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Geometric and Harmonic means based priority dispatching rules for single machine scheduling problems

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Abstract: This work proposes two new priority dispatching rules (PDRs) for solving single machine scheduling problems. These rules are based on the geometric mean (GM) and harmonic mean (HM) of the processing time (PT) and the due date (DD) and they are referred to as GMPD and HMPD respectively. Performance of the proposed PDRs is evaluated on the basis of five measures/criteria i.e. Total Flow Time (TFT), Total Lateness (TL), Number of Late Jobs (TNL), Total Earliness (TE) and Number of Early Parts (TNE). It is found that GMPD performs better than other PDRs in achieving optimal values of multiple performance measures. Further, effect of variation in the weight assigned to PT and DD on the combined performance of TFT and TL is also examined which reveals that for deriving optimal values of TFT and TL, weighted harmonic mean (WHMPD) rule with a weight of 0.105 outperforms other PDRs. The weighted geometric mean (WGMPD) rule with a weight of 0.37 is found to be the next after WHMPD followed by the weighted PDT i.e. WPDT rule with a weight of 0.76.

Key words: job sequencing, priority dispatching rule, single machine scheduling, geometric mean of the processing time and due date (GMPD), harmonic mean of the processing time and due date (HMPD).

1. Introduction

Production scheduling refers to the planning of the manufacturing process and it is an important activity as it leads to enhancement in the productivity of the system, reduction in job lateness, increased utilization of the machine etc. (Doh *et al.*, 2013; Geiger and Uzsoy, 2008; Pinedo, 2009). Apart from manufacturing processes, scheduling finds its application in various other areas like operating systems where it is used for memory allocation of processor, in service industries for operator allotment etc. (Baharom *et al.,* 2015; Lee *et al.,* 2020; Munir *et al.,* 2008; Rafsanjani and Bardsiri, 2012). However, it is imperative in production scheduling due to its undeviating impact on the profitability of a company (Kadipasaoglu *et al.,* 1997; Prakash *et al.,* 2011). The effectiveness of a schedule is very much affected by the priority dispatching rule (PDR) used for job sequencing (Hussain and Ali, 2019; Krishnan *et al.,* 2012; Waikar *et al.,* 1995). These rules offer priority to one or more jobs over other jobs to improve certain performance measures of the system. The well-known PDRs found in the literature are as follows (Forrester, 2006; Pinedo, 2009):

- First Come First Served (FCFS): A job that arrives first will be given the highest priority for processing.
- Shortest Processing Time (SPT): A job having the least processing time (PT) will be processed first.
- Earliest Due Date (EDD): A job with a least due date (DD) will attain the highest priority and will be processed first.

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- SLACK: A slack time of each job is computed which is the difference between the remaining time and the PT. Subsequently, a job with the least slack value will be processed first.
- Critical Ratio (CR): A ratio of time remaining until the DD andPT is calculated which is identified as a critical ratio. Consequently, a job with the least critical ratio will be processed first.

The choice of PDR has a significant effect on the overall performance of the system (Waikar *et al.,* 1995). However, it was observed that no single PDR is capable of optimizing all the performance measures (Chan *et al.,* 2003; Dominic *et al.,* 2004; Đurasević and Jakobović, 2018). Hence, the current era of research in scheduling is focused on the development of new PDRs to achieve optimal values of more than one performance measures (da Silva *et al.,* 2019; Kanet and Li, 2004; Lu *et al.,* 2012).

Holthaus and Rajendran (1997) presented two new PDRs using additive and alternative strategies to combine process time and work content in the next queue and based on their simulation study they reported that this rule performed well to minimize the average flow time. Jayamohan and Rajendran (2007) proposed five new PDRs by combining different rules and showed that the proposed rules performed well for minimizing mean flow time, mean tardiness and percentage of tardy jobs. Dominic *et al*. (2004) examined the effectiveness of three rules developed by combining FIFO, SPT and most work remaining (MWK) PDRs using ARENA 4.0 simulation software and found that the combined rule provided better results as compared to other rules. Vinod and Sridharan (2008) conducted a simulation study to evaluate five new set up oriented PDRs under different experimental conditions. The results of their study showed that the proposed rules provided better results over seven standard rules. Hamidi (2016) proposed a priority dispatching rule where in the author took average of the PT and the DD of the jobs and termed this rule as PDT and suggested that the job with the least PDT value should be processed first. It is found from the literature that the PDT rule gives optimal results for multiple performance measures as compared to FCFS, SPT, EDD, SLACK and CR (Hamidi, 2016). It is observed from literature that researchers have made attempt to combine two or more standard PDRs to develop new PDRs and based on the simulation studies it has been established that the combined rules perform well as compared to standard PDRs.

