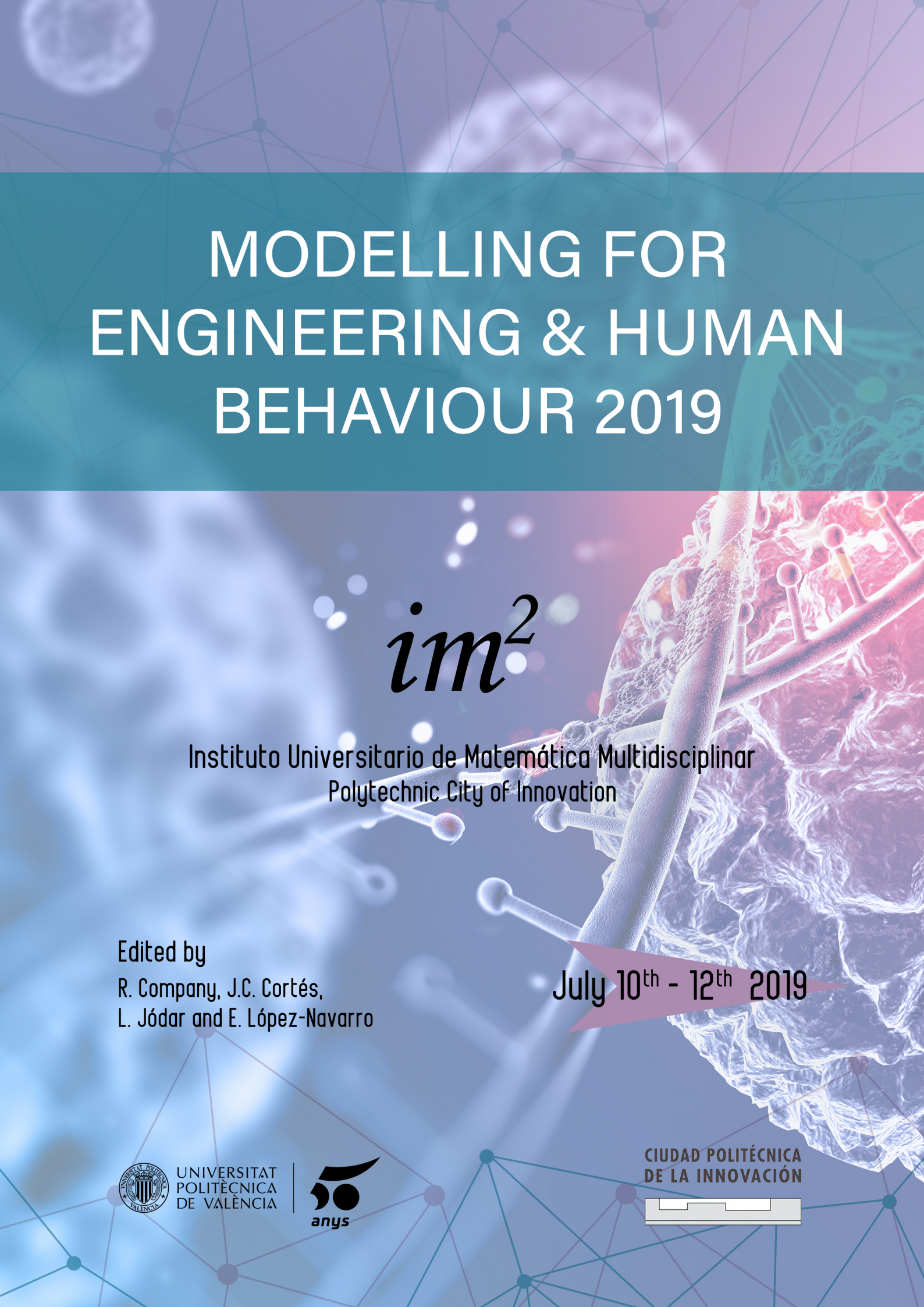
MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR 2019



im^2

Instituto Universitario de Matemática Multidisciplinar
Polytechnic City of Innovation

Edited by
R. Company, J.C. Cortés,
L. Jódar and E. López-Navarro

July 10th - 12th 2019

UNIVERSITAT POLITÈCNICA DE VALÈNCIA
50 anys
CIUDAD POLITÉCNICA DE LA INNOVACIÓN

Modelling for Engineering & Human Behaviour 2019

València, 10 – 12 July 2019

This book includes the extended abstracts of papers presented at XXIst Edition of the Mathematical Modelling Conference Series at the Institute for Multidisciplinary Mathematics "Mathematical Modelling in Engineering & Human Behaviour".

