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The quantum brain model

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1 Introduction

A macroscopic spatio-temporal model of the brain dynamics is presented. It is here called as the spatio-temporal response model (STRM), and also the *quantum brain model*, due to its eigenvalues and eigenfunctions quantization are quantized due due the boundary conditions. Thus, the quantized prediction of the spatio-temporal brain activity (STBA) from a known initial one is possible with this model. Its mathematical structure is a generalization of the temporal response model (TRM) that predicts the temporal brain activity (TBA) as a consequence of several stimuli [1]:

$$\left.\begin{array}{c} \frac{dy(t)}{dt} = \bar{a}(\bar{b}-y(t)) + \sum_i \bar{p}_i \cdot s_i(t) \cdot y(t) - \sum_i \bar{q}_i \cdot \int_{t_0}^{t} e^{\frac{x-t}{\bar{\tau}_i}} \cdot s_i(x) \cdot y(x) dx \\ y\left(t_0\right) = y_0 \end{array}\right\} \quad (1)$$

In (1), t is the time, and $y(t)$, $\bar{b}$ and y_0 are respectively the TBA, its tonic level and its initial value. The TBA is measured with the psychological variable called as General Factor of Personality (GFP) [1]. Besides, $s_i(t)$, $i = 1, 2, \ldots, n$, are the different stimuli, which can be of different natures: the amount of non-consumed drug by cells, a sound, a view, etc., which can hold different mathematical temporal functions. In addition, $\bar{a}(\bar{b} - y(t))$ is the *homeostatic control*, i.e., the cause of the fast recovering of the tonic level $\bar{b}$, being $\bar{a}$ the *homeostatic control power* of this control; $\bar{p}_i \cdot s_i(t) \cdot y(t)$ are the different *excitation effects*, which tend to increase the temporal brain activity, being $\bar{p}_i$ the *excitation effect powers*; $\bar{q}_i \cdot \int_0^t e^{\frac{x-t}{\bar{\tau}_i}} \cdot s_i(x) \cdot y(x)dx$ are the different *inhibitor effects*, which tend to decrease the temporal brain activity and are the cause of the its slow recovering, being $\bar{q}_i$ the *inhibitor effect powers* and being $\bar{\tau}_i$ the *inhibitor effect delays*.

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2 The spatio-temporal response model or quantum brain

The STRM is obtained as a generalization of the TRM. To do this, consider in Eq. (1) that the TBA variable $y(t)$ must be substituted by a function that represents the STBA as a spatial-density depending on the time t and on the three spatial rectangular variables $\mathbf{r} = (x_1, x_2, x_3)$. Then, the time derivative in Eq. (1) must be a partial time derivative. The $\Psi(t, \mathbf{r})$ be the STBA variable, thus, the starting hypothesis is that:

$$y(t) = \left(\iiint_D \Psi^2(t, \mathbf{r}) d\mathbf{r} \right)^{1/2} = (\Psi(t, \mathbf{r}), \Psi(t, \mathbf{r}))^{1/2} \tag{2}$$

In (2), D is the integration domain that depends on the brain geometry considered, and "(,)" represents the inner product. In addition, the spatial dynamics in (1) is introduced as a diffusion term through a Laplacianne function of $\Psi(t, \mathbf{r})$:

$$\frac{\partial \Psi(t, \mathbf{r})}{\partial t} = a \cdot (\omega(\mathbf{r}) - \Psi(t, \mathbf{r})) + \sum_i p_i \cdot s_i(t) \cdot \Psi(t, \mathbf{r}) - \sum_i q_i \cdot \int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \Psi(x, \mathbf{r}) dx + \sigma \nabla^2 \Psi(t, \mathbf{r}) \tag{3}$$