Further, the presented literature demonstrated that PDRs based on combination of PT and DD have been developed by using additive strategy i.e. PDT rule. However, there is lack of studies or no study which has combined the PT and DD using geometric and harmonic means or using similar methods. With this motivation, this work attempts to combine the PT and DD using multiplicative and reciprocal strategy and proposes two new PDRs i.e. GMPD and HMPD. To the best of authors knowledge, these rules have not been proposed yet in literature. Further, researchers have used single machine scheduling problem (SMSP) and simulation based study to compare the effectiveness of the newly developed PDRs over standard rules (Cheng and Kahlbacher, 1993; Tyagi *et al.,* 2016). Therefore, a simulation study is also conducted in this work to evaluate the effectiveness of the proposed rules over FCFS, SPT, EDD, SLACK, CR and PDT on the basis of five performance measures viz. Total Flow Time (TFT), Total Lateness (TL), Total Number of Late parts (TNL), Total Earliness (TE) and Total Number of Early parts (TNE). Hamidi (2016) also proposed weighted PT and DD total (WPDT) rule in which weights are assigned to PT and DD to obtain better performance measures. Keeping this in view, the weighted geometric mean of PT and DD (WGMPD) and weighted harmonic mean of PT and DD (WHMPD) are also proposed in this paper. It is found from the simulation study that the proposed rules give better results as compared to the standard rules as well as PDT rule. Rest of the paper is organized as follows: Section 2 describes how the proposed GMPD and HMPD rules are derived. Section 3 presents a case study in which a sample problem using the standard rules as well as the proposed rules is solved. Section 4 puts forth the simulation study in which eleven thousand randomly generated problems are solved and the results so obtained are compared for different PDRs. Section 5 illustrates the effect of weight variation in the proposed WGMPD and WHMPD on TFT and TL. Finally, section 6 presents the conclusion of the present study. To provide a better understanding of the various symbols used in this manuscript, a nomenclature is provided in Table 1.

2. Proposed rules

A single machine scheduling problem (SMSP) requires effective scheduling of ‘n’ jobs say J1, J2, J3,.........., Jn on a single machine. Several PDRs can be used to schedule these jobs. Among

Table 1. Nomenclature.

Symbol	Meaning
i	Job number
n	Total number of jobs
Ji	Name of the job i
pti	Processing time of job Ji
ddi	Due date of job Ji
Fi	Flow time of job Ji
Li	Lateness of job Ji
Ei	Earliness of job Ji
PDR	Priority dispatching rule
GM	Geometric mean
HM	Harmonic mean
PT	Processing time
DD	Due date
GMPD	Geometric mean of processing time and due date
HMPD	Harmonic mean of processing time and due date
TFT	Total flow time
TL	Total lateness
TNL	Number of late jobs
TE	Total earliness
TNE	Number of early parts
WGMPD	Weighted geometric mean
WHMPD	Weighted harmonic mean
FCFS	First come first served
SPT	Shortest processing time
EDD	Earliest due date
CR	Critical ratio
MWK	Most work remaining

them, the most preferable are SPT and EDD as they possess advantages over others in terms of specific performance measures. SPT rule prioritizes the jobs based on PT which results in small flow times. Whereas, in EDD rule, jobs with least DD are processed first which results in reduced lateness. As SPT rule is purely based on the PT and EDD rule is solely based on the DD of the job, priorities defined on the combined basis of PT and DD might be more effective for optimal results of multiple performance measures. In this regard, Hamidi (2016) made an attempt and proposed the PDT rule, where additive strategy was used to combine the PT and DD. Since, multiplicative and reciprocal strategy has not been examined to combine the different PDRs, this work proposes two new PDRs to solve a SMSP based on the multiplicative and reciprocal strategies. In the first rule, geometric mean of the PT and DDof the jobs is taken, and it is termed as GMPD which is based on the multiplicative strategy. According to the GMPD, top priority is given to a job for which the value of GMPD is the minimum. The second rule considers harmonic mean of the PT and DD of the jobs and it is referred to as HMPD that is based on the reciprocal strategy. A job with the least value of the HMPD is processed first according to this rule.

