

Document downloaded from:

<http://hdl.handle.net/10251/176436>

This paper must be cited as:

Floris, I.; Madrigal-Madrigal, J.; Sales Maicas, S.; Calderón García, PA.; Adam, JM. (2020). Twisting measurement and compensation of optical shape sensor based on spun multicore fiber. *Mechanical Systems and Signal Processing*. 140:1-9.  
<https://doi.org/10.1016/j.ymsp.2020.106700>



The final publication is available at

<https://doi.org/10.1016/j.ymsp.2020.106700>

Copyright Elsevier

Additional Information

# Twisting measurement and compensation of Optical Shape Sensor based on Spun Multicore Fiber

Ignazio Floris<sup>a,b</sup>, Javier Madrigal<sup>b</sup>, Salvador Sales<sup>a</sup>, Pedro A. Calderón<sup>b</sup>,  
Jose M. Adam<sup>b\*</sup>

<sup>a</sup>*ITEAM, Universitat Politècnica de València, Camino de Vera s/n, Valencia, 46022, Spain*

<sup>b</sup>*ICITECH, Universitat Politècnica de València, Camino de Vera s/n, Valencia, 46022, Spain*

---

## Abstract

This study presents a novel method for twisting compensation in optical shape sensing and first reports on twisting sensing by using Fiber Bragg Gratings (FBG) inscribed in a spun Multicore Fiber (MCF). A simple approach, based on Saint-Venant's Torsion Theory for homogeneous circular cylinders, was developed to calculate the fiber twisting from the longitudinal strain sensed in the cores and reconstruct three-dimensional shape taking into account and compensating the twisting, by enhancing the mathematical formulation of the previous approaches. Then, a 44mm-long pre-twisted fiber optic shape sensor was produced in the Institute for Telecommunications and Multimedia Applications (iTEAM) of the UPV, by writing four FBGs in a spun multicore fiber (diameter of 125.1  $\mu\text{m}$ ) with a pre-twisting of 64.9 rotation/meter. Finally, a series of experiments were performed to validate the method and evaluate the sensor performance. The outcomes of the experiments, perfectly in accordance with the theory, prove the elastic behavior of the sensor, even at high levels of deformation. Moreover, the proposed Spun-MCF-based Shape Sensor resulted able to measure twisting with a sensitivity of 0.23  $\text{pm}/^\circ$  and accuracy of 4.81 $^\circ$  within a wide dynamic range of  $\pm 270^\circ$  ( $\pm 6136.4^\circ/\text{m}$ ). These new results can notably enhance the accuracy of fiber optic shape sensors and lead to the design of new fiber geometries and sensors.

**Keywords** *Spun Multicore Fiber; Twisted Multicore Fiber; Optical Shape Sensing;; Twisting Sensing; Optical Fiber Sensor; Distributed Sensing*

---

## 1. Introduction

Optical Fiber Sensors (OFS) have undergone a tremendous expansion over the last few decades in several different fields [1], such as engineering [2,3], industrial [4], medical [5], chemical [6,7] and biological [8,9]. The principal reasons behind this substantial growth are their considerable advantages over their electrical counterparts, including compactness, lightweight, electrically passive operation, resistance to harsh environments, and multiplexing capabilities. Besides OFSs have an inherent capability to sense a variety of measurands (as defined by [10]) in continuous development, such as strain [11,12], temperature [7], moisture [13], vibrations [14], chemical agents [9], and many others [15], using the optical fiber itself as a sensor.

One of the current frontier of the fiber-optic sensing technologies is shape sensing [16], which has been an area of great interest for many researchers and consists in the possibility to dynamically track position and shape of an optical fiber. Fiber Optic Shape Sensors (FOSS) consist of optical Multicore Fibers (MCF) (or sometimes multi-fiber cables, with the same section geometry but larger

---

\* Corresponding author.

E-mail address: joadmar@upv.es (J.M. Adam)

39 core spacing) capable of sensing multidimensional curvature along the sensor's length, by  
40 comparing the longitudinal strain detected in different cores, and, hence, reconstructing shape [17].

41 Since curvature sensing using Fiber Bragg Gratings (FBG) inscribed in different cores of a MCF  
42 was first reported almost 20 years ago [18], a lot of progress has been made in this new branch. The  
43 two-dimensional shape of a multicore fiber was estimated based on distributed strain measurement  
44 [19,20]. An innovative approach for the three-dimensional shape reconstruction of a multicore  
45 optical fiber based on the numerical integration a set of Frenet-Serret equations was proposed by  
46 Moore et al. [21]. Fiber-optic shape sensing based on FBGs written in a MCF was employed in  
47 catheters, needles and minimally invasive surgery systems [5,22,23]. Tunnel monitoring was  
48 performed using multicore fiber displacement sensor [24]. Barrera et al. demonstrated a multicore  
49 optical fiber shape sensors suitable for use under gamma radiation [25].

