 0, 0), (0, 0, 1)) = I$.

Assume that $D_0(x, N_0(x)) = I$, we have $|x_1 - x_3| + \frac{I}{3} x_2 = I$. Since $|x_1 - x_3| + \frac{I}{3} x_2 \leq (x_1 + x_3) + x_2 \leq I$.

Therefore, $|x_1 - x_3| + \frac{I}{3} x_2 = (x_1 + x_3) + x_2 = I$. We obtain that $x_2 = 0$ and $|x_1 - x_3| = I$. Thus,

$x \in \{0_{p'}, I_{p'}\}$. Assume that $D_0(x, N_S(x)) = I$, we have $|x_1 - x_3| + \frac{I}{3} |x_2 - x_4| = I$. Since

$|x_1 - x_3| + \frac{I}{3} |x_2 - x_4| \leq (x_1 + x_3) + (x_2 + x_4) = I$. We obtain that $x_2 = x_4 = 0$ and $|x_1 - x_3| = I$. Thus,

$x \in \{0_{P'}, I_{P'}\}$. Assume that $D_0(x, N_0(x)) = 0$, we have $|x_l - x_3| + \frac{1}{3}x_2 = 0$. Hence, $x_2 = 0$ and $x_l = x_3$.

Assume that $D_0(x, N_S(x)) = 0$, we have $|x_l - x_3| + \frac{1}{3}|x_2 - x_4| = 0$. Hence, $x_2 = x_4$ and $x_l = x_3$.

– We have $D_0((0, 0, 0), (0, a, 0)) = \frac{a}{3}$,

$$D_0((0, 0, 0), (a, 0, 0)) = D_0((0, 0, 0), (0, 0, a)) = \frac{2a}{3}, \text{ and}$$

$$D_0((a, 0, 0), (0, 0, a)) = D_0((a, 0, 0), (0, a, 0)) = D_0((0, 0, a), (0, a, 0)) = a.$$

Remark 5. The order “ \sqsubseteq ” on P^* corresponds to the following order on $\text{PFS}(X)$:

$$A \sqsubseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \eta_A(x) \leq \eta_B(x), \nu_A(x) \geq \nu_B(x), \forall x \in X. \quad (11)$$

Remark 6. The picture fuzzy distance measure on P^* corresponds to the picture fuzzy distance measure on $\text{PFS}(X)$, i.e., for all $A, B \in \text{PFS}(X = \{x_1, x_2, \dots, x_m\})$, we have the picture fuzzy distance measure D_0 between A and B as follows:

$$D_0(A, B) = \frac{1}{3m} \sum_{i=1}^m \left(|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + \left| \max\{\mu_A(x_i), \nu_B(x_i)\} - \max\{\nu_A(x_i), \mu_B(x_i)\} \right| \right). \quad (12)$$

Proposition 3. Consider the picture fuzzy distance measure D_0 in Eq. (10), the picture fuzzy t-norms $T_{i(i=0, \dots, 5)}$ in Example 2, the picture fuzzy t-conorms $S_{i(i=0, \dots, 5)}$ in Example 3, and the picture fuzzy negation N_0 in Example 1. Let A and B be two picture fuzzy sets on the universe $X = \{x_1, x_2, \dots, x_m\}$. Then, we have the following properties:

– $D_0(A \cap_{T_2} B, B) \geq \max\{D_0(A \cap_{T_i} B, A \cap_{T_j} B), D_0(A \cap_{T_k} B, B)\}$,

$$D_0(A \cap_{T_2} B, A) \geq \max\{D_0(A \cap_{T_i} B, A \cap_{T_j} B), D_0(A \cap_{T_k} B, A)\},$$

$$D_0(\overline{A \cap_{T_2} B}^{N_0}, \overline{A}^{N_0}) \geq \max\{D_0(\overline{A \cap_{T_i} B}^{N_0}, \overline{A \cap_{T_j} B}^{N_0}), D_0(\overline{A \cap_{T_k} B}^{N_0}, \overline{A}^{N_0})\},$$

$$D_0(\overline{A \cap_{T_2} B}^{N_0}, \overline{B}^{N_0}) \geq \max\{D_0(\overline{A \cap_{T_i} B}^{N_0}, \overline{A \cap_{T_j} B}^{N_0}), D_0(\overline{A \cap_{T_k} B}^{N_0}, \overline{B}^{N_0})\},$$

$$\forall (i, j) \in \{(x, y) \mid x, y = 0, \dots, 5\} \setminus \{(4, 5)\} \text{ and } k = 0, 1, 3, 4, 5.$$

$$\begin{aligned}
 - \quad & D_0(A \cup_{S_5} B, A) \geq \max\{D_0(A \cup_{S_5} B, A \cup_{S_j} B), D_0(A \cup_{S_k} B, A)\}, \\
 & D_0(A \cup_{S_5} B, B) \geq \max\{D_0(A \cup_{S_5} B, A \cup_{S_j} B), D_0(A \cup_{S_k} B, B)\}, \\
 & D_0(\overline{A \cup_{S_5} B}^{N_0}, \overline{A}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_5} B}^{N_0}, \overline{A \cup_{S_j} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \overline{A}^{N_0})\}, \\
 & D_0(\overline{A \cup_{S_5} B}^{N_0}, \overline{B}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_5} B}^{N_0}, \overline{A \cup_{S_j} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \overline{B}^{N_0})\},
 \end{aligned}$$