$$\Psi(t_0, \mathbf{r}) = \phi(\mathbf{r}) \tag{4}$$

Note in (3) that the tonic level $\bar{b}$ in (1) has been substituted by $\omega(\mathbf{r})$, i.e., a spatial function, unknown by the moment. In addition, σ is the diffusion coefficient, here considered positive-valued, while the other parameters are also positive-valued and conserve the same meanings than in (1), i.e., a instead $\bar{a}$, p_i instead $\bar{p_i}$, q_i instead $\bar{q_i}$ and τ_i instead $\bar{\tau_i}$. However, they are related in a way provided below. The initial condition Eq. (4) must be provided through the spatial distribution of brain activity in the instant $t = t_0$. In addition, the boundary conditions must be also provided, but they depend on the brain geometry considered. They are provided in Section 4 for an idealized box-brain geometry, which considers that the spatial flow through the brain walls cancels. Observe that the spatio-temporal response model provided by Eqs. (3) and (4) can be considered as a generalization of the cable model for a pulse translation on a neuron axon [2], from an only spatial direction to the three spatial dimensions of the brain.

3 Analytical solution of the spatio-temporal response model: the idealized box-brain

Eq. (3) is not separable due to it is a non-homogeneous equation as a consequence of the term $a \cdot \omega(\mathbf{r})$ (known as the non-homogeneous equation source). However, this problem can be overcome by the *method of eigenfunction expansions*. This method considers the solutions of the *associated homogeneous spatio-temporal response model* for a function $\Psi_h(t, \mathbf{r})$, which does not have the source:

$$\frac{\partial \Psi_h(t, \mathbf{r})}{\partial t} = -a \cdot \Psi_h(t, \mathbf{r}) + \sum_i p_i \cdot s_i(t) \cdot \Psi_h(t, \mathbf{r}) - \sum_i q_i \cdot \int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \Psi_h(x, \mathbf{r}) dx + \sigma \cdot \nabla^2 \Psi_h(t, \mathbf{r}) \tag{5}$$

Then, Eq. (5) is so separable by a product:

$$\Psi_h(t, \mathbf{r}) = \rho(t) \cdot \Omega(\mathbf{r}) \tag{6}$$

whose substitution in (5) provides:

$$\frac{\rho'(t)}{\rho(t)} + a - \sum_i p_i \cdot s_i(t) + \frac{1}{\rho(t)} \sum_i q_i \int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \rho(x) dx = \sigma \cdot \frac{\nabla^2 \Omega(r)}{\Omega(r)} \tag{7}$$

In order to Eq. (7) holds, both members of the equation must be a constant. Let λ be this constant. The temporal part of Eq. (7) does not play any role in the solution of the non-homogeneous Eq. (3). However, from the spatial part of (7):

$$\nabla^2 \Omega(\mathbf{r}) = \frac{\lambda}{\sigma} \Omega(\mathbf{r}) \tag{8}$$

Eq. (8) is the Helmoltz equation, which can be solved by separating variables for several coordinate systems, and it is fundamental in the solution of Eq. (3). The solution considered for Eq. (8) is the use of the rectangular coordinates for an idealized box-brain geometry of dimensions L_1 (length, from back to forebrain), L_2 (width, from side to side of brain) and L_3 (height, from down to up brain):

$$\mathbf{r} = (x_1, x_2, x_3) \in [0, L_1] \times [0, L_2] \times [0, L_3] \tag{9}$$

Thus, separating variables in Eq. (8) as: $\Omega(\mathbf{r}) = \Omega_1(x_1) \cdot \Omega_2(x_2) \cdot \Omega_3(x_3)$ and subsequently dividing by the product $\Omega_1(x_1) \cdot \Omega_2(x_2) \cdot \Omega_3(x_3)$:

$$\frac{1}{\Omega_1} \frac{d^2\Omega_1}{dx_1^2} + \frac{1}{\Omega_2} \frac{d^2\Omega_2}{dx_2^2} + \frac{1}{\Omega_3} \frac{d^2\Omega_3}{dx_3^2} = \frac{\lambda}{\sigma} \tag{10}$$

