INVESTIGATION OF METAL BUILT-UP COLUMNS
PART II: RESULTS

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Abstract: In the frame of the large parametrical study metal built-up columns made from steel and made of aluminum alloy were investigated. The second order theory is used for the analysis of the battened and laced built-up columns under combined compression and bending. The bottom columns ends are fixed and the upper ones are free in the case of in-plane buckling. At the column base the translation and rotation are fixed, at the column top translation and rotation are free in the case of in-plane buckling. Translation is fixed and rotation is free at both column ends in out-of-plane buckling. The built-up columns are considered as the columns with effective bending and smeared shear stiffness with a local bow imperfection amplitude \( \varepsilon_0 = L/500 \).

Keywords: Built-up column, Stability, Steel, Aluminum alloy, Shear effects

1. Introduction

More general cases of metal built-up columns made of steel and aluminum alloy are investigated in the paper. The influence of both initial imperfections is analyzed: the global initial sway imperfection at real column and the local bow imperfection at equivalent column. The comparisons of the numerical results of: the first order theory with the second order one without and with influence of the shear deformation. The results of both: analytical solutions and computer program are compared. Accuracy of the solution of the equivalent column with effective bending stiffness and smeared shear stiffness is verified by the solution of the column, which consists of real discrete
components: chords, battens or diagonals and posts. Importance of the column parameters \( y, \varepsilon, \alpha_c \) and \( k_0 \) is shown.

The formulae, which are used in the calculations, are given in Part I, [1].

2. Verification of the built-up column

The calculation of the in-plane buckling resistance of the built-up column with relevant sway or bow initial imperfection according to Eurocodes [2], [3] consists from several steps:

(i) the calculation of the internal forces and deformations of the chords: (1) on the model having the effective bending stiffness and equivalent (smeared) shear stiffness or (2) on the model of discrete frame structure with real stiffness. This calculation may be done (1) with the formulae given in Eurocodes [2], [3] or (2) with the help of theory of the second order, which results may be obtained (2a) with the exact formulae (e.g. Rubin’s ones) or (2b) with the appropriate computer program. If the possibility (1) is chosen it is consequently necessary to calculate the internal forces in the built-up column components: in chords and in battens or lacings (diagonals and posts);

(ii) the chords;

(iii) the built-up column components;

(iv) the connections of the components to the chords; and

(v) the maximum deformation.

Verification of the out-of-plane buckling resistance is necessary to perform too.

3. Results of the part of the parametrical study of the model with equivalent column

The cross-sections of the investigated battened and laced built-up columns are given in Figure 1a and Figure 1b, respectively.

![Figure 1. Cross-section of the built-up columns made of steel S355, a) battened built-up column, b) laced built-up column](image)

3.1. Battened built-up column made of steel
The geometrical equivalent global initial sway imperfection is taken according to [2]. The column is made of steel S355. The yield strength is $f_y = 355$ MPa, the safety factors are $\gamma_{M0} = 1.0$, $\gamma_{M1} = 1.0$. The columns cross-section is drawn in Figure 1a. The design values of the external actions applied at the top of the column with the length $L = 7.2$ m are given in Figure 2. The horizontal force $H_{Ed, tot}$ consists of two parts: the external force $H_{Ed} = 22$ kN and the replacement of the global initial sway imperfection by equivalent horizontal force $N_{Ed}\Phi = 850$ kN/268.328 = 3.205 kN. The formulae (26)-(31) in [1] of the Rubin’s analytical solution of the differential equation (15) [1] are used in the calculation of the column internal forces. They are calculated for the values of the parameters defined in (16) [1]:

a) $\gamma = 1.0$ and $\varepsilon = 1.0$ in the case of the first order theory without effect of the shear forces;

b) $\gamma = 1.0$ and $\varepsilon = 0.863$ in the case of the second order theory without effect of the shear forces; and

c) $\gamma = 1.118$ and $\varepsilon = 0.863$ in the case of the second order theory with effect of the shear forces.

The values of the internal forces were compared with the results of the computer program IQ 100 [4] and it was found that the differences are negligible (Table 1). The values of the top deflections given in Figure 2 were calculated with IQ 100 (Table 2).

