

Case Study on Students' Mathematical Modelling Processes considering the Achievement Level

Estudio de caso sobre los procesos de modelización matemática de los estudiantes teniendo en cuenta el nivel de rendimiento

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Abstract

Heterogeneity can be taken into account in the classroom by using self-differentiating tasks. From a mathematical-didactic point of view, modelling tasks are of particular interest in this regard because of this. The paper focuses on the heterogeneity dimension of mathematical achievement and modelling competence of 15-year-old students. First results of a case study will be described in how far the processes of a modelling task vary with regard to the aspect of achievement levels.

La heterogeneidad puede tenerse en cuenta en el aula mediante el uso de tareas de autodiferenciación. En este sentido, desde el punto de vista matemático-didáctico, las tareas de modelización tienen un interés especial. El artículo se centra en la dimensión de heterogeneidad del rendimiento matemático y la competencia de modelización de los alumnos de 15 años. Se describirán los primeros resultados de un estudio de caso en el que se analiza en qué medida los procesos de una tarea de modelización varían con respecto al aspecto de los niveles de rendimiento.

Keywords: Natural Differentiation, Mathematical Modelling, Mathematical Performance, Qualitative Content Analysis

Palabras clave: Diferenciación natural, modelización matemática, rendimiento matemático, análisis de contenido cualitativo

1. Introduction

In recent years people, especially teachers, register to a growing extent among the general public, but that heterogeneity is increasing in society especially among teachers and thus also in schools (Scherer and Krauthausen, 2010). Consequently, the perception is that learning groups are becoming increasingly heterogeneous. In contrast, the Ministry for School and Continuing Education of the federal state of North Rhine-Westphalia (Ministerium für Schule und Weiterbildung des Landes Nordrhein-Westfalen) demands that the diversity of learners should be in the focus of every education planning discussion (MSW NRW, 2016). As a result, there is the challenge for teachers to take into account the individual learning needs of each learner. Differentiation has to be made possible in regular lessons with class sizes of around 30 students, although the teacher's workload should not have to increase tremendously. The resources of teachers are limited. Therefore, this is accompanied by the demand that differentiation in everyday teaching has to be effectively implemented with the available resources. One idea or possibility to master this heterogeneity in the classroom are self-differentiating tasks such as modelling tasks (Borromeo Ferri, 2018; Maaß, 2007).

In order to investigate this assumption, this paper examines the extent to which the processing of a modelling task differs between students of unequal abilities. For this purpose, the theoretical background of self-differentiating tasks and mathematical modelling will be explained first. This is then followed by the research design of a case study with a specific sampling. The results of three cases will be presented and finally discussed.

2. Literature Review

Self-differentiating tasks, according to Büchter und Leuders (2005), are tasks that allow individual approaches to the problem. They are tasks in which different solutions are possible and thus allow for processing according to individual abilities.

Such self-differentiating tasks are preferred to a level structure of the worksheet for students and serve to implement the principle of differentiation (Schnell and Prediger, 2017). The corresponding design principle, which originates from software development for younger students, can be characterised by a low-threshold approach with concomitant possibilities of achieving a high standard (Shade, 1991). Indeed, Maaß (2005) was able to show in her study that the openness of modelling tasks enables students to develop solutions according to their abilities. It is interesting to note that strong students prefer more demanding approaches, while weaker students choose easier ways. Modelling tasks are thus self-differentiating (Borromeo Ferri, 2018, p. 58), partly because they help to find multiple solutions (Achmetli and Schukajlow, 2019). Bergman Ärlebäck and Bergsten (2010) describe as a feature of Fermi tasks, which can also be regarded as special modelling tasks, their self-differentiating nature. This means that the problem can be worked on and solved in various stages and on different levels of complexity.

Modelling tasks therefore receive special attention when we are dealing with heterogeneous learning groups. Reilly (2017), for example, describes an example of a modelling task that was used in two very heterogeneous classes. It could be shown that the role that such tasks play for students who have to deal with mathematics is of great importance, especially for students with special needs. Scott-Wilson et al. (2017) were even able to show advantages for disabled students when working on modelling tasks. The effects of learning by mathematical modelling

tasks on students with disabilities have been investigated. The daily mathematics lessons were replaced by a series of modelling tasks for one month. The results showed evidence of commitment and meaningful mathematical learning.