50 Regrettably, due to the high flexibility, FOSSs are subject to twisting, in addition to bending and  
51 longitudinal strain. The twisting notably reduces the shape sensors' accuracy, causing significant  
52 uncertainty in the bending direction determination, when performing 3D shape sensing.  
53 Notwithstanding the vast amount of research conducted on shape sensing, this problem has not been  
54 addressed yet in an adequate manner. Tan et al. developed a torsion sensor based on inter-core mode  
55 coupling by tapering a multicore seven-core fiber [27]. Notwithstanding the innovation of this  
56 approach, it cannot be employed for shape sensing, since the fiber structure becomes  
57 inhomogeneous due to the tapering. Askins et al. first investigated the twisting of optical fibers using  
58 a tether fiber [26]. Despite its remarkable novelty, such research is not representative of the  
59 overwhelming majority of fiber optic shape sensors, consisting of MCF, and does not provide any  
60 information about the behavior of these sensors at high levels of twisting deformation, since only  
61 analyzed a limited dynamic range of twisting rotation, between  $\pm 600^\circ/\text{m}$ . Twisting measurement  
62 using MCF is an extremely arduous task due to the littleness of the state of strain generated, as a  
63 result of the small core spacing. In order to overcome this limit, the multicore fiber could be pre-  
64 twisted to increase the twisting sensibility. In this respect, recent progresses in fabrication techniques  
65 have made possible the manufacturing of spun/twisted multicore fiber with very small spin pitch,  
66 20mm (50 turn/m) [28] and even 15.4 mm (64.9 turn/m) [29,30], although their performance have  
67 not been investigated in depth. Nevertheless, an in-depth study focused on the performance of this  
68 new special multicore fiber is still missing.

69 The present work shows an innovative method to compensate the twisting of FOSSs and, thereby,  
70 enhances the accuracy in 3D shape reconstruction in presence of fiber twisting, by employing a spun  
71 multicore fiber (also called twisted multicore fiber), with one of the most used geometry for sensing  
72 applications: the seven-core fiber. The novelty of this research comes from: 1) the development of  
73 new and simple mathematical approach to calculate the twisting of multicore fiber based on the  
74 Saint-Venant's Torsion Theory for homogeneous circular cylinders; 2) the enhancement of one of  
75 the most used approaches for three-dimensional shape reconstruction in order to take into account  
76 the fiber twisting and automatically compensate it; 3) an experimental study carried out to  
77 successfully corroborate the mathematical approach and to evaluate the performance of optical  
78 shape sensor manufactured employing an innovative multicore spun fiber. Therewith, this research  
79 first demonstrates twisting sensing based on spun MCF shape sensor and illustrates the strong  
80 influence of the core spacing and spin pitch on the accuracy of pre-twisted multicore sensor in  
81 twisting sensing, laying the foundation for the design of new fiber geometries and new sensors.

## 82 **2. Shape sensing with twisting compensation**

83 This section illustrates an innovative approach, based on the Saint-Venant's Torsion Theory for  
84 homogeneous circular cylinders, to calculate the multicore fiber twisting from the longitudinal

85 strain of sensed in fiber cores and presents an enhancement of the method, proposed by Moore and  
 86 Rogge [21], in order to reconstruct shape in presence of fiber twisting using spun MCF.

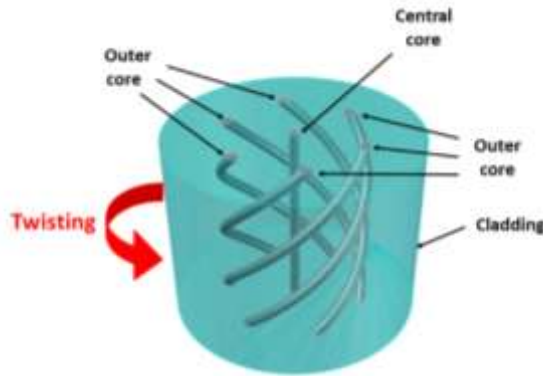
87 *2.1. Twisting sensing*

88 An optical fiber, subject to pure torsion/external twisting, which must not be confused with the  
 89 geometric torsion  $\tau$ , can be studied as a circular cylinder that has one fixed end and the other rotates  
 90 of an angle  $\theta$ . Hypothesizing that the fiber has perfectly elastic behavior, plane sections remain  
 91 plane, radii remain straight and cross sections remain plane and circular, it is possible to apply the  
 92 Saint-Venant's Torsion Theory for homogeneous circular cylinders [31]. Thereby, the sensor is in  
 93 a state of pure shear and the shear strain  $\gamma$  in an element of the sensor is given by Eq. (1).

94 
$$\gamma = r \frac{d\theta}{ds} \tag{1}$$

95 where  $r$  is the radial distance of the element from the sensor axis and  $d\theta/ds$  represents the rate of  
 96 change of the angle of twisting. Since every cross section has the same radius and is subjected to  
 97 the same torque, the angle  $\theta(s)$  varies linearly between extremities and  $d\theta/ds$  is constant.  
 98 Furthermore, the shear strain varies linearly with  $r$ , from zero at the centerline to a peak value at the  
 99 free surface.

100 To calculate the longitudinal strain in a core, distant  $r$  from the axis, it can be considered that the  
 101 cores, initially straight, become circular helices ( $d\theta/ds$  is constant) due to twisting, as shown in Fig.  
 102 1.



103 **Fig. 1.** Twisted multicore seven-core fiber.

104 The length of a core in a twisted multicore fiber can be calculated as the length of a circular helix  
 105 by Eq. (2):

106 
$$x = \sqrt{h^2 + r^2\theta^2} \tag{2}$$

107 where  $x$  is the length of the core,  $h$  is the length of the sensor and  $r$  is the distance of the core  
 108 from the sensor axis and the reciprocal distance between the closest outer cores, generally called  
 109 core-to-core spacing or, simply, core spacing.

110 Thus, the longitudinal strain of the cores due to twisting can be calculated from the definition of  
 111 strain (Eq. (3)).