I.S.B.N.: 978-84-09-16428-8
Version: 18/11/19
Report any problems with this document to ellona1@upvnet.upv.es.

Edited by: R. Company, J. C. Cortés, L. Jódar and E. López-Navarro.
Credits: The cover has been designed using images from kjpargeter/freepik.

Instituto Universitario de Matemática Multidisciplinar

This book has been supported by the European Union through the Operational Program of the [European Regional Development Fund (ERDF) / European Social Fund (ESF)] of the Valencian Community 2014-2020. [Record: GJIDI/2018/A/010].

GENERALITAT VALENCIANA
Conselleria d'Hisenda i Model Econòmic

UNIÓ EUROPEA
Fons Europeu de Desenvolupament Regional
Una manera de fer Europa

Contents

A personality mathematical model of placebo with or without deception: an application of the Self-Regulation Therapy . . . 1

The role of police deterrence in urban burglary prevention: a new mathematical approach . . . 9

A Heuristic optimization approach to solve berth allocation problem . . . 14

Improving the efficiency of orbit determination processes . . . 18

A new three-steps iterative method for solving nonlinear systems . . . 22

Adaptive modal methods to integrate the neutron diffusion equation . . . 26

Numerical integral transform methods for random hyperbolic models . . . 32

Nonstandard finite difference schemes for coupled delay differential models . . . 37

Semilocal convergence for new Chebyshev-type iterative methods . . . 42

Mathematical modeling of Myocardial Infarction . . . 46

Symmetry relations between dynamical planes . . . 51

Econometric methodology applied to financial systems . . . 56

New matrix series expansions for the matrix cosine approximation . . . 64

Modeling the political corruption in Spain . . . 70

Exponential time differencing schemes for pricing American option under the Heston model . . . 75

Chromium layer thickness forecast in hard chromium plating process using gradient boosted regression trees: a case study . . . 79

Design and convergence of new iterative methods with memory for solving nonlinear problems . . . 83

Study of the influence falling friction on the wheel/rail contact in railway dynamics . . . 88

Extension of the modal superposition method for general damping applied in railway dynamics . . . 94

Predicting healthcare cost of diabetes using machine learning models . . . 99

Sampling of pairwise comparisons in decision-making . . . 105

A multi-objective and multi-criteria approach for district metered area design: water operation and quality analysis . . . 110

Updating the OSPF routing protocol for communication networks by optimal decision-making over the k-shortest path algorithm . . . 118

Optimal placement of quality sensors in water distribution systems . . . 124

Mapping musical notes to socio-political events . . . 131

Comparison between DKGA optimization algorithm and Grammar Swarm surrogated model applied to CEC2005 optimization benchmark . . . 136

The quantum brain model . . . 142

Probabilistic solution of a randomized first order differential equation with discrete delay . . . 151

A predictive method for bridge health monitoring under operational conditions . . . 155

Comparison of a new maximum power point tracking based on neural network with conventional methodologies . . . 160

Influence of different pathologies on the dynamic behaviour and against fatigue of railway steel bridges . . . 166

Statistical-vibratory analysis of wind turbine multipliers under different working conditions . . . 171

Analysis of finite dimensional linear control systems subject to uncertainties via probabilistic densities . . . 176

Topographic representation of cancer data using Boolean Networks . . . 180

Trying to stabilize the population and mean temperature of the World . . . 185

Optimizing the demographic rates to control the dependency ratio in Spain . . . 193

An integer linear programming approach to check the embodied CO_2 emissions of the opaque part of a façade . . . 199

Acoustics on the Poincaré Disk . . . 206

Network computational model to estimate the effectiveness of the influenza vaccine *a posteriori* . . . 211

The key role of Liouville-Gibbs equation for solving random differential equations: Some insights and applications . . . 217

