Optimizing the demographic rates to control the dependency ratio in Spain

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1 Introduction

The decline in the birth rate and the increase in longevity are a fact in the developed countries and, a growing trend in the developing countries, so the implications of these facts on future well-being are fundamental, in particular the impact on the population pyramid and, even more, on the dependency rate.

According to the studies on EU-28 [3] the proportion of people of working age is decreasing, while the relative number of retired people is increasing. Demographers warn that this is due to a decrease in births [1,2], but not only this demographic phenomenon affects this increase in the dependency rate. For example, migration control is an essential tool. Particularly, an organization such as the Department of Economic and Social Affairs of UN warns that only a fifteen per cent of the Governments control their current immigration to address their population ageing, and only a a thirteen per cent deal with the problem of the long-term population decline [3].

Underlying these facts, a problem arises: what would be the appropriate birth and migration happening for a society such that, within a reasonable period, its dependency ratio changes its trend?

The aim of this work is to adapt the demographic model presented by [4] to solve the described problem. The model modifications here presented include considering the death and migration rates as control variables, which obligates to change some model parts. This new model has been validated for the case of Spain in its deterministic and stochastic formulations. Finally, the model is used to determine the future evolution of the birth, death and migration rates in Spain

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in order to decrease the dependency ratio. The evolution of these demographic phenomena, which are considered optimal, is calculated by using strategies and scenarios.

2 Demographic Model

The new demographic model (see [4] to appreciate the changes), written in its continuous form, is constituted by the following equations:

$$\frac{\partial w_i(t,x)}{\partial t}+c\frac{\partial w_i(t,x)}{\partial x} = (-d_i(x)\cdot grde_i(t,x)+f_i(x)\cdot gryn_i(t,x)-g_i(x)\cdot grem_i(t,x))\cdot w_i(t,x) \quad (1)$$

$$w_i(t,0) = birt(t)\cdot\frac{b_i(t)}{b_1(t)+b_2(t)}\cdot\int_0^{+\infty}(w_1(t,x)+w_2(t,x))dx\cdot\int_0^{+\infty}\bar{b}_i(x)\cdot w_2(t,x)dx \quad (2)$$

$$w_i(t_0,x) = u_i(x) \quad (3)$$

Where, $i = 1$ represents men and $i = 2$ women.

Eq. (1) is a von Foerster-McKendrick equation that determines the dynamics of population density depending on time and age, $w_i(t,x)$, where $d_i(x)$, $f_i(x)$ and $g_i(x)$ represent respectively the death, immigration and emigration rates, as a function of age. Also, $grde_i(t,x)$, $gryn_i(t,x)$ and $grem_i(t,x)$ are respectively the growth rates for each previous demographic phenomena, as functions of age and time.

Eq. (2) represents the boundary condition, that is, births at $x = 0$. In this equation, $\frac{b_i(t)}{b_1(t)+b_2(t)}$ is the proportion of men or women born (according to $i = 1$ or 2, respectively), that is, births per sex ($b_i(t)$) divided by the total number of births ($b_1(t)+b_2(t)$); $birt(t)$ is the birth rate, i.e., the total numbers of births ($b_1(t)+b_2(t)$) divided by the total population; and $\bar{b}_i(x)$ is the ratio between the fertility rate and births.

Eq. (3) is the initial condition, that is, the initial population density, $u_i(x)$, at $t = t_0$.

Some simplifying hypotheses are made on Eqs. (1) and (2) (similarly to those made in [4]) in order to introduce the death, emigration and immigration rates temporarily defined. Thus, the modifications introduced in the model are the following.

$$d_i(x)\cdot grde_i(t,x) \approx \bar{d}_i(x)\cdot\frac{d_i(t)}{d_1(t)+d_2(t)}\cdot deat(t)\cdot popt(t) \quad (4)$$

$$f_i(x)\cdot gryn_i(t,x) \approx \bar{f}_i(x)\cdot\frac{y_i(t)}{y_1(t)+y_2(t)}\cdot immi(t)\cdot popt(t) \quad (5)$$

$$g_i(x)\cdot grem_i(t,x) \approx \bar{g}_i(x)\cdot\frac{e_i(t)}{e_1(t)+e_2(t)}\cdot emig(t)\cdot popt(t) \quad (6)$$