112  
 113

$$\varepsilon = \frac{x-h}{h} = \frac{\sqrt{h^2+r^2\theta^2}-h}{h} \quad (3)$$

As it can be noticed from (3), the longitudinal strain is remarkably influenced by the core spacing, length of the sensor and angle of twisting being the same. Moreover, the central core is not affected by the twisting, since its axis coincides with the sensor axis. Fig. 2 shows the variation of longitudinal strain due to twisting of an outer core in relation to core spacing and twisting angle per meter.

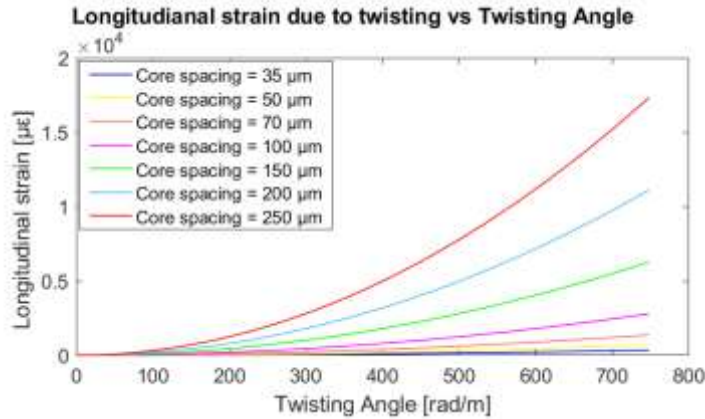


Fig. 2. Longitudinal strain due to twisting in relation to core spacing and twisting angle.

When performing three-dimensional shape sensing, the sensor is not only subject to twisting but also to bending, axial strain and thermal expansion. In this case, the component of strain due to twisting,  $\varepsilon_{twist}$ , can be calculated as defined in Eq. (4).

$$\varepsilon_{twist} = \frac{\sum_{i=1}^n \varepsilon_i^{outer}}{(n-1)} - \varepsilon_{central} \quad (4)$$

where  $\varepsilon_i^{outer}$  is the strain of the  $i$ -th outer core,  $\varepsilon_{central}$  is the strain of the central core and  $n$  is the number of cores.

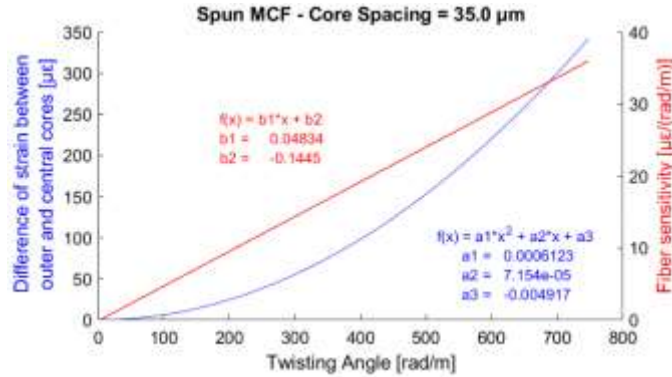
Eq. (4) is valid for the seven-core section geometry studied in this research and for all the section geometries with a central core and several outer cores, equidistant from the sensor axis and with equal angular spacing between them, such as the four-core and five-core section geometries with, respectively, 3 and 4 outer cores and 1 central core.

Finally, the angle of twisting can be calculated from the strain sensed in the cores, combining (3) and (4), as defined in Eq. (5).

$$\theta = \sqrt{\frac{h^2 \varepsilon_{twist}^2 + 2h^2 \varepsilon_{twist}}{r^2}} \quad (5)$$

It has to be pointed out that the longitudinal strain generated in the outer cores by the twisting of a non-twisted multicore fiber does not depend on direction of rotation. In other words, the twisting of a non-twisted MCF of an angle  $-\theta$  or  $+\theta$  always produces a positive longitudinal strain variation of the outer cores, making impossible to distinguish the sense of twisting rotation from the strain detected in the cores. To avoid this problem, a spun/pre-twisted MCF has to be used. Thereby, a twisting rotation in the direction concordant with the pre-twisting rotation produces an elongation

142 of the outer cores, while, a twisting in the opposite direction, produces shortening. Furthermore,  
 143 using a spun multicore fiber is also possible to increase the sensor sensitivity to twisting, as shown  
 144 in Fig. 3 for a multicore fiber with core spacing of 35  $\mu\text{m}$ .  
 145



146 **Fig. 3.** Longitudinal strain due to twisting and fiber sensitivity to twisting in relation twisting angle for a multicore  
 147 fiber with core spacing of 35  $\mu\text{m}$ .

148 **2.2. Shape reconstruction with twisting compensation**

149 Fiber optic shape sensors are OFSSs with multiple cores and embedded strain sensors. Under the  
 150 Kirchhoff's rod hypotheses [32], the longitudinal strain of the fiber and the three-dimensional  
 151 curvature along fiber's length can be calculated from the strain sensed by the cores, using the Eqs.  
 152 (6), (7), and (8) [17,33–35].

$$153 \begin{cases} \varepsilon_{long} = \sum_{i=1}^n \varepsilon_i / n \\ \kappa_x = \sum_{i=1}^n x_i \varepsilon_i / \sum_{i=1}^n x_i^2 \\ \kappa_y = \sum_{i=1}^n y_i \varepsilon_i / \sum_{i=1}^n y_i^2 \end{cases} \quad (6)$$

$$154 |\kappa| = \sqrt{\kappa_x^2 + \kappa_y^2} \quad (7)$$

$$155 \alpha = \tan^{-1}(\kappa_x / \kappa_y) \quad (8)$$

156 where  $\varepsilon_{long}$  is the longitudinal strain,  $\kappa_x$ ,  $\kappa_y$  and  $|\kappa|$  are the two components of curvature and the  
 157 curvature magnitude,  $x_i$ ,  $y_i$  and  $\varepsilon_i$  are, respectively, the coordinates and the strain of the  $i$ -th core,  
 158 and  $\alpha$  is the bending direction angle, which defines the bending direction.

