

# MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR 2019

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Instituto Universitario de Matemática Multidisciplinar  
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Edited by

R. Company, J.C. Cortés,  
L. Jódar and E. López-Navarro

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# Contents

A personality mathematical model of placebo with or without deception: an application of the Self-Regulation Therapy .....	1
The role of police deterrence in urban burglary prevention: a new mathematical approach .....	9
A Heuristic optimization approach to solve berth allocation problem .....	14
Improving the efficiency of orbit determination processes .....	18
A new three-steps iterative method for solving nonlinear systems .....	22
Adaptive modal methods to integrate the neutron diffusion equation .....	26
Numerical integral transform methods for random hyperbolic models .....	32
Nonstandard finite difference schemes for coupled delay differential models .....	37
Semilocal convergence for new Chebyshev-type iterative methods .....	42
Mathematical modeling of Myocardial Infarction .....	46
Symmetry relations between dynamical planes .....	51
Econometric methodology applied to financial systems .....	56
New matrix series expansions for the matrix cosine approximation .....	64
Modeling the political corruption in Spain .....	70
Exponential time differencing schemes for pricing American option under the Heston model .....	75
Chromium layer thickness forecast in hard chromium plating process using gradient boosted regression trees: a case study .....	79
Design and convergence of new iterative methods with memory for solving nonlinear problems .....	83
Study of the influence falling friction on the wheel/rail contact in railway dynamics ..	88
Extension of the modal superposition method for general damping applied in railway dynamics .....	94
Predicting healthcare cost of diabetes using machine learning models .....	99

Sampling of pairwise comparisons in decision-making .....	105
A multi-objective and multi-criteria approach for district metered area design: water operation and quality analysis .....	110
Updating the OSPF routing protocol for communication networks by optimal decision-making over the k-shortest path algorithm .....	118
Optimal placement of quality sensors in water distribution systems .....	124
Mapping musical notes to socio-political events .....	131
Comparison between DKGa optimization algorithm and Grammar Swarm surrogated model applied to CEC2005 optimization benchmark .....	136
The quantum brain model .....	142
Probabilistic solution of a randomized first order differential equation with discrete delay .....	151
A predictive method for bridge health monitoring under operational conditions .....	155
Comparison of a new maximum power point tracking based on neural network with conventional methodologies .....	160
Influence of different pathologies on the dynamic behaviour and against fatigue of railway steel bridges .....	166
Statistical-vibratory analysis of wind turbine multipliers under different working conditions .....	171
Analysis of finite dimensional linear control systems subject to uncertainties via probabilistic densities .....	176
Topographic representation of cancer data using Boolean Networks .....	180
Trying to stabilize the population and mean temperature of the World .....	185
Optimizing the demographic rates to control the dependency ratio in Spain .....	193
An integer linear programming approach to check the embodied $CO_2$ emissions of the opaque part of a façade .....	199
Acoustics on the Poincaré Disk .....	206
Network computational model to estimate the effectiveness of the influenza vaccine <i>a posteriori</i> .....	211

# Sampling of pairwise comparisons in decision-making

J. Benítez<sup>b1</sup>, S. Carpitella<sup>‡</sup>, A. Certa<sup>‡</sup> and J. Izquierdo<sup>‡</sup>

(b) Instituto Universitario de Matemática Multidisciplinar,  
Universitat Politècnica de València,

(‡) FluIng - Instituto Universitario de Matemática Multidisciplinar,  
Universitat Politècnica de València,

(‡) Università degli Studi di Palermo.

## 1 Introduction

Various decision-making techniques rely on pairwise comparisons (PCs) between the involved elements. Traditionally, PCs are provided by experts or relevant actors, and compiled into pairwise comparison matrices (PCMs).

In highly complex problems, the number of elements to be compared may be very large. One of the issues limiting PC applicability to large-scale decision problems is the so-called curse of dimensionality, that is, many PCs need to be elicited from an actor, or built from a body of information.

In general, when applied to a set of  $n$  elements to be compared, the number of PCs that have to be made is  $n(n-1)/2$ . When the information in the comparison matrix is complete, the priorities can be obtained. This is the case of decision-making with complete information. However, if there are missing entries due to uncertainty or lack of information, decision-making must be performed from the available incomplete information. The authors have addressed the issue of incomplete information in [5, 6], and have characterized the consistent completion of a PCM using graph theory in [6].

In this contribution, we claim that less than that number of comparisons may be suitable to develop sound decision-making. There is a trivial solution providing a lower bound for the sample size: just produce  $n-1$  PCs, for example comparing one element with the others. It can be shown that this is equivalent to give directly the priority vector. Here we reduce the number of pairwise comparisons in a decision-making problem by selecting just a sample of  $n$  PCs that are able to provide balanced and unbiased (incomplete) information that still produces consistent and robust decisions. Both the size of the sample and its distribution within the PCM are of interest.

