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ORIGINAL RESEARCH PAPER

Improving the energy efficiency of slightly inductive three-phase three-wire linear systems through single-phase capacitors banks

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Abstract
Recent low-voltage, three-phase distribution networks carry inefficient power, which can be attributed to the reactive power, imbalance of linear loads, and increase in non-linear loads. Researchers have mainly focused on developing active filters to improve the quality of electrical energy. However, in most cases, compensating for the reactive power is sufficient, considering cost and quality. Active filters are expensive, thus, passive compensators are more appealing. These devices are composed of single-phase or three-phase capacitor banks that act on the reactive power consumed by the load. The operation of these devices has been sufficiently validated owing to their long-term use. For unbalanced power, compensators that contain coils are used. A methodology to obtain compensators comprising only single-phase capacitors for the inductive reactive power consumed by the load is presented here; if designed appropriately, these compensators can compensate for a part of or the entire unbalanced power, resulting in greater efficiency in the transfer of electrical energy.

1 | INTRODUCTION

The quality of electrical power supply has always been concerning to researchers, especially low-voltage three-phase systems, whether they consist of three or four wires. These systems generally function in an unbalanced manner, which is attributed to both loads and voltages, manifesting as an increase in the total apparent power with respect to the ideal power of a balanced system characterised by the positive-sequence active power. Therefore, these imbalances are translated into unbalanced power owing to the appearance of negative-sequence and zero-sequence voltages and currents. Thus, these unbalanced powers constitute the inefficiencies of the system [1–4]. These powers are apparently usually lower in the three-phase three-wire systems than those in the four-wire systems because there is neither a zero-sequence voltage nor current and most of the connected loads are three-phase. Studies regarding these powers and their physical significance have been conducted [5–8].

Technological advances also contribute to other inefficient power owing to the use of non-linear loads, such as variable speed drives (widely used in industry) and arc welding equipment, among others. Furthermore, there are domestic loads that generate this type of power, especially those that use switched power supplies for operation, such as computers, printers, and battery chargers. This inefficient power was classified as harmonic power [8], which has also been discussed widely by the scientific community [9]. Therefore, active filters have been developed to reduce the effects of harmonic components [10].

Thus, as additional power is involved in the transfer of electrical energy, the total apparent power that the network must supply to a load increases [11, 12].

Therefore, researchers have focused on the development of elements to compensate for or reduce inefficient power. These studies can be generally categorised into the following two types of studies: compensators consisting of passive elements, also known as reactive power compensators (RPC), and compensators consisting of active elements, also known as advanced compensator type switching power converters (SPC), or active filters.

TABLE 1 Compensation methods in balanced circuits

Balanced circuits	The best passive compensation method
Highly inductive	By method [14]
Slightly inductive	Method MLL (Minimum Loss Line) [17]
Capacitive	Method SBC (Sinusoidal Balanced Current) [17]
Resistive	There is nothing to compensate

TABLE 2 Compensation methods in unbalanced circuits

Unbalanced circuits	The best passive compensation method
Highly inductive	By method [14]
Slightly inductive	By method [14], but it uses some coil
Capacitive	By method [14], but it uses some coil
Resistive	By method [14], but it uses some coil

If both technologies are compared, the SPC type compensators are significantly better than the RPC type because they are smaller, less noisy, have more functions to increase the quality of energy, are more precise, and are faster.

However, most networks do not require all the functions of these compensators. For typical functions, such as load balancing and power factor correction, studies [13] have indicated that SPC devices are between 30% and 35% higher in cost compared to that of the RPCs.

Therefore, the authors will focus on the first group, that is, on the compensation of unbalanced powers in three-phase three-wire linear systems with unbalanced voltages and loads by utilizing passive elements. Therefore, single-phase capacitor banks are used.

In 2019, the authors presented a study [14] in which the classical methodology was improved by extending its application to three-phase systems with unbalanced voltages without considering the characteristics of the load. In this study, the most important contributions and trends are analysed. All the studies analysed use three passive elements to form the compensator for the negative-sequence current consumed by the load. As a result, there are always three elements in the compensator configuration, two elements of one nature and another of an opposite nature (inductive or capacitive).

Finally, models using single-phase capacitor banks have been presented [15, 16]. These compensators are interesting because capacitors are widely used in the compensation of inductive reactive energy, either as scalable batteries or in their more advanced form, called static VAR compensators (SVCs). The proper functioning of these devices has been sufficiently proven. However, these studies are applicable to highly inductive loads but they were not demonstrated with slightly inductive loads.

Table 1 presents a summary of the current state of the energy efficiency improvement in balanced three-phase three-wire systems using passive compensators.

In these ideal cases (balanced circuits), all the methods coincide in the final result because there are only positive sequences for both voltage and current. However, the same does not occur for unbalanced circuits, a summary of which is shown in Table 2.

The vast majority of electrical systems are either highly or slightly inductive. This study is focused on the latter and a methodology to compensate for the reactive power and negative-sequence current consumed by the load is presented, only with the capacitors.

In this paper, two new procedures are proposed to improve the energy efficiency of three-wire linear electrical systems with unbalanced voltages and/or loads. The first procedure compensates for the negative-sequence current consumed by the load based on the use of one or two capacitors. The second procedure is for the development of a compensating circuit for the reactive power consumed by the load, and part of the negative-sequence current that it consumes. This compensator will be formed, in principle, by using one or two capacitors. This achieves a minimal current that circulates through the power supply network for the correct operation of a load without using coils in the compensators. To determine the values of the passive elements that compose the compensating circuits, it is only necessary to obtain the line-to-line voltages and line currents. These parameters are easily measurable on a bus. Thus, it is not necessary to obtain the state of the upstream network or the value and connection method of the downstream loads.

The power expressions used are presented in Section 2, which is followed by the development of methodology for obtaining the capacitors to be placed in the compensator of the negative-sequence of the current consumed by the load in Section 3. The facility is also presented to extend the procedure to the use of the remaining passive elements. Finally, the terms highly inductive circuits and slightly inductive circuits are defined.

In Section 4, slightly inductive circuits are analysed, and another methodology is developed to obtain the capacitors required to compensate for the reactive power of the load and minimise the negative-sequence current consumed by the load. In Section 5, several cases are analysed considering the methodologies presented, as well as the efficiency of the system. The results are compared to the MLL and SBC reactive power compensation strategies, and to the methodology according [14]. Finally, Section 6 presents the conclusions of this study.

2 | EXPRESSIONS OF ELECTRICAL POWER

Figure 1 demonstrates an unbalanced linear load that is connected to a three-phase three-wire system with generally unbalanced voltages.