159 Once calculated the fiber twisting along the fiber, its effects can be compensated by applying the  
 160 superposition principle and correcting the bending direction angle in each instrumented section  
 161 according to Eq. (9).

$$162 \alpha' = \alpha - \theta \quad (9)$$

163 where  $\alpha'$  is the compensated bending direction angle.

164 Hence, by mean of interpolation or curve fitting [21,36], the functions of curvature,  $\kappa(s)$ , and  
 165 torsion,  $\tau(s)$ , along the fiber can be determined respectively from curvature and from the bending  
 166 direction angle (see Eqs. (9) and (10)).

$$167 \quad \tau(s) = \frac{d\alpha'}{ds} \quad (10)$$

168 Finally, the 3D shape of the sensors can be obtained through numerical integration of the Frenet-  
169 Serret formulas (Eq. (11))[21]:

$$170 \quad \begin{bmatrix} \mathbf{T}' \\ \mathbf{N}' \\ \mathbf{B}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{bmatrix} \quad (11)$$

171 where  $\mathbf{T}$ ,  $\mathbf{N}$  and  $\mathbf{B}$  are, respectively, the tangent, normal and binormal vectors.

172 It is worth noting that the approach here presented is valid for several optical strain sensing  
173 technologies, including FBGs, Brillouin and Rayleigh scattering, and for different number of cores  
174 and sensor geometries, including multi-fiber shape sensors and multicore fiber shape sensors.  
175 Nonetheless, this research is only focused on multicore fiber shape sensors, since they are the most  
176 used [5,17,21,23], thanks to their compactness, ease of handling and availability, being employed  
177 for communication purposes.

178 It is also worth pointing out that, when it is valid the assumption of absence or negligibility of  
179 external twisting/torsion, its compensation can be omitted, for instance when the shape sensor is  
180 fastened to a rigid support, although this would limit its handiness and compactness and restrict its  
181 possible applications. Whereas, in general cases, if not compensated, the twisting leads to  
182 remarkable uncertainty in the determination of the bending direction and drastically reduces the  
183 accuracy of the sensor in performing three-dimensional shape sensing.

### 184 3. Spun fiber optic shape sensor fabrication

185 A pre-twisted fiber optic shape sensor was produced in the Institute of Telecommunications and  
186 Multimedia Applications (iTEAM) of the *Universitat Politècnica de València* (UPV) by writing  
187 Fiber Bragg Gratings in a spun 7-core multicore fiber (see Fig. 1) with a spin pitch of 15.4 mm (64.9  
188 rotation/meter), manufactured and provided by FIBERCORE Ltd. [29,37]. The fiber had seven  
189 single-mode cores (mode field diameter of 6.4  $\mu\text{m}$  and numerical aperture of 0.2) with doubly  
190 symmetric configuration (60° of angular spacing and core spacing of 35  $\mu\text{m}$ ) and a cladding  
191 diameter of 125.1  $\mu\text{m}$ .

192 The spun multicore fiber was hydrogen-loaded for 14 days at ambient temperature and at a  
193 pressure of 20 bars with the purpose of improving the photosensitivity. Afterward, four FBG were  
194 inscribed in a 44mm long portion of the fiber by means of a 244 nm CW frequency-doubled argon-  
195 ion laser with 60 mW output power using the phase-mask method [38]. The size of the laser beam  
196 was adjusted in order to reach all cores and the inscription was carried out simultaneously in the  
197 seven cores.

### 198 4. Experimental setup

199 Fig. 4 illustrates the experimental setup. The pre-twisted fiber optic shape sensor was fastened  
200 with two fiber rotators, situated at 44 mm distance from one another and assembled on multi-axis  
201 stages for nano-positioning.