13

New matrix series expansions for the matrix cosine approximation

Emilio Defez [♭1], Javier Ibáñez[♮], José M. Alonso[♮], Jesús Peinado[♯] and Pedro Alonso-Jordá[*]

(♭) Institut Universitari de Matemàtica Multidisciplinar,
Universitat Politècnica de València,
(♮) Instituto de Instrumentación para Imagen Molecular,
Universitat Politècnica de València,
(♯) Departamento de Sistemas Informáticos y Computación,
Universitat Politècnica de València,
(∗) Grupo Interdisciplinar de Computación y Comunicaciones,
Universitat Politècnica de València.

1 Introduction and notation

The computation of matrix trigonometric functions has received remarkable attention in the last decades due to its usefulness in the solution of systems of second order linear differential equations. Recently, several state-of-the-art algorithms have been provided for computing these matrix functions, see [1–4], in particular for the matrix cosine function.

Among the proposed methods for the approximate computation of the matrix cosine, two fundamental ones stand out: those based on rational approximations [1, 5–7], and those related to polynomial approximations, using either Taylor series developments [8,9] or serial developments of Hermite matrix polynomials [10]. In general, polynomial approximations showed to be more efficient than the rational algorithms in tests because they are more accurate despite a slightly higher cost.

Bernoulli polynomials and Bernoulli numbers have been extensively used in several areas of mathematics (an excelent survey about Bernoulli polynomials and its applicacions can be found in [11]).

In this paper, we will present a new series development of the matrix cosine in terms of the Bernoulli matrix polynomials. We are going to verify that its use allows obtaining a new and competitive method for the approximation of the matrix cosine.

The organization of the paper is as follows: In Section 2, we will obtain two serial developments of the matrix cosine in terms of the Bernoulli matrix polynomials. In Section 3, we will present

[1]e-mail: edefez@imm.upv.es

the different numerical tests performed. Conclusions are given in Section 4.

Throughout this paper, we denote by $\mathbb{C}^{r\times r}$ the set of all the complex square matrices of size r. Besides, we denote I as the identity matrix in $\mathbb{C}^{r\times r}$. A polynomial of degree m is given by an expression of the form $P_m(t) = a_m t^m + a_{m-1}t^{m-1} + \cdots + a_1 t + a_0$, where t is a real variable and a_j, for $0 \leq j \leq m$, are complex numbers. Moreover, we can define the matrix polynomial $P_m(B)$ for $B \in \mathbb{C}^{r\times r}$ as $P_m(B) = a_m B^m + a_{m-1}B^{m-1} + \cdots + a_1 B + a_0 I$. As usual, the matrix norm $\|\cdots\|$ denotes any subordinate matrix norm; in particular $\|\cdots\|_1$ is the usual $1-$norm.

2 On Bernoulli matrix polynomials

The Bernoulli polynomials $B_n(x)$ are defined in [12, p.588] as the coefficients of the generating function

$$g(x,t) = \frac{te^{tx}}{e^t - 1} = \sum_{n\geq 0} \frac{B_n(x)}{n!} t^n \ , \ |t| < 2\pi, \tag{1}$$

where $g(x,t)$ is an holomorphic function in $\mathbb{C}$ for the variable t (it has an avoidable singularity in $t = 0$). Bernoulli polynomials $B_n(x)$ has the explicit expression

$$B_n(x) = \sum_{k=0}^{n} \binom{n}{k} B_k x^{n-k}, \tag{2}$$

where the Bernoulli numbers are defined by $B_n = B_n(0)$. Therefore, it follows that the Bernoulli numbers satisfy

$$\frac{z}{e^z - 1} = \sum_{n\geq 0} \frac{B_n}{n!} z^n \ , \ |z| < 2\pi, \tag{3}$$

where

$$B_0 = 1, B_k = -\sum_{i=0}^{k-1} \binom{k}{i} \frac{B_i}{k+1-i}, k \geq 1. \tag{4}$$

Note that $B_3 = B_5 = \cdots = B_{2k+1} = 0$, for $k \geq 1$. For a matrix $A \in \mathbb{C}^{r\times r}$, we define the $m - th$ Bernoulli matrix polynomial by the expression

$$B_m(A) = \sum_{k=0}^{m} \binom{m}{k} B_k A^{m-k}. \tag{5}$$

We can use the series expansion

$$e^{At} = \left(\frac{e^t - 1}{t}\right) \sum_{n\geq 0} \frac{B_n(A)t^n}{n!} \ , \ |t| < 2\pi, \tag{6}$$

to obtain approximations of the matrix exponential. A method based in (6) to approximate the exponential matrix has been presented in [13].

