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Additional Information

Time-dependent model for unidirectional composite with a viscoelastic matrix

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Abstract

Modelling of time-dependent properties of unidirectional fiber reinforced plastics on the basis of structural mechanics of composite materials and mechanics of hereditary media has been proposed in this research. The constitutive equations of unidirectional fiber reinforced plastics have been obtained using the Volterra correspondence principle and the algebraic properties of resolvent operators. Modelling of viscoelastic properties of a unidirectional composite material on the basis of EDT-10 epoxy resin has been shown.

Keywords: Creep, relaxation, constitutive equation, hereditary operator, resolvent operator, elastic viscoelastic correspondence principle

1. Introduction

The unidirectional layer is a base for thin-walled structures of layered polymer composites. Therefore, it is of great importance to model its mechanical properties by the properties of its components. Since the mechanical properties of unidirectional polymer composites are time dependent, the general laws of structural mechanics are inadequate to describe the composite behaviour under time varying loading. In structural mechanics, elastic properties of unidirectional composite material are commonly modelled by a body with alternating layers of matrix and fiber and on the base of micromechanics of the lamina stress-strain relations are established [1]. The development of structural models to predict time-dependent properties of unidirectional polymer matrix composites has been undertaken [2-4]. In [2], a homogenization cell filled with a transversally elastic fiber and a viscoelastic isotropic matrix was considered. The constitutive equation for the matrix was chosen in the form of a nonlinear dependence of the strain rate on the power functions of time and stress. In [3], a three-dimensional micromechanical model is proposed based on different representations of the homogenization cell, which have different values of the volume content of fibers and determine the relations of the hereditary type of anisotropic body. The theoretical foundations of linear viscoelasticity, including consideration of continuous and discrete creep and relaxation spectra are described in [4-8]. A comprehensive method for constructing constitutive relations based on test results during short-term and long-term tests have been proposed in [9]. As it was shown in [10, 11] time-dependent properties of unidirectional composites to a great extent are determined by viscoelastic behaviour of polymer matrix and its bonding with the fibres. Using the correspondence principle [4, 5] and the relations of structural mechanics, the relations for modelling of viscoelastic properties of unidirectional composite can be obtained. Along with the Laplace transform, an interrelation between elastic and viscoelastic characteristics can be established using operator correspondence principle based on relations of mechanics of hereditary solids [4]. Using the operator approach, the viscoelastic properties can be described with the aid of hereditary operators having algebraic properties allowing to reduce the relations to the explicit form [4]. The algebra of resolvent hereditary operators was elaborated in [4]. For a kernel of the resolvent operator, the following functions can be used: power function (Abel's kernel), exponential function, Rabotnov fraction-exponential function, linear combination of exponential or fraction-exponential functions [4, 6]. A new special exponential-type kernel for modelling the viscoelastic properties is given in [7].

2. Model description

2.1. **Structural mechanics relationships**

The relations for the technical elastic constants expressed through the elastic properties of the components and their volume fraction are given in [1]. The expressions for the effective characteristics are as follows:

$$E_1 = E_f\psi + E_m(1-\psi) \qquad (1)$$

$$\frac{1}{E_2} = \frac{\psi}{E_f} + \frac{1-\psi}{E_m} \qquad (2)$$

where $E_1,\ E_2$ are elastic characteristics of the unidirectional fiber reinforced composite, E_f, E_m are fiber and matrix moduli, ψ is fiber volume fraction.

2.2. **Theory of resolvent operators**

The operator form of the linear constitutive hereditary equation under uniaxial loading can be written as follows

$$E\varepsilon = \sigma + K^*\sigma = (1+K^*)\sigma, \qquad (3)$$

where action of hereditary operator on stress σ takes the following form: $K^*\sigma = \int_0^t K(t-\tau)\sigma(\tau)d\tau$, $K(t)$ is kernel of the operator.

The constitutive equation (3) can formally be inversed and written in the following form

$$\sigma = E(\varepsilon - R^*\varepsilon) = E(1 - R^*)\varepsilon \tag{4}$$

Operator R^* is resolvent in relation to operator K^*. Substituting the expression for σ from (4) into (3) yields the equation for interrelation between the initial operator and its resolvent

$$(1 + K^*)^{-1} = 1 - R^* \tag{5}$$

For Abel's kernel: $K(t) = I_\alpha(t) = \frac{t^\alpha}{\Gamma(1+\alpha)}$, $-1 < \alpha < 0$, the expression for Abel's operator is given by $I_\alpha^* \cdot 1 = I_\alpha^* = \frac{\beta t^{1+\alpha}}{\Gamma(2+\alpha)}$, 1 is a unity Heaviside function, Γ is gamma-function. The resolvent of Abel's operator is Rabotnov fraction-exponential function can be written as

$$R^* = Z_\alpha^*(-\beta) \cdot 1 = t^{1+\alpha} \sum_{n=0}^{\infty} \frac{(-\beta t^{1+\alpha})^n}{\Gamma[1+(1+\alpha)(1+n)]} \tag{6}$$

Series (6) converges at $\beta > 0$.