2.1. The geometric mean of processing time and due date (GMPD)

For two numbers say '*a*' and '*b*' the mathematical formula used to calculate geometric mean is √(ab). Considering an SMSP of '*n*' jobs J_1, J_2, J_3,.........., J_n with deterministic processing time pt_i and due date dd_i of job J_i, the priority function for GMPD rule is defined according to Equation (1).

$$GM_i = \sqrt{pt_i \times dd_i} \tag{1}$$

Hence, for each job, the value of priority function is calculated and the job with the least GM_i value will be processed first.

2.2. The harmonic mean of processing time and due date (HMPD)

The harmonic mean is related to the arithmetic mean in a manner that it is reciprocal of the arithmetic mean. PDR based on the arithmetic mean of PT and DD i.e. PDT has already been developed and established to provide better performance than other standard PDRs (Hamidi, 2016). Hence, it was supposed that PDR based on harmonic mean might be able to give better results. In this regard, the harmonic mean of PT and DD (HMPD) rule is proposed in this work. The priority function for HMPD rule is used as defined by Equation (2).

$$HM_i = \frac{2 \times pt_i \times dd_i}{pt_i + dd_i} \tag{2}$$

Hence, priorities to jobs are defined based on HM_i value. A job with a minimum value of HM_i will be most preferable for processing over other jobs.

3. Case Study

In this section, a sample problem is solved using the proposed PDRs along with other rules to examine the effectiveness of the proposed PDRs for solving SMSP. The sample problem consists of ten jobs that need to be sequenced for processing on a machine. Each job has a deterministic PT in the interval of 1 to 15 days which is determined using uniform distribution. The DD of each job is also considered to be deterministic;in the interval ranging from 3×PT of the job and 60 days, and it is determined using the same distribution (uniform distribution). The PT and the DD of each job for the sample problem considered in this work are shown in Table 2.

Table 2. PT and DD of jobs.

Job	PT (days)	DD (days)
J_1	9	34
J_2	5	53
J_3	15	50
J_4	1	28
J_5	8	56
J_6	14	45
J_7	3	15
J_8	10	30
J_9	9	32
J_{10}	4	36

The effectiveness of the PDRs is examined on the basis of commonly used performance measures i.e. flow time, lateness, number of late parts, earliness and number of early parts (Oyetunji, 2009). It may be noted that either maximum or average or total values of these performance measures may be used. In the present work, total values of the performance measures are considered for the evaluation of the PDRs. The performance measures considered in this work are described as follows:

- Total flow time (TFT): It represents the sum of time each job spends in the system and it is determined using Equation (3).

$$TFT = \sum_{i=1}^{n} F_i \tag{3}$$

where, $F_i=F_{i-1}+pt_i$. pt_i and F_i indicate the PT and flow time of job J_i respectively.

- Total lateness (TL): It measures the total amount of lateness to the jobs and it is computed using Equation (4).

$$TL = \sum_{i=1}^{n} L_i \tag{4}$$

where, L_i=max(0,F_i–dd_i). dd_i and L_i represents the DD and lateness of job J_i respectively.

- Total number of late parts (TNL): If a job is completed after its DD, it is said to be late. TNL is the measure that indicates the count of the late parts i.e.

- Total earliness (TE): The difference between the DD and flow time when a job is completed before its DD is regarded as the earliness of a job otherwise earliness of a job is defined as 0. Total earliness is defined by Equation (5).

$$TE = \sum_{i=1}^{n} E_i \tag{5}$$

where, E_i=max(0,dd_i–F_i). dd_i and E_i represent the DD and earliness of job J_i respectively.

- Total number of early parts (TNE): The count of the jobs which are early is regarded as TNE i.e., where n is the number of jobs.

The sample problem shown in Table 1 is solved using different PDRs viz. FCFS, SPT, EDD, SLACK, CR, PDT, GMPD and HMPD. The sequence of jobs for processing on a machine so obtained is depicted in Table 3.

Table 3. Sequence of jobs using different PDRs.