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<sup>1</sup>e-mail: jbenitez@mat.upv.es

We address this research within a linearization theory developed by the authors [2] based on optimizing the consistency of reciprocal matrices.

## 2 Problem statement and solution

If there are  $n$  alternatives, the expert must build an  $n \times n$  reciprocal matrix, and therefore, produce  $n(n-1)/2$  PCs. If  $n$  is large,  $n(n-1)/2$  is also large and the expert can be easily tired and lose the necessary concentration. For example, if  $n = 10$  (which is not very large), then  $n(n-1)/2 = 45$ , and a survey of 45 questions may be tedious, strenuous and time-consuming. In contrast, if the expert is asked to fill fewer entries, the survey will become more friendly and, arguably, more reliable.

### 2.1 Problem

Here we focus on the incomplete  $n \times n$  reciprocal matrix  $B$ , where only entries  $b_{12}, b_{23}, \dots, b_{n-1,n}, b_{n1}$  are known. For example, for size  $6 \times 6$ ,

$$B = \begin{bmatrix} 1 & b_{12} & \star & \star & \star & b_{n1}^{-1} \\ b_{12}^{-1} & 1 & b_{23} & \star & \star & \star \\ \star & b_{23}^{-1} & 1 & b_{34} & \star & \star \\ \star & \star & b_{34}^{-1} & 1 & b_{45} & \star \\ \star & \star & \star & b_{45}^{-1} & 1 & b_{56} \\ b_{n1} & \star & \star & \star & b_{56}^{-1} & 1 \end{bmatrix}. \quad (1)$$

The next result characterizes when  $B$  can be completed to be consistent.

**Theorem 1** *Let  $B \in M_n$  be a reciprocal incomplete matrix with known entries  $b_{12}, b_{23}, \dots, b_{n-1,n}, b_{n1}$ .*

(i) *Matrix  $B$  admits a consistent completion if and only if*

$$b_{12} b_{23} \cdots b_{n-1,n} b_{n1} = 1. \quad (2)$$

(ii) *If  $B$  admits a consistent completion, then it is unique, say  $C$ , and  $C$  satisfies the following condition: if  $(B)_{ij}$  is unspecified and  $i < j$ , then  $(C)_{ij} = b_{i,i+1} b_{i+1,i+2} \cdots b_{j-1,j}$ .*

Let's check the performance of this approach. Let  $A$  be a (fully known) reciprocal matrix. We can find  $X_A$ , the closest consistent matrix to  $A$  by using the formula given in [4]. Also, from the incomplete matrix  $B$  defined as in the statement of Theorem 1, supposing that  $B$  satisfies the criterion given in this theorem, we can easily compute  $C$ . The next example compares matrices  $X_A$  and  $C$  and calculates the distance between both.

**Example 1** *Let*

$$A = \begin{bmatrix} 1 & 2 & 2 & 8 \\ 1/2 & 1 & 4 & 1/2 \\ 1/2 & 1/4 & 1 & 1 \\ 1/8 & 2 & 1 & 1 \end{bmatrix}. \quad (3)$$

This matrix  $A$  is not consistent (e.g.,  $\text{rank}(A) > 1$ , see [3, Theorem 1]). The Perron eigenvalue is  $\lambda_{\max} \simeq 4.84$ , and (see [7])  $\text{CI}(A) = (\lambda_{\max} - 4)/(4 - 1) \simeq 0.279$  and  $\text{CI}(A)/\text{RI}_4 \simeq 0.314 > 0.1$ ; the consistency of  $A$  is not acceptable (Saaty's criterion); here  $\text{RI}_4 = 0.89$  is the random index for  $4 \times 4$  matrices.

Using the formula given in [4], we have

$$X_A \simeq \begin{bmatrix} 1 & 2.38 & 4 & 3.36 \\ 0.42 & 1 & 1.68 & 1.41 \\ 0.25 & 0.59 & 1 & 0.84 \\ 0.29 & 0.71 & 1.19 & 1 \end{bmatrix}.$$

To apply Theorem 1, let us consider the following incomplete reciprocal matrix

$$B = \begin{bmatrix} 1 & 2 & \star & 8 \\ 1/2 & 1 & 4 & \star \\ \star & 1/4 & 1 & 1 \\ 1/8 & \star & 1 & 1 \end{bmatrix}.$$

Since (2) holds, then there exists a unique consistent completion, namely

$$C = \begin{bmatrix} 1 & 2 & b_{12}b_{23} & 8 \\ 1/2 & 1 & 4 & b_{23}b_{34} \\ (b_{12}b_{23})^{-1} & 1/4 & 1 & 1 \\ 1/8 & (b_{23}a_{34})^{-1} & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 8 & 8 \\ 1/2 & 1 & 4 & 4 \\ 1/8 & 1/4 & 1 & 1 \\ 1/8 & 1/4 & 1 & 1 \end{bmatrix}.$$

Now the distance between  $X_A$  and  $C$  is  $d(X_A - C) = \|X_A - C\|_F = \text{tr}(X_A C^T) \simeq 2.4992$ ,  $\text{tr}(\cdot)$  being the trace operator.