In order to Eq. (10) holds, each member of the addition must be a constant. These constants must be negative-valued to obtain an oscillatory dynamics, thus let $-k_i^2$ be, $i = 1, 2, 3$, these constants:

$$\frac{1}{\Omega_i} \frac{d^2\Omega_i}{dx_i^2} = -k_i^2, \qquad i = 1, 2, 3. \tag{11}$$

And from (10) and (11):

$$\lambda = -\sigma(k_1^2 + k_2^2 + k_3^2). \tag{12}$$

Also, from (11):

$$\Omega_i(x_i) = A_i \cos(k_i x_i) + B_i \sin(k_i x_i), \qquad i = 1, 2, 3. \tag{13}$$

being A_i and B_i ($i = 1, 2, 3$) arbitrary constants.
With the boundary conditions that the spatial flow through the brain walls cancels in Eq. (4):

$$\left.\frac{\partial \Psi_h(t, \mathbf{r})}{\partial x_i}\right|_{x_i=0} = 0; \quad \left.\frac{\partial \Psi(t, \mathbf{r})}{\partial x_i}\right|_{x=L_i} = 0 \quad i = 1, 2, 3. \tag{14}$$

which provide, from Eq. (13):

$$B_i = 0; \quad \sin(k_i L_i) = 0 \to k_i L_i = n_i \pi \to k_i = \frac{\pi}{L_i} n_i; \quad n_i = 1, 2, \ldots, +\infty; \quad i = 1, 2, 3 \tag{15}$$

Eq. (15) represents the quantization of the eigenvalues of the associated homogeneous spatio-temporal response model, as function of three positive integers. Note that the integers are restricted to vary in the range $n_i = 1, 2, \ldots, +\infty$ ($i = 1, 2, 3$), thus, the constants k_i are as well positive-valued. On a hand, $k_i = 0$ has not physical sense, and the integers varying in the range $n_i = -1, -2, \ldots, -\infty$ $(i = 1, 2, 3)$ will duplicate unnecessarily the solutions. In addition, the separating constant λ from Eq. (12) becomes quantized, which can be rewritten as $\lambda_{n_1n_2n_3}$ (from now onwards the expression $n_i = 1, 2, \ldots, +\infty$ is over understood):

$$\lambda_{n_1n_2n_3} = -\sigma\pi^2 \left(\left(\frac{n_1}{L_1}\right)^2 + \left(\frac{n_2}{L_2}\right)^2 + \left(\frac{n_3}{L_3}\right)^2 \right) \tag{16}$$

As a consequence of (16), the solution of Eq. (8) is a superposition of the eigenfunctions:

$$\bar{\Omega}_{n_1n_2n_3}(\boldsymbol{r}) = \prod_{i=1}^{3} \sin\left(\frac{\pi \cdot n_i}{L_i} x_i\right) \tag{17}$$

Note that the eigenfunction Eq. (17) define an orthogonal base, due to:

$$\begin{aligned}(\bar{\Omega}_{n_1n_2n_3}(\mathbf{r}), \bar{\Omega}_{m_1m_2m_3}(\mathbf{r})) &= \iiint_D \bar{\Omega}_{n_1n_2n_3}(\mathbf{r}) \cdot \bar{\Omega}_{m_1m_2m_3}(\mathbf{r}) d\mathbf{r} \\ &= \prod_{i=1}^{3} \int_0^{L_i} \sin(\frac{\pi n_i}{L_i} x_i) \cdot \sin(\frac{2\pi m_i}{L_i} x_i) dx_i \\ &= \frac{L_1L_2L_3}{8} \delta_{n_1m_1}\delta_{n_2m_2}\delta_{n_3m_3}.\end{aligned} \tag{18}$$

Thus, the corresponding orthonormal base is given by the eigenfunctions:

$$\Omega_{n_1n_2n_3}(\mathbf{r}) = \frac{\bar{\Omega}_{n_1n_2n_3}(\mathbf{r})}{\left(\bar{\Omega}_{n_1n_2n_3}, \bar{\Omega}_{n_1n_2n_3}\right)^{1/2}} = \left(\frac{8}{L_1 \cdot L_2 \cdot L_3}\right)^{1/2} \prod_{i=1}^{3} \sin\left(\frac{\pi \cdot n_i}{L_i} x_i\right). \tag{19}$$