From (6), we obtain the following expression for the matrix cosine and sine:

$$\left.\begin{aligned} \cos(A) &= (\cos(1)-1)\sum_{n\geq 0}\frac{(-1)^n B_{2n+1}(A)}{(2n+1)!} + \sin(1)\sum_{n\geq 0}\frac{(-1)^n B_{2n}(A)}{(2n)!}, \\ \sin(A) &= \sin(1)\sum_{n\geq 0}\frac{(-1)^n B_{2n+1}(A)}{(2n+1)!} - (\cos(1)-1)\sum_{n\geq 0}\frac{(-1)^n B_{2n}(A)}{(2n)!}. \end{aligned}\right\} \tag{7}$$

Note that unlike the Taylor (and Hermite) polynomials that are even or odd, depending on the parity of the polynomial degree n, the Bernoulli polynomials do not verify this property. Thus, in the development of $\cos(A)$ and $\sin(A)$, all Bernoulli polynomials are needed (and not just the even-numbered ones).

Replacing in (6) the value t for it and $-it$ respectively and taking the arithmetic mean, we obtain the expression

$$\sum_{n\geq 0}\frac{(-1)^n B_{2n}(A)}{(2n)!}t^{2n} = \frac{t}{2\sin\left(\frac{t}{2}\right)}\left(\cos\left(tA-\frac{t}{2}I\right)\right)\ ,\ |t|<2\pi. \tag{8}$$

Taking $t=2$ in (8) it follows that

$$\cos(A) = \sin(1)\sum_{n\geq 0}\frac{(-1)^n 2^{2n} B_{2n}\left(\frac{A+I}{2}\right)}{(2n)!}, \tag{9}$$

Note that in formula (9) only even grade Bernoulli's polynomials appear.

3 Numerical Experiments

Having in mind expressions (7) and (9), two different approximations are given to compute cosine matrix function.

To test the proposed method and the two distinct approximations, and to compare them with other approaches, the following algorithms have been implemented on MATLAB R2018b:

- *cosmber*. New code based on the new developments of Bernoulli matrix polynomials (formulae (7) and (9)). The maximum value of m to be used is $m=36$, with even and odd terms.
- *cosmtay*. Code based on the Taylor series for the cosine [8]. It will provide a maximum value of $m=16$, considering only the even terms, which would be equivalent to $m=32$ using the even and odd terms.
- *cosmtayher*. Code based on the Hermite series for the cosine [10]. As mentioned before, it will provide a maximum value of $m=16$.
- *cosm*. Code based on the Padé rational approximation for the cosine [7].

The following sets of matrices have been used:

a) Diagonalizable matrices. The matrices have been obtained as $A = V \cdot D \cdot V^T$, where D is a diagonal matrix (with complex or real values) and matrix V is an orthogonal matrix, $V = H/16$, where H is a Hadamard matrix. We have $2.18 \leq \|A\|_1 \leq 207.52$. The matrix cosine is exactly calculated as $\cos(A) = V \cdot \cos(D) \cdot V^T$.

b) Non-diagonalizables matrices. The matrices have been computed as $A = V \cdot J \cdot V^{-1}$, where J is a Jordan matrix with complex eigenvalues with module less than 10 and random algebraic multiplicity between 1 and 5. Matrix V is a random matrix with elements in the interval $[-0.5, 0.5]$. We have $1279.16 \leq \|A\|_1 \leq 87886.4$. The matrix cosine is exactly calculated as $\cos(A) = V \cdot \cos(J) \cdot V^{-1}$.

c) Matrices from the Matrix Computation Toolbox [14] and from the **Eigtool Matlab package** [15]. These matrices have been chosen because they have more varied and significant characteristics.

