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Metodología para el cálculo de los cimientos de una máquina rotativa que soporta cargas dinámicas incluido el arranque transitorio
A methodology for the calculation of the foundation of a rotary machine supporting dynamic loads including the transient starting

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***Resumen*— A menudo, los ingenieros resuelven problemas en relación con estructuras y cimientos desde el punto de vista de la estática estructural. Nada tan lejos de la realidad cuando finalmente, en la estructura o los cimientos, se instala una máquina. Las cargas producidas por las máquinas cambian con el tiempo y no serán constantes. Las partes que formaron una máquina generalmente se mueven y transmiten a la estructura cargas dinámicas que cambian con el tiempo. Pensar en cargas dinámicas significa considerar la variable "tiempo" para calcular una base o una estructura. Una parte de la energía desperdiciada por la máquina se transforma en radiación de la vibración de la máquina y se transmite al suelo (Richart et al., 1970). Durante el transitorio para obtener la velocidad nominal de la máquina, el sistema puede cruzar su "frecuencia natural" y colapsar por un exceso de amplitud de vibración (Richart et al., 1970; Arya et al., 1979; Chowdhury & Dasgupta, 2009). Las ecuaciones diferenciales de D'Alambert basadas en la analogía de Lysmer & Richart (1966) se aplicaron en el dominio del tiempo para estudiar el movimiento vertical, deslizamiento y balanceo (Barkan, 1962) de la base del conjunto - máquina de bloque inercial. Las ecuaciones diferenciales se integraron con un esquema de pasos de tiempo (Chowdhury & Dasgupta, 2009), el método β de Newmark (1959), obteniendo la amplitud de vibración, velocidad, aceleración y fuerza en el modo de operación transitoria y permanente. La metodología se aplicó a una máquina rotativa que funciona a 3.000 r.pm. con un bloque inercial y una base de bloque, un problema de 3 masas con 37 variables. El suelo, sus parámetros e impedancia se calculan aplicando la Norma ACI 351.3R-04 (2004). Las cargas dinámicas se calcularon de acuerdo con la norma ACI 351.3R-04, las normas API estándar 613 (Arya et al., 1979) y la norma ISO 1940/1 (2003). Se desarrolló un programa MATLAB para resolver las ecuaciones diferenciales D'Alambert y obtener la amplitud de vibración, velocidad, aceleración y fuerza cambiando la velocidad de la máquina durante los primeros 3.000 segundos desde 0 a 3.000 segundos con diferentes funciones de arranque (Rodriguez et al., 2010). El programa generó soluciones aleatorias de las 37 variables. El programa permitió corregir restricciones a la solución calculada. Se aplicó un conjunto de reglas al modo de operación transitorio y permanente de la máquina (Rodriguez et al., 2010). Los límites, extraídos de la Norma ISO, de la amplitud de vibración, velocidad,**

aceleración y fuerza en el modo de operación transitoria y permanente se aplicaron para obtener la solución correcta. Finalmente, esta metodología permite aplicar metaheurísticas para optimizar el costo de la fundación.

Palabras Clave— Cargas dinámicas; Cimientos; Transitorio; Vibraciones.

Abstract*— Often engineers solve problems in relationship with structures and foundations from the point of view of structural statics. Nothing so far of the reality when finally, on the structure or the foundation, is installed a machine. Loads produced by machines change with time and will not be constant. The parts that made a machine are usually moving and they transmit to the structure dynamics loads which change with time. Thinking in dynamics loads means consider the variable "time" to calculate a foundation or a structure. A part of the energy wasted by the machine is transformed in radiation from the vibration of the machine and transmitted to the soil (Richart et al., 1970). During the transient to get the nominal speed of the machine, the system can cross its "natural frequency" and collapse by an excess of amplitude of vibration (Richart et al., 1970; Arya et al., 1979; Chowdhury & Dasgupta, 2009). D'Alambert differential equations based in the Lysmer's analogy (Lysmer & Richart, 1966) were applied in the time domain to study the vertical movement, sliding and rocking (Barkan, 1962) of the ensemble foundation – inertial block – machine. Equations differentials were integrated with a time-step scheme (Chowdhury & Dasgupta, 2009), the Newmark's β method (Newmark, 1959), getting the amplitude of vibration, speed, acceleration and strength in the transient and in the permanent operation mode. Methodology was applied to a rotary machine working at 3.000 r.pm. with an inertial block and a block foundation, a 3-mass problem with 37 variables. The ground, its parameters and impedance are calculated applying the Norma ACI 351.3R-04 (2004). Dynamic loads were calculated in accordingto ACI Norm 351.3R-04, API Norms Standard 613 (Arya et al., 1979) and ISO Norm 1940/1 (2003). A MATLAB program was developed to solve the D'Alambert differential equations and get the amplitude of vibration, speed, acceleration and strength changing the speed of the machine during the first 3.000 seconds since 0 to 3.000 seconds with different starting functions (Rodriguez et al., 2010). Random solutions of the 37 variables were generated by the program. The program allowed to fix constraints to the solution calculated. A set of rules were applied to the transient and the permanent operation mode of the machine (Rodriguez et al., 2010). Limits, extracted from the ISO Norm, of the amplitude of vibration, speed, acceleration and strength in the transient and in the permanent operation mode were applied to get the right solution. Finally, this methodology permits to applied metaheuristics to optimize the cost of the foundation.

Index Terms— Dynamic loads; Foundations, Transient, Vibrations.

I. INTRODUCTION

The study of the dynamics loads that generate a machine working is a part of the civil engineering that has been in developing since the XIX century. Join to the born of the machines appears on the foundations and structures that support the machines vibrations due to the imperfections in the making them.