In these equations, the proportions of deaths, immigration or emigration for men or women, ($\frac{d_i(t)}{d_1(t)+d_2(t)}$, $\frac{y_i(t)}{y_1(t)+y_2(t)}$, $\frac{e_i(t)}{e_1(t)+e_2(t)}$, respectively) (according to $i = 1$ or 2, respectively) are considered, that is, deaths, immigration and emigration per sex ($d_i(t)$, $y_i(t)$ and $e_i(t)$) divided by the total number of deaths, immigration or emigration. Finally, $\bar{d}_i(x)$, $\bar{f}_i(x)$ and $\bar{g}_i(x)$ are the ratios between the different demographic rates (functions of age) and deaths, immigration and emigration respectively in $t = 0$. Note that, $popt(t)$ can be calculated by the model as:

$$popt(t) = \int_0^{+\infty} (w_1(t,x) + w_2(t,x))dx \tag{7}$$

With these considerations on the initial model, the following equations are obtained:

$$\begin{aligned}\frac{\partial w_i(t,x)}{\partial t} + c\frac{\partial w_i(t,x)}{\partial x} = \Big(\big(- \bar{d}_i(x) \cdot \frac{d_i(t)}{d_1(t)+d_2(t)} \cdot deat(t) + \bar{f}_i(x) \cdot \frac{y_i(t)}{y_1(t)+y_2(t)} \cdot inmi(t) \\ - \bar{g}_i(x) \cdot \frac{e_i(t)}{e_1(t)+e_2(t)} \cdot emig(t)\big) \cdot \int_0^{+\infty} (w_1(t,x) + w_2(t,x))dx\Big) \cdot w_i(t,x)\end{aligned} \tag{8}$$

$$w_i(t,0) = birt(t) \cdot \frac{b_i(t)}{b_1(t)+b_2(t)} \cdot \int_0^{+\infty} (w_1(t,x) + w_2(t,x))dx \cdot \int_0^{+\infty} (\bar{b}_i(x) + w_2(t,x))dx \tag{9}$$

$$w_i(t_0,x) = u_i(x) \tag{10}$$

3 Model Validation

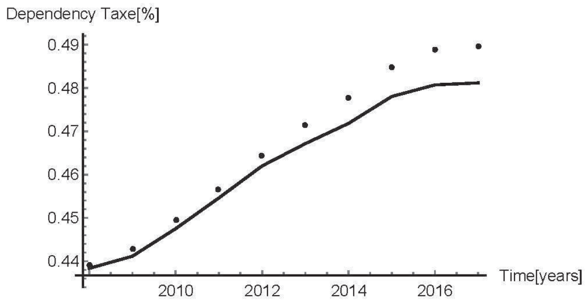
The validation of the model is performed for Spain in the 2007-2017 period, i.e., for those years whose information is available in the World Data Bank [5]. The obtained data are also used to fit input variables to time.

Although the model has been written as a set of differential and functional equations, the solutions have been calculated with the Euler Method, following [6, 7], which explain that the Euler Method is more adequate to solve such equations. In the case of the integral in Eq. (9), it is calculated through the Simpson Composite Rule. This approach results in a set of finite difference equations that has been programmed in Visual Basic 6.0 using Sigem [8, 9].

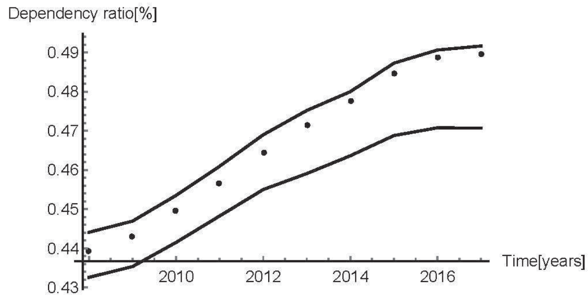
The corresponding validation has been performed like in [4]. On the one hand, the deterministic formulation of the model is validated through the determination coefficients and the random residuals tests. The real and simulated data are plotted in Figs. 1a and 2a. On the other hand, the stochastic formulation is also validated by checking that the historical data fall between the minimum and maximum simulated values (Figs. 1b and 2b). The validation process is considered successful because the determination coefficients, R^2, are very high, and the maximum relative error does not exceed 4.51% in any case. In the case of the stochastic validation, all the real data are within the 99% generated confidence interval.