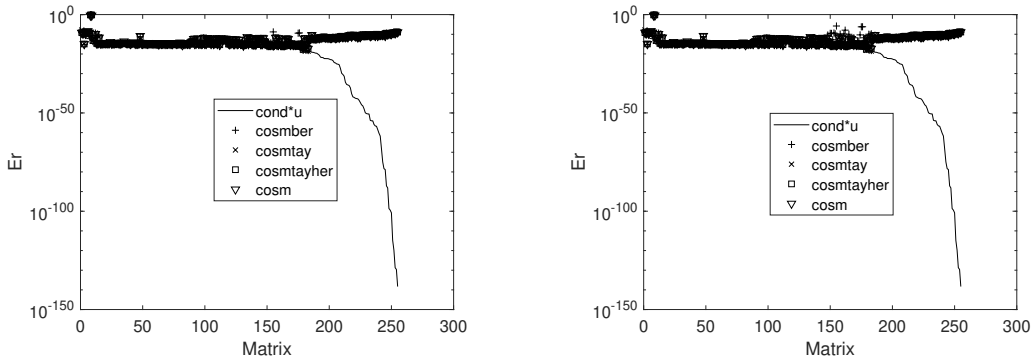
In the numerical test, we used 259 matrices of size 128×128: 100 from the diagonalizable set, 100 from the non-diagonalizable set, 42 from Matrix Computation Toolbox and 17 from Eigtool Matlab package. Results are given in Tables 1 and 2. The rows of each table show the percentage of cases in which the relative errors of `cosmber` (Bernoulli) is lower, greater or equal than the relative errors of `cosmtay` (Taylor), `cosmtayher` (Hermite) and `cosm` (Padé). Graphics of the Normwise relative errors and the Performance Profile are given in Figures 1 and 2. The total number of matrix products was: 3202 (*cosmber*), 2391 (*cosmtay*), 1782 (*cosmtayher*) and 3016 (*cosm*). Recall that in the Bernoulli implementation, the maximum value of m to be used was $m = 36$ considering all the terms and, in the rest of algorithms, was $m = 32$ but just having into account the even terms.

$E(cosmber) < E(cosmtay)$	55.60%
$E(cosmber) > E(cosmtay)$	44.40%
$E(cosmber) = E(cosmtay)$	0%
$E(cosmber) < E(cosmtayher)$	50.97%
$E(cosmber) > E(cosmtayher)$	49.03%
$E(cosmber) = E(cosmtayher)$	0%
$E(cosmber) < E(cosm)$	76.83%
$E(cosmber) > E(cosm)$	23.17%
$E(cosmber) = E(cosm)$	0%

Table 1: Using approximation (7)

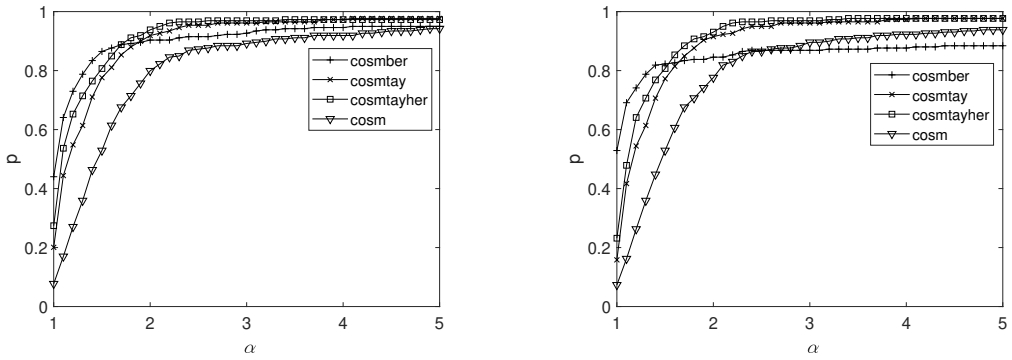
$E(cosmber) < E(cosmtay)$	65.64%
$E(cosmber) > E(cosmtay)$	34.36%
$E(cosmber) = E(cosmtay)$	0%
$E(cosmber) < E(cosmtayher)$	60.62%
$E(cosmber) > E(cosmtayher)$	39.38%
$E(cosmber) = E(cosmtayher)$	0%
$E(cosmber) < E(cosm)$	73.75%
$E(cosmber) > E(cosm)$	26.25%
$E(cosmber) = E(cosm)$	0%

Table 2: Using approximation (9)



(a) Using formula (7) (b) Using formula (9)

Figure 1: Normwise relative errors.