Such that, from Eqs. (8) and (19):

$$\nabla^2 \Omega_{n_1n_2n_3}(\mathbf{r}) = \frac{\lambda_{n_1n_2n_3}}{\sigma} \Omega_{n_1n_2n_3}(\mathbf{r}) \tag{20}$$

$$(\Omega_{n_1n_2n_3}(\mathbf{r}), \Omega_{m_1m_2m_3}(\mathbf{r})) = \delta_{n_1m_1}\delta_{n_2m_2}\delta_{n_3m_3} \tag{21}$$

$$\Omega(\mathbf{r}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \Omega_{n_1n_2n_3}(\mathbf{r}) = \left(\frac{8}{L_1 \cdot L_2 \cdot L_3}\right)^{1/2} \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \prod_{i=1}^{3} \sin\left(\frac{2\pi \cdot n_i}{L_i} x_i\right) \tag{22}$$

That is, $\frac{\lambda_{n_1n_2n_3}}{\sigma}$ are the eigenvalues of the operator ∇^2 with associated eigenfunctions $\Omega_{n_1n_2n_3}$. These eigenfunctions are fundamental to find the solutions of the non-homogeneous spatio-temporal response model given by Eq. (3) by the following expansions:

$$\begin{aligned}\Psi(t, \mathbf{r}) &= \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \eta_{n_1n_2n_3}(t) \cdot \Omega_{n_1n_2n_3}(\mathbf{r}) \\ &= \left(\frac{8}{L_1 \cdot L_2 \cdot L_3}\right)^{1/2} \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} \eta_{n_1n_2n_3}(t) \cdot \prod_{i=1}^{3} \sin\left(\frac{\pi \cdot n_i}{L_i} x_i\right)\end{aligned} \tag{23}$$

$$\omega(\mathbf{r}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} C_{n_1n_2n_3}\cdot\Omega_{n_1n_2n_3}(\mathbf{r})$$
$$= \left(\frac{8}{L_1\cdot L_2\cdot L_3}\right)^{1/2}\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty} C_{n_1n_2n_3}\cdot\prod_{i=1}^{3}\sin\left(\frac{\pi\cdot n_i}{L_i}x_i\right) \tag{24}$$

Such that:

$$\eta_{n_1n_2n_3}(t) = (\Psi(t,\mathbf{r}),\Omega_{n_1n_2n_3}(\mathbf{r})) = \left(\frac{8}{L_1\cdot L_2\cdot L_3}\right)^{1/2}\prod_{i=1}^{3}\int_0^{L_i}\Psi(t,\mathbf{r})\cdot\sin\left(\frac{2\pi\cdot n_i}{L_i}x_i\right)dx_i \tag{25}$$

$$C_{n_1n_nn_3} = (\omega(\mathbf{r}),\Omega_{n_1n_2n_3}(\mathbf{r})) = \left(\frac{8}{L_1\cdot L_2\cdot L_3}\right)^{1/2}\prod_{i=1}^{3}\int_0^{L_i}\omega(\mathbf{r})\cdot\sin\left(\frac{2\pi\cdot n_i}{L_i}x_i\right)dx_i \tag{26}$$

In the beginning, the two sides of Eq. (3) are multiplied by $\Omega_{m_1m_2m_3}(\mathbf{r})$ and taken the inner product (,):

$$\eta'_{n_1n_2n_3}(t) = \left(-a+\lambda_{n_1n_2n_3}+\sum_i p_i\cdot s_i(t)\right)\eta_{n_1n_2n_3}(t)$$
$$-\sum_i q_i\cdot\int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}}\cdot s_i(x)\cdot\eta_{n_1n_2n_3}(x)dx + a\cdot C_{n_1n_2n_3} \tag{27}$$