Appropriate aids in complex modelling situations are proposed as a possible pedagogical means for working with modelling tasks in heterogeneous learning groups (Stender and Kaiser, 2015). Heterogeneous group work is a pedagogical practice that has proven promising in providing all pupils with the necessary learning opportunities to develop mathematical skills (Staples, 2008).

3. Theoretical Background

In this paper, mathematical modelling means “the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution” (Niss et al. 2007, p.12). These activities are usually described with the help of a modelling cycle. There are various detailed modelling cycles (Borromeo Ferri, 2006). For our analyses, we used the more detailed modelling cycle from Blum and Leiss (2007), which is presented in the following example task. “Good modelling tasks” have special properties (Reit and Ludwig, 2015) that support this self-differentiating character of a task. They are open, i.e. they allow different approaches to the task, solution and thus different results (Siller and Greefrath, 2020; Maaß, 2010). So, for the task to be worked on at different levels there will not be one correct result. There are, however, other properties of modelling tasks that support the self-differentiating character. Good modelling tasks include an *authentic* context (Siller and Greefrath, 2020; Maaß, 2010), allowing for learners to develop a positive attitude towards mathematics (Maaß, 2007). The low achievers will lose their fear of abstract mathematics and the high achievers will be encouraged to deal with the tasks on a high level in order to develop an authentic solution (Maaß, 2007). The realistic contexts (Siller and Greefrath, 2020; Maaß, 2010), which should be relevant to the learners’ present or future life, enable learners to use their everyday knowledge to find the solution (Maaß, 2007). Knowledge from everyday life can, for example, serve as a basis for reference value. Furthermore, modelling tasks can stimulate various activities when solving them. The more and the more clearly, sub-competencies (Kaiser, 2007) are addressed, the greater the possibility for students to find their own solutions. Hence we can summarise some properties of modelling tasks that prove useful for the self-differentiating properties (Greefrath, Siller and Ludwig, 2017, p. 936):

Openness: Is there more than one possibility to solve the problem (solution variety)?

Authenticity: Is the factual context authentically related to the actual situation?

Relevance: Is the factual context relevant to the students (factual problem)?

Sub-competencies: Which sub-competencies of modelling are required for dealing with the problem?

4. Modelling Task Christmas Tree

The modelling task used in this study meets these requirements. The context of the task is the Christmas market in Muenster, where a Christmas tree in front of a church in the city

centre is to be decorated. To this end, a new chain of lights has to be purchased (see Figure 1). The students' task is to determine the length of the chain of lights and the number of light bulbs needed.

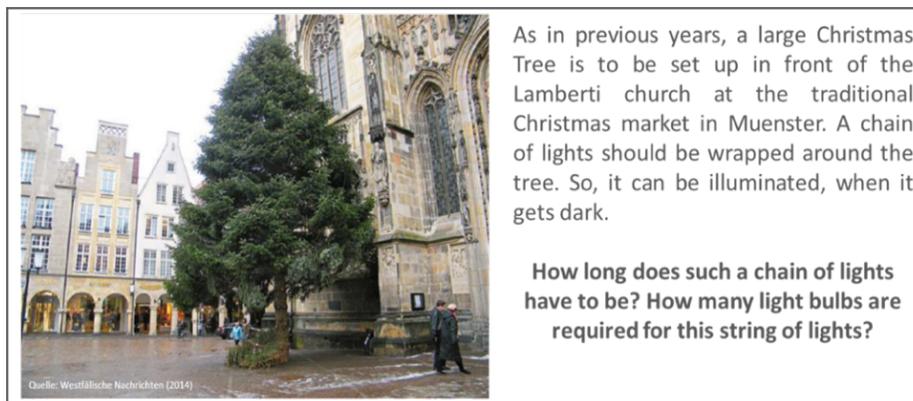


Figure 1 – Modelling Task Christmas tree. This task was developed as part of a master thesis by Jannis Kommnick

This task is under-determined when, for example, there is lacking information on the size of the tree, requiring that the size has to be estimated or determined by values of reference sizes. Basic knowledge can be used, e.g. the height of a person walking near the Christmas tree. This openness concerning various possible solutions assures that the task can be carried out on different levels. Authenticity is given because the organisers of the Christmas market really have to deal with that issue. A certain relevance is given by the fact that tree decorating has a long-standing tradition in Germany. Therefore, many students will be familiar with this activity and apply use their everyday knowledge to find a solution. Furthermore, data collection took place shortly before Christmas. To solve the task, an entire modelling process must be run through. An exemplary solution to this “Christmas tree”-task is presented in the following in reference to the modelling cycle by Blum and Leiss (2007) (see Figure 2).