FCFS	SPT	EDD	SLACK	CR	PDT	GMPD	HMPD
J_1	J_4	J_7	J_7	J_8	J_7	J_4	J_4
J_2	J_7	J_4	J_8	J_7	J_4	J_7	J_7
J_3	J_{10}	J_8	J_9	J_9	J_8	J_{10}	J_{10}
J_4	J_2	J_9	J_1	J_1	J_{10}	J_2	J_2
J_5	J_5	J_1	J_4	J_4	J_9	J_9	J_5
J_6	J_1	J_{10}	J_6	J_6	J_1	J_8	J_9
J_7	J_9	J_6	J_{10}	J_{10}	J_2	J_1	J_1
J_8	J_8	J_3	J_3	J_3	J_6	J_5	J_8
J_9	J_6	J_2	J_2	J_2	J_5	J_6	J_6
J_{10}	J_3	J_5	J_5	J_5	J_3	J_3	J_3

It may be noted that for PDT rule, the jobs are sequenced based on the minimum value of the sum of PT and DD. For GMPD rule job sequence is defined based on the minimum value of priority function defined in Equation (1) and for HMPD rule Equation (2) is used to define the job sequence. Further, the values of performance measures are computed when the jobs are sequenced with these PDRs. The performance so obtained is shown in Table 4.

It can be seen from Table 4 that (i) the least TFT (306 days) is obtained when the jobs are sequenced using SPT rule or HMPD rule, (ii) the minimum value of TL (47 days) is obtained when the jobs

Table 4. Performance measures for different PDRs.

PDR	TFT (days)	TL (days)	TNL	TE (days)	TNE
FCFS	444	168	6	103	4
SPT	306	72	4	145	6
EDD	375	59	4	63	5
SLACK	410	73	6	42	4
CR	417	73	6	35	4
PDT	339	47	4	87	6
GMPD	311	55	4	123	6
HMPD	306	70	4	143	6

are sequenced using PDT rule, (iii) the number of late jobs is the least i.e. 4 when either of the SPT, EDD, PDT, GMPD or HMPD rule is used, (iv) maximum TE (145 days) is observed when SPT rule is used,and (v) the number of jobs which are completed before the DD i.e. TNE is maximum when either of the SPT, EDD, PDT, GMPD or HMPD is utilized. These results are in line with the results reported in the literature that no single rule is the best for all the performance measures (Dominic *et al.,* 2004). However, an attempt has been made to identify a PDR which might be effective in achieving optimal values of all performance measures. For this purpose, the best performance value for a measure is identified among the considered PDRs. Based on the best performance value, the percentage deviation in the performance of each PDR is calculated. It is to be mentioned that for TFT, TL and TNL, maximum values whereas for TE and TNE minimum values is regarded as the best performance value. In this way, a PDR with the best value for a specific performance measure will get a percentage deviation of 0% and a PDR with least average percentage over all the performance measure will be the best PDR. The percentage deviation of each performance measure for each PDR are depicted in Table 5.

Table 5. Percentage deviation of the performance measures.

PDR	TFT (%)	TL (%)	TNL (%)	TE (%)	TNE (%)	Average % deviation
FCFS	45.10	257.45	50.00	28.97	33.33	82.97
SPT	0.00	53.19	0.00	0.00	0.00	10.64
EDD	22.55	25.53	0.00	56.55	16.67	24.26
SLACK	33.99	55.32	50.00	71.03	33.33	48.73
CR	36.27	55.32	50.00	75.86	33.33	50.16
PDT	10.78	0.00	0.00	40.00	0.00	10.16
GMPD	1.63	17.02	0.00	15.17	0.00	6.77
HMPD	0.00	48.94	0.00	1.38	0.00	10.06

It is observed from Table 5 that GMPD, HMPD and PDT are the top three rules with least deviations. Hence, it can be inferred that the mean based PDRs outperform the standard PDRs. Further, among the considered PDRs, the deviation from the best is the least for GMPD rule. Hence, it can be concluded that the best PDR for deriving optimal values of the five performance measures i.e. TFT, TL, TNL, TE, and TNE is GMPD. The next in the row is found to be HMPD. Subsequently, the next best rule is observed to be the PDT rule. PDT rule has also been reported to perform better than FCFS, EDD, SLACK, and CR for obtaining optimal values of the multiple performance measures (Hamidi, 2016). Based on the values of average percentage deviation (Table 5) the importance of rest of the PDRs in decreasing order is EDD>SLACK>CR>FCFS.

