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Strip-pair comparison method for building threshold color-difference model: theoretical model validation

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Abstract: This paper presents a method for developing color-difference models near a threshold, based on the serial exploration method described by Torgerson [Theory and Methods of Scaling; Wiley & Sons (1958); Chap. 7], involving the construction of color-control strips of patches arranged in arrays of $2 \times$ n, where n is the number of pairs in the strip. The patches in the lower row should be calorimetrically identical, while the color of the patches in the upper row should vary progressively in constant steps of CIELAB color difference along selected color space vector directions. Prospective observers are instructed to indicate the patch pair number for which they begin to perceive a slight color difference between corresponding patches. The frequency data obtained from the observers was used to build a threshold color-difference model. The intention was to validate the method with theoretical data to determine the effect of the precision with which the strips are constructed, on the accuracy of the estimated parameters. Theoretical frequency data was generated using the CIE94 color difference formula, whose associated color discrimination ellipsoid parameters are very easy to determine, associated with a hypothetical logistic psychometric curve for different color centers. The proposed method allows to determine color discrimination parameters with a precision nearby 4% and an accuracy of 3% with respect to the simulated theoretical parameters, for color samples generated with a standard deviation of ΔE^{*}_{ab}=0.2 of the superimposed error around the ideal color difference of pairs of patches.

1. Introduction

The topic of color difference has been extensively studied. In 1942, MacAdam [1], Brown [2–3], and Wyszecki and Fielder [4] attempted to determine the precision of self-luminous color matching for normal trichromats. Later, Rich, Billmayer, and Howe [5] aimed to determine the color-difference ellipsoids for surface color samples. A number of other authors [6–14] in a variety of fields, using an assortment of materials and methods, obtained the color-difference datasets required for the development and improvement of color-difference equations. Currently, the most commonly used formulas for calculating color differences in industrial applications are the CIELAB, CMC, CIE94, CIEDE2000, and CAM02 formulas. The majority of these formulas were derived from the datasets obtained through different psychometric experiments, in which the observers are asked about their subjective sensation regarding the color difference between a number of different samples and a reference. The grey-scale [11] and pass-fail methods are the most commonly used ones; the latter has two variants, a constant-stimuli variant [8–10,13–14] (also known as pair comparison or the anchor-pair method) and a threshold variant [5,11].

The performance of color-difference datasets and formulas, according to different fitting criteria (PF/3 [15], STRESS [16], and Pearson's correlation coefficient [17]), have been verified numerous times [17–24].

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However, in the course of color-difference research, numerous problems have arisen, such as the significant differences in the shape, size, and orientation of the ellipsoids [4,11]; poor fitting [22]; differences in the results depending on the method used [20,23] (grey-scale or pair comparison); and the dependency on the results of the anchor pair used in constant stimuli experiments [24]. These issues prompted the International Commission on Illumination (CIE), which maintains color-difference committees, to repeatedly call for a coordinated effort by the research community [25,26] and for the submission of color datasets to improve the color-difference formulas.

Additionally, the construction of color-difference datasets is a difficult and tedious process because of the large number of factors affecting color-difference perception. The process is also variable, which requires each pair to be judged a large number of times to obtain statistically significant results.

To accelerate the collection of color-difference data, an alternative to the grey-scale and pair-comparison methods is proposed in section 2, called Strip-Pair Comparison Method (SCM), which is based on the serial exploration method described by Torgerson [27] for scaling general psychophysical attributes. This technique, as described in part *a* of section 2, is based on the construction of color strips consisting of pairs of patches whose color difference increases in defined directions within the selected color space, in a scheme of directions similar to those proposed by Alman and Berns [13–14] and Brusola et al. [28]. In principle, the metrics determined using the SCM are applicable only in the determination of small color differences near the threshold. However, SCM can be easily adapted by using an anchor pair. Model checking of the proposed SCM is performed in part *b* of the methods section. In part *b* we verify the correct performance of SCM with theoretical frequency data generated from CIELAB color differences of the patch pairs in the strips with and without the presence of noise.

2. Methods

We have subdivided this section into two parts. In part a we define SCM and in part b we propose a methodology for checking SCM. Part *a* includes subsections from 2.1 to 2.4 and part *b* includes subsections from 2.5 to 2.7.

a. *SCM definition*

2.1. Generation of color strips

The basic premise of the proposed method is the generation of strips of patches arranged in arrays of $2 \times n$. Figure 1 shows the example where $n = 10$.