202  
203  
204  
205

206  
207

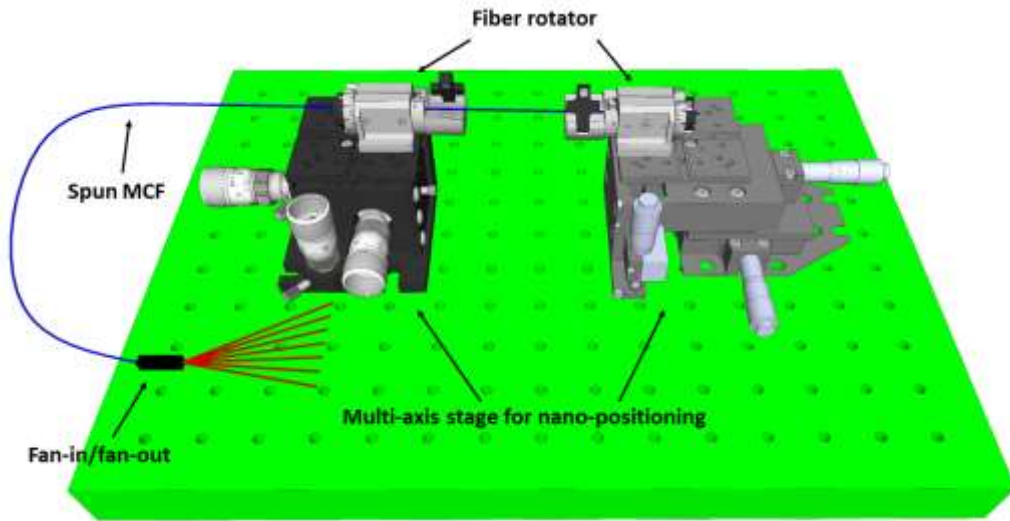


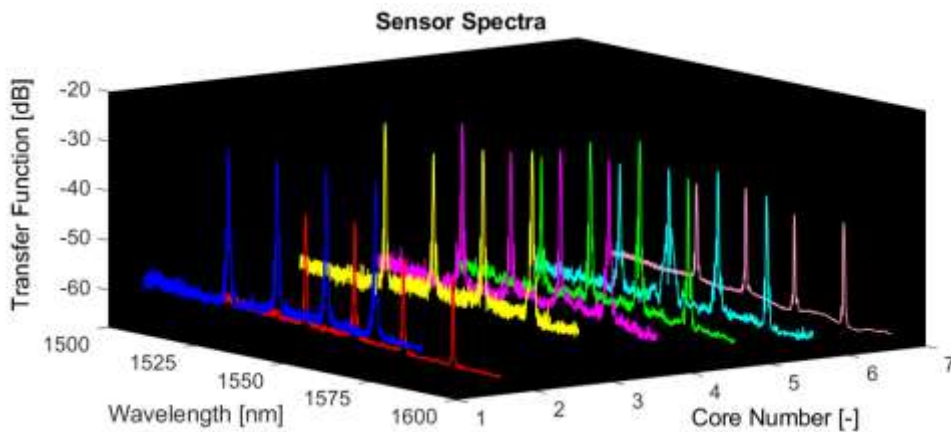
Fig. 4. Experimental setup.

208  
209  
210  
211  
212  
213  
214  
215

First, the left fiber rotator was turned from  $0^\circ$  to  $270^\circ$  in the direction coherent with the pre-twisting and, then, moved back to the initial position. This sense of rotation is defined positive because stretches the outer cores. Thereafter, the right rotator was rotated between  $0^\circ$  and  $-270^\circ$  in the negative sense of rotation, which shortens the outer cores. The sensor was interrogated by using a Static Optical Sensing Interrogator (sm125) combined with a Channel Multiplexer (sm041) (Micron Optics).

216 **5. Results and discussion**

217 The pre-twisted multicore shape sensor was placed on the fiber rotators and the core spectra were  
218 recorded. Then, the FBG peaks were tracked during the experiments and their wavelength shifts  
219 were calculated and converted into strain, dividing them by a gauge factor value of  $1.2 \text{ pm}/\mu\epsilon$ ,  
220 obtained from several tensile tests and in accordance with the literature [39]. The spectra of the  
221 seven cores are plotted in Fig. 5.



222



223 **Fig. 5.** Core spectra (the central core is the core number 1, while the cores from 2 to 7 are outer cores ordered in  
 224 clockwise direction).

225 With the aim of evaluating the accuracy of the sensor in measuring twisting, the strain of the  
 226 cores was tracked, while the sensor was being twisted in the positive and negative sense of rotation,  
 227 considering as positive the direction of rotation that elongates the outer cores. The values of twisting  
 228 angle applied during the experiments and the resulting values of strain of the outer core due to  
 229 twisting, calculated in accordance with the theoretical approach presented in Subsection 2.1, are  
 230 listed in Table 1.

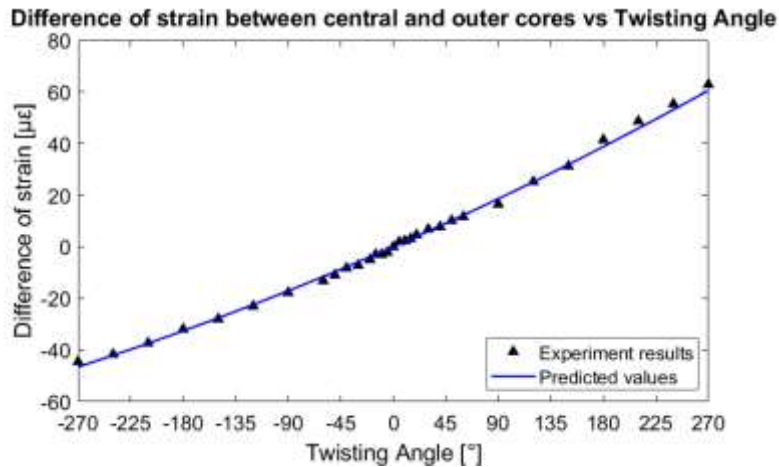
231 **Table 1.** Inputs and outputs of the experiments according to the theoretical approach presented in Subsection 2.1.

Test n°	Twisting rotation [°]	Twisting and pre-twisting rotation [rad]	Fiber spin pitch [mm]	Outer cores strain variation due to twisting and pre-twisting [με]	Outer cores strain variation due to twisting [με]
0	0	17.95	15.40	101.95	0.00
1	5	18.04	15.33	102.95	0.99
2	10	18.13	15.25	103.95	1.99
3	15	18.21	15.18	104.95	3.00
4	20	18.30	15.11	105.96	4.00
5	30	18.48	14.96	107.99	6.03
6	40	18.65	14.82	110.04	8.08
7	50	18.82	14.69	112.11	10.15
8	60	19.00	14.55	114.19	12.24
9	90	19.52	14.16	120.57	18.62
10	120	20.05	13.79	127.13	25.18
11	150	20.57	13.44	133.86	31.90
12	180	21.09	13.11	140.76	38.80
13	210	21.62	12.79	147.83	45.88
14	240	22.14	12.49	155.08	53.12
15	270	22.66	12.20	162.50	60.55
0	0	17.95	15.40	101.95	0.00
1	-5	17.86	15.48	100.96	-0.99
2	-10	17.78	15.55	99.98	-1.97
3	-15	17.69	15.63	99.00	-2.95
4	-20	17.60	15.71	98.03	-3.93
5	-30	17.43	15.86	96.09	-5.86
6	-40	17.25	16.02	94.18	-7.78
7	-50	17.08	16.19	92.28	-9.67
8	-60	16.90	16.35	90.41	-11.55
9	-90	16.38	16.88	84.89	-17.06
10	-120	15.86	17.43	79.55	-22.40
11	-150	15.33	18.03	74.39	-27.57
12	-180	14.81	18.67	69.39	-32.56
13	-210	14.29	19.35	64.57	-37.38
14	-240	13.76	20.09	59.93	-42.03
15	-270	13.24	20.88	55.45	-46.50

