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Improving the efficiency of orbit determination processes

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In Classical Mechanics, the solution of the two-body problem with known initial values is well defined and finds many applications. For instance, it can be applied to approximate the position of an Earth satellite. Nowadays, achieving a certain precision in the calculation of the position of artificial satellites around the Earth is fundamental. However, as the Earth-satellite system is in practice affected by numerous other bodies, a differential correction is often necessary. It is also necessary to know, for example, the position of the International Space Station (ISS) at a given moment or the position of each of the satellites that make up the Global Navigation Satellite System (GNSS).

The problem of n-gravitational bodies is fundamental for Positional Astronomy, as it studies the motion of celestial bodies subjected to forces derived from Newton's gravitation law. An example is the motion of planets around the Sun or the motion of artificial satellites around the Earth. This problem is simplified by dispensing with the gravitational action of stars, planets and other distant celestial bodies, since it is small due to its distance. In addition, the considered stars are replaced by other material points with the same mass and whose position coincides with the corresponding center of gravity. This replacement, proposed by Newton, is justified by the classical theorems of the theory of potential.

Now, it is known that if $n \geq 3$, the problem does not have an analytical solution, so it would be necessary to resort to approximate solutions and the theory of perturbations. That is why we will focus on the two-body problem, that is, $n = 2$. In this case the problem has an analytical solution, i.e., is integrable. Performing three observations in three given times and using the solution of two-body problem, the initial position of the satellite can be adjusted. Nevertheless, we aim to correct the orbit, getting an improved position using high order iterative methods and measuring only angle coordinates.

The coordinate system used is the absolute equatorial one, which has as reference plane the celestial equator, which is an extension of the plane of the terrestrial equator to the celestial sphere, and as direction the celestial poles (North, NCP, and South, SCP, in Figure 1), which are an extension of the Earth's axes. So, the right ascension α and declination δ coordinates

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yield the position any celestial body at an specific instant in the celestial sphere. Let us remark that the right ascension α is measured on the celestial equator with the Vernal point γ as starting position. This point is one of the intersection points between celestial equator and the ecliptic (extension to the celestial sphere of the orbit of the Earth around the Sun), that apparently crosses the Sun at Spring Equinox (seen from the Earth).

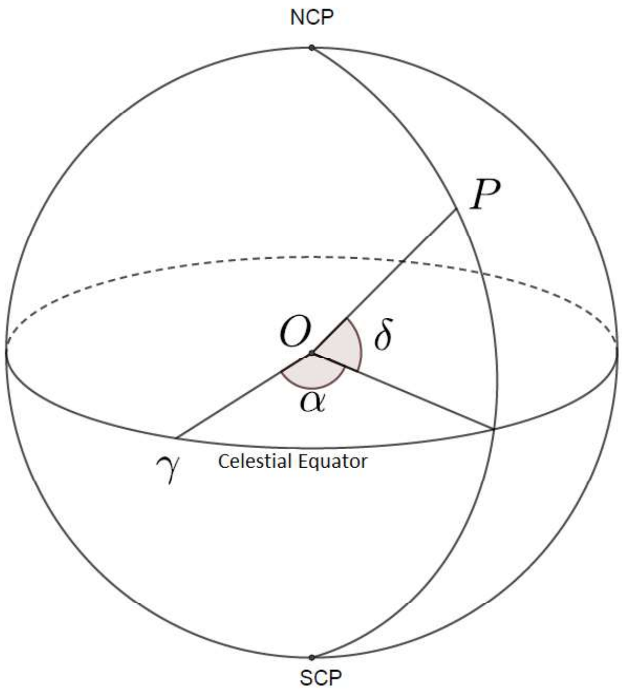

Figure 1: Right Ascension and Declination coordinates

Our next aim is to perform an algorithm (see Figure 2 allowing us to calculate the position of an artificial satellite around the Earth, at a given instant t, known two initial positions X_0 and X_1 in the instants t_0 and t_1. From the Gauss method of the areas, it is possible to obtain, from this information, the speed of the star at instant t_0. Thus, we have the position and velocity of the satellite at instant t_0, which are the initial conditions of the differential equation describing the two-body problem. Solving this equation by the classical method, one obtains the position and velocity of the star in an instant of time t.