The initial conditions for the integro-differential Eq. (27) are given by Eq. (25) in $t=t_0$ and by Eq. (4):

$$\eta_{n_1n_2n_3}(t_0) = (\Psi(t_0,\boldsymbol{r}),\Omega_{n_1n_2n_3}(\boldsymbol{r})) = (\phi(\mathbf{r}),\Omega_{n_1n_2n_3}(\mathbf{r}))$$
$$= \left(\frac{8}{L_1\cdot L_2\cdot L_3}\right)^{1/2}\prod_{i=1}^{3}\int_0^{L_i}\phi(\mathbf{r})\cdot\sin\left(\frac{2\pi\cdot n_i}{L_i}x_i\right)dx_i \tag{28}$$

In conclusions: the solution of the spatio-temporal response model given by Eqs. (3) and (4) is provided by the expansion Eq. (23), where $\eta_{n_1n_2n_3}(t)$ is given by Eq. (27), with initial conditions Eq. (28), and $C_{n_1n_2n_3}$ by Eq. (26). Note that the functions $\Omega_{n_1n_2n_3}(\mathbf{r})$ are given by Eq. (19), considering the geometric idealized case of a box-brain.

4 Steady solution of the spatio-temporal response model

The steady solution of the spatio-temporal response model $\Psi^{(s)}(\mathbf{r})$ is provided as $t\to+\infty$, which corresponds for the idealized case that no stimuli are influencing on brain, i.e., when $s_i(t)=0$, in Eq. (3):

$$\Psi^{(s)}(\mathbf{r}) = \lim_{t\to+\infty}\Psi(t,\mathbf{r}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\lim_{t\to+\infty}\eta_{n_1n_2n_3}(t)\cdot\Omega_{n_1n_2n_3}(\mathbf{r}) \tag{29}$$

In this case, when $s_i(t)=0$, Eq. (27) becomes:

$$\eta'_{n_1n_2n_3}(t) + (a-\lambda_{n_1n_2n_3})\,\eta_{n_1n_2n_3}(t) = a\cdot C_{n_1n_2n_3} \tag{30}$$

Eq. (30) has a simple analytical solution:

$$\begin{aligned}\eta_{n_1n_2n_3}(t) &= \mathrm{e}^{-(a-\lambda_{n_1n_2n_3})(t-t_0)}\left(\eta_{n_1n_2n_3}(t_0) + a\cdot k_{n_1n_2n_3}\int_{t_0}^{t}\mathrm{e}^{\left(a-\lambda_{n_1n_2n_3}\right)(x-t_0)}dx\right)\\ &= \eta_{n_1n_2n_3}(t_0)\,\mathrm{e}^{-\left(a-\lambda_{n_1n_2n_3}\right)(t-t_0)} + \frac{a\cdot k_{n_1n_2n_3}}{\left(a-\lambda_{n_1n_2n_3}\right)}\left(1-\mathrm{e}^{-\left(a-\lambda_{n_1n_2n_3}\right)(t-t_0)}\right)\end{aligned} \tag{31}$$

Note in Eq. (31) that, from Eq. (16), $a-\lambda_{n_1n_2n_3} > 0$ due to $\lambda_{n_1n_2n_3} < 0$. In addition, it tends to the steady state $\eta^{(s)}_{n_1n_2n_3}(t)$ as $t\to+\infty$:

$$\eta^{(s)}_{n_1n_2n_3} = \frac{a\cdot C_{n_1n_2n_3}}{\left(a-\lambda_{n_1n_2n_3}\right)}. \tag{32}$$