232

233

234 Fig. 6 shows the comparison between the predicted values and the outcomes of the experiments.  
 235 The experiments results are perfectly consistent with the outputs of the theoretical approach,  
 236 proving that the hypotheses of the Saint-Venant's Torsion Theory hold in this problem and that the  
 237 fiber has elastic behavior even at high values of deformation. The sensor was able to detect the fiber  
 238 twisting with an average sensitivity of  $0.23 \text{ pm}/^\circ$  and an accuracy of  $4.81^\circ$  within a wide dynamic  
 239 range of  $\pm 270^\circ$  ( $\pm 6136.4^\circ/\text{m}$ ), while the a maximum error was of  $13.53^\circ$ .



240 Fig. 6. Comparison between predicted values and experiment outcomes.

241

242

243 It has to be highlighted that, even though the fiber has a perfectly elastic behavior, the  
 244 relationship between strain and twisting angle is a nonlinear and monotonically increasing function,  
 245 as shown by Eq. (5) and Fig. 6. In the light of this, the sensitivity of the sensor to twisting as well  
 246 as its accuracy grow with the increasing twisting and pre-twisting rotations (decreasing spin pitch).  
 247 Consequently, the performance of the sensor can be improved by increasing the pre-twisting  
 248 rotation, in addition to increasing the core spacing (see Eq. (9)). Moreover, it has to be emphasized  
 249 that no temperature compensation was necessary, since the distance between the cores of the spun  
 250 multicore fiber is extremely small ( $35 \text{ }\mu\text{m}$ ) and, therefore, it can be assumed that the temperature is  
 constant in the section.

## 251 6. Conclusions

252 This paper reports on experimental study carried out to investigate the performance of an  
 253 innovative spun-MCF-based fiber optic shape sensor in twisting sensing and presents a simple  
 254 method enhance the accuracy in shape sensing, by compensating the fiber twisting.

255 The outcomes of the experiments, perfectly consistent with the theory, first demonstrate that  
 256 optical shape sensors based on spun multicore fiber are efficiently able to sense twisting and prove  
 257 the perfectly elastic behavior of the fiber, even at high levels of twisting deformation, assumption  
 258 underpinning the Saint-Venant's Torsion Theory. Altogether, the sensor, long  $44 \text{ mm}$ , was able to  
 259 sense twisting with a sensitivity of  $0.23 \text{ pm}/^\circ$  and accuracy of  $4.81^\circ$  within a wide dynamic  
 260 range of  $\pm 270^\circ$  ( $\pm 6136.4^\circ/\text{m}$ ).

261 In addition, this research, by defining the influence of core spacing and spin pitch on the accuracy  
 262 of pre-twisted fiber optic shape sensor in twisting sensing, lays the foundations for the design of a  
 263 new generation of optical shape sensors.

264 **Acknowledgments**

265 This work was carried out within the ITN-FINESSE framework, funded by the European  
266 Union's Horizon 2020 Research and Innovation Program under the Marie Skłodowska-Curie  
267 Action Grant Agreement N° 722509. It was also supported by the Ministry of Economy and  
268 Competitiveness of Spain under DIMENSION TEC2017-88029-R.