4. Simulation study

It is likely that results obtained for the sample problem solved in the previous section might change when the parameters of the sample problem i.e. PT and DD is changed. Therefore, it is indeed vital to examine the changes that may occur in the results when these parameters are altered. For this purpose, a simulation study was conducted in which eleven thousand problems were generated randomly and in each problem ten jobs were considered. Both PT (ranging from 1 to 10 days) and DD (in the interval of 3 x PT to 60 days) of each job was deterministically generated using uniform distribution. The performance measures were computed for each problem and the average results obtained for the eleven thousand problems were used to compare different PDRs. The average of TFT taken over eleven thousand solved problems using different PDRs is shown in Figure 1.

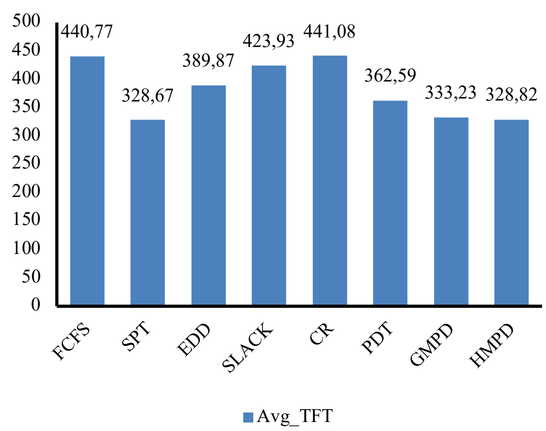

Figure 1. Average TFT of eleven thousand problems.

It is evident from Figure 1 that the least value of TFT (328.67 days) is obtained when job sequencing is done using SPT rule and this result is in line with the result reported in literature (Lu *et al.,* 2012; Tyagi *et al.,* 2016). Further, mean based PDRs i.e. GMPD, HMPD and PDT perform better than other PDRs except SPT. Among the three mean based PDRs, HMPD is found to give better results for TFT followed by GMPD and PDT.

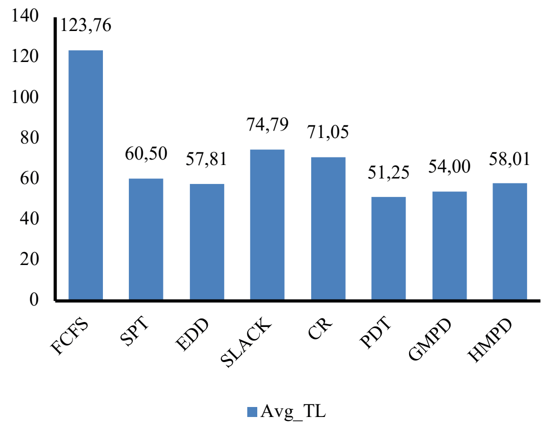

Figure 2. Average TL of eleven thousand problems.

Figure 2 depicts average value of TL for the eleven thousand solved problems using different PDRs and it reveals that PDT rule performs better than other PDRs as the average TL value for this rule is minimum i.e. 51.25 days. The proposed GMPD rule is next in the row as it results in average total lateness of 54 days. Performance of rest of the PDRs in descending order as observed from Figure 2 is EDD>HMPD>SPT>CR>SLACK>FCFS.

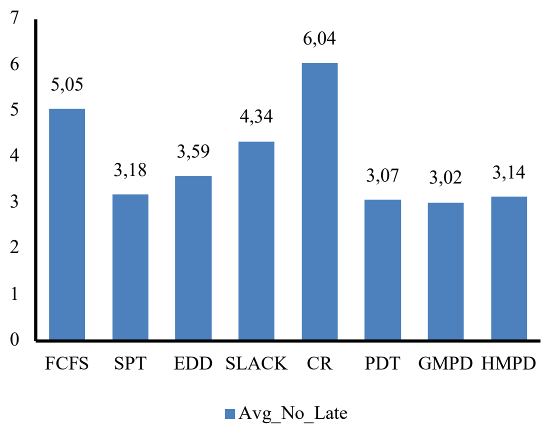

Figure 3. Average TNL of eleven thousand problems.

Results for the number of late parts for different PDRs are depicted in Figure 3 which clearly shows that performance of the GMPD is the best as it provides minimum number of late parts i.e. 3.02 and this rule is followed by HMPD, PDT, SPT, EDD, SLACK, FCFS, and CR.

