# Table of Contents

**Dedicatorias** ..... vii

**Abstract** ..... 1

**Resumen** ..... 3

**Resum** ..... 5

**1 Introduction** ..... 7
- 1.1 Examples of application of complex networks ..... 8
- 1.2 A perspective on divisibility networks ..... 9
- 1.3 Objectives ..... 11
- 1.4 Preliminaries ..... 12
  - 1.4.1 What is a network? ..... 12
  - 1.4.2 Basic notation and network measures ..... 12
- 1.5 Network Models ..... 20
  - 1.5.1 Random Networks ..... 20
  - 1.5.2 Small world networks ..... 28
  - 1.5.3 Scale-Free Networks ..... 31
  - 1.5.4 Hierarchical networks ..... 34
- 1.6 Network in number theory ..... 38
  - 1.6.1 Divisibility networks ..... 38
  - 1.6.2 Network analyses of conjectures in number theory ..... 43
  - 1.6.3 Networks based on Fibonacci numbers ..... 46

**2 Divisibility patterns within Pascal divisibility networks (Journal article)** ..... 49

Abstract ..... 49
- 2.1 Introduction ..... 49
- 2.2 Network analysis ..... 51
  - 2.2.1 Degree distribution ..... 51
  - 2.2.2 Local clustering coefficient ..... 54
  - 2.2.3 Global clustering coefficient ..... 57
  - 2.2.4 Assortative coefficient ..... 58
  - 2.2.5 Average path length ..... 58

2.3 Conclusions . . . . . 59

**3 Divisibility networks of the rational numbers in the unit interval (Journal article)** . . . . . 61
Abstract . . . . . 61
3.1 Introduction . . . . . 62
3.2 Matherials and Methods . . . . . 63
3.2.1 Networks $G(\mathcal{A}_n^1)$ . . . . . 64
3.2.2 Networks $G(\mathcal{A}_n^2)$ . . . . . 65
3.2.3 Networks $G(\mathcal{B}_n^1)$ . . . . . 65
3.2.4 Networks $G(\mathcal{B}_n^2)$ . . . . . 66
3.3 Results . . . . . 67
3.3.1 Degree distribution . . . . . 67
3.3.2 Density and Sparsity . . . . . 69
3.3.3 Local clustering coefficient . . . . . 70
3.3.4 Network topologies . . . . . 72
3.4 Conclusions . . . . . 74

**4 Discussion of the results and conclusions** . . . . . 77
4.1 Discussion of the results . . . . . 77
4.2 Future work . . . . . 78

**References** . . . . . 81