The individual vertically oriented strips are composed of pairs of color patches with no visible boarder in between. The strips increase in color difference ΔE^*_{ab}, in n - 1 steps, from near null color difference (the pair of patches labeled #1) to the maximum value selected (the pair of patches labeled #10). These strips should be printed in such a way that the color differences, for each pair of patches, increase in the specified vector directions within the CIELAB color space, trying to maintain a consistent color for one of the patches in each pair, according to the color center selected, while varying the other patch as described. In this paper, we propose the same pattern of directions as Brusola et al. [28], which is shown in Fig. 2, however others can be chosen.

Theoretically, strips could be used in this method with different orientation schemes, number of patches and maximum color difference. However, due to the limitations of current color reproduction systems, we propose to construct the strips with the direction scheme shown in Fig. 2, a maximum color difference of 4 CIELAB units and n=10 pairs of patches per strip. In this way, taking into account that with the current printing systems it is possible to achieve a repeatability around ΔE^*_{ab}=0.2 [29] and that the approximate jump between two consecutive patches on the strip would be 0.4, we could obtain a sample distribution similar to the to that

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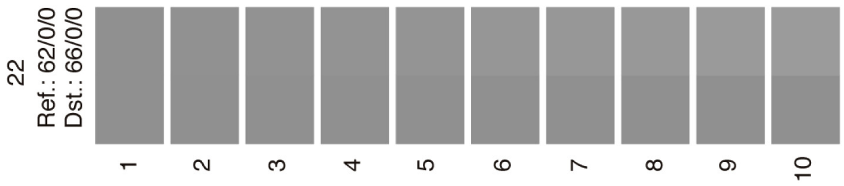

Fig. 1. Example of a strip used in the proposed method. Lower patches (left patches if the strip is vertically oriented) should be printed or displayed with the same CIELAB coordinates as the chosen color center. Upper patches (right patches if the strip is vertically oriented) should be printed or displayed so that ΔE^*_{ab} color differences increase with respect to the upper patches, from near zero to the maximum value, from left to right at approximately constant steps in a selected vector direction of the CIELAB color space. The example shown corresponds to the vector direction #22 (ΔL^* positive axis vector direction), according to the vector direction scheme shown in Fig. 2, for a gray color center in the CIELAB coordinates [62/0/0], $\Delta E^*_{ab} max = 4$ for the tenth pair of patches, and increasing CIELAB color differences between intermediate patches of ΔE^*_{ab} *step* $\approx 4/9$.

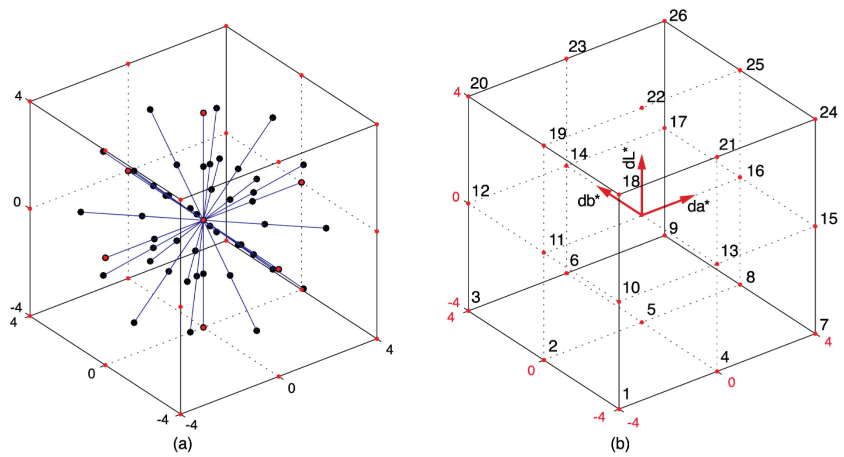

Fig. 2. Pattern scheme of color differences along selected directions within the color space proposed by Brusola et al. [28]. Figure 2(a) shows the pattern scheme for 26 vector directions, 3 steps and $\Delta E^*_{ab} max = 4$. Figure 2(b) shows the identification labels for every vector direction, where red dots indicate a point through which every vector direction must pass.

shown in Fig. 2 not excessively affected by noise. The maximum value of color difference in the strips of 4 is suggested because for most of the color difference formulas the difference of 1 given by the formula corresponds to color differences ΔE^*_{ab} lower than 4. However, other values could be used after taking into account the precision and accuracy of the color reproduction system used.