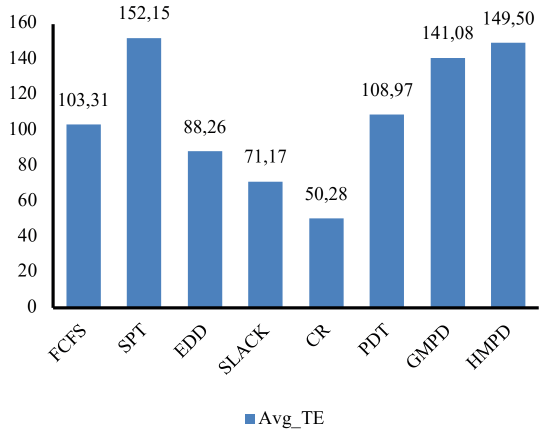

Figure 4. Average TE of eleven thousand problems.

Figure 4 depicts results of the simulation study for TE for different PDRs which clearly show that maximum TE is observed when jobs are sequenced using SPT rule. Next to SPT rule is the HMPD and then the other PDRs follow. The decreasing order of the PDRs is SPT>HMPD>GMPD>PDT>FCFS> EDD>SLACK>CR.

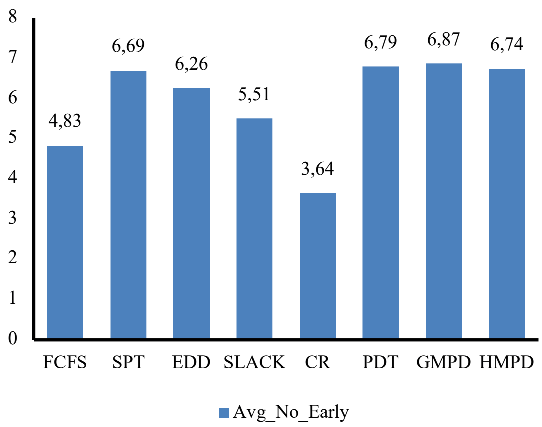

Figure 5. Average TNE of eleven thousand problems.

The performance measure TNE is used to examine the count over all the parts which are completed on or before the DD. Figure 5 depicts the average value of the TNE for different PDRs obtained from simulation study. It is evident from Figure 5 that GMPD rule is better than other PDRs as the average value of TNE for this rule is maximum i.e. 6.87. The other two mean based PDRs i.e. PDT and HMPD come next to GMPD. The sequence of PDRs in decreasing order is GMPD>PDT>HMPD>SPT> EDD>SLACK>FCFS>CR.

The results obtained from the simulation study show that no single rule gives the best results for all the performance measures. However, it is observed that the results derived from using mean based PDRs are best for a few performance measures and for others although the results are not the best but they are close to the best one. Thus, based on the results of the simulation study it is suggested that mean based PDRs should be used to obtain optimal results for all the performance measures. To investigate the effect of PDRs on all performance measures taken together, all PDRs are ranked separately based on the results of the simulation study. The ranking results for different PDRs with respect to the performance measures are shown in Table 6.

Table 6. Ranking results for PDRs.

	Rank based on performance measures					Average
PDR	TFT	TL	TNL	TE	TNE	Rank
FCFS	7	8	7	5	7	6.8
SPT	1	5	4	1	4	3
EDD	5	3	5	6	5	4.8
SLACK	6	7	6	7	6	6.4
CR	8	6	8	8	8	7.6
PDT	4	1	2	4	2	2.6
GMPD	3	2	1	3	1	2
HMPD	2	4	3	2	3	2.8

It is evident from Table 6 that average rank of the mean based PDRs i.e. GMPD, PDT, and HMPD is higher than other PDRs. Further, among the three mean based PDRs, the proposed GMPD rule is found to be the best (as its average rank is 2) followed by PDT and HMPD for achieving optimal results for multiple performance measures. Thus, the proposed GMPD rule is the most promising rule as compared to other PDRs and it should be implemented to obtain compromised or optimal results for multiple performance measures of single machine scheduling problems. Next to GMPD is the PDT and then HMPD and therefore, it is found that mean based PDRs perform better as far as scheduling of jobs on a single machine is concerned.