$\forall (i, j) \in \{(x, y) \mid x, y = 0, 1, 3, 4, 5\} \setminus \{(1, 3)\}$ and $k = 0, 1, 3, 4$.

$$\begin{aligned}
 - \quad & D_0(A \cup_{S_2} B, A) \geq \max\{D_0(A \cup_{S_2} B, A \cup_{S_j} B), D_0(A \cup_{S_k} B, A)\}, \\
 & D_0(A \cup_{S_2} B, B) \geq \max\{D_0(A \cup_{S_2} B, A \cup_{S_j} B), D_0(A \cup_{S_k} B, B)\}, \\
 & D_0(\overline{A \cup_{S_2} B}^{N_0}, \overline{A}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_2} B}^{N_0}, \overline{A \cup_{S_j} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \overline{A}^{N_0})\}, \\
 & D_0(\overline{A \cup_{S_2} B}^{N_0}, \overline{B}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_2} B}^{N_0}, \overline{A \cup_{S_j} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \overline{B}^{N_0})\},
 \end{aligned}$$

$\forall i, j = 0, 2, 3, 4$, and $k = 0, 3, 4$.

$$\begin{aligned}
 - \quad & D_0(A \cap_{T_i} B, A \cup_{S_j} B) \geq D_0(A \cap_{T_0} B, A \cup_{S_0} B) \text{ and} \\
 & D_0(\overline{A \cap_{T_i} B}^{N_0}, \overline{A \cup_{S_j} B}^{N_0}) \geq D_0(\overline{A \cap_{T_0} B}^{N_0}, \overline{A \cup_{S_0} B}^{N_0}), \forall i, j = 0, \dots, 5.
 \end{aligned}$$

Proof. These properties are proved as follows. Firstly, we see that for all $x, y \in [0, 1]$,

$$\max(0, x + y - 1) \leq xy \leq \min(x, y) \quad \text{and} \quad \min(1, x + y) \geq x + y - xy \geq \max(0, x + y - 1). \text{ Hence,}$$

$$(\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), \min(1, x_3 + y_3)) \sqsupseteq (x_1 y_1, x_2 y_2, x_3 + y_3 - x_3 y_3). \text{ This means } T_2 \sqsupseteq T_1.$$

Similarly, we obtain that $T_2 \sqsupseteq T_3 \sqsupseteq T_4 \sqsupseteq T_1 \sqsupseteq T_0$ and $T_2 \sqsupseteq T_3 \sqsupseteq T_5 \sqsupseteq T_1 \sqsupseteq T_0$. Hence,

$$A \cap_{T_2} B \subseteq A \cap_{T_3} B \subseteq A \cap_{T_4} B \subseteq A \cap_{T_1} B \subseteq A \cap_{T_0} B \subseteq A,$$

$$A \cap_{T_2} B \subseteq A \cap_{T_3} B \subseteq A \cap_{T_5} B \subseteq A \cap_{T_1} B \subseteq A \cap_{T_0} B \subseteq A,$$

$$A \cap_{T_2} B \subseteq A \cap_{T_3} B \subseteq A \cap_{T_4} B \subseteq A \cap_{T_1} B \subseteq A \cap_{T_0} B \subseteq B, \text{ and}$$

$$A \cap_{T_2} B \subseteq A \cap_{T_3} B \subseteq A \cap_{T_5} B \subseteq A \cap_{T_1} B \subseteq A \cap_{T_0} B \subseteq B.$$

Since D_0 is the picture fuzzy distance measure, thus

$$D_0(A \cap_{T_2} B, A) \geq \max\{D_0(A \cap_{T_1} B, A \cap_{T_1} B), D_0(A \cap_{T_k} B, A)\} \text{ and}$$

$$D_0(A \cap_{T_2} B, B) \geq \max\{D_0(A \cap_{T_1} B, A \cap_{T_1} B), D_0(A \cap_{T_k} B, B)\},$$

$\forall (i, j) \in \{(x, y) \mid x, y = 0, \dots, 5\} \setminus \{(4, 5)\}$ and $k = 0, 1, 3, 4, 5$. Furthermore, we have

$$\bar{A}^{N_0} = \left\{ (x : N_0((\mu_A(x), \eta_A(x), \nu_A(x)))) \mid x \in X \right\} = \left\{ (x : (\nu_A(x), 0, \mu_A(x))) \mid x \in X \right\}, \otimes.$$