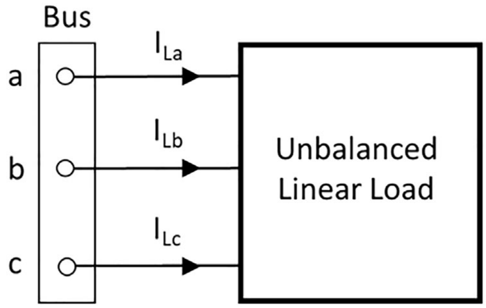

FIGURE 1 Unbalanced three-phase three-wire linear system

According to Buchholz, the total apparent power on the bus is determined by Equation (1).

$$S_T = 3\sqrt{\left(V_+^2 + V_-^2 + V_0^2\right)\left(I_+^2 + I_-^2 + I_0^2\right)}$$
$$= \sqrt{S^2 + S_{uT}^2} = \sqrt{P^2 + Q^2 + S_{uT}^2} \quad (1)$$

where S_{uT} is the total unbalance apparent power caused by the voltages and currents of different sequences. Its value is obtained by using Equation (2).

$$S_{uT}^2 = S_{u+}^{-}2 + S_{u-}^{+}2 - 2\left(P_+P_-\right) - 2\left(Q_+Q_-\right) \quad (2)$$

here

- S_{u+}^{-} indicates the apparent power resulting from the positive-sequence voltage and negative-sequence current;
- S_{u-}^{+} indicates the apparent power resulting from the negative-sequence voltage and positive-sequence current;
- P_+ y P_-, are the positive and negative-sequence active powers, respectively;
- Q_+ y Q_-, are the positive and negative-sequence reactive powers, respectively.

The values of these apparent powers are calculated from (3–4).

$$S_{u+}^{-} = 3V_+I_- \quad (3)$$

$$S_{u-}^{+} = 3V_-I_+ \quad (4)$$

We would like to highlight that when obtaining measurements on the bus of a three-phase three-wire system, there is no zero-component of the current and the zero-component of the generator voltage cannot be determined because only line-to-line voltages can be measured.

3 | NEGATIVE-SEQUENCE CURRENT COMPENSATION. HIGHLY AND SLIGHTLY INDUCTIVE CIRCUITS

Assuming that the circuit shown in Figure 1 is powered by an unbalanced system in voltages whose values have been obtained

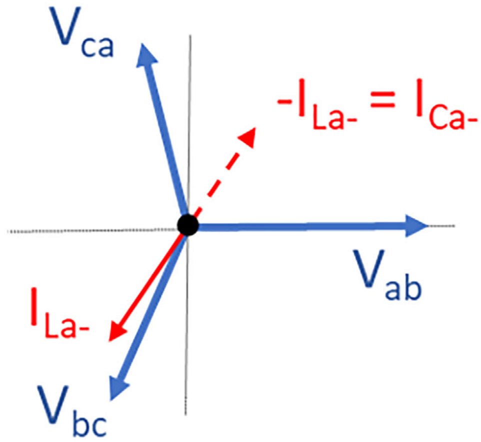

FIGURE 2 Unbalanced voltage system that feeds the load shown in Figure 1

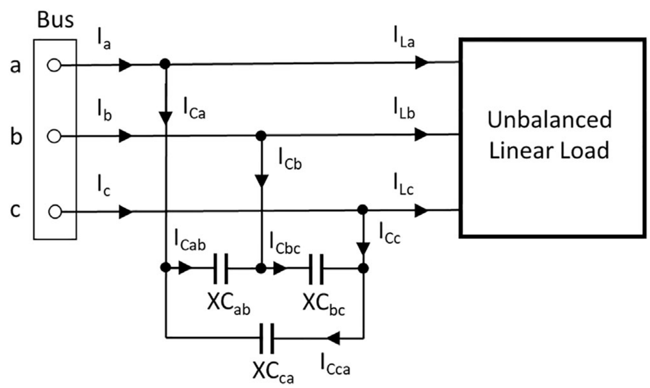

FIGURE 3 Circuit presented in Figure 1 with capacitors supplied with line-to-line voltages

on the bus, a negative-sequence line current is apparent. The fundamental component of this current ($\boldsymbol{I}_{La-}$) may be in any quadrant of its vector diagram. Assuming that this current $\boldsymbol{I}_{La-}$ is in the third quadrant, as indicated in Figure 2, capacitors that consume a negative-sequence current whose fundamental component ($\boldsymbol{I}_{Ca-}$) is of equal value in the modulus to $\boldsymbol{I}_{La-}$, but 180° out of phase, must be added for compensation, as shown in Figure 2.

As shown in Figure 3, currents will flow through the capacitors placed between each two phases $\boldsymbol{I}_{Cab}$, $\boldsymbol{I}_{Cbc}$ and $\boldsymbol{I}_{Cca}$.

The vector diagram shown in Figure 4 is obtained if we transfer these currents to the vector diagram presented in Figure 2.

The negative fundamental component of this system of currents consumed by the capacitors ($\boldsymbol{I}_{Cab-}$) must provide a negative-sequence line current equal to the sought current $\boldsymbol{I}_{Ca-}$, thus compensating for the negative component of the load $\boldsymbol{I}_{La-}$. Therefore, the Fortescue transform must be considered. The value of $\boldsymbol{I}_{Cab-}$ is given by Equation (5).

$$\boldsymbol{I}_{Cab-} = \frac{\boldsymbol{I}_{Cab} + \boldsymbol{a}^2\boldsymbol{I}_{Cbc} + \boldsymbol{a}\,\boldsymbol{I}_{Cca}}{3} \quad (5)$$

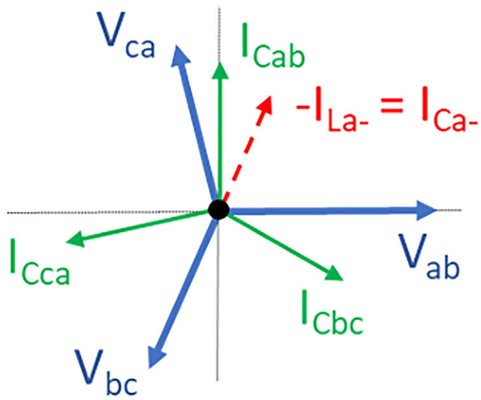

FIGURE 4 Resulting currents when connecting the three capacitors placed between the phases

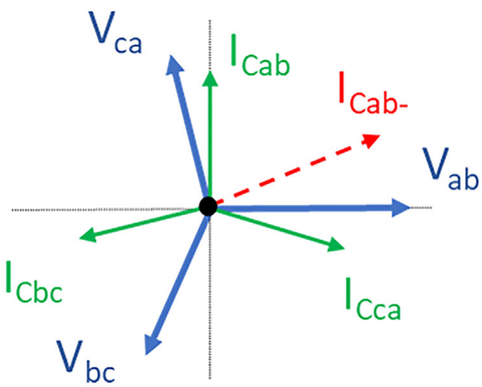

FIGURE 5 Vector diagram resulting for obtaining $\boldsymbol{I_{Cab-}}$

In addition, the relationship between the negative fundamental component of the phase current $\boldsymbol{I_{Cab-}}$, with the negative fundamental component of the line current $\boldsymbol{I_{Ca-}}$, as shown in Equation (6), will also be considered.

$$\boldsymbol{I_{Ca-}} = \frac{\boldsymbol{I_{Cab-}}}{\sqrt{3}} e^{30j} \tag{6}$$

Therefore, the ratio is shown by (7).

$$\boldsymbol{I_{Cab-}} = -\sqrt{3}\,\boldsymbol{I_{Ca-}}e^{-30j} \tag{7}$$

Thus, the current phasor consumed by the capacitors must be relocated, and the current $\boldsymbol{I_{Cab-}}$ must be obtained. As a result, the vector diagram in Figure 5 is obtained.