5. Weighted GMPD and HMPD rule

It has been shown in the previous section that the PDRs based on three different strategies i.e. additive, multiplicative and reciprocal provides better results, in terms of the optimal performance measures of the system, as compared to other PDRs. It is realized that in these strategies, the weight component of both numbers is same. It is very likely that the weighted mean based PDRs where the weight components of the PT and DD of jobs are different may provide a better sequence of jobs and therefore, it is matter of investigation. Consequently, the performance of weighted form of PDT, GMPD, and HMPD is compared in this section. The weighted PT and DD (WPDT) rule has already been developed and reported in the literature (Hamidi, 2016). Hence, the weighted geometric mean of PT and DD (WGMPD) and weighted harmonic mean of PT and DD (WHMPD) rules are proposed in this work. The function used to determine the priority of the jobs using WPDT, WGMPD and WHMPD is given in Equation (6), Equation (7), and Equation (8) respectively.

$$PD_i = w \cdot pt_i + (1-w)dd_i \tag{6}$$

$$WGM_i = pt_i^{w} \cdot dd_i^{1-w} \tag{7}$$

$$WHM_i = \frac{1}{\frac{w}{pt_i} + \frac{(1-w)}{dd_i}} \tag{8}$$

where, w represents weight

A job with the least value of WGM_i and WHM_i is processed first when the priority is determined using WGMPD and WHMPD respectively.

The motivation behind the development of weighted PDRs is to combine SPT and EDD rules to obtain optimal multiple performance measures. SPT and EDD rules have been considered as they possess advantage over other PDRs in terms of small flow times and reduced lateness respectively. Therefore, in this section, the weight used in WPDT, WGMPD, and WHMPS is varied from 0 to 1 with an increment of 0.1 and the values of two performance measures i.e. TFT and TL are compared. As the unit of measurement TFT and TL is different, it is necessary to normalize their values so as to bring their values on a common scale. Among the several methods of normalization, the min-max normalization method is used. The mathematical formula used in min-max normalization method is given in Equation (9).

$$x_i^n = \frac{x_i - \min_i x_i}{\max_i x_i - \min_i x_i} \tag{9}$$

where, x_i and x_i^n represent the original and normalized value of the i^{th} attribute.

The normalized value of TFT and TL with varying weights for WPDT rule is shown in Figure 6. It may be noted that when the value of weight is 0, it represents EDD rule as the job sequencing is done based on DD only. When the weight is set at 1, job sequencing is done based on SPT rule and when the weight is 0.5 it represents PDT rule. It is evident from Figure 6 that for EDD rule ($w=0$), the values of both performance measures i.e. TFT and TL are high which is not desirable. When $w=1$ i.e.for SPT rule, value of TFT is minimum (almost zero) but the value of TL issignificantly high. Further, for PDT rule i.e. when $w=0.5$, the value of TL is minimum, and also the value of TFT is smaller than that of EDD but higher than SPT. As the WPDT rule was developed with an aim to obtain optimal values of TFT and TL, the graph shown in Figure 6 supports the fact that WPDT rule can give optimal results for both the performance measures i.e. TFT and TL. It can also be observed from Figure 6 that better results can be obtained if a weight of 0.76 is considered in the WPDT as the two performance measures intersect at this point which suggests that their values are optimal. The normalized value of TFT and TL when weight is 0.76 is found to be 0.20.

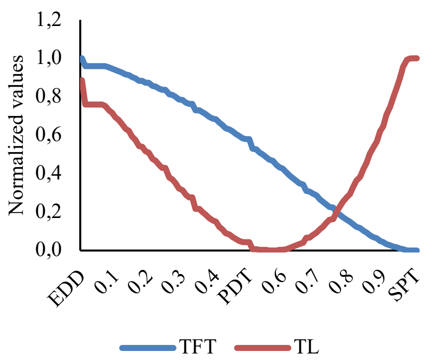

Figure 6. Variation in normalized values of TFT and TL with weight for WPDT rule.

The variation of normalized values of TFT and TL for WGMPD with varying weights is shown in Figure 7. The pattern observed in this case is similar to that of the WPDT rule as shown in Figure 6. It can be seen from Figure 7 that for $w=0$, TFT and TL values are high, but for SPT ($w=1$), TFT value is minimum but TL value is reasonably high. For GMPD rule ($w=0.5$), the values of TFT and TL are in between that of SPT and EDD. Further, intersection of TFT and TL is obtained when the weight of 0.37 is assigned. Hence, it can be concluded that for GMPD rule, the weight of 0.37 will result in optimal values of TFT and TL which is 0.16.

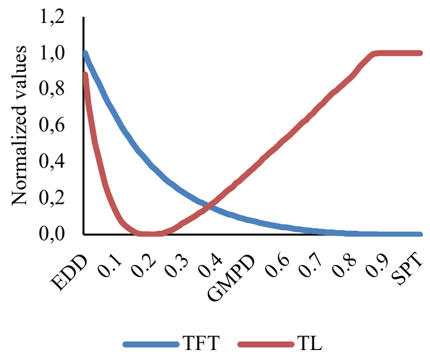

Figure 7. Variation in normalized values of TFT and TL with weight for WGMPD rule.

