GAP SOLITONS IN ACOUSTIC LAYERED MEDIA

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Abstract
In this work we study numerically the existence of gap solitons in an acoustic media. To approach the problem a set of coupled-mode equations are given. In order to obtain a solitary wave in a real media, it is necessary of two phenomena, dispersion and nonlinearity. Acoustic media, usually can present nonlinearity but also low dispersions. To increase dispersion, it is proposed to use a multi-layered medium, a kind of sonic crystal in 1D, that it is demonstrated to have high dispersion near some frequency bands. In this kind of media, it is possible to observe soliton waves.

Keywords: Layered media, Phononinc, Harmonic Generation, Gap Soliton.

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1 Introduction

Wave propagation is common to many fields of physics such as mechanics, optics or acoustics. In the case of acoustics, high intensity waves present specific features such as distortion, shock formation or harmonic generation. These phenomena have been studied since the second half of 18th century [1]. Common acoustic media are characterized by a quadratic nonlinearity and very weak dispersion.

During last decades, a class of artificial materials, the so-called sonic crystals, has received increasing interest, owing to their ability of manipulating sound wave propagation. They consist of a periodic arrangement of scatterers embedded in a host medium. Due to periodicity, sonic crystals present *band gaps*, that is ranges of frequency where waves are not allowed to propagate. At the edges of the bands near the band gaps, waves propagate but experience high dispersion.

There is a recent interest in studying the propagation of intense waves through periodic acoustic media [2]. The interplay between nonlinearity and dispersion leads to several phenomena including selective harmonic generation and propagation of solitary waves. In fact, in the band gap, waves are not allowed to propagate in linear regime, but under certain conditions, solitary waves can propagate without changing its shape. These waves have been called *gap solitons*. In the case of quadratic media, such solutions take the name of simultons, since localization affects both to the fundamental wave and its second harmonic.

In this work, we study a 1D periodic system, formed by the periodic repetition of a set of two layers of different fluid materials, producing a 1D periodic modulation of the physical properties.

2 Coupled Mode Theory

The propagation of acoustic waves in the multilayered media is described by a second-order nonlinear wave equation, also known as the lossless-Westervelt equation [1],

$$\frac{\partial^2 p}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho c^4}\frac{\partial^2 p^2}{\partial t^2} \tag{1}$$

where p is the acoustic pressure field, c is the space-dependent sound velocity, ρ is the fluid density and β the nonlinearity parameter.

From (1), we have derived a set of coupled-mode equations for the envelopes of the acoustic pressure profiles, analogous to the ones that appear in the optical case [3]. Our analytical approach consists in reducing the nonlinear wave equation for pressure field (1) to a set of first order partial differential, coupled-mode equations. After some assumptions we get the system (2),

$$\begin{aligned} \frac{1}{v_1}\frac{\partial A_1^{\pm}}{\partial t} \pm \frac{\partial A_1^{\pm}}{\partial z} &= i\delta k_1 A_1^{\pm} + i m_1 A_1^{\mp} + i\gamma_1 A_2^{\pm}\left(A_1^{\pm}\right)^* \\ \frac{1}{v_2}\frac{\partial A_2^{\pm}}{\partial t} \pm \frac{\partial A_2^{\pm}}{\partial z} &= i\delta k_2 A_2^{\pm} + i m_2 A_2^{\mp} + i\gamma_2\left(A_1^{\pm}\right)^2 \end{aligned} \tag{2}$$

where t and z denotes the time and spatial dimensions, respectively, A_1 and A_2 are the envelopes (slow variation) of first and second harmonic, respectively, of near Bragg frequency solutions, and (±) denotes the forward and backward fields. The other parameters denote: phase velocity v, detuning δk, modulation of the medium m, and nonlinear parameter γ, for first and second harmonic (see (3) for further details).

3 Results

In this work, we reproduce numerically the existence of acoustic gap solitons (simultons) in a 1D layered acoustic media in the frame of the coupled-mode theory. In our numerical model we have supposed an homogeneous linear medium for $z \in [-70,-40]$, where backward reflection and second harmonic can propagate but can not be generated by forward field ($\delta k = m = \gamma = 0$); followed by a modulated medium for $z \in [-40,0]$, where $\delta k = -0.9$, $m = 1$ and $\gamma = 1$; then the symmetric media is added.
Under this assumptions we can exite a solitary wave solution. In fact, if we inject a pair of beams with opposite directions in the boundaries of the system, we can also excite a stationary gap simulton, where energy remains at the same point during time.