It is easy to prove the following lemma: If $A \subseteq B$, then $\bar{B}^{N_0} \subseteq \bar{A}^{N_0}$. Thus,

$$D_0(\overline{A \cap_{T_2} B}^{N_0}, \bar{A}^{N_0}) \geq \max\{D_0(\overline{A \cap_{T_1} B}^{N_0}, \overline{A \cap_{T_1} B}^{N_0}), D_0(\overline{A \cap_{T_k} B}^{N_0}, \bar{A}^{N_0})\} \text{ and}$$

$$D_0(\overline{A \cap_{T_2} B}^{N_0}, \bar{B}^{N_0}) \geq \max\{D_0(\overline{A \cap_{T_1} B}^{N_0}, \overline{A \cap_{T_1} B}^{N_0}), D_0(\overline{A \cap_{T_k} B}^{N_0}, \bar{B}^{N_0})\},$$

$\forall (i, j) \in \{(x, y) \mid x, y = 0, \dots, 5\} \setminus \{(4, 5)\}$ and $k = 0, 1, 3, 4, 5$.

Secondly, we have $S_0 \sqcap S_4 \sqcap S_3 \sqcap S_5$, $S_0 \sqcap S_4 \sqcap S_3 \sqcap S_2$, and $S_0 \sqcap S_4 \sqcap S_1 \sqcap S_5$. Hence,

$$A \subseteq A \cup_{S_0} B \subseteq A \cup_{S_4} B \subseteq A \cup_{S_3} B \subseteq A \cup_{S_5} B,$$

$$B \subseteq A \cup_{S_0} B \subseteq A \cup_{S_4} B \subseteq A \cup_{S_3} B \subseteq A \cup_{S_5} B,$$

$$A \subseteq A \cup_{S_0} B \subseteq A \cup_{S_4} B \subseteq A \cup_{S_1} B \subseteq A \cup_{S_5} B, \text{ and}$$

$$B \subseteq A \cup_{S_0} B \subseteq A \cup_{S_4} B \subseteq A \cup_{S_1} B \subseteq A \cup_{S_5} B.$$

Therefore $D_0(A \cup_{S_5} B, A) \geq \max\{D_0(A \cup_{S_1} B, A \cup_{S_1} B), D_0(A \cup_{S_k} B, A)\},$

$$D_0(A \cup_{S_5} B, B) \geq \max\{D_0(A \cup_{S_1} B, A \cup_{S_1} B), D_0(A \cup_{S_k} B, B)\},$$

$$D_0(\overline{A \cup_{S_5} B}^{N_0}, \bar{A}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_1} B}^{N_0}, \overline{A \cup_{S_1} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \bar{A}^{N_0})\}, \text{ and}$$

$$D_0(\overline{A \cup_{S_5} B}^{N_0}, \bar{B}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_1} B}^{N_0}, \overline{A \cup_{S_1} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \bar{B}^{N_0})\},$$

$\forall (i, j) \in \{(x, y) \mid x, y = 0, 1, 3, 4, 5\} \setminus \{(1, 3)\}$ and $k = 0, 1, 3, 4$.

Now, we have $A \subseteq A \cup_{S_0} B \subseteq A \cup_{S_4} B \subseteq A \cup_{S_3} B \subseteq A \cup_{S_2} B$ and

$$B \subseteq_* A \cup_{S_0} B \subseteq_* A \cup_{S_1} B \subseteq_* A \cup_{S_2} B \subseteq_* A \cup_{S_3} B \subseteq_* A \cup_{S_4} B.$$

Thus, for all $i, j = 0, 2, 3, 4$, and $k = 0, 3, 4$, we have

$$D_0(A \cup_{S_2} B, A) \geq \max\{D_0(A \cup_{S_1} B, A \cup_{S_1} B), D_0(A \cup_{S_k} B, A)\},$$

$$D_0(A \cup_{S_2} B, B) \geq \max\{D_0(A \cup_{S_1} B, A \cup_{S_1} B), D_0(A \cup_{S_k} B, B)\},$$

$$D_0(\overline{A \cup_{S_2} B}^{N_0}, \overline{A}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_1} B}^{N_0}, \overline{A \cup_{S_1} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \overline{A}^{N_0})\}, \text{ and}$$

$$D_0(\overline{A \cup_{S_2} B}^{N_0}, \overline{B}^{N_0}) \geq \max\{D_0(\overline{A \cup_{S_1} B}^{N_0}, \overline{A \cup_{S_1} B}^{N_0}), D_0(\overline{A \cup_{S_k} B}^{N_0}, \overline{B}^{N_0})\}.$$

Finally, we see that $A \cap_{T_2} B \subseteq_* A \cap_{T_0} B \subseteq_* A \subseteq_* A \cup_{S_0} B \subseteq_* A \cup_{S_5} B$.

Thus, we obtain that $D_0(A \cap_{T_2} B, A \cup_{S_5} B) \geq D_0(A \cap_{T_0} B, A \cup_{S_0} B)$ and the remaining inequalities of

Proposition 3.