![Figure 2. Steel built-up column; Geometry, actions and distributions of the internal forces $N_x(x)$, $V_y(x)$, $M_z(x)$ and top deflexion $w(7.2$ m) [5]](image-url)
### Table 1. Comparisons of M-values calculated with formula (1) [1] with IQ 100 [2] ones

<table>
<thead>
<tr>
<th>Quantity calculated by</th>
<th>Theory</th>
<th>$S_i$ [kN]</th>
<th>Section $x$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M(x)$ [kNm]</td>
<td></td>
<td></td>
<td>7.2</td>
</tr>
<tr>
<td>formula 2nd order</td>
<td>8175.3</td>
<td>0</td>
<td>151.1929</td>
</tr>
<tr>
<td>IQ 100 2nd order</td>
<td>8175.3</td>
<td>0</td>
<td>151.193</td>
</tr>
<tr>
<td>formula 2nd order</td>
<td>$\infty$</td>
<td>0</td>
<td>128.8514</td>
</tr>
<tr>
<td>IQ 100 2nd order</td>
<td>$\infty$</td>
<td>0</td>
<td>128.851</td>
</tr>
<tr>
<td>formula 1st order</td>
<td>$\infty$</td>
<td>0</td>
<td>90.738</td>
</tr>
<tr>
<td>IQ 100 1st order</td>
<td>$\infty$</td>
<td>0</td>
<td>90.738</td>
</tr>
</tbody>
</table>

Note: $M$-values calculated with formula (1) [1] given in Figure 2 are rounded.

### Table 2. The $w$-values calculated with IQ 100

<table>
<thead>
<tr>
<th>Theory</th>
<th>$S_i$ [kN]</th>
<th>Section $x$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>7.2</td>
</tr>
<tr>
<td>$w(x)$ [mm]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd order</td>
<td>8175.3</td>
<td>108.4</td>
</tr>
<tr>
<td>2nd order</td>
<td>$\infty$</td>
<td>64.0</td>
</tr>
<tr>
<td>1st order</td>
<td>$\infty$</td>
<td>46.9</td>
</tr>
</tbody>
</table>

### 3.2. Laced built-up column made of steel

The steel cross-section is given in Figure 1b. The design values of the external actions applied at the top if the column with the length $L = 7.2$ m are given in Figure 3. The formulae (26)-(31) [1] of the Rubin’s analytical solution of the differential equation given in (15) [1] are used again in the calculation of the internal forces. They are calculated for the values of the parameters defined in (16) [1]:

- a) $\gamma = 1.0$ and $\varepsilon = 1.0$ in the case of the first order theory without effect of the shear forces;
- b) $\gamma = 1.0$ and $\varepsilon = 0.418$ in the case of the second order theory without effect of the shear forces; and
- c) $\gamma = 1.031$ and $\varepsilon = 0.418$ in the case of the second order theory with effect of the shear forces.

The differences between formulae results and the computer program IQ 100 [4] are again negligible. The values of the top deflections given in Figure 3 were calculated with IQ 100.

### 3.3. Laced built-up column made of aluminum alloy

The column is made of aluminum alloy EN-AW 6061-T6, buckling class A. The material properties are: the yield strength $f_0 = 240$ MPa; the material safety factor $\gamma_m = 1.1$; Young modulus $E = 70$ GPa. The cross-section Class 3 is extruded I section with flanges $120 \times 12$ mm; section height $240$ mm; web thickness $9$ mm and fillet radius $16$ mm. The lacing consists of diagonals $L 40 \times 4$ mm and posts $L 40 \times 4$ mm, Class 4 cross-sections. The design values of the external actions applied at the top of the column top and other geometrical details are given in Figure 4 for the model having the real
length $L$ and global sway initial imperfection and in Figure 5 for the model having the length $2L$ and local bow initial imperfection.

Figure 3. Steel laced built-up column. Geometry, actions and distributions of the internal forces $N_x(x), V_y(x), M_z(x)$ [6]

Figure 4. The results of the aluminum equivalent member method [6] for the model having the real length $L$ and global sway initial imperfection. The upper values in bold relate to the theory of the second order with shear effect, the ones without shear effect are below them and the values of the theory of the first order are in the brackets.