As shown in the vector diagram in Figure 5, placing a capacitor between phases a-b and another one between phases c–a is sufficient for obtaining the current $\boldsymbol{I_{Cab-}}$. Its value is defined by the currents $\boldsymbol{I_{Cab}}$ and $\boldsymbol{I_{Cca}}$; thus, it is sufficient to propose Equation (8).

$$I_{Cab} + I_{Cca} = I_{Cab-} \tag{8}$$

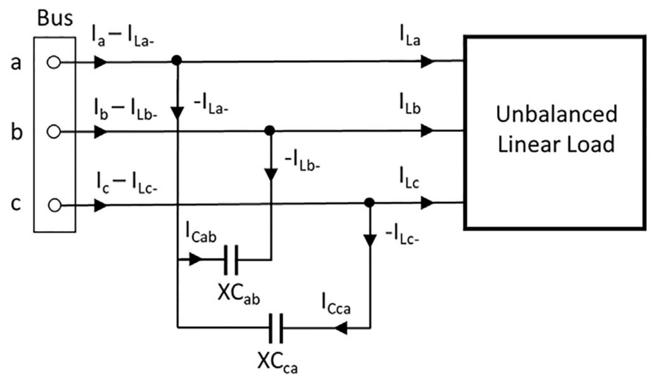

FIGURE 6 Circuit of Figure 1 with the negative-current compensator formed by capacitors

The system of equations shown in (9) is obtained by decomposing (8) into a real part and an imaginary part.

$$\left.\begin{aligned} I_{Cab}cos\beta_{I_{Cab}} + I_{Cca}cos\,\beta_{I_{Cca}} &= I_{Cab-}\,cos\beta_{I_{Cab-}} \\ I_{Cab}sin\beta_{I_{Cab}} + I_{Cca}sin\,\beta_{I_{Cca}} &= I_{Cab-}\,sin\beta_{I_{Cab-}} \end{aligned}\right\} \tag{9}$$

As shown in (9), the only unknown variables are the moduli of the capacitor currents, I_{Cab} and I_{Cca}. This is because the angles of currents $\beta_{I_{Cab}}$and $\beta_{I_{Cca}}$ are known, given that they will be 90° ahead of the angles of their respective voltages.

Thus, the values of the reactance of the capacitors to be placed are those indicated in (10).

$$X_{Cab} = \frac{V_{ab}}{I_{Cab}}\ ;\ X_{Cca} = \frac{V_{ca}}{I_{Cca}} \tag{10}$$

Using the aforementioned, it is possible to compensate the negative-sequence current consumed by the load in a manner that it will no longer be delivered by the network on the bus, originating the circuit shown in Figure 6.

As indicated by the procedure presented, it is also simple to calculate the resulting capacitors if the current $\boldsymbol{I_{La-}}$ is in another quadrant. Furthermore, if this current coincided with the opposing currents of one of the capacitors to be placed, it will only be necessary to place a capacitor to form the compensator.

3.1 | Study of negative-sequence current compensation

The proposed method is easily applicable to the remaining of the passive elements because other elements such as resistors (R), coils (L), capacitors (C), or a combination can be used to comply with Equation (8). As shown in Figure 4, all the currents were added owing to the placement of the resistors and coils in all phases; we moved them according to expressions (5) and (7), and named $\boldsymbol{I_{ab-}}$ as the negative fundamental component, obtaining the vector diagram shown in Figure 7.

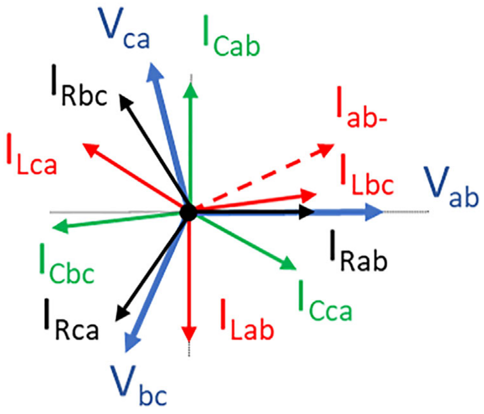

FIGURE 7 Currents resulting from the different passive elements placed between phases *A, B, C*

TABLE 3 Various alternatives to configure the compensator

	Elements to connect		
Alternatives	V_{ab}	V_{bc}	V_{ca}
1	R	–	R
2	–	L	L
3	R	–	L
4	R and C	–	–
5	–	R and L	–
6	–	R	C
7	C	L	–

Figure 7 indicates that there are several combinations that can be used to obtain $\boldsymbol{I_{ab-}}$. For example, in this case, in addition to the proposed capacitors, all the combinations shown in Table 3 are also valid.

As shown in Table 3, there are seven more combinations of passive elements to compensate for the negative-sequence current drawn by the load. However, the most notable elements are capacitors because they do not consume active power, rather assist in compensating for the reactive power consumed by the load, and have demonstrated sufficient long-term operation.

3.2 | Highly and slightly inductive circuits

Although determining the parameters that can establish this definition is complex, the authors have established the following criteria:

- When the capacitor or two capacitors are obtained according to the aforementioned procedure, it consumes less capacitive reactive power than the inductive reactive power of the load (Q_{LTotal}), indicating the circuit is highly inductive. This is shown in Equation (11) as follows:

$$\left.\begin{matrix}(Q_{Cab}+Q_{Cbc})\\(Q_{Cab}+Q_{Cca})\\(Q_{Cbc}+Q_{Cca})\end{matrix}\right\} < Q_{LTotal} \tag{11}$$

- When the circuit is slightly inductive, the circuit becomes capacitive by placing the capacitor or capacitors that compensate for the negative component of the load current. This can be observed in Equation (12) as follows:

$$\left.\begin{matrix}(Q_{Cab}+Q_{Cbc})\\(Q_{Cab}+Q_{Cca})\\(Q_{Cbc}+Q_{Cca})\end{matrix}\right\} > Q_{LTotal} \tag{12}$$

When the circuit is highly inductive, three additional capacitors are needed to compensate for the inductive reactive power of the load, in addition to the capacitors for compensating the negative component of the load current. In this case, the ideal solution is provided by the solution developed in a previous study [14]. As a result, a compensator for both inefficiencies was obtained, which consisted of three capacitors.