And from Eq. (29):

$$\begin{aligned}\Psi^{(s)}(\mathbf{r}) &= \lim_{t\to+\infty}\Psi(t,\mathbf{r}) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\eta^{(s)}_{n_1n_2n_3}\cdot\Omega_{n_1n_2n_3}(\mathbf{r}) = a\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\frac{C_{n_1n_2n_3}}{\left(a-\lambda_{n_1n_2n_3}\right)}\Omega_{n_1n_2n_3}(\mathbf{r})\\ &= \left(\frac{8}{L_1\cdot L_2\cdot L_3}\right)^{1/2}\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\frac{a\cdot C_{n_1n_2n_3}}{\left(a-\lambda_{n_1n_2n_3}\right)}\prod_{i=1}^{3}\sin\left(\frac{\pi\cdot n_i}{L_i}x_i\right).\end{aligned} \tag{33}$$

5 Relationship between the spatio-temporal response model and the temporal response model

The relationship of the solutions of both models, taking into account Eqs. (2) and (23):

$$\begin{aligned}y^2(t) = (\Psi(t,\mathbf{r}),\Psi(t,\mathbf{r})) &= \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\sum_{m_1=1}^{\infty}\sum_{m_2=1}^{\infty}\sum_{m_3=1}^{\infty}\left(\Omega_{n_1n_2n_3}(\mathbf{r})\right)\Omega_{m_1m_2m_3}(\boldsymbol{r}))\cdot\eta_{n_1n_2n_3}(t)\\ &\cdot\eta_{m_1m_2m_3}(t) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\eta^2_{n_1n_2n_3}(t)\end{aligned} \tag{34}$$

That is:

$$y(t) = \left(\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}\eta^2_{n_1n_2n_3}(t)\right)^{1/2} \tag{35}$$

From Eq. (38) the hypothesis of the isolated existence of the time functions $y_{n_1n_2n_3}(t)$ can be state as the projection:

$$y_{n_1n_2n_3}(t) = \left(\Psi_{n_1n_2n_3}(t,\mathbf{r}),\Psi(t,\mathbf{r})\right)^{1/2} = \eta_{n_1n_2n_3}(t) \tag{36}$$

Such that:

$$\Psi_{n_1n_2n_3}(t,\mathbf{r}) = y_{n_1n_2n_3}(t)\cdot\Omega_{n_1n_2n_3}(\mathbf{r});\quad y(t) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\sum_{n_3=1}^{\infty}y_{n_1n_2n_3}(t) \tag{37}$$

Due to $\eta_{n_1n_2n_3}(t)$ hold the integro-differential Eq. (27), with initial conditions Eq. (28), the functions $y_{n_1n_2n_3}(t)$ hold the equations:

$$y'_{n_1n_2n_3}(t) = \left(-a + \lambda_{n_1n_2n_3} + \sum_i p_i \cdot s_i(t)\right) y_{n_1n_2n_3}(t) - \sum_i q_i \cdot \int_0^t e^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot y_{n_1n_2n_3}(x)dx + a \cdot C_{n_1n_2n_3} \tag{38}$$

$$y_{n_1n_2n_3}(t_0) = \left(\frac{8}{L_1 \cdot L_2 \cdot L_3}\right)^{1/2} \prod_{i=1}^{3} \int_0^{L_i} \phi(\mathbf{r}) \cdot \sin\left(\frac{2\pi \cdot n_i}{L_i} x_i\right) dx_i \tag{39}$$

The corresponding temporal steady states of $y_{n_1n_2n_3}(t)$ are, from Eq. (32):

$$y^{(s)}_{n_1n_2n_3} = \frac{a \cdot C_{n_1n_2n_3}}{(a - \lambda_{n_1n_2n_3})} \tag{40}$$

In fact, reorganizing Eq. (38) to obtain the mathematical structure of Eq. (1):

$$\begin{aligned} y'_{n_1n_2n_3}(t) = (a - \lambda_{n_1n_2n_3}) \left(\frac{a \cdot C_{n_1n_2n_3}}{a - \lambda_{n_1n_2n_3}} - y_{n_1n_2n_3}(t)\right) + \sum_i p_i \cdot s_i(t) \cdot y_{n_1n_2n_3}(t) \\ - \sum_i q_i \cdot \int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot y_{n_1n_2n_3}(x)dx \end{aligned} \tag{41}$$