4 Model application

In the application case, the aim is to minimize the dependency ratio. This minimization decreases the pressure on the productive population. Thus, the dependency ratio is defined as

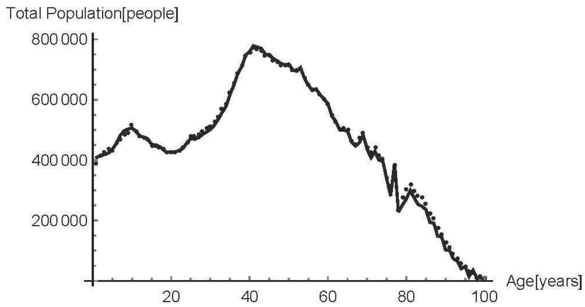


(a) Deterministic validation, $R^2 = 0.997542$

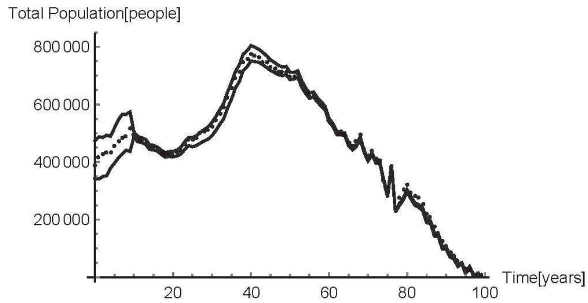


(b) Stochastic validation

Figure 1: Dependency ratio for Spain in the 2008-2017 period.



(a) Deterministic validation, $R^2 = 0.998135$



(b) Stochastic validation

Figure 2: Total population for Spain in 2017.

the dependent population (population with ages from 0 to 15 and with 65 or more) divided by the productive population (population with ages from 16 to 64). That is:

$$obje(t) = \frac{\sum_i (\int_0^{x_m} w_i(t,x)dx + \int_{x_M}^{+\infty} w_i(t,x)dx)}{\sum_i \int_{x_m}^{x_M} w_i(t,x)dx} \tag{11}$$

In Eq. (11), x_m is the minimum working age, generally $x_m = 15$, and x_M the retirement age, generally $x_M = 65$.

The method, that has been used to find the evolution of the input variables (control variables) that minimize the dependency ratio, that is, the objective variable $obje(t)$ is to determine strategies over control variables (see Table 1).

Control variable	SS1	SS2	SS3	SS4
Birth Rate	↑	↑	↑	↑
Emigration Rate	↑	↓	↑	↓
Immigration Rate	↑	↓	↓	↑

Table 1: Strategies to minimize the dependency ratio. ↑: to increase 5% the tendency; ↓ to decrease 5% the tendency.

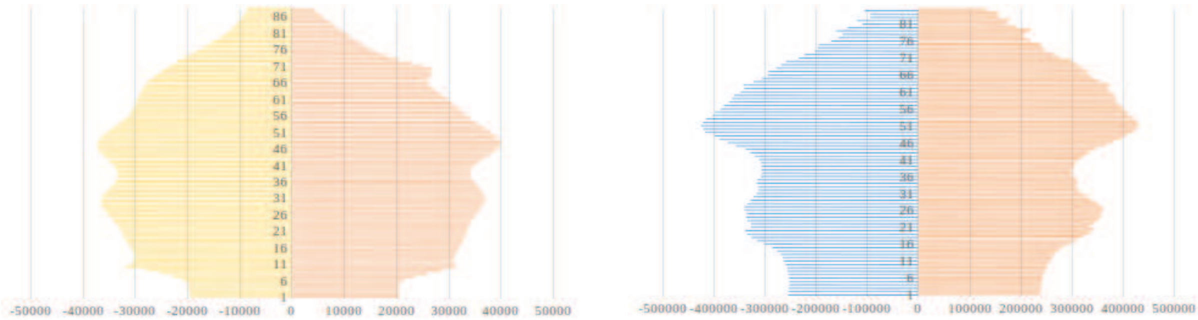
For the application case here presented (the case of Spain), the time t runs in the 2018-2027 period and, the corresponding deterministic model formulation results are shown in Table 2.

year	SS1	SS2	SS3	SS4
2018	0.4891209	0.488799	0.4919379	0.4863304
2019	0.4886366	0.4885366	0.4918719	0.4855817
2020	0.489179	0.4895301	0.4931213	0.4858498
2021	0.4872227	0.4879565	0.4917752	0.4837045
2022	0.4847987	0.4860044	0.4901535	0.4809992
2023	0.4832554	0.4850131	0.4894081	0.4791768
2024	0.4808911	0.4831095	0.4877837	0.476468
2025	0.4779461	0.4807993	0.4857836	0.473359
2026	0.4763169	0.4797138	0.4851585	0.4713179
2027	0.4748512	0.4790213	0.4790213	0.4695968

Table 2: Values of the dependency ratio, $obje(t)$, for the case of Spain in the 2018-2028 period.

To reduce the dependency ratio, i.e. to get more people in working ages with respect to those in non-working ages, it is necessary to apply the SS4 and to modify the trend of the demographic control variables on those terms: increasing the birth and immigration rates and, reducing the deaths (Table 1).

In this situation, the Spain population pyramid has changed as Fig. 3 shows.



(a) 2017 (b) 2018

Figure 3: Pyramid population, female (right) and male (left) population for Spain.

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