When placing the capacitors to compensate for the negative sequence of the load current, the inductive reactive power of the load is also compensated; the resulting capacitors with the procedure proposed in this section coincide with those obtained using the procedure [14].

Finally, when the circuit is not highly inductive, the solution proposed by the procedure in [14] involves the use of one or more coils. In Section 4, the authors propose a new procedure using single-phase capacitor banks to compensate for both inefficiencies. This allows the line currents flowing through the network to be minimal, which is less than the currents obtained with the reactive power compensation method called MLL. This indicates that the losses owing to the Joule effect in the power line are minimal.

4 | ENERGY EFFICIENCY IMPROVING IN SLIGHTLY INDUCTIVE CIRCUITS USING CAPACITORS

The procedure established in Section 3 may be used by adding another condition to the initial Equation (8). This new condition must indicate that the sum of the capacitive reactive powers consumed by the capacitors is equal to the inductive reactive power consumed by the load; the system of equations for the example in Section 3 is shown in Equation (13).

$$\left.\begin{matrix}\boldsymbol{I_{Cab}}+\boldsymbol{I_{Cca}}=\boldsymbol{I_{Cab-}}\\Q_{Cab}+Q_{Cca}=Q_{Ltotal}\end{matrix}\right\} \tag{13}$$

However, although the inductive reactive power of the load is compensated, the maximum reduction in the negative component of the current is not obtained, which is why the authors present the procedure developed below.

Considering the example shown in Figure 6, two capacitors must be calculated, which must meet the following two conditions: The reactive power consumed should be equal to the inductive reactive power of the load, and the sum of the fundamental component of the negative-sequence current consumed by the capacitors $\boldsymbol{I_{Cab-}}$ and that consumed by the load $\boldsymbol{I_{Lab-}}$ must be a minimum ($\boldsymbol{I_{ab-min}}$). This provides the system of equations shown in Equation (14).

$$\left.\begin{array}{c}\boldsymbol{I_{Cab-}} + \boldsymbol{I_{Lab-}} = \dfrac{\boldsymbol{I_{Cab}}}{3} + \dfrac{\boldsymbol{I_{Cca}}}{3}a + \boldsymbol{I_{Lab-}} = \boldsymbol{I_{ab-min}} \\ Q_{Cab} + Q_{Cca} = Q_{LTotal}\end{array}\right\} \quad (14)$$

By developing the equations of system (14), we obtain Equation (15).

$$Q_{Cab}\left(\frac{1}{3V_{ab}}e^{j(\varphi_{Vab}+90)} - \frac{1}{3V_{ca}}e^{j(\varphi_{Vca}+90+120)}\right) + \boldsymbol{I_{Lab-}} + \frac{Q_{Ltotal}}{3V_{ca}}e^{j(\varphi_{Vca}+90+120)} = \boldsymbol{I_{Cab-min}} \quad (15)$$

All the values, except for the reactive power that must be consumed by the capacitor to be placed between phases A and B, can be determined by Equation 15. Equation (16) is obtained by simplifying (15).

$$Q_{Cab}(a + jb) + (c + jd) = \boldsymbol{I_{Cab-\min}} \quad (16)$$

where

$$a = Re\left[\frac{1}{3V_{ab}}e^{j(\varphi_{Vab}+90)} - \frac{1}{3V_{ca}}e^{j(\varphi_{Vca}+90+120)}\right]$$

$$c = Re\left[\boldsymbol{I_{Lab-}} + \frac{Q_{Ltotal}}{3V_{ca}}e^{j(\varphi_{Vca}+90+120)}\right]$$

$$b = Im\left[\frac{1}{3V_{ab}}e^{j(\varphi_{Vab}+90)} - \frac{1}{3V_{ca}}e^{j(\varphi_{Vca}+90+120)}\right]$$

$$d = Im\left[\boldsymbol{I_{Lab-}} + \frac{Q_{Ltotal}}{3V_{ca}}e^{j(\varphi_{Vca}+90+120)}\right]$$

When determining the minimum value of $f(Q_{Cab})$, it can be expressed as shown in Equation (17).

$$f\ (Q_{Cab}) = \left(a\,Q_{Cab} + c\right) + j\left(b\,Q_{Cab} + d\right) \quad (17)$$

The value of the modulus of $f(Q_{Cab})$ is now proposed, as shown in Equation (18).

$$\left|f\ (Q_{Cab})\right| = \sqrt{(a\,Q_{ab} + c)^2 + (b\,Q_{ca} + d)^2} \quad (18)$$

To determine the minimum value of (18), it is necessary to determine the minimum of the radicand. Naming $f_M(Q_{Cab})$ to the modulus of the radicand, we obtain Equation (19).

$$f_M\ (Q_{Cab}) = (a\,Q_{cab} + c)^{\ 2} + (b\,Q_{Cca} + d)^2 \quad (19)$$

By developing Equation (19), Equation (20) is obtained.

$$f_M\ (Q_{Cab}) = Q_{Cab}^{\ 2}\ \left(a^2 + b^2\right) + 2Q_{Cab}\,(ac + bd) + c^2 + d^2 \quad (20)$$

With respect to Q_{Cab}, we obtain Equation (21) by deriving Equation (20).

$$f'_M\ (Q_{Cab}) = 2Q_{Cab}\left(a^2 + b^2\right) + 2\,(ac + bd) \quad (21)$$

By equating Equation (21) to zero and solving it, Equation (22) is obtained.

$$Q_{Cab} = \frac{-2\,(ac + bd)}{2\,(a^2 + b^2)} \quad (22)$$

Finally, to verify that it is a minimum value, we apply the second derivative and determine that it is greater than 0, which indicates that it meets the minimum value condition. This is indicated by Equation (23).

$$f''_M\ (Q_{Cab}) = 2\left(a^2 + b^2\right) > 0 \text{ it meets with the minimum value condition} \quad (23)$$

The reactive powers that each capacitor must consume are determined by solving the aforementioned Equation (22). Thus, once the values of the line-to-line voltages are known, it is easy to calculate the reactance and capacities of these capacitors.

The placement of these capacitors minimises the current delivered by the network to the bus because they compensate for the reactive power consumed by the load and for the maximum possible negative-sequence current without using coils.

5 | PRACTICAL CASES

Three types of circuits were analysed to validate the findings of this study. The first, second, and third circuit was highly inductive, slightly inductive, and between being highly and slightly inductive, respectively, as defined in Section 3.

In the three practical cases, the results obtained using the method proposed in this paper, called Method "D" are analysed and compared with other existing methods (Methods A, B and C), where

- Method "A" was developed in [14]. In highly inductive circuits, its results are excellent. The big problem is that in the compensation of slightly inductive circuits, one or two coils are used. To find the solution, this method needs to know the line-to-line voltages and line currents.