269 **References**

- 270 [1] D. Tosi, E. Schena, C. Molardi, S. Korganbayev, Fiber optic sensors for sub-centimeter spatially resolved  
271 measurements: Review and biomedical applications, *Opt. Fiber Technol.* 43 (2018) 6–19.  
272 doi:10.1016/j.yofte.2018.03.007.
- 273 [2] J. He, Z. Zhou, O. Jinping, Optic fiber sensor-based smart bridge cable with functionality of self-sensing, *Mech.*  
274 *Syst. Signal Process.* 35 (2013) 84–94. doi:10.1016/j.ymsp.2012.08.022.
- 275 [3] N. Roveri, A. Carcaterra, A. Sestieri, Real-time monitoring of railway infrastructures using fibre Bragg grating  
276 sensors, *Mech. Syst. Signal Process.* 60–61 (2015) 14–28. doi:10.1016/j.ymsp.2015.01.003.
- 277 [4] H. Alian, S. Konforty, U. Ben-Simon, R. Klein, M. Tur, J. Bortman, Bearing fault detection and fault size  
278 estimation using fiber-optic sensors, *Mech. Syst. Signal Process.* 120 (2019) 392–407.  
279 doi:10.1016/j.ymsp.2018.10.035.
- 280 [5] A. Beisenova, A. Issatayeva, D. Tosi, C. Molardi, Fiber-Optic Distributed Strain Sensing Needle for Real-Time  
281 Guidance in Epidural Anesthesia, *IEEE Sens. J.* 18 (2018) 8034–8044. doi:10.1109/JSEN.2018.2865220.
- 282 [6] A. Sultangazin, J. Kusmangaliyev, A. Aitkulov, D. Akilbekova, M. Olivero, D. Tosi, Design of a Smartphone  
283 Plastic Optical Fiber Chemical Sensor for Hydrogen Sulfide Detection, *IEEE Sens. J.* 17 (2017) 6935–6940.  
284 doi:10.1109/JSEN.2017.2752717.
- 285 [7] J. Madrigal, D. Barrera, S. Sales, Refractive Index and Temperature Sensing Using Inter-Core Crosstalk in  
286 Multicore Fibers, *J. Light. Technol.* 37 (2019) 4703–4709. doi:10.1109/JLT.2019.2917629.
- 287 [8] M. Piestrzyńska, M. Dominik, K. Kosiel, M. Janczuk-Richter, K. Szot-Karpińska, E. Brzozowska, L. Shao, J.  
288 Niedziółka-Jonsson, W.J. Bock, M. Śmietana, Ultrasensitive tantalum oxide nano-coated long-period gratings  
289 for detection of various biological targets, *Biosens. Bioelectron.* 133 (2019) 8–15.  
290 doi:10.1016/j.bios.2019.03.006.
- 291 [9] Y. Zhao, X. Zhou, X. Li, Y. Zhang, Review on the graphene based optical fiber chemical and biological sensors,  
292 *Sensors Actuators B Chem.* 231 (2016) 324–340. doi:10.1016/j.snb.2016.03.026.
- 293 [10] I. Gertsbakh, *Measurement Theory for Engineers*, in: *Meas. Theory Eng.*, Springer Berlin Heidelberg, Berlin,  
294 Heidelberg, 2003. doi:10.1007/978-3-662-08583-7\_1.
- 295 [11] P. Li, Z. Zhang, J. Zhang, Simultaneously identifying displacement and strain flexibility using long-gauge fiber  
296 optic sensors, *Mech. Syst. Signal Process.* 114 (2019) 54–67. doi:10.1016/j.ymsp.2018.05.005.
- 297 [12] Z. Zhou, W. Liu, Y. Huang, H. Wang, H. Jianping, M. Huang, O. Jinping, Optical fiber Bragg grating sensor  
298 assembly for 3D strain monitoring and its case study in highway pavement, *Mech. Syst. Signal Process.* 28  
299 (2012) 36–49. doi:10.1016/j.ymsp.2011.10.003.
- 300 [13] M. Mieloszyk, W. Ostachowicz, Moisture contamination detection in adhesive bond using embedded FBG  
301 sensors, *Mech. Syst. Signal Process.* 84 (2017) 1–14. doi:10.1016/j.ymsp.2016.07.006.
- 302 [14] P.L. Fuhr, P.J. Kajenski, D.L. Kunkel, D.R. Huston, A subcarrier intensity modulated fiber optic sensor for  
303 structural vibration measurements, *Mech. Syst. Signal Process.* 7 (1993) 133–143. doi:10.1006/mssp.1993.1003.
- 304 [15] D. Zheng, Z. Cai, I. Floris, J. Madrigal, W. Pan, X. Zou, S. Sales, X.I.Z. Ou, S.A.S. Ales, Temperature-  
305 insensitive optical tilt sensor based on a single eccentric-core fiber Bragg grating, *Opt. Lett.* (n.d.).  
306 doi:10.1364/OL.99.099999.
- 307 [16] M. Amanzadeh, S.M. Aminossadati, M.S. Kizil, A.D. Rakić, Recent developments in fibre optic shape sensing,  
308 *Meas. J. Int. Meas. Confed.* 128 (2018) 119–137. doi:10.1016/j.measurement.2018.06.034.