In Figure 1, we can see how both beams are generated at t=50 and propagates in opposite directions through the linear medium (not second harmonic is generated). When they arrive at the interface between both media it is generated a second harmonic, part of the energy is reflected and the other travels through the medium. This part travels without dispersion (even though the medium it is dispersive) and both harmonics travel together with slower velocity than it would travel separated, this is a simulton.

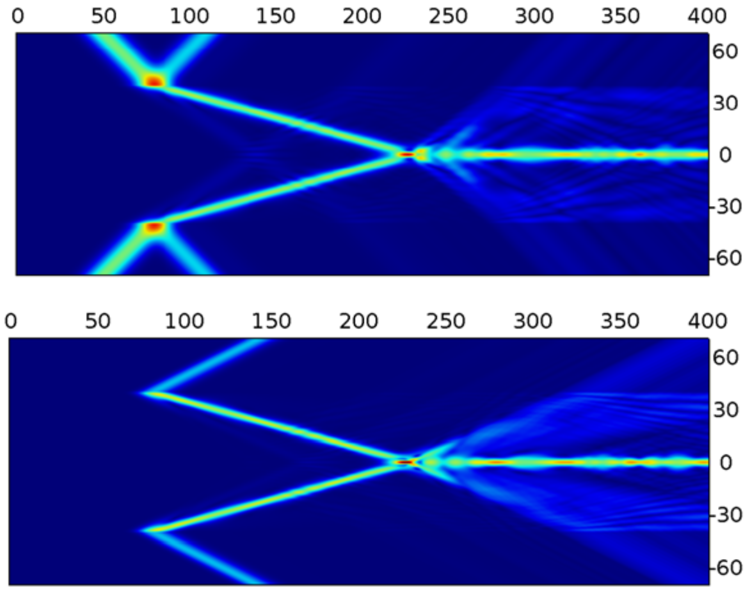

Figure 1 – Stationary gap simulton: (up) amplitude of envelope of first harmonic, (bottom) amplitude of the envelope of second harmonic.

At final stage, both simultons collide at the center of the domain and they produce a stationary gap soliton. As we can see in Figure 2, a cut of the map among z=0 is presented. It is observed how energy remains in that point during time.

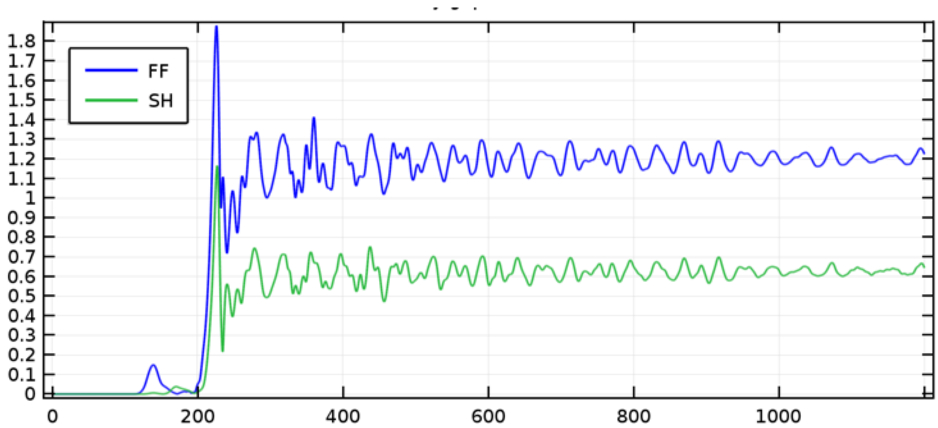

Figure 2 – Stationary gap simulton among z=0: (blue line) first harmonic (fundamental frequency), and (red line) second harmonic generated by the first one.

4 Conclusion

It is demonstrated that solitary waves solutions can be excited in the frame of coupled mode theory in optical devices (3). In addition a set of similar equations are given for the acoustical problem and a set of physical properties can be induced in order to demonstrate this kind of waves. The next stage of this work is to demonstrate it in a real experimental set-up choosing the proper materials to do a multi-layered media.

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References

[1] Hamilton, M. F., & Blackstock, D. T. (Eds.). Nonlinear acoustics. *Academic press San Diego*, Vol 237, 1998.

[2] Jiménez, N., Mehrem, A., Picó, R., García-Raffi, L. M., & Sánchez-Morcillo, V. J. Nonlinear propagation and control of acoustic waves in phononic superlattices. *Comptes Rendus Physique*, Vol 17 (5), 2016, pp 543-554.

[3] Conti, C., Assanto, G., & Trillo, S. Parametric gap solitons in quadratic media. *Optics express*, Vol 3 (11),1998, pp 389-404.