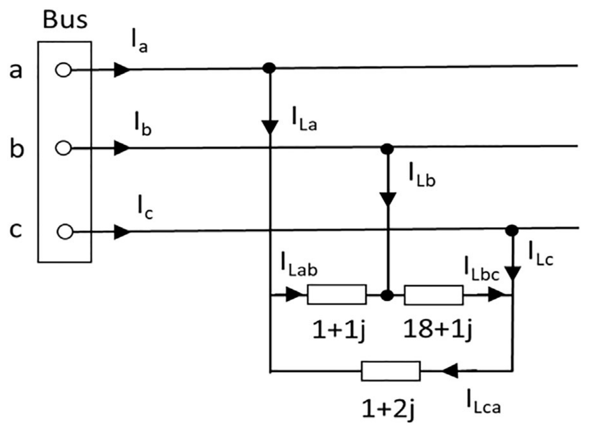

FIGURE 8 Circuit Case 1

TABLE 4 Line-to-line voltages and line currents on the bus. Case 1

Ph.	V_{zz} (V)		Ph.	I_z (A)	
zz	Modulus	Angle	z	Modulus	Angle
ab	376.0	0	a	354.4	−75.6
bc	392.3	−115	b	260.4	139.6
ca	413.3	121	c	206.5	57.67

- Method “B”, called MLL (Minimum Loss Line), is cited in [17] and it is used as a comparative method in [17]. To find the solution you need to know the line-to-line voltages and the internal impedances of the loads. For this reason, the values of the impedances of the loads in practical cases are known. In the rest of the methods it is not necessary, including the method proposed in this paper.
- Method “C”, called SBC (Sinusoidal Balanced Current), is cited in [17] and it is used as a comparative method in [17]. To find the solution, this method needs to know the line-to-line voltages and line currents.

The Orcad Pspice 9.2 software was used in the simulation of the practical cases.

5.1 | CASE 1: Highly inductive circuit

To analyse this case, the three-phase three-wire circuit is shown in Figure 8.

It should be noted that it is unnecessary to determine the load values because the voltages and currents are obtained by measuring on the bus. However, to simulate the examples, the values of the loads were set. For this case, the line-to-line voltages and line currents in the RMS values are shown in Table 4.

A decomposition of the line currents using the direct Fortescue transform provides the positive and negative sequence values shown in Table 5.

The first step is to determine how inductive the circuit is based on the criteria outlined in Section 3. For this, the negative sequence line current consumed by the load will be considered, but 180° out of phase. According to the vector diagram in Figure 5, this current can be achieved by including a capacitor between phases a and b and another capacitor between phases c and a. Solving the system of Equations (9), the currents that will circulate through these capacitors are the following:

$$\mathbf{I_{Cab}} = 164.97\ e^{j90}\ A \quad \mathbf{I_{Cca}} = 9.29\ e^{j210.6}\ A$$

The reactive powers consumed by these capacitors are given by:

$$Q_{Cab} = V_{ab}\ \ I_{Cab} = 62029.9\ VAr$$

$$Q_{Cca} = V_{ca}\ \ I_{Cca} = 3838.5\ VAr$$

If $Q_{Cab} + Q_{Cca}$ is compared with the total reactive power consumed by the load Q_{LTotal}, it is observed that $(Q_{Cab} + Q_{Cca}) < Q_{LTotal}$, therefore, this circuit is considered highly inductive according to (11).

Following the method proposed in Section 4, the values of the two capacitors of the compensator that compensate for the total reactive power of the load would be determined.

Using the system of Equations (13) and applying Equation (22), the reactive powers of the capacitors that compensate for the total reactive power are the following:

$$Q_{Cab} = 96505.01\ VAr \quad Q_{Cca} = 42976.1\ VAr$$

Therefore, the capacitive reactance values of the capacitors are given by:

$$X_{Cab} = 1.465\ \Omega \quad X_{Cca} = 3.974\ \Omega$$

Table 6 shows the reactance values of the proposed method “Method D” and the rest of the methods to be compared (Methods A, B and C).

TABLE 5 positive-sequence and negative-sequence line currents on the bus

Ph.	I_{zL+} (A)		I_{zL-} (A)	
z	Modulus	Angle	Modulus	Angle
a	264.7	−80.1	92.74	−62.9
b	264.7	160	92.74	57.11
c	264.7	39.9	92.74	177.1

TABLE 6 Impedances

Method	Z_{ab} (Ω)	Z_{bc} (Ω)	Z_{ca} (Ω)
A	−j1.634	−j6.235	−j6.041
B	−j2.000	−j325.0	−j2.500
C	−j3.269	−j3.269	−j3.269
D	−j1.465		−j3.974

TABLE 7 Powers, power factor, and efficiency factor

	P (kW)	*Q* (kVAr)	*SuT* (kVA)	*ST* (kVA)	*PF*	*EF*
Load	113.371	139.481	66.051	191.496	0.631	0.592
A	113.371	0	6.241	113.543	1.000	0.998
B	113.371	0	60.897	128.691	1.000	0.881
C	113.371	−3.110	66.051	131.246	1.000	0.864
D	113.371	0	43.333	121.370	1.000	0.934

The consumed powers, power factor (PF), and efficiency factor (EF) are listed in Table 7. The PF and EF factors are determined by Equations (24) and (25).

$$PF = \cos\left(arc\ tg\frac{Q}{P}\right) \tag{24}$$

$$EF = \frac{P}{ST} \tag{25}$$

The following was determined by analysing Tables 6 and 7:

- The ideal solution is proposal A; that is, the method proposed by the authors in [14]. All the inductive reactive power consumed by the load is compensated, as well as its negative-sequence current. The fact that the resulting unbalance power (S_{uT}) value is due to the negative-sequence voltage presented by the network must be considered.
- With proposal D, the entire reactive power of the load ($PF = 1$) and part of the negative current from the network are eliminated. However, the efficiency factor was lower than that with using method A.
- Using proposal B (MLL), a *PF* of unity value is achieved; however, by compensating for a lower negative-sequence current than the current consumed by the load, a lower *EF* than with method A is achieved.
- Finally, by using proposal C, a method similar to method B was obtained, where a reactive power of a capacitive sign remained, although the *PF* value was practically unity. This reactive power corresponds to a negative sequence of voltages and currents. In this case, none of the negative-sequence current consumed by the load was compensated, and is reflected in the *EF*, which is less than that with proposal A.

The line currents using the most suitable method (A) have the temporal form shown in Figures 9 and 10, before and after connecting the capacitors, respectively.

As can be seen, a balanced current system has been obtained with which the negative component is zero. This can be better appreciated in Table 8, where their RMS values are presented.