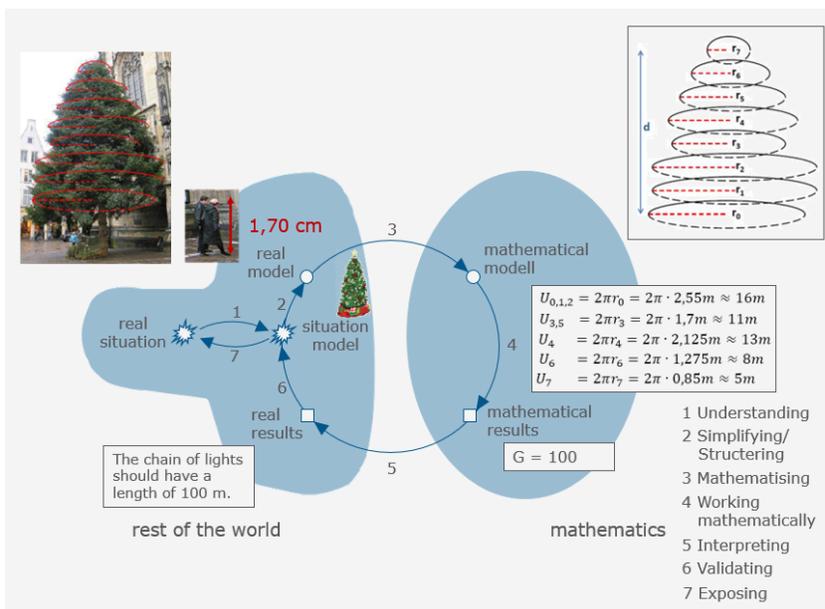


Figure 2 – Example solution according to Blum and Leiss (2007, p.225)

The real situation is represented by the task, which must be understood in the first step. Understanding creates a mental model of the given situation, which is characterised by personal associations and does not yet contain any distinction between unimportant and important information. Here, for example, memories of the last Christmas may be included. The differentiation of important and unimportant information takes place in a step of simplifying and structuring. Only information necessary for processing remains, such as a reference value to determine a scale for calculating the size in reality. Solution-relevant information is organised and assumptions are made to simplify reality. The real model is then translated into a mathematical model by identifying mathematical objects, such as circles and introducing mathematical notations. This mathematical model can now be used for mathematical work. The mathematical result obtained in this way must be interpreted in the context of the facts. In this modelling process, a length of 100 metres was determined for the string of lights. This real result is then validated, i.e. it is checked whether the result appears plausible in terms of magnitude, whether it has the right unit, whether the assumptions make sense and the chosen model is a reasonable one. After this examination, the initial question can be answered. This exemplary solution represents only one of many possible solutions.

5. Research Design

There are several facets of heterogeneity, but since performance heterogeneity is omnipresent in the everyday classroom, the focus of this study was set on it. The self-differentiating nature of modelling tasks suggests that students with different levels of mathematical achievement solve modelling tasks in various ways. We are interested in what these differences may be in detail. The following research question therefore arises:

How do the modelling processes of students working on the Christmas Tree Task differ in terms of general mathematical achievement?

The design shown below was developed to give a first idea of how this question might be answered in the context of a broader study in the future.

5.1. Participants and Procedure

This study was conducted in December of 2019. Twenty-one high school students participated in the survey. The participants were approximately 15 years old. All twenty-one students accomplished the performance test DEMAT 9 (Schmidt, Ennemoser and Krajewski, 2013), so that the performance spectrum of a class could be represented (see Figure 1). The DEMAT 9 test is a curriculum-based procedure for the assessment of general mathematical competencies in year 9. The test provides a reliable, economic and curricular valid record of mathematics performance and therefore enables the economic testing of large samples.