4. An Application of the Picture Fuzzy Distance Measure for Controlling Network Power Consumption

4.1. The Problem and the Solution

The interconnection network is important in the parallel computer systems. Saving interconnection network power is always interested, researched and becoming more and more urgent in the current technological era. In order to achieve high performance, the architectural design of the interconnection network requires an effective power saving mechanism. The aim of this mechanism is to reduce the network latency (the average latency of a message) and the percentages between the number of links that are kept switched on by the saving mechanism and the total number of links [25]. As a simplified way of understanding, this is a matter of optimizing the number of links opened in a networking system. This is a decision-making problem for the trunk link state.

In 2010, Alonso et al. introduced the power saving mechanism in regular interconnection network [25]. This model dynamically increases or reduces the number of links that compose a trunk link. This is done by measuring network traffic and dynamically turning these individual links on or off based on a u_{on} / u_{off} threshold policy with keeping at least one operational link (see *Fig. 1* and *Fig. 2*).

The two parameters u_{on} and u_{off} are designed based on different requirements of mechanism aggressiveness (controlled by the value $u_{avg} = (u_{on} + u_{off}) / 2$) and mechanism responsiveness (controlled by the difference $u_{on} - u_{off}$).

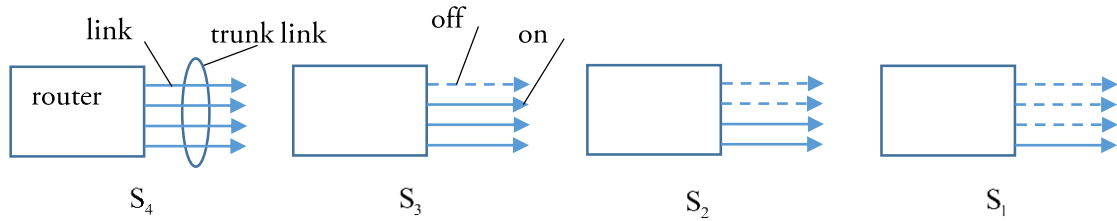


Fig. 1. Four trunk link states.

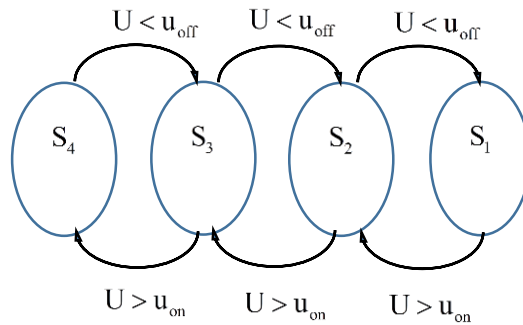


Fig. 2. The operational mechanism of switches.

In order to avoid the possibility of cyclic state transitions that makes the system become unstable, the following restrictions hold in the selection u_{on} and u_{off} :

$$0 < u_{off} \leq \frac{u_{on}}{2} \leq \frac{U_{max}}{2}. \tag{13}$$

Thus, the different values of u_{off} and u_{on} that satisfy Eq. (13) are stifferly chosen in order to achieve different goals of responsiveness and aggressiveness for the power saving mechanism. In 2015, they continue to study and modify power consumption control in fat-tree interconnection networks based on the static and dynamic thresholds policies [26]. In general, this threshold policy is hard because it is without any fuzzy approaches, parameter learning and optimizing processes.

In 2017, Phan et al. [27] proposed a new method in power consumption estimation of network-on-chip based on fuzzy logic [27]. However, this fuzzy logic system based on Sugeno model is too rudimentary and the parameters here are chosen according to the authors' quantification. In this paper, aiming to replace the above threshold policy in decision making problem for the trunk link state, we propose a higher-level fuzzy system based on the proposed single-valued neutrosophic distance measure in Section 3.

4.2. The Adaptive Neuro Picture Fuzzy Inference System (ANPFIS)

In this subsection, an ANPFIS based on picture fuzzy distance measure is introduced to decision making problems. ANPFIS is a modification and combination between ANFIS [28], picture fuzzy set, and picture fuzzy distance measure. Hence, ANPFIS operates based on the picture fuzzification and defuzzification

processes, the picture fuzzy operators and distance measure, and the learning capability for automatic picture fuzzy rule generation and parameter optimization. The model is showed as in the *Fig. 3*.