2.2. Visual assessment

Every strip, numbered according to the diagram shown in Fig. 2, should be presented multiple times to a significant number of observers. The number of observers and assessments should be tuned when working with real data. Observers should be normal color vision, according to the Farnsworth-Munsell 100-tone test or equivalent.

The observers should be instructed to detect the pair of patches, on each strip, for which they notice the first transition from no noticeable color difference to a just noticeable color difference (JND) and should indicate the JND pair. In this method, all of the pairs of patches on one strip are presented simultaneously, recording one pair number per trial and per strip, rather than reporting an opinion on each pair of patches when observed individually, as performed when using the pair comparison or grey-scale methods. The observers can freely select the orientation of the strips during the test if warrantied uniformity of illumination within that orientation. The evaluation should be performed under typical standardized CIE 116 [30] conditions (uniform neutral grey with $L* = 50$ background; 1000 lux lighting illuminance, with D65 lighting color temperature; > 4 ° subtended visual angle sample size; normal color vision observer; in edge contact sample separation; and 0 - 5 ΔE^*_{ab} sample color difference magnitude) for printed samples. For displayed samples, the evaluation should be performed under ISO 3664 conditions for appraisal of images displayed on color monitors (D65 white point, luminance of the white point ≥ 160 cd/m2, ambient illumination shall be low enough to get less than 1/4 of the monitor white point luminance on a perfect reflecting diffuser placed at the position of the faceplate of the monitor with the monitor switched off, neutral grey for background and surround). However, these reference conditions can be changed when studying parametric effects.

The only reference condition dissimilar to the pair comparison or gray-scale methods is the presentation of all of the patches in a strip simultaneously rather than pair-by-pair. The findings from the investigation of the influence of this condition on the results is presented in [29].

The result of the repeated visual assessments made by the observers should be a table of the absolute frequencies for each pair of patches for all of the strips, whose sum should equal the total number of trials.

2.3. Tolerance T50 computations for every vector direction

Suppose $\boldsymbol{r_{ik}}$ represents the number of times observers designated pair k as the *JND* pair of the strip i, and $\boldsymbol{dE_{ik}}$ represents the corresponding CIELAB color difference. Then, as reported by Torgerson [27] we can compute the upper ($\boldsymbol{L_u}$) and lower ($\boldsymbol{L_l}$) threshold limits and the point of subjective equality or threshold of 50% probability (***T50***), for every strip. This computation can be done using equations (1) - (3).

$$L_{ui} = \frac{1}{N_i}\sum_{k=1}^{n} r_{ik} dE_{ik} \tag{1}$$

$$L_{li} = \frac{1}{N_i}\sum_{k=1}^{n} r_{ik} dE_{ik-1} \tag{2}$$

$$T_{50i} = (L_{ui} + L_{li})/2 \tag{3}$$

where $\boldsymbol{N_i}$ is the total number of assessments for strip i. $\boldsymbol{L_u}$ values are calculated as the mean of the $\Delta\,\mathrm{E}^*_{\mathrm{ab}}$ of the pair designated as the *JND* pair by the observers. $\boldsymbol{L_l}$ values are calculated as the mean of the $\Delta\,\mathrm{E}^*_{\mathrm{ab}}$ of the pair previous to the *JND* pair, e.g. the contiguous pair to the *JND* pair with no perceived difference in color. Therefore, in Eq. (2), we multiply the frequency values ($\boldsymbol{r_{ik}}$) by the color-difference ($\Delta\,\mathrm{E}^*_{\mathrm{ab}}$) of the previous pair of patches, under the assumption that color differences increase as k is increased. To implement Eq. (2), we assume $\Delta E^*_{ab} = 0$ for a hypothetical pair of patches previous to k = 1. This is not a very critical assumption if we design the strips so that the *JND* pairs are in the intermediate patches of the strips.