5.2 | CASE 2: Slightly inductive circuit

To analyse this case, the three-phase three-wire circuit is shown in Figure 11.

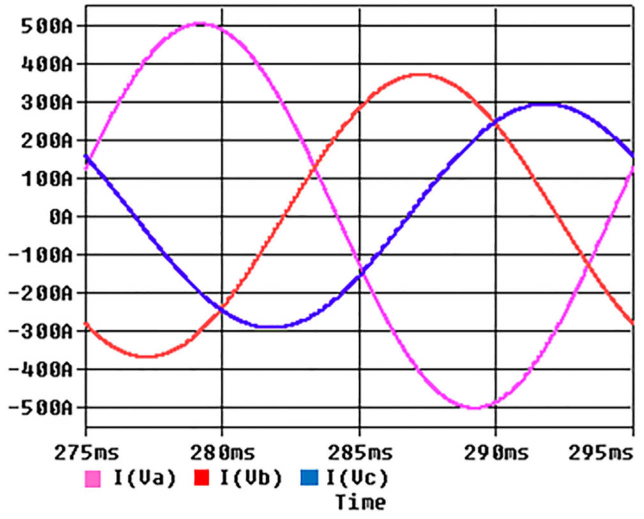

FIGURE 9 Line currents before compensation. Case 1

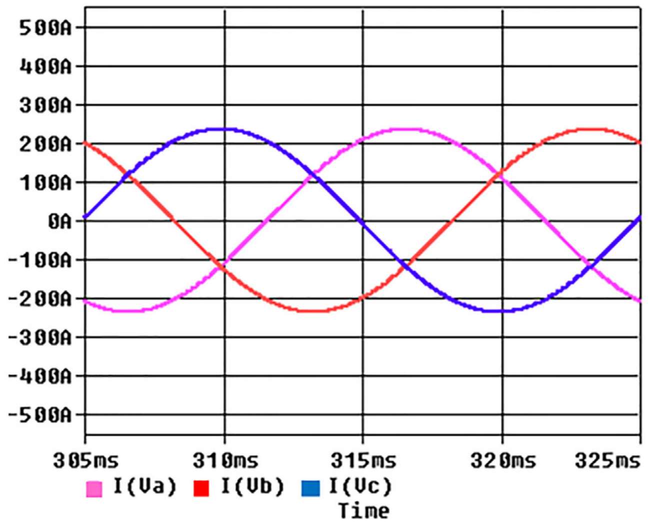

FIGURE 10 Line currents after compensation. Case 1

TABLE 8 positive-sequence and negative-sequence line currents on the bus after compensation

Ph.	I_{zL+} (A)		I_{zL-} (A)	
z	Modulus	Angle	Modulus	Angle
a	166.31	−28.1	0.000	−13.5
b	166.31	−148	0.000	107
c	166.31	92	0.000	−133

In this case, the line-to-line voltages and line currents in the RMS values are shown in Table 9.

The decomposition of the line currents using the direct Fortescue transform provides the positive and negative sequence values shown in Table 10.

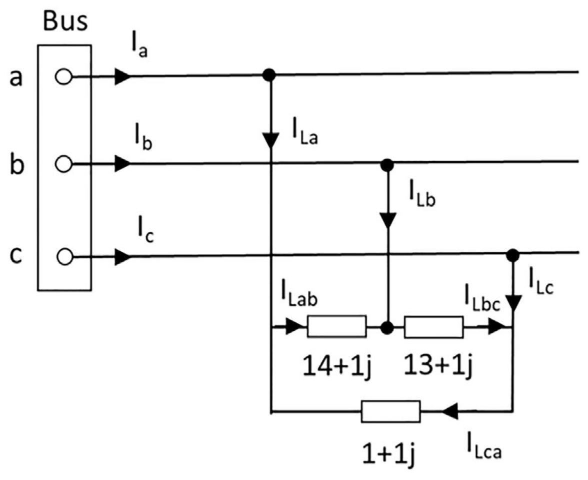

FIGURE 11 Circuit Case 2

TABLE 9 Line-to-line voltages and line currents on the bus. Case 2

Ph.	V_{zz} (V)		Ph.	I_z (A)	
zz	Modulus	Angle	z	Modulus	Angle
ab	376.0	0.000	a	288.7	−99
bc	392.3	−115	b	48.07	−150
ca	413.3	121	c	321.4	74.2

TABLE 10 positive-sequence and negative-sequence line currents on the bus

Ph.	I_{zL+} (A)		I_{zL-} (A)	
z	Modulus	Angle	Modulus	Angle
a	194.1	−67.8	159.1	−139
b	194.1	172.2	159.1	−18.6
c	194.1	52.21	159.1	101.4

As in the previous case, the first step is to determine how inductive the circuit is based on the criteria outlined in section 3. For this, the negative sequence line current consumed by the load will be considered, but 180° out of phase. According to the vector diagram in Figure 5, this current can be achieved by including a capacitor between phases a and b and another capacitor between phases c and a. Solving the system of Equations (9), the currents that will circulate through these capacitors are the following:

$$\boldsymbol{I_{Cab}} = 206.8\ e^{j90}\ A\ \ \boldsymbol{I_{Cca}} = 310\ e^{j210.6}\ A$$

The reactive powers consumed by these capacitors are given by:

$$Q_{Cab} = V_{ab}\ \ I_{Cab} = 77743.4\ VAr$$

$$Q_{Cca} = V_{ca}\ \ I_{Cca} = 128096.9\ VAr$$

TABLE 11 Impedances

Method	Z_{ab} (Ω)	Z_{bc} (Ω)	Z_{ca} (Ω)
A	−j3,700	j3,863	−j1,927
B	−j197,00	−j170,00	−j2,000
C	−j5,501	−j5,501	−j5,501
D	−j6,397		−j2,631

TABLE 12 Powers, power factor, and efficiency factor

	P (kW)	*Q* (kVAr)	*SuT* (kVA)	*ST* (kVA)	*PF*	*EF*
Load	107.218	87.025	101.405	171.325	0.776	0.626
A	107.218	0	5.903	107.380	1.000	0.998
B	107.218	0	65.893	125.847	1.000	0.852
C	107.218	2.298	101.405	147.594	1.000	0.726
D	107.218	0	54.115	120.101	1.000	0.893

If $Q_{Cab} + Q_{Cca}$ is compared with the total reactive power consumed by the load Q_{LTotal}, it is observed that $(Q_{Cab} + Q_{Cca}) > Q_{LTotal}$, therefore, this circuit is considered slightly inductive according to (12).

Following the method proposed in Section 4, the values of the two capacitors of the compensator that compensate for the total reactive power of the load would be determined.