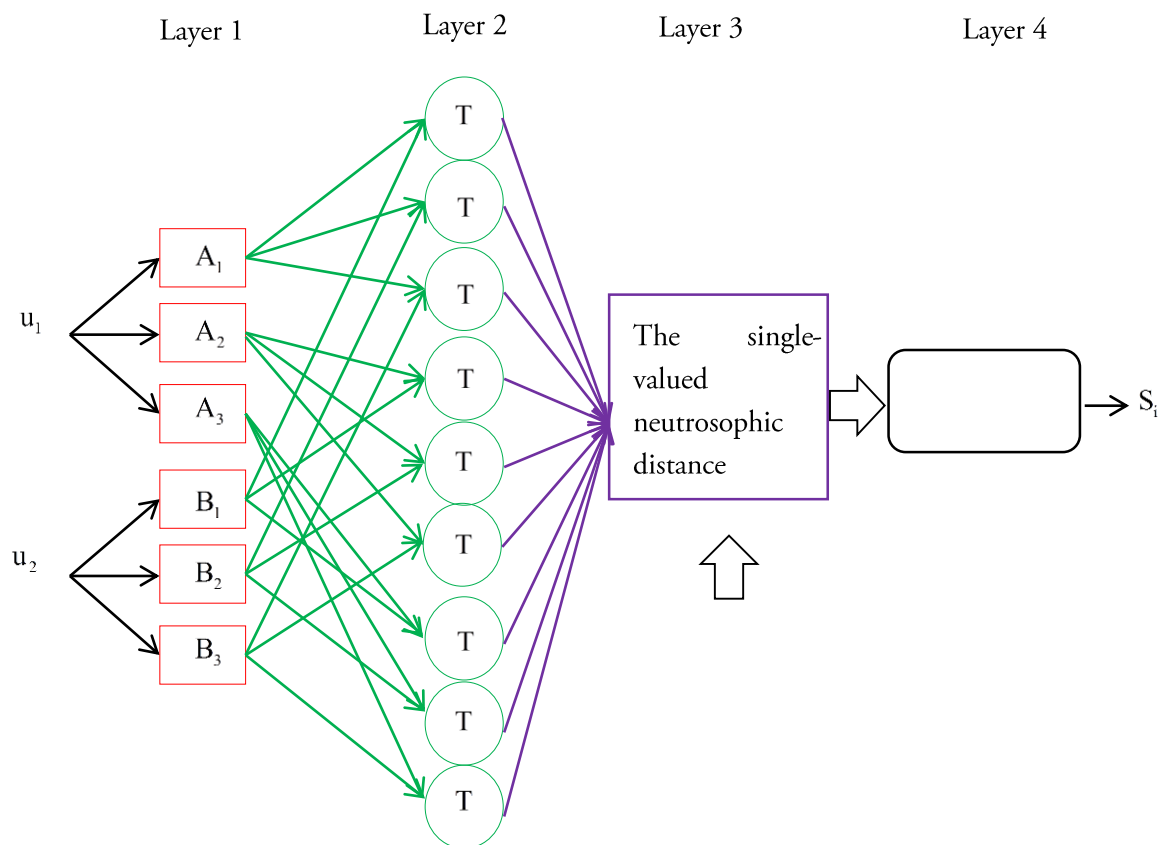


Fig. 3. The proposed ANPFIS decision making model.

The model has the inputs are number values and the output $S_i, i \in \{1, \dots, n\}$ is the chosen solution. ANPFIS includes four layers as follows:

Layer 1-Picture Fuzzification. Each input value is connected to three neurons O_i^l , in other words is fuzzified by three corresponding picture fuzzy sets named “High”, “Medium”, and “Low”. We use the Picture Fuzzy Gaussian Function (PFGF): the PFGF is specified by two parameters. The Gaussian function is defined by a central value m and width $k > 0$. The smaller the k , the narrower the curve is. Picture fuzzy Gaussian positive membership, neutral, and negative membership functions are defined as follows

$$\mu(x) = \exp\left(-\frac{(x-m)^2}{2k^2}\right),$$

$$v(x) = c_1(1 - \mu(x)), \quad (c_1 \in [0, 1]), \text{ and}$$

$$\eta(x) = c_2(1 - \mu(x) - v(x)), \quad (c_2 \in [0, 1]), \text{ where the parameters } m \otimes \text{ and } k \triangleright \text{ are trained.}$$

Layer 2-Automatic Picture Fuzzy Rules. The picture fuzzy t-norm T (see *Definition 7* and *Example 2*) is used in this step in order to establish the IF-THEN picture fuzzy rules, i.e., the links between the neuros O_i^1 of Layer 1 and the neuros O_k^2 of Layer 2 as follows

“If O_i^1 is x and O_j^1 is y then O_k^2 is $T(x, y)$.”

For examples $T(x, y) = T_1^\lambda(x, y)$, where [18]

$$T_1^\lambda(x, y) = \left(\frac{x_1 y_1}{\lambda_1 + (1 - \lambda_1)(x_1 + y_1 - x_1 y_1)}, \frac{x_2 y_2}{\lambda_2 + (1 - \lambda_2)(x_2 + y_2 - x_2 y_2)}, \left(x_3^{\lambda_3} + y_3^{\lambda_3} - x_3^{\lambda_3} y_3^{\lambda_3} \right)^{\frac{1}{\lambda_3}} \right), \text{ here } x, y \in P^*,$$

and the parameters $\lambda_1, \lambda_2, \lambda_3 \in [1, +\infty)$ are trained.

Layer 3 – Calculate the difference to the samples. The difference between the input I and the sample K is calculated by the proposed picture fuzzy distance measure D_0 in *Eq. (10)* as follows

$$D_0(I, K) = \frac{1}{3m} \sum_{i=1}^m \omega_i \cdot \left\{ |\mu_1(x_i) - \mu_2(x_i)| + |\eta_1(x_i) - \eta_2(x_i)| + |v_1(x_i) - v_2(x_i)| \right. \\ \left. + \left| \max\{\mu_1(x_i), v_2(x_i)\} - \max\{\mu_2(x_i), v_1(x_i)\} \right| \right\},$$

where, m is the number of attribute neuro values and $\omega_{(i=1, \dots, m)}$ are the trained weights.