2.4. Coefficients of discrimination ellipses or ellipsoids

Once we calculate the *T50* values, we can determine the coefficients of the chromaticity-discrimination ellipses and the coefficients of the color discrimination ellipsoids in Eq. (4), if we

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assume that major axis of the ellipsoid is parallel to a^*-b^* plane, or Eq. (5), following reported procedures [11,31,32], by minimizing Eq. (6)

$$\Delta e_i^2 = g_1 \Delta a_i^2 + g_2 \Delta b_i^2 + g_3 \Delta L_i^2 + 2 g_4 \Delta a_i \Delta b_i \tag{4}$$

$$\Delta e_i^2 = g_1 \Delta a_i^2 + g_2 \Delta b_i^2 + g_3 \Delta L_i^2 + 2 g_4 \Delta a_i \Delta b_i + 2 g_5 \Delta a_i \Delta L_i + 2 g_6 \Delta b_i \Delta L_i \tag{5}$$

$$Z = \sum_i (\Delta V_{T50} - \Delta e_i)^2 \tag{6}$$

Where ΔV_{T50} is the constant value of visual difference assigned to the *T50* tolerance at threshold (in this paper we have used $\Delta V_{T50} = 1$ to check the model) and $\Delta a_i^*,\ \Delta b_i^*,\ \Delta L_i^*$ are the corresponding CIELAB coordinates differences in every strip i at *T50*.

b. *SCM checking*

2.5. Frequency data from hypothetical perfect color data

To verify the proposed method, we generated theoretical data using the CIE94 color difference formula. We selected this color difference formula because, of the weighted ones, it is the one with which the determination of the coefficients of color discrimination ellipsoids are easiest to perform. It should be noted that we are trying to verify whether the method is capable of estimating the correct ellipsoid coefficients, not trying to evaluate any of the color difference formulas. To verify the model, we only require accurate frequency data generated from assumed discrimination ellipsoid coefficients and an assumed psychometric curve to check the degree of fit between the parameter prediction given by the method and the initial assumed values.

Table 1 presents the theoretical data generated for the CIE94 color-difference formula, for the 26 vector directions shown in Fig. 2, for strips of 10 pairs with a maximum ΔE^*_{ab}=4 for the last patches in every strip, around the red color center (L*=44, a*=37, b*=23), and logistic psychometric curve proposed by Robertson [25] with parameters: $\alpha = \beta = 2$. The psychometric curve chosen is given by Eq. (7).

$$p = \frac{1}{1 + e^{(\alpha - \beta \Delta E)}} \tag{7}$$

The frequency data can be generated using the following process:

Step 1. Calculate the color difference ΔE for the assumed color-difference model (ΔE_{94} for the theoretical data shown in Table 1) *on each strip patch pair*.

Step 2. Determine the probability parameter (p) of the associated binomial distribution depending on the psychometric curve chosen (Eq. (7) for the generated data shown in Table 1).

Step 3. For every pair of patches, generate a random value for the frequency, fit to a binomial distribution with probability p and N repetitions ($N = 100$ in the case shown in Table 1).

Step 4. Set pairs with a frequency of less than 50% to 0, set the remainder to 1, and record the number of the pair that first transitions from 0 to 1. A frequency of less than 50% would mean that the majority of observers did not perceive a color difference, while a frequency greater than 50% would mean the majority did perceive a color difference.

Step 5. Repeat steps 1–4 (*Nreps* = *100* in the example in Table 1) and accumulate the frequency data for every pair.The result is the simulated data for the visual assessment indicated in section 2.2.

2.6. Frequency data from the randomized color data

In practice, it is difficult to print or display a color stimulus with the desired CIELAB coordinates. There is always an error that increases or decreases depending on the control exercised over the printing or displaying process. To simulate this situation and evaluate its impact on the accuracy of the estimations of the model parameters, we propose the method to generate the theoretical

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Table 1. Theoretical data generated from the CIE94 color-difference formula around the red color center with CIELAB coordinates (*L^*= 44, a^*= 37, b^*= 23*), and logistic psychometric curve given by Eq. (7) with $\alpha = \beta = 2$, for the 26 vector directions shown in Fig. 1. r_{ik} is the hypothetical observed absolute frequency for patch *k* on strip *i* for a total of N = 100 repetitions, and ΔL^*, Δa^*, and Δb^* are the differences in the CIELAB coordinates of the final pair of patches in every strip, with a maximum color difference of ΔE^*_{ab}=4