Using the system of Equations (13) and applying Equation (22), the reactive powers of the capacitors that compensate for the total reactive power are the following:

$$Q_{Cab} = 22098.5\ VAr\ \ Q_{Cca} = 64926.4\ VAr$$

Therefore, the capacitive reactance values of the capacitors are given by:

$$X_{Cab} = 6.397\ \Omega\ \ X_{Cca} = 2.631\ \Omega$$

Table 11 shows the reactance values of the proposed method "Method D" and the rest of the methods to be compared (Methods A, B and C).

The following was determined by analysing Tables 11 and 12:

- An ideal solution would be obtained using method A; however, a coil ($\boldsymbol{Z_{bc}}$) is required. Therefore, the best solution for using only capacitors is provided by proposal D, where two capacitors are used, one between phases a and b, and the other between phases a and c.
- Method B compensates for the reactive power of the load; however, it compensates for a lower negative-sequence current of the load than that of method D, which results in a lower *EF*.
- Finally, a lower *EF* than that presented by proposal D is obtained using proposal C.

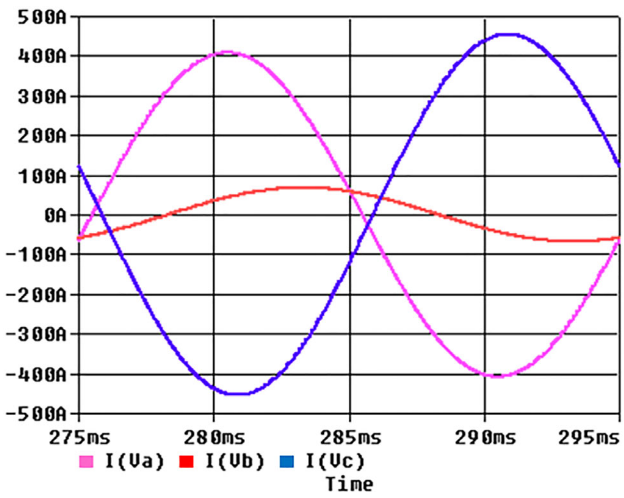

FIGURE 12 Line currents before compensation. Case 2

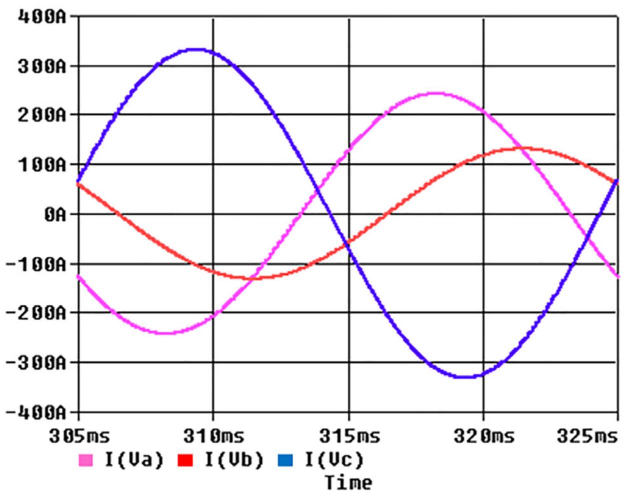

FIGURE 13 Line currents after compensation. Case 2

TABLE 13 Line currents after compensation

Ph.	I_z (A)	
z	Modulus	Angle
a	171.3	−58.6
b	92.86	−116.5
c	234.2	101.8

Using the most suitable method (D), the line currents have the temporal form shown in Figures 12 and 13 before and after connecting the capacitors, respectively.

Figures 12 and 13 demonstrate that the line currents are more balanced after placing the two capacitors. The RMS values are listed in Table 13.

The symmetric components of the line currents in Table 13 are listed in Table 14. Table 14 indicates that the negative component is not completely reduced (see Table 10); although it is reduced more than that using any other method (without using coils).

TABLE 14 positive-sequence and negative-sequence line currents on the bus after compensation

Ph.	I_{zL+} (A)		I_{zL-} (A)	
z	Modulus	Angle	Modulus	Angle
a	152.5	−27.9	87.7	−121
b	152.5	92.1	87.7	118.7
c	152.5	147,9	87.7	−1.27

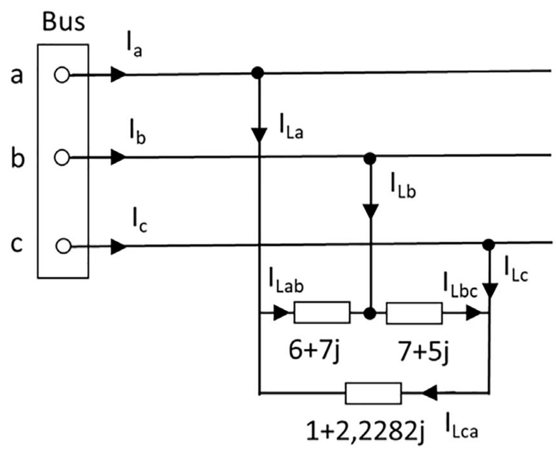

FIGURE 14 Circuit Case 3

TABLE 15 Line-to-line voltages and line currents on the bus. Case 3

Ph.	V_{zz} (V)		Ph.	I_z (A)	
zz	Modulus	Angle	z	Modulus	Angle
ab	376	0	a	183.5	−113
bc	392,3	−115	b	66.77	172.7
ca	413,3	120.6	c	211.3	49.5

5.3 | CASE 3: Circuit that is between being highly and slightly inductive

To analyse this case, the three-phase three-wire circuit is shown in Figure 14.

In this case, the line-to-line voltages and line currents in the RMS values are shown in Table 15.

The decomposition of the line currents using the direct Fortescue transform provides the positive and negative-sequence values shown in Table 16.

As in the previous cases, the first step is to determine how inductive the circuit is based on the criteria outlined in section 3. For this, the negative sequence line current consumed by the load will be considered, but 180° out of phase. According to the vector diagram in Figure 5, this current can be achieved by

TABLE 16 Positive-sequence and negative-sequence line currents on the bus

Ph.	I_{zL+} (A)		I_{zL-} (A)	
z	Modulus	Angle	Modulus	Angle
a	143.6	−86.6	83.56	−162
b	143.6	153.4	83.56	−42
c	143.6	33.4	83.56	78

TABLE 17 Impedances

METHOD	Z_{ab} (Ω)	Z_{bc} (Ω)	Z_{ca} (Ω)
A	−j7.564		−j2.543
B	−j12,143	−j14.8	−j2.677
C	−j5,566	−j5.566	−j5,566
D	−j7,564		−j2.543

including a capacitor between phases a and b and another capacitor between phases c and a. Solving the system of Equations (9), the currents that will circulate through these capacitors are the following:

$$\mathbf{I_{Cab}} = 49.71\, e^{j90}\, A \quad \mathbf{I_{Cca}} = 162.5\, e^{j210.6}\, A$$

The reactive powers consumed by these capacitors are given by:

$$Q_{Cab} = V_{ab} \quad \mathbf{I_{Cab}} = 18690.7\ VAr$$

$$Q_{Cca} = V_{ca} \quad \mathbf{I_{Cca}} = 67154.8\ VAr$$

If $Q_{Cab} + Q_{Cca}$ is compared with the total reactive power consumed by the load Q_{LTotal}, it is observed that $(Q_{Cab} + Q_{Cca}) \cong Q_{LTotal}$, therefore, this circuit is on the border between the definition of a highly inductive and slightly inductive circuit.