Layer 4-Picture Defuzzification. In this final step, we point out the minimum difference value in all values received from Layer 3,

$$\text{Min } D_0(I, K) = D_0(I, K_i).$$

Then, the output value of the ANPFIS is the solution S_i which is corresponding to the sample K_i .

4.3. Application of the ANPFIS algorithm in Controlling Network Power Consumption

In this part, we present the installation of ANPFIS algorithm in the trunk link state Controller of interconnection network.

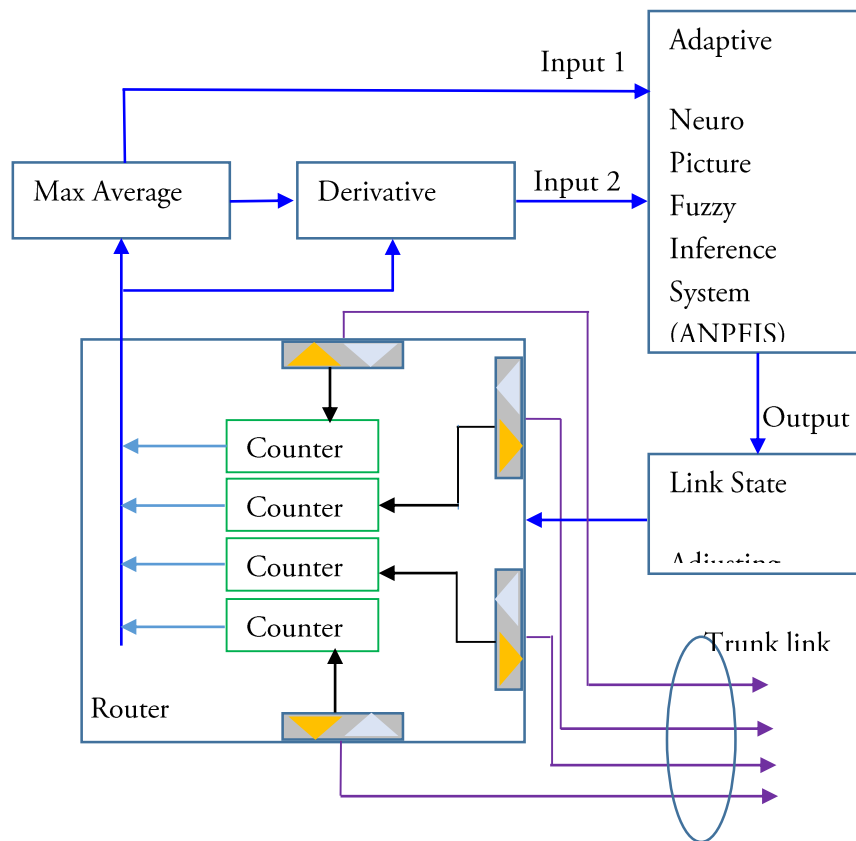


Fig. 4. The architecture of the trunk link state controller based on ANPFIS.

Fig. 4 describes the architecture of the trunk link state Controller based on ANPFIS. This Controller is developed from the previous architecture which is proposed by Phan et al. in 2017 for network-on-chip [27]. For details, each router input port will be equipped with a traffic counter. These counters count the data flits passing through the router in certain clock cycles based on the corresponding response signals from the router. The flits through the router is counted in a slot of time. When the counting finish, the traffic through the corresponding port will be calculated [27]. Each port of the router is connected with a Counter, then there are four average values of the traffic.

- The Max Average (MA) block receives the values of traffic from the counters which are connected with the routers ports. It compares these values and chose the maximum value for Input 1 of the ANPFIS.
- The Derivative (DER) block calculates the derivative of traffics obtained from the counters. This value is defined as an absolute value of the traffics change in a unit of time. This value is determined according to the maximum traffic value decided by MA block. After that, the DER gives it to the Input 2 of the ANPFIS for further processes.

The value domain of Input 1 and Input 2 is from 0 to the maximum bandwidth value. They are normed into $[0, 1]$ by Min Max normalization.

- Through the ANPFIS block, the received Output is the trunk link state $S_i, i \in \{1, 2, 3, 4\}$. The received new state are adjusted by the Link State Adjusting block.

5. Experiments on Real-World Datasets

5.1. Experimental Environments

In order to evaluate performance, we test the ANPFIS method on the real datasets of the network traffic history taken from the UPV (Universitat Politècnica de València) university with related methods. The descriptions of the experimental dataset are presented in *Table 1*.

Table 1. The descriptions of the experimental dataset.

No. elements (checking-cycles)	16.571	
No. attributes	2 (MA, DER)	
The normalized value domain of attributes	MA	DER
	[0,1]	[-1,1]
No. classes (No. link states)	4	

We compare the ANPFIS method against the methods of Hung (M2012) [21], Junjun et al. (M2013) [22], Maheshwari et al. (M2016) [23], Ngan et al. (H-max) [8], and ANFIS [28] in the Matlab 2015a programming language. The Mean Squared Error (MSE) degrees of these methods are given out to compare their performance.