Strip	Δa^*	Δb^*	ΔL^*	r_{i1}	r_{i2}	r_{i3}	r_{i4}	r_{i5}	r_{i6}	r_{i7}	r_{i8}	r_{i9}	r_{i10}
1	-2.31	-2.31	-2.31	0	0	0	14	81	5	0	0	0	0
2	-2.83	0.00	-2.83	0	0	0	66	34	0	0	0	0	0
3	-2.31	2.31	-2.31	0	0	0	59	41	0	0	0	0	0
4	0.00	-2.83	-2.83	0	0	0	79	21	0	0	0	0	0
5	0.00	0.00	-4.00	0	0	16	84	0	0	0	0	0	0
6	0.00	2.83	-2.83	0	0	0	80	20	0	0	0	0	0
7	2.31	-2.31	-2.31	0	0	0	56	44	0	0	0	0	0
8	2.83	0.00	-2.83	0	0	0	63	37	0	0	0	0	0
9	2.31	2.31	-2.31	0	0	0	9	86	5	0	0	0	0
10	-2.83	-2.83	0.00	0	0	0	0	0	3	39	54	4	0
11	-4.00	0.00	0.00	0	0	0	0	2	44	53	1	0	0
12	-2.83	2.83	0.00	0	0	0	2	73	25	0	0	0	0
13	0.00	-4.00	0.00	0	0	0	1	41	57	1	0	0	0
14	0.00	4.00	0.00	0	0	0	0	38	60	2	0	0	0
15	2.83	-2.83	0.00	0	0	0	2	71	27	0	0	0	0
16	4.00	0.00	0.00	0	0	0	0	1	29	61	9	0	0
17	2.83	2.83	0.00	0	0	0	0	0	2	29	55	14	0
18	-2.31	-2.31	2.31	0	0	0	11	85	4	0	0	0	0
19	-2.83	0.00	2.83	0	0	1	67	32	0	0	0	0	0
20	-2.31	2.31	2.31	0	0	0	56	44	0	0	0	0	0
21	0.00	-2.83	2.83	0	0	0	82	18	0	0	0	0	0
22	0.00	0.00	4.00	0	0	15	85	0	0	0	0	0	0
23	0.00	2.83	2.83	0	0	0	82	18	0	0	0	0	0
24	2.31	-2.31	2.31	0	0	0	56	44	0	0	0	0	0
25	2.83	0.00	2.83	0	0	0	65	35	0	0	0	0	0
26	2.31	2.31	2.31	0	0	0	8	85	7	0	0	0	0

Table 2. A comparison between the ellipsoid coefficients of the underlying model (CIE94) was used to generate the frequency data and those obtained using SCM for selected CIE color centers, where g_i is the coefficient of the 50% probability discrimination ellipsoids; a, b, and c are the principal semi-axis lengths of the ellipsoids; θ is the angle between the projection of the major axis of the discrimination ellipsoid and the positive semi-axis a*; and σ is the standard deviation of the parameters, estimated using a Monte Carlo method.

Col.	[L^* a^* b^*]	Model	g_1	g_2	g_3	g_4	g_5	g_6	a	b	c	θ (deg.)
Red	[44 37 23]	CIE94	0.1842	0.2956	1.0000	-0.1128	0.0000	0.0000	2.96	1.65	1.00	31.87
		SCM	0.1960	0.3164	1.0731	-0.1202	-0.0025	0.0030	2.90	1.64	0.99	31.81
		2σ	0.014	0.022	0.0702	0.0136	0.047	0.05	0.14	0.06	0.03	0.31
Yellow	[87 -7 47]	CIE94	0.3357	0.1067	1.0000	0.0349	0.0000	0.0000	3.14	1.71	1.00	98.47
		SCM	0.3430	0.1111	1.0193	0.0355	0.0002	0.0000	3.08	1.69	0.99	98.52
		2σ	0.0025	0.0010	0.0048	0.0013	0.0032	0.0028	0.01	0.01	0.002	0.29
Green	[56 -32 0]	CIE94	0.1681	0.4567	1.0000	0.0000	0.0000	0.0000	2.44	1.48	1.00	180
		SCM	0.1730	0.4646	1.0176	0.0002	0.0000	0.0000	2.40	1.47	0.99	180
		2σ	0.0014	0.0028	0.049	0.0021	0.0030	0.031	0.01	0.01	0.002	0.4
Blue	[36 5 -31]	CIE94	0.4548	0.1791	1.0000	0.0457	0.0000	0.0000	2.37	1.46	1.00	279
		SCM	0.4605	0.1851	0.9961	0.0633	0.0004	0.0006	2.42	1.45	1.00	282
		2σ	0.0033	0.0018	0.0053	0.0022	0.0034	0.0035	0.01	0.01	0.01	0.43
Grey	[62 0 0]	CIE94	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.00	1.00	1.00	0
		SCM	1.0334	1.0328	1.0336	0.0007	-0.0002	-0.0002	0.99	0.98	0.98	0
		2σ	0.0078	0.0086	0.0083	0.0069	0.0047	0.0058	0.01	0.01	0.01	-

Table 3. Results obtained for the parameters of the color discrimination ellipsoids for theoretical color differences, as described in subsection 2.1, randomized by a normal noise $\mathrm{N}(0, 0.2)$, as indicated in subsection 2.6.