Following the method proposed in Section 4, the values of the two capacitors of the compensator that compensate for the total reactive power of the load would be determined.

Using the system of Equations (13) and applying Equation (22), the reactive powers of the capacitors that compensate for the total reactive power are the following:

$$Q_{Cab} = 18690.9\ VAr \quad Q_{Cca} = 67155.1\ VAr$$

Therefore, the capacitive reactances values of the capacitors are given by:

$$X_{Cab} = 7.564\ \Omega \quad X_{Cca} = 2.543\ \Omega$$

Table 17 shows the reactance values of the proposed method "Method D" and the rest of the methods to be compared (Methods A, B and C)

TABLE 18 Powers, power factor, and efficiency factor

	P (kW)	*Q* (kVAr)	*SuT* (kVA)	*ST* (kVA)	*PF*	*EF*
Load	53.173	85.846	51.659	113.426	0.527	0.469
A	53.173	0	2.927	53.254	1	0.998
B	53.173	0	13.525	54.866	1	0.969
C	53.173	2.104	51.659	74.165	0.999	0.717
D	53.173	0	2.297	53.254	1	0.998

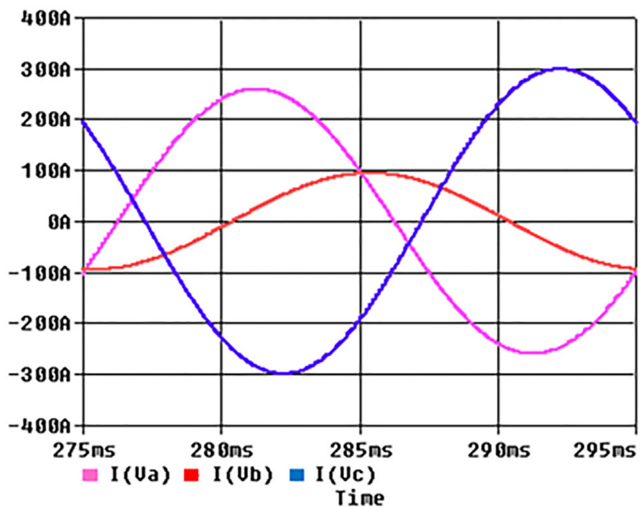

FIGURE 15 Line currents before compensation. Case 3

TABLE 19 Positive-sequence and negative-sequence line currents on the bus after compensation

Ph.	I_{zL+} (A)		I_{zL-} (A)	
z	Modulus	Angle	Modulus	Angle
a	78.004	−28.1	0	164.9
b	78.004	−148	0	−75.1
c	78.004	91.9	0	44.9

The following is determined by analysing Tables 17 and 18. Here, proposal A presents an impedance $\mathbf{Z_{bc}}$, which can be considered as an infinite value (open circuit). Therefore, the solutions proposed by methods A and D coincide. In this case, the two methods offer a compensator that is formed by the two capacitors, which make it is possible to compensate for the reactive power and negative-sequence current consumed by the load.

Using any of the most suitable methods, the line currents have the time form shown in Figures 15 and 16, before and after connecting the capacitors, respectively.

A balanced currents system is obtained, similar to the highly inductive circuit. This indicates that the negative-sequence current is zero. This can be better appreciated in Table 19, where the RMS values are presented.

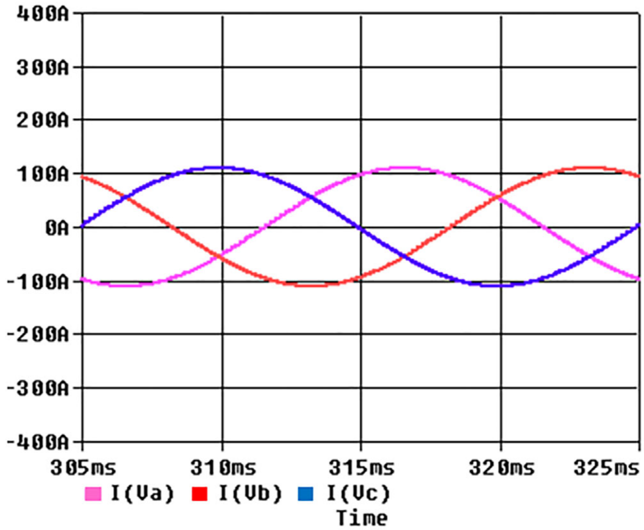

FIGURE 16 Line currents after compensation. Case 3

6 | CONCLUSION

This study presents a new procedure for the design of passive compensators for inefficiencies owing to the negative-sequence current and positive-sequence reactive current consumed by the load using single-phase capacitor banks. This procedure is valid for three-phase three-wire systems with unbalanced voltages and loads, regardless of the characteristics of the load, and it is only necessary to determine the voltages between the phases and line currents. These parameters are easily measurable on a bus.

Two new procedures have been introduced to compensate for the indicated inefficiencies. The first, developed in Section 3, focuses on compensating for the negative current consumed by the load. It is used to classify circuits as highly inductive and slightly inductive. The second procedure, developed in Section 4, focuses on the compensation of reactive power and part of the negative-sequence current, in a manner that the maximum compensation for the said inefficiencies is obtained. This allows the line current flowing through the network to be minimal; therefore, the losses in the network will be minimal as well. To achieve this, a distinction is made between the circuits with highly inductive and low inductive loads. Depending on the behaviour of the load, a compensation strategy that guarantees the most efficient solution among the established procedures has been defined, and it uses only single-phase capacitor banks.

Three types of circuits were analysed to verify the proposed method. The most common methods were used along with this proposed and then compared. For highly inductive loads, the most efficient procedure using passive elements corresponds to that presented by the authors in a previous study [14]. For slightly inductive loads, the new procedure proposed, which is referred to as method D in the previous section, ensures the minimum negative-sequence current and the total compensation of reactive power, achieving the maximum energy efficiency of the system without using coils.

This verifies the validity of the established procedure to obtain a passive compensator based on banks of single-phase capacitors in three-phase three-wire and slightly inductive circuits.

Finally, it should be noted that the MLL and SBC compensation strategies present inferior results because they are only aimed at compensating the inductive reactive power of the load. Using them can cause the negative sequence current consumed by the load to decrease or increase, and consequently impact the efficiency of the network.

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CONFLICT OF INTEREST STATEMENT

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

DATA AVAILABILITY STATEMENT

Data sharing not applicable

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