(a) Using formula (7) (b) Using formula (9)

Figure 2: Performance Profile.

4 Conclusions

In general, the implementation based on the new Bernoulli series (9) is more accurate than (7), comparing it with the one based on the Taylor series, algorithm (`cosmtay`) and Hermite series, algorithm (`cosmtayher`), and the one based in Padé rational approximation, algorithm (`cosm`).

Acknowledgement

This work has been partially supported by Spanish Ministerio de Economía y Competitividad and European Regional Development Fund (ERDF) grants TIN2017-89314-P and by the Programa de Apoyo a la Investigación y Desarrollo 2018 of the Universitat Politècnica de València (PAID-06-18) grants SP20180016.

References

[1] Serbin, S.M. and Blalock, S.A., An algorithm for computing the matrix cosine. *SIAM Journal on Scientific Computing*, 1(2): 198–204, 1980.

[2] Dehghan, M. and Hajarian, M., Computing matrix functions using mixed interpolation methods. *Mathematical and Computer Mod- elling*, 52(5-6): 826–836, 2010.

[3] Higham, N.J., *Functions of Matrices: Theory and Computation*. SIAM, Philadelphia, PA, USA, 2008.

[4] Alonso-Jordá, P., Peinado, J., Ibáñez, J., Sastre, J. and Defez, E., Computing matrix trigonometric functions with GPUs through Matlab. *The Journal of Supercomputing*, pages 1–14, 2018.

[5] Tsitouras, Ch. and Katsikis, V., Bounds for variable degree ratio- nal L_∞approximations to the matrix cosine. *Computer Physics Communications*, 185(11): 2834–2840, 2014.

[6] Serbin, S.M., Rational approximations of trigonometric matrices with application to second-order systems of differential equations. *Applied Mathematics and Computation*, 5(1): 75–92, 1979.

[7] Al-Mohy, A.H., Higham, N.J. and Relton, S.D., New algorithms for computing the matrix sine and cosine separately or simultaneously. *SIAM Journal on Scientific Computing*, 37(1): A456–A487, 2015.

[8] Sastre, J., Ibáñez, J., Alonso-Jordá, P., Peinado, J. and Defez, E., Two algorithms for computing the matrix cosine function. *Applied Mathe- matics and Computation*, 312: 66–77, 2017.

[9] Sastre, J., Ibáñez, J., Alonso-Jordá, P., Peinado, J. and Defez, E., Fast Taylor polynomial evaluation for the computation of the matrix cosine. *Journal of Computational and Applied Mathematics*, 354: 641–650, 2019.

[10] Defez, E., Ibáñez, J., Peinado, J., Sastre, J. and Alonso-Jordá, P., An efficient and accurate algorithm for computing the matrix cosine based on new Hermite approximations. *Journal of Computational and Applied Mathematics*, 348: 1–13, 2019.

[11] Kouba, O., Lecture Notes, Bernoulli Polynomials and Applications. *arXiv preprint arXiv*:1309.7560, 2013.

[12] WJ Olver, F., W Lozier, D., F Boisvert, R. and W Clark, C., *NIST handbook of mathematical functions hardback and CD-ROM*. Cambridge University Press, 2010.

[13] Defez, E., Ibáñez, J., Peinado, J., Alonso-Jordá, P. and Alonso, J.M., Computing matrix functions by matrix Bernoulli series. In *19th International Conference on Computational and Mathematical Methods in Science and Engineering (CMMSE-2019)*, From 30th of Juny to 6th of July 2019. Poster presented at some conference, Rota (Cádiz), Spain.

[14] Higham, N.J., *The test matrix toolbox for MATLAB (Version 3.0)*. University of Manchester Manchester, 1995.

[15] Wright, TG., Eigtool, version 2.1. URL: `web.comlab.ox.ac.uk/pseudospectra/eigtool`, 2009.