5.2. The Quality

The MSE degree of the ANPFIS method are less than those of other methods. The specific values are expressed in *Table 2*.

Table 2. The performance of the methods.

Method	M2012 [21]	M2013 [22]	M2016 [23]	H-max [8]	ANFIS [28]	ANPFIS
MSE	0.2009	0.2606	0.2768	0.1259	0.2006	0.0089

Fig. 5 clearly show the difference between the performance values of six considered algorithms. In *Fig. 5*, the blue columns illustrate the MSE values of the methods. It can be seen that the columns of the other methods are higher than that of the ANPFIS method. That means the accuracy of the proposed method is better than that of the related methods on the considered dataset.

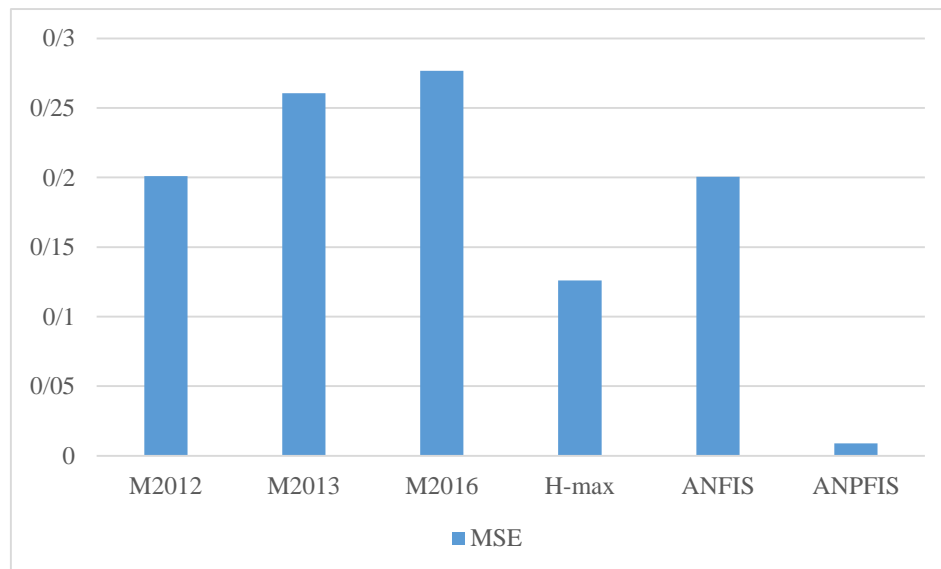


Fig. 5. The MSE values of 6 methods.

6. Conclusion

The neutrosophic theory increasingly attracts researchers and is applied in many fields. In this paper, a new single-valued neutrosophic distance measure is proposed. It is also a distance measure between picture fuzzy sets and is a development of the H-max measure which was introduced by Ngan et al. [8]. Further, an Adaptive NPFIS based on the proposed measure is shown and applied to the decision making for the link states in interconnection networks. The proposed model is tested on the real datasets taken from the UPV university. The MSE value of the proposed methods is less than that of other methods.

Acknowledgment

This work has been supported by the Spanish Ministerio de Economía y Competitividad (MINECO) and the European ERDF under grants PID2019-105903RB-100 and RTI2018-098156-B-C51. The author (R.T. Ngan) also would like to thank the Erasmus+ Mobility Program (2016-1-ES01-KA107-023453) for supporting her work.

References

- [1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8(3), 338-353.
- [2] Khorshidi, H. A., & Nikfalazar, S. (2017). An improved similarity measure for generalized fuzzy numbers and its application to fuzzy risk analysis. *Applied soft computing*, 52, 478-486.
- [3] Liang, D., & Xu, Z. (2017). The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets. *Applied soft computing*, 60, 167-179.
- [4] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning— I. *Information sciences*, 8(3), 199-249.
- [5] Coupland, S., & John, R. (2008). New geometric inference techniques for type-2 fuzzy sets. *International journal of approximate reasoning*, 49(1), 198-211.
- [6] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy sets Syst.*, 20(1), 87-96.