The resolvent of sum of fraction-exponential or exponential functions are reduced to consequent solving of some equations and system of linear equations [4]. It is possible to inverse the sum of fraction-exponential functions only in the case of the same value of the parameter of singularity α. The general expression (5) for the series of resolvent operators is given by

$$\left(1 + \sum_{i=1}^{n} k_i Z_\alpha^*(-\beta_i)\right)^{-1} = 1 - \sum_{s=1}^{n} m_s Z_\alpha^*(-\gamma_s) \tag{7}$$

Multiplying $\left(1 + \sum_{i=1}^{n} k_i Z_\alpha^*(-\beta_i)\right)\left(1 - \sum_{s=1}^{n} m_s Z_\alpha^*(-\gamma_s)\right)$ and using formula for resolvent operators multiplication $Z_\alpha^*(x) Z_\alpha^*(y) = \frac{1}{x-y}[Z_\alpha^*(x) - Z_\alpha^*(y)]$ the equations for determining parameters m_s and γ_s give the following relations

$$1 + \sum_i \frac{k_i}{\beta_i - \gamma_s} = 0,\ 1 + \sum_s \frac{m_s}{\beta_i - \gamma_s} = 0,$$

Since the exponential operator is a special case of Rabotnov fraction-exponential operator formula (7), it is also applicable for deriving a resolvent of the sum of exponential operators.

2.3. Creep compliance and relaxation modulus

Time-dependent properties of unidirectional composite material are modelled similarly to the elastic-viscoelastic correspondence principle [4]. The expressions of structural mechanics used in this paper are correct under the following assumptions: full bonding of the components; components are in uniaxial stress state, i.e. stresses arising from difference in a Poisson's ratio are negligible. For simplicity, it is assumed that fibres are elastic. Estimation of the effective viscoelastic properties can be obtained by substituting the following characteristics with the hereditary operator and relaxation modulus and creep compliance of polymer matrix can be written as:

$$E_m^* = E_m(1 - R_m^*) \tag{8}$$

$$\frac{1}{E_m^*} = \frac{1}{E_m}(1 + K_m^*), \tag{9}$$

where E_m is an instantaneous value of the matrix modulus, which can be estimated through long term data processing or under short term tests in which it can be reduced to an acceptable level of the amount of time-dependent strain. Operator $R_m^*$is resolvent in relation to K_m^*. Let us take a consideration that time-dependent properties of unidirectional composite are defined by viscoelastic properties of polymer matrix. Substituting (8), (9) into relations (1), (2) the explicit operator form of the effective relaxation moduli and creep compliances of unidirectional composites can be established.

Substituting expression (8) in (1) the operator expression for hereditary modulus in longitudinal direction yields

$$E_1^* = E_f\psi + E_m^*(1-\psi) = E_1(1-\lambda_1 R_m^*), \quad (10)$$

where $\lambda_1 = \frac{E_m(1-\psi)}{E_1}$.

Just as the relaxation modulus using the operator representation for creep compliance of matrix: $\frac{1}{E_m^*} = \frac{1}{E_m}(1+K_m^*)$, the hereditary operator relation for creep compliance of the layer can be written as:

$$\frac{1}{E_2^*} = \frac{\psi}{E_f} + \frac{1-\psi}{E_m^*} = \frac{1}{E_2}(1+\lambda_2 K_m^*), \quad (11)$$

where $\lambda_2 = \frac{E_2(1-\psi)}{E_m}$.

Note that in absence of time-dependent properties the expressions derived degenerate into the relations of structural mechanics of composites.

