



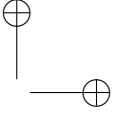
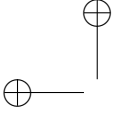
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Synthesis of the Complete Inverse Kinematic Model of Non-Redundant Open-Chain Robotic Systems Using Groebner Basis Theory

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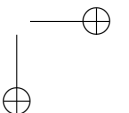
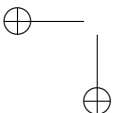
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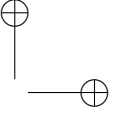
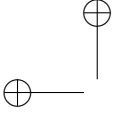


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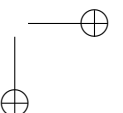
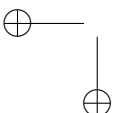


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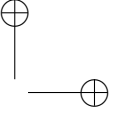
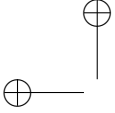
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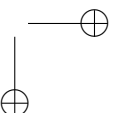
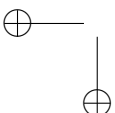
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