Figure 8 shows variation in the values of TFT and TL with varying weights of WHPD rule. Once again, a similar pattern of variation as that of WPDT and WGMPD is observed in this case too. It is evident from Figure 8 that a weight of 0.105 results in the optimum values i.e. 0.135 of both TFT and TL as the two curves intersects at this point.

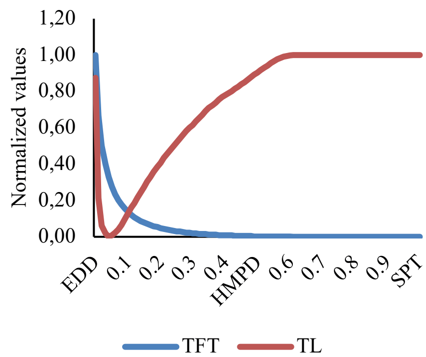

Figure 8. Variation in normalized values of TFT and TL with weight for WHMPD rule.

From the analysis of the weight variation in the WPDT, WGMPD, and WHPD presented above, it is found that a weight of 0.76 used in the WPDT results in the optimum values of both TFT and TL which is 0.20. Further, a weight of 0.37 employed in the WGMPD rule and a weight of 0.105 used in the WHMPD rule leads to the optimum values of TFT and TL which are 0.16 and 0.135 respectively. Further, it is also found that among the three weighted mean based rules, WHMPD with a weight of 0.105 produces results better than WGMPD with the weight of 0.37 and WPDT with weight 0.76 as optimum values of TFT and TL are minimum i.e. 0.135.

6. Conclusion

Scheduling is imperative in manufacturing systems as it directly affects the systems' performance. For a single machine scheduling problem, priority dispatching rules are used to define the sequence of jobs to be processed as they help in enhancing the performance of the system. Among the standard PDRs i.e. FCFS, SPT, EDD, SLACK and CR, SPT performs better as far as minimum flow time is required and EDD rule is found to be promising for minimizing the lateness of the jobs. Further, it was realized that combining these rules might be able to perform better and hence, several combined rules were developed and proposed. However, it is found that most of these rules are based on the additive strategy. Therefore, an attempt was made in this study to examine the effectiveness of the combined rules based on the multiplicative and reciprocal strategy. In this regard two PDRs, first considering multiplicative strategy and second with the reciprocal strategy have been proposed in this work. The first rule named as geometric mean of PT and DD (GMPD) is based on multiplicative strategy and the other i.e. harmonic mean of PT and DD (HMPD) is based on reciprocal strategy. Five performance measures viz. Total flow time (TFT), Total Lateness (TL), Number of Late parts (TNL), Total Earliness (TE) and Number of Early parts (TNE) were used to examine the effectiveness of the proposed rules. This study demonstrates the application of heuristic and metaheuristic algorithms to deal with best PDR. The major conclusions drawn from the present work are as follows:

- SPT rule is found to the best rule for minimizing the TFT of the jobs.
- PDT rule performs better than other PDRs as far as minimum TL is required.
- The proposed rule GMPD results in the least number of late parts compared to other PDRs.
- For maximizing the total earliness, the SPT rule is found to give better results.
- The maximum number of early parts is observed when the GMPD rule is used.
- For optimal performance,it is found that PDRs based on different combination strategies, i.e. additive, multiplicative or reciprocal perform better than other PDRs. Further, among the three strategy based PDRs, GMPD rule performs better than others followed by PDT and HMPD.
- In weighted PDRs, optimal values of multiple performance measures are found when a weight of 0.105 is used in WHMPD followed by WGMPD with a weight of 0.37 and WPDT with a weight of 0.76.

The findings of this study suggest that GMPD rule is the best rule among the considered PDRs. However, there are shortcomings of this work which could be considered in future work. The comparison of the new rules has been done with six rules. However, there are various other rules which can also be compared to find the effectiveness of the proposed rules. Further, in this work the multiplicative strategy and reciprocal strategy were used to combine PT and DD which can also be used to combine process time and work content as done by Holthaus and Rajendran (1997). The results of the study are limited for a single machine scheduling problems. Hence, it can be extended further for multiple machines and flexible manufacturing systems.

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