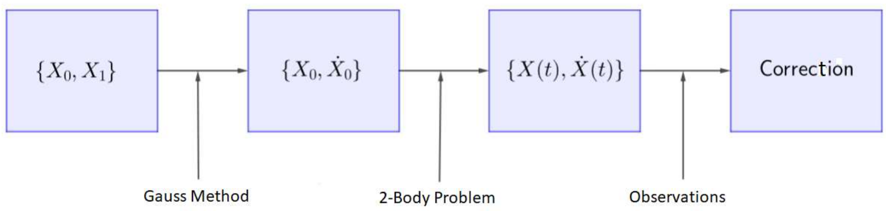

Figure 2: Orbit Correction Process

These calculations are only approximations, as only the Earth-satellite system is being taken into account. The gravitational action of the Moon, the Sun or the other planets of the Solar

System, for example, is not being considered. Nor are being taken into account the modifications that may be being made to the satellite's own trajectory, which surely would not correspond with the calculations made. That is why, if a certain precision is required, a correction is necessary.

The differential correction method used is based on least squares procedure [3] and use angular observations, since they are easier to obtain. These will be declination, δ and right ascension, α. Each of them will be observed at different times $t_0, t_1, \ldots, t_n$, with $\in \mathbb{N}$. These will be, respectively, $\alpha_0^O, \alpha_1^O, \ldots, \alpha_n^O$ right ascensions, and $\delta_0^O, \delta_1^O, \ldots, \delta_n^O$ declinations, where the superscript O represents that they are observed magnitudes. These data are grouped in a single vector λ^O. These data have been obtained from a NASA database, so we also have access to the real positions and velocities we want to estimate with the correction process, which allows us to check the efficiency of our procedure.

The aim is to compare these observations with the calculations made. The function that calculates the declination and right ascension in those instants of time, given an initial position vector X and velocity vector $\dot{X}$, is denoted by F. Therefore, we want to minimize

$$\|\lambda^O - F(X, \dot{X})\|_2^2$$

For getting this aim, it is necessary to find a solution to the following system of equations:

$$\nabla F(X, \dot{X})^T (\lambda^O - F(X, \dot{X})) = 0. \quad (1)$$

This system can be solved, for example, by the multidimensional extension of Steffensen's method, which allows us to avoid the calculation and evaluation of Jacobian matrices, needed for example when Newton's method is used. The initial guess is that obtained through the two-body problem, in order to ensure convergence. It is verified that the correction improves considerably the result obtained only with the problem of the two bodies, since we compare it with the real data of NASA.

So, when convergence is available, the following questions arise: is it possible to reduce the calculation time? Could it be done in a single iteration? To increase the number of observations, would improve numerical calculations? Perhaps it would be better to use a more efficient numerical method to ensure faster convergence? Is it important that observations are made in an instant of time close to the time you want to calculate?

To answer these questions, several studies are conducted to improve the correction:

- Reduction of the calculation time: some simplifications made in equation (1) are justified so that the system of equations can be solved more quickly.
- Increasing of the number of observations used: by a practical analysis it is concluded that four observations are the optimal amount for both right ascension and declination.
- Separation between observations: it follows that the separation between the observations made is very important, as the closer you are to the instant of time in which you want to obtain the position of the star, the better the results will be. In fact, from a certain distance, the results may not converge.

- Implementation of a high order numerical method: a higher order numerical method (than Steffensen's one) is implemented, also free of Jacobian matrices, but with fourth-order of convergence. With it, convergence is achieved in the first iteration, so it can be deduced that the correction process has been improved.

References

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