Color	[L^* a^* b^*]	Model	g_1	g_2	g_3	g_4	a	b	c	θ (deg.)
Red	[44 37 23]	CIE94	0.184	0.296	1.000	-0.113	2.96	1.65	1.00	31.9
		SCM	0.191	0.315	1.023	-0.123	2.96	1.60	0.99	31,8
		2σ	0.032	0.040	0.169	0.028	0.23	0.10	0.08	5.5
Yellow	[87 -7 47]	CIE94	0.336	0.107	1.000	0.035	3.14	1.71	1.00	98.5
		SCM	0.356	0.109	1.006	0.041	3.14	1.66	1.00	99.0
		2σ	0.055	0.011	0.149	0.029	0.20	0.13	0.07	5.8
Green	[56 -32 0]	CIE94	0.179	0.457	1.000	0.000	2.44	1.48	1.00	180.0
		SCM	0.179	0.499	1.023	0.000	2.39	1.42	0.99	180.1
		2σ	0.022	0.085	0.191	0.055	0.15	0.12	0.10	9.9
Blue	[36 5 -30]	CIE94	0.455	0.179	1.000	0.046	2.37	1.46	1.00	279.0
		SCM	0.485	0.191	1.039	0.052	2.36	1.42	0.98	279.6
		2σ	0.084	0.027	0.167	0.048	0.19	0.13	0.08	8.5
Grey	[62 0 0]	CIE94	1.000	1.000	1.000	0.000	1.00	1.00	1.00	0.0
		SCM	1.085	1.103	1.076	-0.010	1.05	0.97	0.90	0.0
		2σ	0.314	0.321	0.263	0.240	0.12	0.08	0.08	-

frequencies described in the previous section. However, this is preceded by a step in which a random error $\boldsymbol{N}(0, \boldsymbol{\sigma_{cd}})$ from a normal distribution with a mean of zero and standard deviation $\boldsymbol{\sigma_{cd}}$, is added to the theoretical color differences of the control strips. The values of $\boldsymbol{\sigma_{cd}}$ studied were 0, 0.1, 0.2, and 0.3, taking into account the fact that using current quality printing systems, values of 0.15 can be achieved.

2.7. Model verification

Once the theoretical data has been generated, randomized or not, we are able to determine the model parameters following the procedures indicated in sections 2.3 and 2.4, and compare them with those of the underlying model from which we have generated the theoretical data.

3. Results

A comparison between the discrimination ellipsoid parameters obtained using the proposed method of strip comparison method (SCM), based on the serial exploration method by Torgerson [27], and the parameters of the underlying color-difference model (CIE94) used to generate the data, is presented in Table 2. The results presented in this table were generated under the assumption that the color data for every pair of patches on the strips perfectly fit the desired CIELAB color differences in all vector directions around the selected color centers (CIE [25,33]), as described in 2.1.

As can be seen from Table 2, the ellipsoid parameters determined using SCM fit quite well the CIE94 ellipsoid parameters for the color centers shown. Additionally, an estimation of the accuracy of every parameter has been obtained using a Monte Carlo method, based on applying SCM over randomly generated frequency data, repeating step 5 of section 2.5 approximately 100 times, and calculating the standard deviation (σ) from the computed parameters. σ has not been calculated for the gray color center for the θ parameter because, in that case, the ellipsoid degenerates to a sphere with no defined principal directions.

The results from the application of the same process to the theoretical color data differences, as described in section 2.1, randomized by an error $\boldsymbol{N}(0, \boldsymbol{\sigma_{cd}} = 0.2)$, as indicated in section 2.6, are presented in Table 3.