The lowest number of points achieved was 14 points and the highest number of points achieved was 35 points out of a maximum of 43 points. The average number of points achieved in the class was 23.76 points. Out of this sample, six students participated in a video study. All of them had a declaration of consent of their legal guardians. The refusal of such a declaration is the reason why, for example, P21 and P3 did not participate in the video study. The students were selected according to their achievement levels, whereby two belonged to the upper achievement level (31 and 35 points, red bars in Figure 3), two to the intermediate achievement

level (22 and 24 points, yellow bars in Figure 3) and two to the lower achievement level (15 and 17 points, green bars in Figure 3). Two students with a similar achievement level worked together. This allowed a natural way of communication. The video study was carried out in a laboratory situation, so that the external influencing factors were minimised. The students had as much processing time as they needed. Three pairs were filmed while independently solving the modelling task Christmas tree (see Figure 1). This task type had not been discussed in class previously.

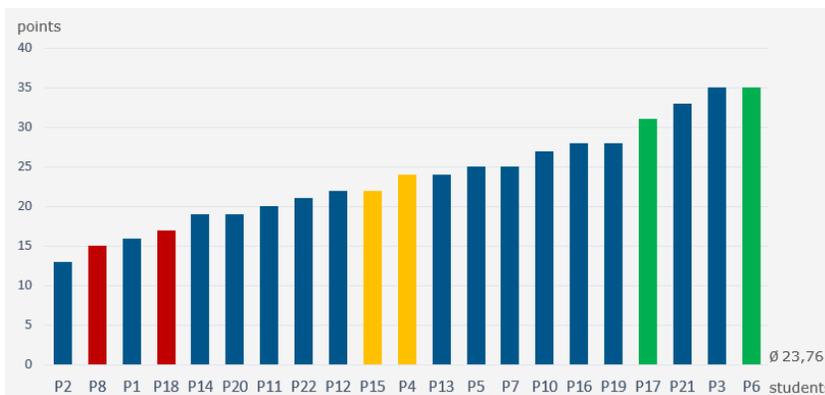


Figure 3 – Sampling DEMAT 9

5.2. Data Analysis

The data to be evaluated included three videos of task processing of the pairs of students. First, the video material was transcribed for the subsequent analysis, so that the transcripts facilitated the evaluation. It is a literal transcript in which non-verbal expressions and actions were also included. Written notes and sketches were integrated into the transcript. Mayring's (2014) qualitative content analysis provided the basis for the evaluation of the data. The deductive category system for the analysis of the modelling processes was based mainly on the following modelling sub-competencies according to Blum and Leiss (2007): Understanding, Simplifying/Structuring, Mathematising, Working Mathematically, Interpreting, Validating and Exposing (see Table 1).

Sub-competency	Description
Understanding	The real situation is reproduced on the basis of the task text.
Simplifying/Structuring	Students identify relevant and irrelevant information from a real problem and make assumptions that simplify reality.
Mathematising	Students translate specific, simplified real situations into mathematical models.
Working mathematically	Students calculate a mathematical result.
Interpreting	Students interpret the mathematical result in reality and interpret the result with regard to the question.
Validating	Students judge the real results obtained in terms of plausibility.
Exposing	Students record their solution in written form

Table 1 – Action-specific description of sub-competencies (Greefrath and Vorhölter, 2016; Siller and Greefrath, 2020)

The data were encoded using time-sampling, i.e. one encoding unit was set to 30 seconds. For this purpose, the sub-competency which dominated within these 30 seconds of processing time was coded (see Reusser, Pauli and Waldis, 2010). The coding involved certain difficulties of delimitation of the sub-competencies, which had to be subject of discussion, to ensure that a satisfactory result could be achieved. To ensure validity and objectivity, 30 per cent of the transcripts were coded by a second independent person. Intercoder realism was satisfactory at a value of 0.78 (Cohens, 1968). Below a transcription excerpt is shown (see Table 2), revealing how the sub-competencies “Simplifying/Structuring” and “Mathematising” were delineated. “Simplifying/Structuring” includes the search for information relevant to the solution. In the subsequent “Mathematising”, mathematical-symbolic representations are introduced, e.g. measuring lengths in the picture.