With initial conditions (28) in (41). Then, by comparing Eqs. (1) and (41), the following equivalences can be derived:

$$\bar{a} \to \bar{a}_{n_1n_2n_3} = a - \lambda_{n_1n_2n_3}; \quad \bar{b} \to \bar{b}_{n_1n_2n_3} = \frac{a \cdot C_{n_1n_2n_3}}{a - \lambda_{n_1n_2n_3}}; \quad \bar{p}_i = p_i; \quad \bar{q}_i = q_i; \quad \bar{\tau}_i = \tau_i \tag{42}$$

In Eq. (42) the parameter values $\bar{a}_{n_1n_2n_3}$ and $\bar{b}_{n_1n_2n_3}$ are quantized, such that, by Eq. (16), $\lambda_{n_1n_2n_3} = -\sigma\pi^2 \left(\left(\frac{n_1}{L_1}\right)^2 + \left(\frac{n_2}{L_2}\right)^2 + \left(\frac{n_3}{L_3}\right)^2\right)$; $n = 1, 2, \ldots, +\infty$. Particularly, note that the quantized $\bar{b}_{\lambda_{n_1n_2n_3}}$ parameter values coincide with the values provided by Eq. (32), such as it must be held by the theory coherence. Note that only these parameters, $\bar{a}_{n_1n_2n_3}$ and $\bar{b}_{n_1n_2n_3}$, which represent biological properties of the brain, are quantized, but not those that are related with the stimuli dynamics, such that $\bar{p}_i$, $\bar{q}_i$ and $\bar{\tau}_i$.

6 Calibration of the STRM

There are several ways to observe experimentally the STBA. One of the most important ways is Neuroimage, which has had to develop the brain mappings, by using the Talairach and MNI coordinates [3] to measure the STBA by measuring the change of some important biological indicators in the brain, such as oxygen, blood, etc. In fact, one of the crucial aims of the Neuroimage technic is the study of the brain resting state [4], which can be identified with the steady state $\Psi^{(s)}(\mathbf{r})$ of Eq. (33). The information that Neuroimage would provide about the mathematical structure of the brain resting state would be a first way to validate the quantum brain model presented. However, in general, to validate the STRM the Neuroimage technic needs the $\phi(\mathbf{r})$ function knowledge, in order to obtain the initial conditions $\eta_{n_1n_2n_3}(t_0)$ through Eq. (28). And the same problems happen with the EEG (electroencephalogram) technic [5], which measures the STBA by the electrical potential.

However, a first result can be provided for the STBA by its relationship with the TBA through the identities Eq. (42). Due to the $\phi(\mathbf{r})$ is unknown, the calibration of Eq. (41) is done with the initial condition of the experimental design presented in the beginning.

One subject consumed 10 mg of methylphenidate, and the GFP was observed every 7.5 minutes during 180 minutes (3 hours), with the 5 adjectives scale, GFP-FAS [6], in the interval [0,50]. The initial condition was also observed before consumption, with value y_0, which is considered as initial condition of Eq. (41) instead the unknown one of Eq. (28). Assuming that no methylphenidate is present in the organism, the temporal function of the methylphenidate [1] is given by:

$$s(t) = \begin{cases} \frac{\alpha \cdot M}{\beta - \alpha}\left(e^{-\alpha \cdot t} - e^{-\beta \cdot t}\right) : \alpha \neq \beta \\ \alpha \cdot M \cdot t \cdot e^{-\alpha \cdot t} : \alpha = \beta \end{cases} \tag{43}$$

The calibration of Eq. (41) by generating random numbers is provided graphically in Figs. 1 and 2.