Figure 3 shows the evolution of the θ parameter, estimated using SCM for the selected CIE color centers, as a function of σ_{cd} (the standard deviation of the random error, $N(0, \sigma_{cd})$), added to the theoretical color-differences in every vector direction, as described in subsection 2.6). The continuous lines in Fig. 3 correspond to the hue angles (h^*_{ab}) of the selected color centers. The dashed lines are the corresponding mean values of the θ parameter, estimated using SCM for a set of 100 random samples, generated by adding a normal random error of 0 mean and σ_{cd} standard deviation to the theoretical color differences. The dotted lines correspond to the upper and lower bounds, computed as the mean value of θ ± 2σ, where σ is the standard deviation of the θ parameters obtained for the 100 random samples. Therefore, the dotted lines define an interval for each σ_{cd}, which comprises approximately 95% of the θ values obtained and whose mean value is centered around the true value (h^*_{ab}), which should be predicted by the model.

As we can see, the SCM model prediction of the θ parameter is quite good. Up to $\sigma_{cd} = 0.2$, the 95% interval predicted is less than ± 10 ° of the mean value for all of the color centers analyzed.

Similarly, Figs. 4 and 5 show the evolution of the a and b parameters, respectively, (the two principal semi-axes of the color discrimination ellipsoid) as a function of σ_{cd}. In this case, we observe an increasing bias for the estimated a or b parameters, as σ_{cd} increases. These figures show a tendency to predict smaller ellipsoids than the true ones, with the difference increasing as σ_{cd} increases.

The reduction in the size of the predicted ellipsoids may be due to the visual assessment procedure. Observers are asked to indicate the first pair of patches on the strip for which they see a

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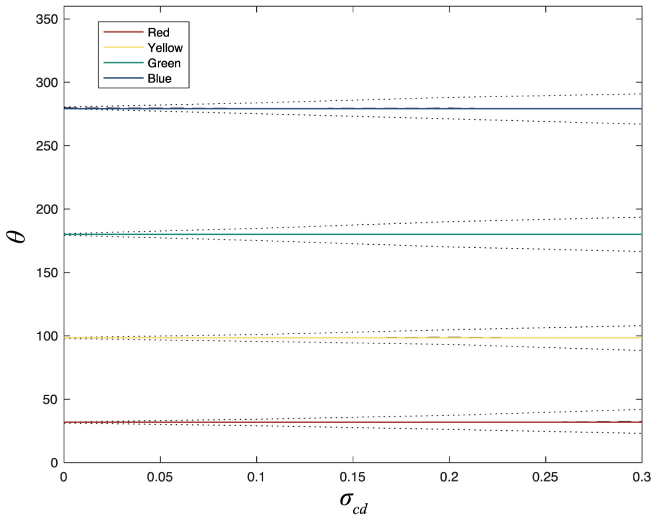

Fig. 3. Evolution of θ as a function of the standard deviation σ_{cd}. Continuous lines correspond to the hue angles (h^*_{ab}) of the selected color centers. The dashed lines (nearly overlapped by continuous lines) are the corresponding mean values of the estimated θ parameter, obtained by SCM for 100 sets of samples, generated randomly, adding $N(0, \sigma_{\mathrm{cd}})$ to the theoretical color differences. The dotted lines correspond to the previously discussed mean value of θ ± 2σ, where the values of σ are the standard deviation of the estimated θ parameter for the set of randomly generated samples.

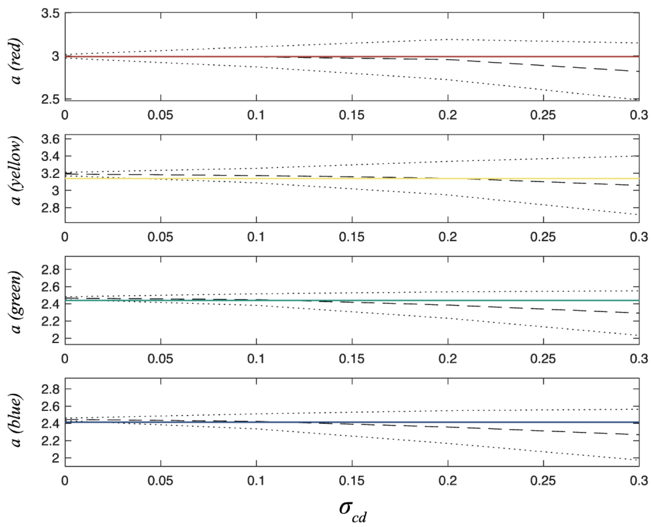

Fig. 4. Evolution of the *a* parameter as a function of the standard deviation σ_{cd}. Continuous lines correspond to the first principal semi-axis of the color discrimination ellipsoid for the selected color centers. The dashed lines are the corresponding mean values of the *a* parameter, estimated using SCM for the 100 sets of samples, generated randomly, adding $N(0, \sigma_{cd})$ to the theoretical color differences. The dotted lines correspond to the previously discussed mean value of *a* ± 2σ, where the values of σ are the standard deviation of the estimations of the *a* parameter.