3. Viscoelasticity of polymer matrix

Let us consider EDT-10 epoxy resin as a matrix of unidirectional composite. The readings of long-term creep in 50000 hours (5.7 years) were given in [10]. Test results show significant creep of the epoxy resin, creep exceeds instantaneous values by several times. It should be noted that exponential operator is a particular case of fraction-exponential operator, i.e. $Z_0^*(-\beta_i)$. The expression of creep kernel was taken as follows:

$$K_m(t-\tau) = \sum_{i=1}^{k} A_i \alpha_i exp[-\alpha_i(t-\tau)] \quad (12)$$

Creep under $\bar{\sigma}$ = const is described by:

$$\varepsilon(t) = \frac{\bar{\sigma}}{E_m}\left[1 + \sum_{i=1}^{k} A_i\left(1 - exp(-\alpha_i t)\right)\right], \tag{13}$$

where E_m is instantaneous Young's modulus equal to 3.3 *GPa*, values of coefficients A_i and α_i are taken from [9] and are given in Table.

$\alpha_i, hour^{-1}$	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
A_i	0.092	0.099	0.1163	0.393	1.815	2.838

Alternative to the kernel as a sum of exponential functions power function (Abel's kernel) can be also taken. In this case creep curve can be described by

$$\varepsilon(t) = \frac{\sigma}{E_m}\left(1 + \frac{k}{\Gamma(2+\alpha)} t^{1+\alpha}\right), \tag{14}$$

where parameters $\alpha = -0.6,\ k = 0.0441\ hour^{-(1+\alpha)}$ have been obtained by processing of experimental data. A comparison of experimental data and analytical creep curves is presented in Fig. 1.