P18:	Because somehow, we don't have any length specifications or measurement specifications. For example, there are also two people here. I don't know how tall they are.	Simplifying/Structuring
P8:	Yes, and also the church, which is still a little way up here. But here, the house, it's definitely bigger. I don't know if five metres is enough. That's about it. And the tree is about eight or so.	
P18:	<i>(Measures distances in the illustration with fingers and ruler)</i> I have now looked approximately like this and, for example, would say like this <i>(points to the person in the foreground of the illustration)</i> and simply put the set square on it, approximately at 2.2/2.3. If you put this here approximately 3.1 or 3.2. That you say, for example, if they are now one metre eighty tall, for example. That you then say that you add about half of that.	Mathematising

Table 2 – Transcription excerpt

6. Results

The case study on mathematical modelling shows that all three pairs of students were able to solve the modelling task according to their performance level. The following outlines the students work processes. For this purpose, the work processes are shown as bars, where a coding unit of 30 corresponds to a box. For the sake of clarity, the bars have been divided into sections of one minute each. The colour coding for the respective sub-competencies can be found in the legend below the illustration.

6.1. Case 1: Low Achievement Level

The pair of students, who are assigned a low performance level after the performance test, spent about 26 minutes working on the task (see Figure 4).

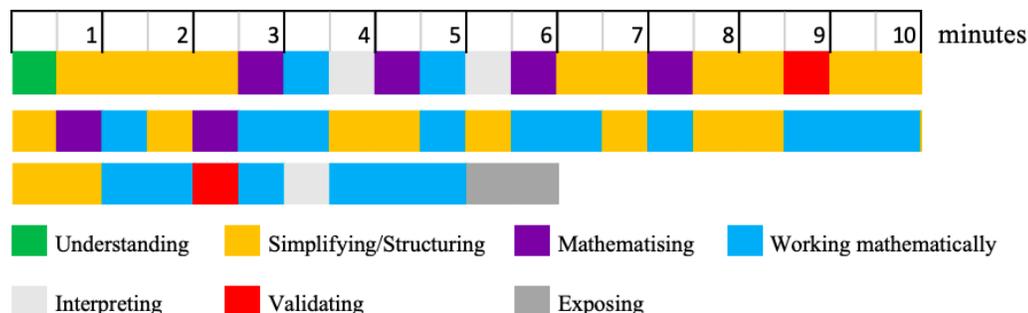


Figure 4 – Process of the Pair of Students with Low Achievement Level (26 min)

The modelling process is characterised by frequent changes between the sub-competencies “Simplifying/Structuring” and “Working Mathematically”. Already at the beginning of the editing process, the pair chose a person depicted in the picture as a reference. However, this selected reference value was not included in the further development process. Validation processes occurred only sporadically. During the mathematical work, mistakes became apparent in the mathematical notation. The rules for the use of the equal sign were disregarded (see Figure 5).

Handwritten mathematical work showing calculations for a Christmas tree problem. The work includes several equations with errors in notation and calculation:

$$5m \cdot 2 = 10m \cdot 4 = 40m + 3 \cdot 8$$
~~$$64m + 2 \cdot 3$$~~

$$64 + 2 \cdot 6 = 76m + 6 = 82m + 3 \cdot 4 = 94m + 4 = 98m$$

$$2 + 3 = 5$$

$$98 + 5 = 103m$$

Figure 5 – Excerpt Low Achievement Level

The students estimated values for the width of the treetop in different section on the basis of the pictured Christmas tree, which they doubled in order to also account for the back not shown in the picture. They correctly carried out the arithmetical operations, but failed to note them down in a mathematically correct form. This continued in the subsequent editing process, especially when determining the number of light bulbs instead of using a division with the remainder. The students did not draw a sketch during the entire editing process.

6.2. Case 2: Intermediate Achievement Level

In total, the modelling process of the students with an intermediate performance level needed 14 and a half minutes, which was the shortest modelling process (see Figure 6).