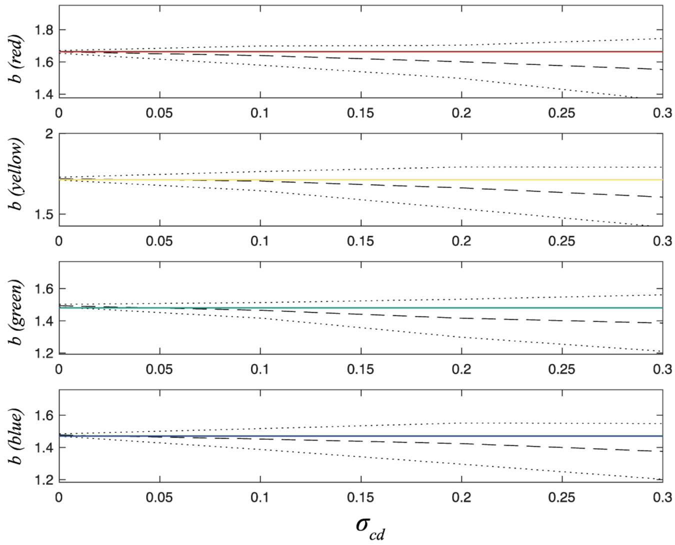

Fig. 5. Evolution of the b parameter as a function of the standard deviation σ_{cd}. Continuous lines correspond to the second principal semi-axis of the color discrimination ellipsoid for the selected color centers. The dashed lines are the corresponding mean values of the b parameter, estimated using SCM for the 100 sets of samples, generated randomly, adding $N(0, \sigma_{cd})$ to the theoretical color differences. The dotted lines correspond to the previously discussed mean value of $b \pm 2\sigma$, where the values of σ are the standard deviation of the estimations of the b parameter

just noticeable difference (labeled the *JND* pair), in order to determine the threshold of perceived color difference. This allows for the accurate detection of the threshold position, provided the pairs of patches are properly ordered, with the color difference of the patches increasing along the strip. However, if the strip has not been printed or displayed with the appropriate precision, the color difference between pairs of patches previous to the correct *JND* pair, can be greater than that of the correct *JND* pair. This means that it is possible that the observer erroneously selects the earlier pair as the *JND* pair, reducing the dimension of the correct threshold position in the direction associated with the strip. On average, for the color centers represented in Figs. 4 and 5, the calculated coefficient of variation (Cv), for the first and second principal axes, goes from around 2% for $\sigma_{cd} = 0.1$, to 4% for $\sigma_{cd} = 0.2$, and to 6% for $\sigma_{cd} = 0.3$, and the bias, calculated as the percentage of the deviation of the estimated mean value from the theoretical value, goes from around 1% for $\sigma_{cd} = 0.1$, to 3% for $\sigma_{cd} = 0.2$, and to 6% for $\sigma_{cd} = 0.3$. However, the higher values for both the Cv and bias specified for $\sigma_{cd} = 0.3$ are still lower than the ranges that can be computed from published color-difference datasets [10–15,18].

4. Conclusions

The color strip comparison method (SCM), based on the serial exploration method proposed by Torgerson [27] to scale general psychophysical attributes, has been successfully verified. Simulations were used to generate frequency data from the CIE94 color difference formula and from the logistic psychometric curve. The model was checked under the assumption that the color differences between pairs of patches in the strips have a superposed random error, $N(0, \sigma_{cd})$, added to the theoretical color difference.

Our experiments showed a good agreement between the theoretical discrimination ellipsoid parameters and the parameters estimated by SCM for strips of 10 pairs of patches with a maximum color difference of $\Delta E^*_{ab} = 4$, even for samples printed or displayed with a superimposed random error of $\sigma_{cd} = 0.2$. Which represents the precision required, for printing or displaying the strips, of approximately 1/2 the theoretical CIELAB color difference increment required in one step of the strips. Precision that is perfectly feasible using today's digital printing systems as is reported in [29].

Taking into account the parametric effects already studied by Brusola et al. [29], we believe that the strip-pair comparison method is a good possible time efficient alternative to classical methods for developing color-difference models.

Disclosures

The authors declare no conflicts of interest.

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