- [7] Chen, S. M., Cheng, S. H., & Lan, T. C. (2016). A novel similarity measure between intuitionistic fuzzy sets based on the centroid points of transformed fuzzy numbers with applications to pattern recognition. *Information sciences*, 343-344, 15-40. doi: 10.1016/j.ins.2016.01.040
- [8] Ngan, R. T., Cuong, B. C., & Ali, M. (2018). H-max distance measure of intuitionistic fuzzy sets in decision making. *Applied soft computing*, 69, 393-425.
- [9] Chen, S. M., & Li, T. S. (2013). Evaluating students' answerscripts based on interval-valued intuitionistic fuzzy sets. *Information sciences*, 235, 308-322. doi: 10.1016/j.ins.2012.12.031
- [10] Smarandache, F. (1999). *A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic, rehoboth*. American Research Press, Rehoboth.
- [11] Ali, M., & Smarandache, F. (2017). Complex neutrosophic set. *Neural computing and applications*, 28(7), 1817-1834.
- [12] Dat, L. Q., Thong, N. T., Ali, M., Smarandache, F., Abdel-Basset, M., & Long, H. V. (2019). Linguistic approaches to interval complex neutrosophic sets in decision making. *IEEE access*, 7, 38902-38917.
- [13] Smarandache, F. (2016). *Degree of dependence and independence of the (sub) components of fuzzy set and neutrosophic set*. Infinite Study.
- [14] Smarandache, F. (2019). *Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set (atanassov's intuitionistic fuzzy set of second type), q-rung orthopair fuzzy set, spherical fuzzy set, and n-hyperspherical fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited)*. Infinite Study.
- [15] Cuong, B. C., & Kreinovich, V. (2013, December). Picture fuzzy sets-a new concept for computational intelligence problems. *2013 third world congress on information and communication technologies (WICT 2013)* (pp. 1-6). IEEE.
- [16] Thong, P. H. (2015). A new approach to multi-variable fuzzy forecasting using picture fuzzy clustering and picture fuzzy rule interpolation method. In *Knowledge and systems engineering* (pp. 679-690). Springer, Cham. doi:10.1007/978-3-319-11680-8_54
- [17] Wei, G., & Gao, H. (2018). The generalized Dice similarity measures for picture fuzzy sets and their applications. *Informatica*, 29(1), 107-124.
- [18] Cong, C. B., Ngan, R. T., & Long, L. B. (2017). Some new de morgan picture operator triples in picture fuzzy logic. *Journal of computer science and cybernetics*, 33(2), 143-164.
- [19] Cuong, B. C., Kreinovitch, V., & Ngan, R. T. (2016, October). A classification of representable t-norm operators for picture fuzzy sets. *2016 eighth international conference on knowledge and systems engineering (KSE)* (pp. 19-24). IEEE.
- [20] Khalil, A. M., Li, S. G., Garg, H., Li, H., & Ma, S. (2019). New operations on interval-valued picture fuzzy set, interval-valued picture fuzzy soft set and their applications. *IEEE access*, 7, 51236-51253.
- [21] Hung, K. C. (2012). Medical pattern recognition: applying an improved intuitionistic fuzzy cross-entropy approach. *Advances in fuzzy systems*, 863549. 1-6. doi: 10.1155/2012/863549
- [22] Mao, J., Yao, D., & Wang, C. (2013). A novel cross-entropy and entropy measures of IFSs and their applications. *Knowledge-based systems*, 48, 37-45.
- [23] Maheshwari, S., & Srivastava, A. (2016). Study on divergence measures for intuitionistic fuzzy sets and its application in medical diagnosis. *Journal of applied analysis and computation*, 6(3), 772-789.
- [24] Ngan, R. T., Cuong, B. C., & Tuan, T. M. (2018, August). Medical diagnosis from images with intuitionistic fuzzy distance measures. *International joint conference on rough sets* (pp. 479-490). Springer, Cham.
- [25] Alonso, M., Coll, S., Martínez, J. M., Santonja, V., López, P., & Duato, J. (2010). Power saving in regular interconnection networks. *Parallel computing*, 36(12), 696-712.
- [26] Alonso, M., Coll, S., Martínez, J. M., Santonja, V., & López, P. (2015). Power consumption management in fat-tree interconnection networks. *Parallel computing*, 48, 59-80.
- [27] Phan, H. P., Tran, X. T., & Yoneda, T. (2017, May). Power consumption estimation using VNOC2. 0 simulator for a fuzzy-logic based low power Network-on-Chip. *2017 IEEE international conference on IC design and technology (ICICDT)* (pp. 1-4). IEEE.
- [28] Çaydaş, U., Haşçalık, A., & Ekici, S. (2009). An adaptive neuro-fuzzy inference system (ANFIS) model for wire-EDM. *Expert systems with applications*, 36(3), 6135-6139.
- [29] Ali, M., Deo, R. C., Downs, N. J., & Maraseni, T. (2018). An ensemble-ANFIS based uncertainty assessment model for forecasting multi-scalar standardized precipitation index. *Atmospheric research*, 207, 155-180.
- [30] Karaboga, D., & Kaya, E. (2019). Adaptive network based fuzzy inference system (ANFIS) training approaches: a comprehensive survey. *Artificial intelligence review*, 52(4), 2263-2293.
- [31] Ali, M., Deo, R. C., Downs, N. J., & Maraseni, T. (2018). An ensemble-ANFIS based uncertainty assessment model for forecasting multi-scalar standardized precipitation index. *Atmospheric research*, 207, 155-180.

- [32] Wang, W., & Xin, X. (2005). Distance measure between intuitionistic fuzzy sets. *Pattern recognition letters*, 26(13), 2063-2069.

Appendix

Source code and datasets of this paper can be found at this link, <https://sourceforge.net/projects/pfdm-datasets-code/>.