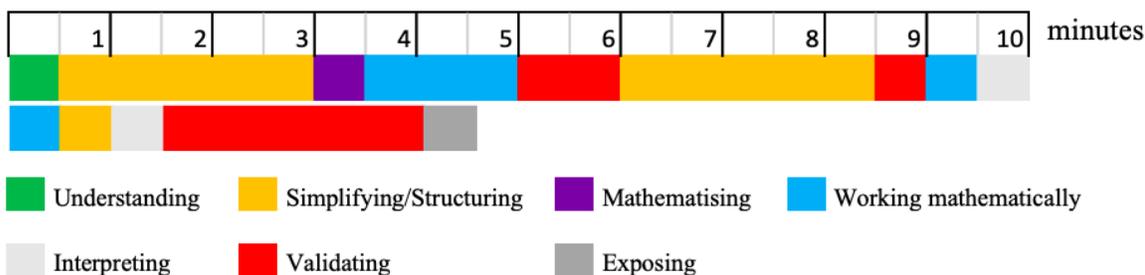


Figure 6 – Process of the Pair of Students with Intermediate Achievement Level (14 min 30 sec)

It contained many simplifications, not much mathematical work is necessary. Validation processes occurred more frequently and in longer phases. The medium-level group was the only group that made sketches. Already at the beginning of the editing process, they mathematised the tree crown as an equilateral triangle, as is shown in the respective sketches (see Figure 7). They immediately mathematised them and, using Pythagoras' theorem, calculated the legs of the equilateral triangle without checking the necessity of the calculation. However, this approach was validated shortly afterwards, as the following transcriptional excerpt shows:

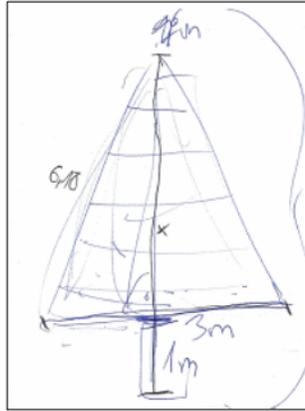


Figure 7 – Sketch 1 Intermediate Achievement Level

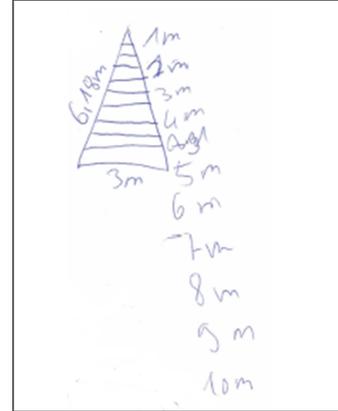


Figure 8 – Sketch 2 Intermediate Achievement Level

P15: “This is really stupid. We have only calculated the side length and not the surface area. That doesn’t even make sense.”

A new approach was developed, which divides the tree crown into horizontal sections and the corresponding lengths they estimate in a highly simplified way, starting at one metre (see Figure 8).

6.3. Case 3: High Achievement Level

The modelling process of the students with high-achievement levels required 27 minutes and was characterised by a very long phase of “simplifying and structuring” at the beginning of the process (see Figure 9). Only at the end of the editing process did they work mathematically.

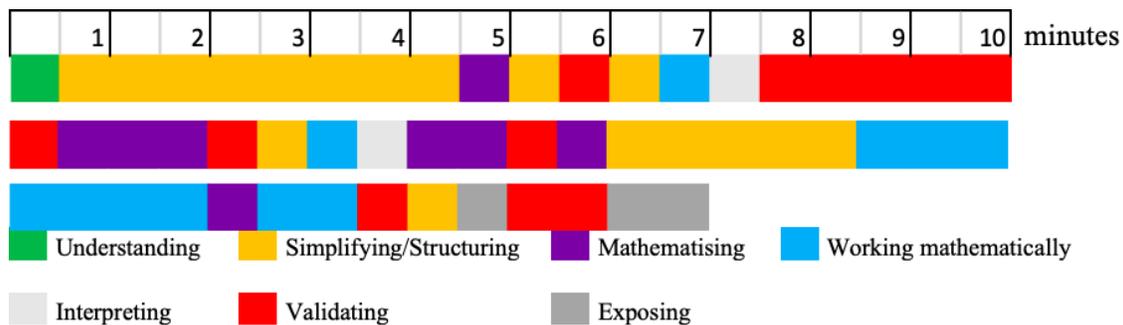


Figure 9 – Process of the Pair of Students with High Achievement Level (27 min)

They always validate the choice of reference size:

P6: Huh, do we have to do that with her (points to person in background), because she is in the background? Look, that’s not in any way proportional. Generally, much smaller. If she were standing there, she’s not just that tall, is she?