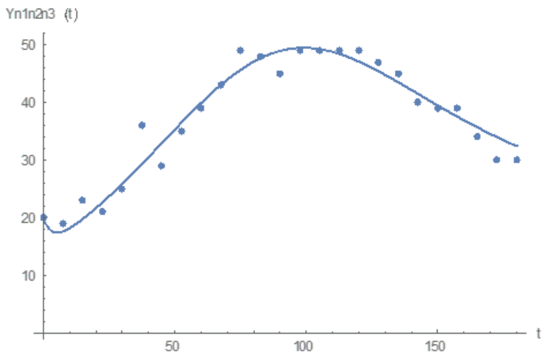

Figure 1: GFP response, $y_{n_1 n_2 n_3}(t)$, to the 10 mg of MF versus time. Experimental values (dots) and theoretical values (line). R2=0.94.

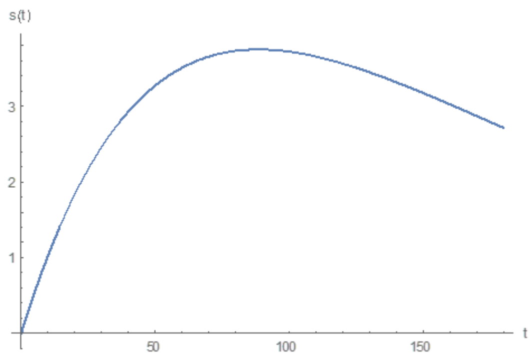

Figure 2: MF evolution s(t) of Eq. 44 in the organism.

The results of the calibration provide the following parameter values: $\alpha = 0.011510165332$; $\beta = 0.011069991532$; $a = 0.312844518371$; $C_{n_1 n_2 n_3} = 15.580863173952$; $p = 0.057535289406$; $q = 0.000000125055$; $\tau = 0.035782907172$; $M = 10.0$; $\sigma = 0.000432176762$; $n1 = 1$; $n2 = 1$; $n3 = 1$. These values permit to obtain the results of the corresponding eigenfunction $\Psi_{n_1 n_2 n_3}(t, \mathbf{r})(n1 = 1; n2 = 1; n3 = 1)$ of Eq. (37). Two graphical representations are provided in Figs. 3 and 4:

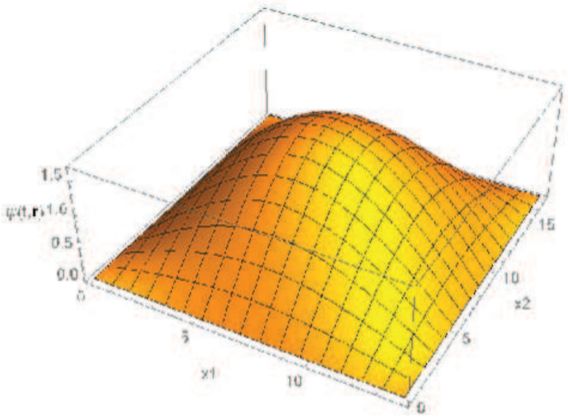

Figure 3: STBA result for $\Psi_{n_1 n_2 n_3}(t, \mathbf{r})$ with n1=1; n2=1; n3=1: t=88.5846 (instant of maximum TBA) for x3=L3/2.

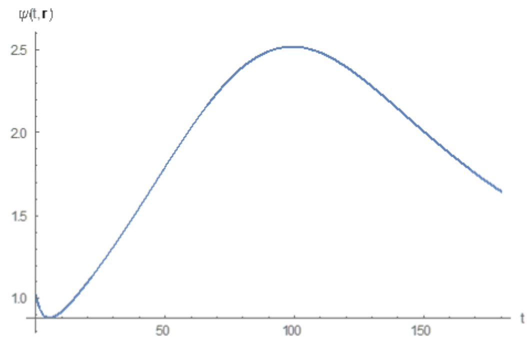

Figure 4: STBA result for $\Psi_{n_1 n_2 n_3}(t, \mathbf{r})$ with n1=1; n2=1; n3=1: $t \in [t_0, T]$ for $\mathbf{r} = (L_1/2, L_2/2, L_3/2)$.

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