The comparisons of the values valid for the model having the real length $L$ and global sway initial imperfection (Figure 4) with the values in upper half of the model having the length $2L$ and local bow initial imperfection (Figure 5) show very
agreement. That means that the both models may be used for the calculation of internal forces. But creating of the correct relevant model drawn in Figure 5 is not so easy.

The comparisons of the results of the analytical solutions in the form of the Rubin’s formulae (26)-(31) [1] given in Figure 2 - Figure 5 with the numerical results of the computer program IQ 100 [4] are again negligible. Both kind of results are almost identical and they confirm the validity of Rubin’s formulae.

\[ N_{ch,Ed} = \frac{N_{Ed}}{2} \pm \frac{M_{II,Ed,max}}{I_{eff, ch}}, \quad D_{Ed} = \frac{V_{II,Ed,max}}{2A}, \quad P_{Ed} = \frac{V_{II,Ed,max}}{n}. \] (1)

The last step is the evaluation of the verification equations according to [2], [3]. It was done in details in [5].
3.5. Comparisons of the most important results

The comparisons of the most important results are for Figure 2 - Figure 3 cases presented in Table 3.

Table 3. Built-up column parameters $\gamma$, $\epsilon$, $\alpha_{cr}$ and $k_{II}$. Indexes $v$, $V$ indicate shear effects.

<table>
<thead>
<tr>
<th>Built-up column parameters with (without) shear effect</th>
<th>Steel S355 Battened, Figure 2</th>
<th>Steel S355 Laced, Figure 3</th>
<th>Aluminum alloy EN-AW 6061-T6 Laced, Figure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = \left[1/(1 - N_{ed}/S_{v})\right]^{0.5}$</td>
<td>1.118 (1.0)</td>
<td>1.006 (1.0)</td>
<td>1.048 (1.0)</td>
</tr>
<tr>
<td>$\epsilon = L_0 \left[\gamma N_{ed}/EI_{eff}\right]^{0.5}$</td>
<td>0.863 (0.816)</td>
<td>0.413 (0.412)</td>
<td>0.455 (0.444)</td>
</tr>
<tr>
<td>$\alpha_{cr,v} = N_{cr,v}/N_{ed}$</td>
<td>2.664 (3.702)</td>
<td>13.327 (14.542)</td>
<td>7.964 (12.506)</td>
</tr>
<tr>
<td>$k_{II,v} = \alpha_{cr,v}/(\alpha_{cr,v} - 1)$</td>
<td>1.601 (1.370)</td>
<td>1.081 (1.074)</td>
<td>1.144 (1.087)</td>
</tr>
<tr>
<td>$k_{II,v} = M_{II,v,max}/M_{I,max}$</td>
<td>1.513 (1.303)</td>
<td>1.083 (1.080)</td>
<td>1.117 (1.095)</td>
</tr>
</tbody>
</table>

The paper [8] explains how to use Unique Global and Local Initial (UGLI) imperfection method. The PhD student [9] had problem to use this method for special non-uniform column in compression and therefore writes about obstacles of this method. In fact there are no obstacles in this method.

The interesting information may be found in [10]-[12].

The calculations and design of built-up columns according to the former Czechoslovak standard ČSN 73 1401:1982 [13] may be found in [14].

4. Results of the part of the parametrical study of the discrete models

It may be interesting to know if the equivalent column model with the effective bending and smeared shear stiffness describes behavior in similar way as the discrete model. The comparison of the maximal axial forces in the chord of built-up column calculated on the model of:

a) the continuum with effective bending and smeared shear stiffness according to [2];

b) the simple frame structures with discrete components, in which the joints are not required to resist moments shows good agreement.

The b) model was calculated twice: b1) the influence of axial tension forces in the tension chord and in the tension diagonals was neglected, b2) the influence of the all axial forces in compressed and also tension components was taken into account to obtain more exact values. The internal forces and deformations of both b) models were calculated with the computer program IQ 100 [4]. The difference between b1-value 1180.6 kN and more exact b2-value 1164.1 kN is not important. The good agreement was achieved also between the a-value 1153.9 kN of the continuum with effective bending and smeared shear stiffness and the more exact b2-value 1164.1 kN of the simple frame structures with discrete components. It was confirmed that calculation of
the internal forces of the equivalent column model may replace the calculation of the internal forces on the discrete model. More details about calculation of internal forces and verification of all components of discrete models of battened and laced built-up columns may be found in [5]-[7].