- 309 [17] I. Floris, J. Madrigal, S. Sales, J.M. Adam, P.A. Calderón, Experimental study of the influence of FBG length on  
310 optical shape sensor performance, *Opt. Lasers Eng.* 126 (2020) 105878. doi:10.1016/j.optlaseng.2019.105878.
- 311 [18] M.J. Gander, W.N. MacPherson, R. McBride, J.D.C. Jones, L. Zhang, I. Bennion, P.M. Blanchard, J.G. Burnett,  
312 a H. Greenaway, Bend measurement using Bragg gratings in multicore fibre, *Electron. Lett.* 36 (2000) 2–3.
- 313 [19] G.A. Miller, C.G. Askins, E.J. Friebele, Shape sensing using distributed fiber optic strain measurements, in:  
314 *Second Eur. Work. Opt. Fibre Sensors, 2004*: p. 528. doi:10.1117/12.566653.
- 315 [20] R.G. Duncan, M.E. Froggatt, S.T. Kreger, R.J. Seeley, D.K. Gifford, A.K. Sang, M.S. Wolfe, High-accuracy  
316 fiber-optic shape sensing, in: K.J. Peters (Ed.), 2007: p. 65301S. doi:10.1117/12.720914.
- 317 [21] J.P. Moore, M.D. Rogge, Shape sensing using multi-core fiber optic cable and parametric curve solutions, *Opt.*  
318 *Express.* 20 (2012) 2967. doi:10.1364/OE.20.002967.
- 319 [22] H. Moon, J. Jeong, S. Kang, K. Kim, Y.-W. Song, J. Kim, Fiber-Bragg-grating-based ultrathin shape sensors  
320 displaying single-channel sweeping for minimally invasive surgery, *Opt. Lasers Eng.* 59 (2014) 50–55.  
321 doi:10.1016/j.optlaseng.2014.03.005.
- 322 [23] F. Khan, A. Denasi, D. Barrera, J. Madrigal, S. Sales, S. Misra, Multi-core Optical Fibers with Bragg Gratings as  
323 Shape Sensor for Flexible Medical Instruments, *IEEE Sens. J. PP* (2019) 1–1. doi:10.1109/JSEN.2019.2905010.
- 324 [24] W.N. MacPherson, M. Silva-Lopez, J.S. Barton, a J. Moore, J.D.C. Jones, D. Zhao, L. Zhang, I. Bennion, N.  
325 Metje, D.N. Chapman, C.D.F. Rogers, Tunnel monitoring using multicore fibre displacement sensor, *Meas. Sci.*  
326 *Technol.* 17 (2006) 1180–1185. doi:10.1088/0957-0233/17/5/S41.
- 327 [25] D. Barrera, J. Madrigal, S. Delepine-Lesoille, S. Sales, Multicore optical fiber shape sensors suitable for use  
328 under gamma radiation, *Opt. Express.* 27 (2019) 29026. doi:10.1364/OE.27.029026.
- 329 [26] C.G. Askins, G.A. Miller, E.J. Friebele, Bend and Twist Sensing in a Multiple-Core Optical Fiber, in:  
330 *OFC/NFOEC 2008 - 2008 Conf. Opt. Fiber Commun. Fiber Opt. Eng. Conf., IEEE, 2008*: pp. 1–3.  
331 doi:10.1109/OFC.2008.4528404.
- 332 [27] F. Tan, Z. Liu, J. Tu, C. Yu, C. Lu, H.-Y. Tam, Torsion sensor based on inter-core mode coupling in seven-core  
333 fiber, *Opt. Express.* 26 (2018) 19835. doi:10.1364/oe.26.019835.
- 334 [28] P.S. Westbrook, K.S. Feder, T. Kremp, T.F. Taunay, E. Monberg, J. Kelliher, R. Ortiz, K. Bradley, K.S. Abedin,  
335 D. Au, G. Puc, Integrated optical fiber shape sensor modules based on twisted multicore fiber grating arrays, in:  
336 I. Gannot (Ed.), 2014: p. 89380H. doi:10.1117/12.2041775.
- 337 [29] L.J. Cooper, A.S. Webb, A. Gillooly, M. Hill, T. Read, P. Maton, J. Hankey, A. Bergonzo, Design and  
338 performance of multicore fiber optimized towards communications and sensing applications, in: S. Jiang, M.J.F.  
339 Dignonnet (Eds.), *Proc. SPIE - Int. Soc. Opt. Eng.*, 2015: p. 93590H. doi:10.1117/12.2076950.
- 340 [30] <https://www.fibercore.com/product/multicore-fiber>, <https://www.fibercore.com/product/multicore-fiber>, (n.d.).
- 341 [31] A.E.H. Love, *A Treatise on the Mathematical Theory of Elasticity*, 1st ed., Cambridge University Press, 1892.  
342 <https://hal.archives-ouvertes.fr/hal-01307751>.
- 343 [32] O.M. O'Reilly, Kirchhoff's Rod Theory, in: *Model. Nonlinear Probl. Mech. Strings Rods Role Balanc. Laws*,  
344 Springer International Publishing, Cham, 2017: pp. 187–268. doi:10.1007/978-3-319-50598-5\_5.
- 345 [33] I. Floris, S. Sales, P.A. Calderón, J.M. Adam, Measurement uncertainty of multicore optical fiber sensors used to  
346 sense curvature and bending direction, *Meas. J. Int. Meas. Confed.* 132 (2019).  
347 doi:10.1016/j.measurement.2018.09.033.
- 348 [34] I. Floris, P.A. Calderón, S. Sales, J.M. Adam, Effects of core position uncertainty on optical shape sensor  
349 accuracy, *Measurement.* 139 (2019) 21–33. doi:10.1016/j.measurement.2019.03.031.
- 350 [35] I. Floris, J.M. Adam, P.A. Calderón, S. Sales, Measurement uncertainty of 7-core multicore fiber shape sensors,  
351 in: K. Kalli, G. Brambilla, S.O. O'Keefe (Eds.), *Seventh Eur. Work. Opt. Fibre Sensors, SPIE, 2019*: p. 22.  
352 doi:10.1117/12.2539421.
- 353 [36] S. Jäckle, J. Strehlow, S. Heldmann, Shape Sensing with Fiber Bragg Grating Sensors, in: H. Handels, T.M.  
354 Deserno, A. Maier, K.H. Maier-Hein, C. Palm, T. Tolxdorff (Eds.), *Bild. Für Die Medizin 2019*, Springer  
355 Fachmedien Wiesbaden, Wiesbaden, 2019: pp. 258–263. doi:10.1007/978-3-658-25326-4\_58.
- 356 [37] <https://www.fibercore.com/product/multicore-fiber>, (n.d.).
- 357 [38] R. Kashyap, *Fiber Bragg Gratings*, 2nd Editio, Elsevier, 2010. <https://www.elsevier.com/books/fiber-bragg->

- 358 gratings/978-0-12-372579-0.  
359 [39] R.J. Black, D. Zare, L. Oblea, Y.-L. Park, B. Moslehi, C. Neslen, On the Gage Factor for Optical Fiber Grating  
360 Strain Gages, Proc. Soc. Adv. Mater. Process Eng. 52, 2008 (2008).  
361 [http://www.cs.cmu.edu/~ylpark/publications/SAMPE\\_08.pdf](http://www.cs.cmu.edu/~ylpark/publications/SAMPE_08.pdf).  
362