P17: (laughs) Right, this is really stupid!

P6: This one maybe? (points to people in the foreground)

P17: That was really stupid. // Oh, man. // Right.

The high-performing pair of students was the only group that measured lengths in the figure and converted them into a kind of scale using a reference size. But they did this for each section individually (see Figure 10).

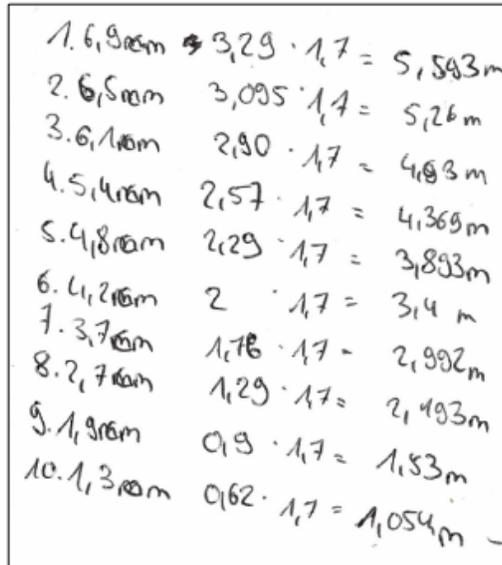


Figure 10 – Excerpt from the High Achievement Level

7. Discussion and Conclusion

The present qualitative study showed interesting results. As the research area is still unexplored, no hypotheses were formulated in advance, but an exploratory approach was applied to the research question. The processing of this open task by the students on their individual performance level confirms the self-differentiating character of this modelling task (Borromero Ferri, 2018; Maaß, 2007). It was shown that all three pairs of students had concluded their solution processes. All sub-competencies could be coded for each pair of students in the course of task processing. Despite different assumptions and mathematical models, the obtained results that can be considered as realistic. Individual modelling processes could be observed during task solving (Borromero Ferri, 2007). The durations of the sub-competencies in the process differ regarding the various student pairs. There were differences with regard to frequency, especially in the sub-competency of validation.

The comparison of the modelling processes clearly shows that the group with a low level of achievement often switches between the sub-competencies ‘simplifying/structuring’ and ‘working mathematically’. They rarely validate. The process of intermediate achievement students shows many simplifications, so that little mathematical work is necessary. The editing process contains longer phases of validation. The high achievement level students ‘simplify and structure’ at the beginning for a very long time, so that they only ‘work mathematically’ at the end of the editing process. They also validate their approach frequently and over a longer period of time. This could indicate that high-achieving students are more reflective.

However, there were also some limitations. The results can only be generalised to a limited extent, as the sample of three pairs of students was small and thus only represented the range of achievement in one class. Only the processes of a specific modelling task were considered. Our findings therefore only refer to this one task. The process of couples were examined, hence only conclusions referring to the process of a couple can be made. The individual editing process may have taken a different course. Therefore, further research with more pairs of students and other modelling tasks will be necessary.

In summary, this case study has shown that modelling tasks are suitable for teaching in heterogeneous performance settings. Of course, a single task cannot resolve the heterogeneity of a class. However, it is easier for the teacher if all students work on one task, instead of several tasks having different levels of difficulty. The low-performing students were not overchallenged, nor were the high-performing students underchallenged. Furthermore, it can be stated that this modelling task can be worked on well in partner work. On the basis of the results, further research interest is being developed to clarify whether a typical pattern of performance levels can be found with a larger sample size. This information would also be of great importance for practical use in school in order to make better use of the self-differentiating properties of modelling tasks in teaching. Since this task has been shown to be suitable for a heterogeneous group of students, it can give indications to what characteristics modelling tasks require in order to be self-differentiating. For example, the question of the targeted promotion of certain types of students is of great value. If the typical behaviour of students showing unequal mathematical achievements in the processing of modelling tasks is confirmed, we should be able to develop targeted aids or solution plans.

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