If (2) does not hold, the matrix  $B$  defined in Theorem 1 has no consistent completion. If we denote by  $\mathcal{C}_n$  the set of  $n \times n$  consistent matrices, then we must find  $D \in M_n$ , a reciprocal completion of  $B$ , such that

$$d(D, \mathcal{C}_n) \leq d(D', \mathcal{C}_n)$$

for any  $D' \in M_n$  reciprocal completion of  $B$ .

We summarize the obtained results in the following theorem.

**Theorem 2** Let  $B \in M_n$  be a reciprocal incomplete matrix with known entries  $b_{12}, b_{23}, \dots, b_{n-1,n}, b_{n1}$ .

- (i) There is a unique reciprocal completion of  $B$ , say  $D$ , such that  $d(D, \mathcal{C}_n) \leq d(D', \mathcal{C}_n)$  for all  $D' \in M_n$  reciprocal completion of  $B$ .
- (ii) There is a unique  $Z \in \mathcal{C}_n$  such that  $d(D, Z) = d(D, \mathcal{C}_n)$ .
- (iii)  $Z = E[\phi_n(\mathcal{L}^\dagger \mathcal{Q} \boldsymbol{\rho})]$ , where  $\boldsymbol{\rho} = (\log b_{12}, \dots, \log b_{n-1,n}, \log b_{n1})^T$ , and matrices  $\mathcal{Q}, \mathcal{L}$  are the Laplacian matrix and the incidence matrix, respectively, of the graph associated to  $B$ .
- (iv) If  $(i, j)$  is an unknown entry of  $B$ , then the  $(i, j)$  entry of  $D$  and  $Z$  are equal.



Note that the oriented graph with  $n$  vertices and edges associated to  $B$  is

$$\{1 \rightarrow 2, 2 \rightarrow 3, \dots, n-1 \rightarrow n, n \rightarrow 1\}.$$

Let's define the following matrix  $J$  directly associated to the structure of  $B$

$$J = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{bmatrix}. \quad (4)$$

A number of considerations enable us to prove the following result.

**Theorem 3** *Let  $B \in M_n$  be a reciprocal incomplete matrix with known entries  $b_{12}, b_{23}, \dots, b_{n-1,n}, b_{n1}$ . Under the notation of Theorem 2, one has*

$$Z = E \left[ \phi_n \left( \frac{1}{2n} \sum_{k=0}^{n-1} (n-2k-1) J^k \boldsymbol{\rho} \right) \right],$$

where the matrix  $J$  is given in (4) and  $\phi_n$  is the linear mapping  $\phi_n : \mathbb{R}^n \rightarrow M_n$  given by  $(\phi_n(\mathbf{v}))_{ij} = v_i - v_j$ .

This expression shows that neither inverses nor pseudo-inverses have to be computed. Also,  $\boldsymbol{\rho}, J\boldsymbol{\rho}, J^2\boldsymbol{\rho}, \dots, J^{n-1}\boldsymbol{\rho}$  are trivial to compute. For example, for  $n = 4$ , one has  $J^0 = J^4 = I_4$ ,

$$J^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad J^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad J^3 = J^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

If  $\boldsymbol{\rho} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)^T$  and  $\hat{\boldsymbol{\rho}} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4)^T$ , then  $J^0\boldsymbol{\rho}$  are the 1, 2, 3, 4 entries of  $\hat{\boldsymbol{\rho}}$ ;  $J^1\boldsymbol{\rho}$  are the 2, 3, 4, 5 entries of  $\hat{\boldsymbol{\rho}}$ ;  $J^2\boldsymbol{\rho}$  are the 3, 4, 5, 6 entries of  $\hat{\boldsymbol{\rho}}$ ; and  $J^3\boldsymbol{\rho}$  are the 4, 5, 6, 7 entries of  $\hat{\boldsymbol{\rho}}$ .

### 3 Conclusions

Making too many comparisons may be strenuous and time-consuming, and lead to wrong and harmful conclusions. It is indispensable to focus on its reduction [1]. There is not a general solution to the problem of finding an optimal sample of PCs to be issued so that  $\text{card}(\text{sample}) < n(n-1) = 2$  and still produce sound DM. We have given a solution in which one compares just the elements of a balanced and unbiased subset of items. The solution, according to Theorem 3, is obtained through elementary, simple calculations.

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