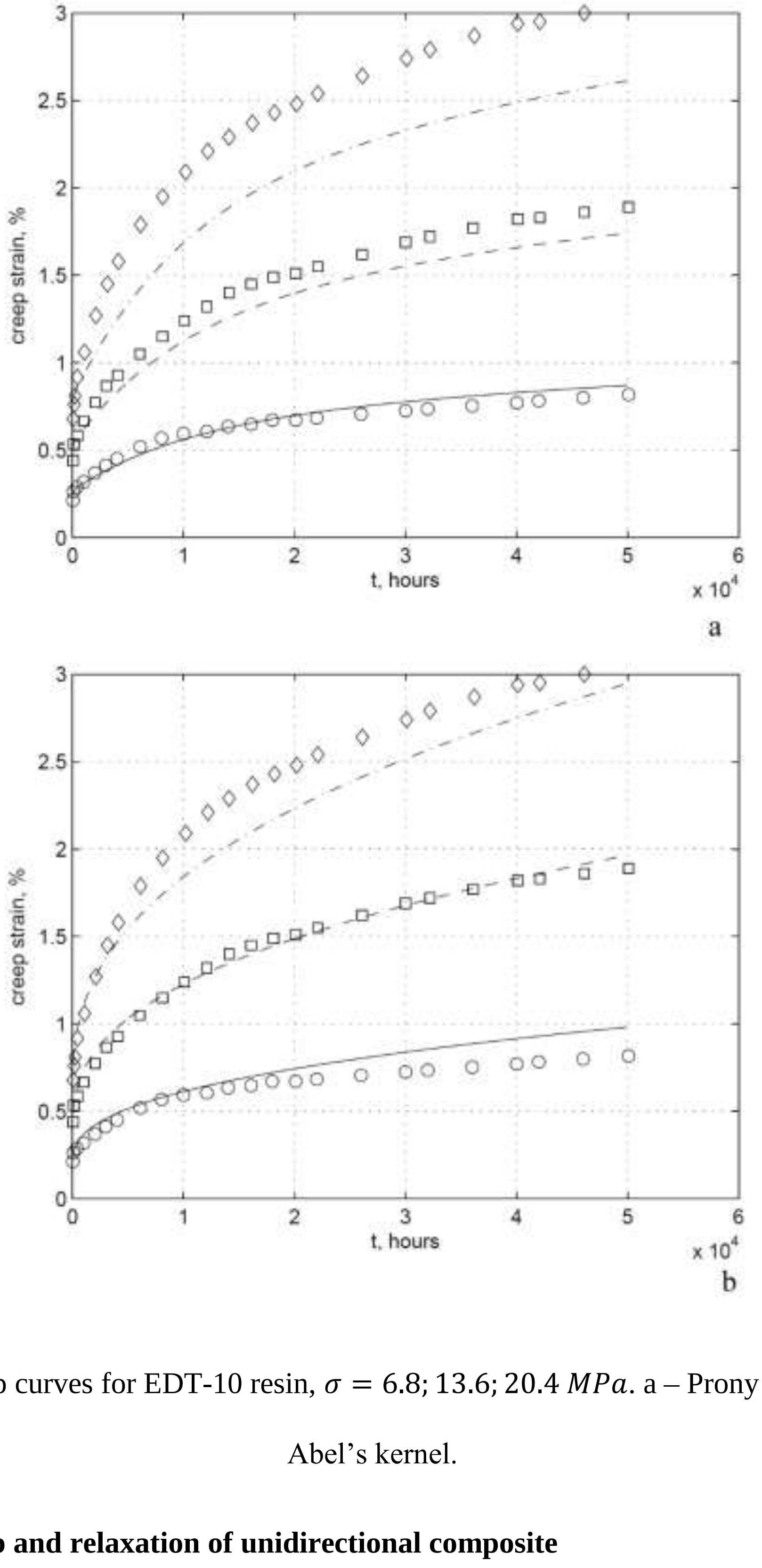

Fig.1. Creep curves for EDT-10 resin, $\sigma = 6.8; 13.6; 20.4\ MPa$. a – Prony series, b – Abel’s kernel.

4. Creep and relaxation of unidirectional composite

Let us consider creep of unidirectional composite under longitudinal uniaxial loading. Creep of unidirectional composite at $\overline{\sigma_1}$=const can be written as

$$\varepsilon_1(t) = \frac{\overline{\sigma_1}}{E_1(1-\lambda_1 R_m^*)} = \frac{\overline{\sigma_1}}{E_1}(1 + \lambda_1 K_m^*). \tag{15}$$

For comparison, it is convenient to present creep curves in the following dimensionless form

$$\tilde{\varepsilon} = \frac{E\varepsilon}{\overline{\sigma}} = 1 + \lambda K_m^*. \tag{16}$$

The relative creep strain in (16) defined by hereditary operator λK_m^* can be compared with relative instantaneous or elastic strain.

Creep of the carbon fiber reinforced plastic, depending on the type of the kernel can be written as follows:

$$\varepsilon_1(t) = \frac{\overline{\sigma_1}}{E_1}(1 + \lambda_1 \sum_{i=1}^{n} A_i[1 - exp(-\alpha_i t)]) \tag{17}$$

$$\varepsilon_1(t) = \frac{\overline{\sigma_1}}{E_1}\left(1 + \lambda_1 \frac{k}{\Gamma(2+\alpha)} t^{1+\alpha}\right) \tag{18}$$

Young's modulus of fiber in transverse direction was taken equal to 6.2 *GPa*. Creep strain in longitudinal direction is negligibly little and on the time scale of 5000 hours is approximately equal to 0.06 of elastic strain and in some cases the model of elastic behaviour can be acceptable [9].

Using relation (11) creep at $\overline{\sigma_2}$ =const can be represented as

$$\varepsilon_2(t) = \frac{\overline{\sigma_2}}{E_2}(1 + \lambda_2 K_m^*) \quad . \tag{19}$$

Creep strain in transverse direction is significant, on the base of 50,000 hours it is about three times more then corresponding elastic strain.

The constitutive equations corresponding to the sum of exponential functions and Abel's kernel can be written as follows

$$\sigma_1 = E_1(1 - \sum_{i=1}^{n} B_i Z_0^*(-\beta_i))\varepsilon_1 \tag{20}$$

$$\sigma_1 = E_1\big(1 - \lambda Z_\alpha^*(-\lambda)\big)\varepsilon \tag{21}$$

The stress relaxation is a special case of equations (20), (21) at $\varepsilon = const.$

The approach also can be used for evaluation of stress redistribution between the components in a unidirectional composite.

Thus using (15), stress in fibres can be estimated as:

$$\sigma_f(t) = \frac{\overline{\sigma_f} E_f}{E_1}(1 + \lambda_1 K_m^*). \tag{22}$$

The stress relaxation in the epoxy resin after some operator transformations can be written as follows:

$$\sigma_m(t) = \frac{\overline{\sigma_1} E_m}{E_1}[1 - (1 - \lambda_1)K_m^*] \tag{23}$$

Under constant stress applied to a unidirectional composite the fibres get uploaded due to relaxation in the polymer matrix. However, in the longitudinal direction the level of the uploading is negligibly small. It should be noted that to a large extent, unidirectional polymer composites manifest the time-dependent properties and physical nonlinearity under shear [4].

5. Conclusion

A structural-phenomenological model for constructing the constitutive equations of a unidirectional composite material based on the algebra of resolvent operators of hereditary mechanics of solids is proposed. It was assumed that the reinforcing fibers follow the law of elasticity, and the polymer matrix is viscoelastic.

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