5. Results of the part of the parametrical study of the FEM models

The results of Finite Element Method (FEM) investigation of the special steel built-up column consisting of four cold formed C-sections jointed in 4 battened planes and cold formed C-sections creating lacing in one middle plane in z direction (Figure 6) were published in [15].

Figure 6. Geometry of CFS built-up column (1 - C-8B; 2 - C-OB; 3 - battens 200x200x4mm), $f_{y} = 500$ MPa, $f_{u} = 550$ MPa [8]
6. Conclusions

The most important parameters of the column are: the shear parameter \( \gamma \), the column parameter \( \varepsilon \), the parameter \( \alpha_{cr} \) and amplification factor \( k_{II} (M_{II} = k_{II}M_{I}) \).

The parameters \( \gamma, \varepsilon, \alpha_{cr} \) and \( k_{II} \) give to the designer very important information about behavior of the built-up columns without doing more complicated exact calculations of the internal forces. The first order analysis may be used according to Eurocodes for the structure if \( \alpha_{cr} \geq 10 \) (15) in elastic (plastic) analysis. For example for the laced built-up column the value of the parameter \( \gamma = 1.006 \approx 1.0 \) indicates that the influence of the shear stress is negligible. The value of the parameter \( \alpha_{cr} = 13.327 > 10 \) indicates that the first order theory may be used and the error will be smaller than 10%. Then the designer may use very simple calculation to obtain maximum bending moment \( M_{\text{max}} = M(x = 0) = M_{\text{Ed,e}} + H_{\text{Ed,tot}} = 350 \text{kNm} + 25.205 \text{kN} \cdot 7.2 \text{m} = 531.47 \text{kNm} \). It is the value, which is comparable with the exact values 575.7 kNm and the error is only 7.7% on the unsafe side.

The results of the large parametrical study of the behavior of steel and aluminum built-up battened and laced columns shown:

(i) it is possible to use model of column with effective bending and equivalent (smeared) shear stiffness;
(ii) for Euler fall I the both models could be accepted: the model having the real length \( L \) and global sway initial imperfection compared and the model of equivalent member method having the length \( 2L \) and local bow initial imperfection;
(iii) shear effects are more pronounced at battened columns;
(iv) in verification of in-plane and out-of-plane buckling resistance the built-up column must be treated as a column under compression and biaxial bending.

Acknowledgements

Project No. 1/0244/20 is supported by the Slovak Grant Agency VEGA.

References


Figures

Figure 1.
Figure 2.

Theory of the 2nd order with shear influence $S_0 = 8175.3$ kN

without shear influence $S_w = \infty$

Theory of the 1st order (values in brackets) $S_\alpha = \infty$
Figure 3.
Figure 4.
Figure 5.
Figure 6.
### Table 1.

<table>
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</tbody>
</table>

Note: $M$-values calculated with formula (1) [1] given in Figure 2 are rounded.
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<table>
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<tr>
<th>w(x)[mm]</th>
<th>Theory</th>
<th>S&lt;sub&gt;v&lt;/sub&gt; [kN]</th>
<th>Section x [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>7.2  3.6  1.2  0</td>
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<tr>
<td></td>
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<td>64.0 19.7 2.4 0</td>
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<td>∞</td>
<td>46.9 14.7 1.8 0</td>
</tr>
</tbody>
</table>
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<tr>
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</tr>
</tbody>
</table>
Legend to figures and tables,

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Figure 3. Steel laced built-up column. Geometry, actions and distributions of the internal forces $N_x(x)$, $V_y(x)$, $M_z(x)$ [6]

Figure 4. The results of the aluminum equivalent member method [6] for the model having the real length $L$ and global sway initial imperfection. The upper values in bold relate to the theory of the second order with shear effect, the ones without shear effect are below them and the values of the theory of the first order are in the brackets

Figure 5. The results of the aluminum equivalent member method [7] for the model having the length $2L$ and local bow initial imperfection. The upper values in bold relate to the theory of the second order with shear effect, the ones without shear effect are below them

Figure 6. Geometry of CFS built-up column (1 - C-8B; 2 - C-OB; 3 - battens 200x200x4mm), $f_{yb} = 500$ MPa, $f_u = 550$ MPa [8]

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