For beginning the studies of the vibration and its influence in the foundations and structures was developed a model of a "simple degree of freedom" with vertical movement combined with the "half-space theory" to make a model of the soil. Reality is more complex. Engineers developed different types of structures and foundations to guarantee that the vibrations create by the machines were dumped for protecting the machines, the people and the environment of them.

During the operation of a machine, in the starting, the machine could cross the resonant frequency, where the amplitude of the displacement is increased out of reasonably limits and damage the machine (Fig. 1).

Models of foundations were changing mainly with the experience and "rule of thumb method" (Arya et al., 1979; ACI Committee 351, 2004) to calculate foundations were applied to build the foundations. Recommendations of the "rule of thumb method" to limit the amplitude of vibration of a machine are focused to increase the volume of the foundation

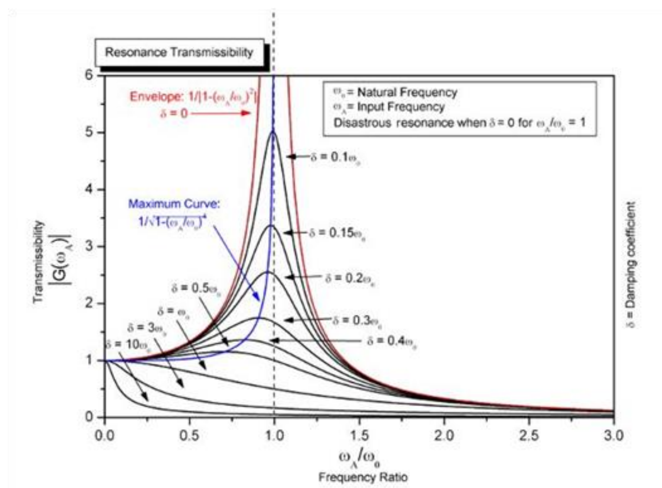

Fig. 1. Resonance **Transmissibility**.

to increase its weight and of course to increase its costs. Optimization of the foundations supporting dynamics loads produced by machines means not to cross or crossing the resonance frequency in the transient operation mode safely and at the lowest cost of the foundation.

A. Optimization of foundations for machines. State of the art

Optimization of the cost of the foundation is proposed in four papers by three authors.

Paper one. Sienkiewicz & Wilczynski, (1993), in their paper 'Minimum-weight design machine foundations under vertical load', proposed the optimization of a single degree of freedom system (S.D.O.F. in advanced) based in the weight of the block foundation. Impedances of the soil are calculated using the Novak model.

The displacement was calculated by the Richart-Whitman's model of condensed parameters. Poisson soil parameter is considered using the values of Richart-Whitman (Richart et al., 1970).

Cost function depends on the tree parameters of the block foundations, length, wide and height. Restrictions considerer were the resonance frequency, vertical displacement, value limits of length and wide of the block and stresses in the soil.

For solving authors consider the problem as a sequence linear programming (SLP). An initial solution is calculated, and small changes are made by Taylor approximation using the Gauss simplex algorithm around the solution. The convergence of the algorithm is not guaranteed if the variables are not limited.Two examples were calculated.

Paper two. Sienkiewicz & Wilczynski (1976), in their paper 'Shape optimization of a dynamically loaded machine foundation coupled to a semi-infinite inelastic medium', proposed the optimization based in the volume of the block foundation.

Machine was under vertical loads and mounted on the block. Soil parameters were with the half-space theory. Vertical, sliding and rocking displacement were considered, in fact tree degrees of freedom. Authors solve the problem in the frequency domain using complex variable.

Optimization function is the volume of the foundation with two variables, length and wide. Height is considered fixed.

Restrictions considered were the vertical displacement, horizontal displacement, value limits of length and wide of the block and stresses in the soil

For solving authors consider the problem as a sequence linear programming (SLP) using Taylor's series around the solution. To guarantee stability and convergence of the algorithm the limits of the variables are fixed. An example of a reciprocating machine was calculated.

Paper three. Silva et al. (2002) in their paper "Optimization of elevated concrete foundations for vibrating machines" proposed a concrete elevated structured used for steam turbines for its optimization.

The dimensions of the structure and the reinforcement steel are the design variables to be optimized. A cost function is defined depending on concrete, reinforcement steel and the shape of the structure.

Restrictions considered are the limits of the material and the stresses on the soil. To calculate the structure a finite elements analysis was employed. Movement equations are developed with a lumped parameters method.

Limits of displacement, speed and acceleration are considered from the

Optimization of the cost function is solved by five numerical methods in the time domain. A numerical example is included. Conclusions were that computation time is too long, about forty days and can be reduced to four days using approximations.

Paper four. Anyagebunam (2011), in the paper Minimum foundation mass for vibration control, optimised the foundation on an elastic half space using lumped parameters designing a minimum weight foundation.

With the Richart-Whitmar's parameters and the Lysmer's analogy equations and operating the equations in function of a parameter "D" of damping. Mass is a function of the parameter D and was derivate to get the minimum mass.

Author considered that the minimum got it was too small for the size of the machine. Size of the foundations is not compatible with the size of the machine. An example is included.

The authors consider only the permanent mode of working the machine, but what happens during the transient mode? Introduction of the variable time in the calculation of the foundation of a machine means analyse transient mode too.

B. Methodology proposed in this paper

Depending on the characteristics and the applications of the machines a type of foundations must be designed to guarantee the correct operation of the machine.

The model proposed for a rotatory machine includes an inertial block (Fig. 2), and a block foundation are designed to reduce the vibration of the ensemble (Arya et al., 1979; Prakash, & Vijay, 1988; Srinivasulu & Vaidyanathan, 1976). So, three masses connected between are consider studying their movement due to the dynamic loads produced by the working of the machine.

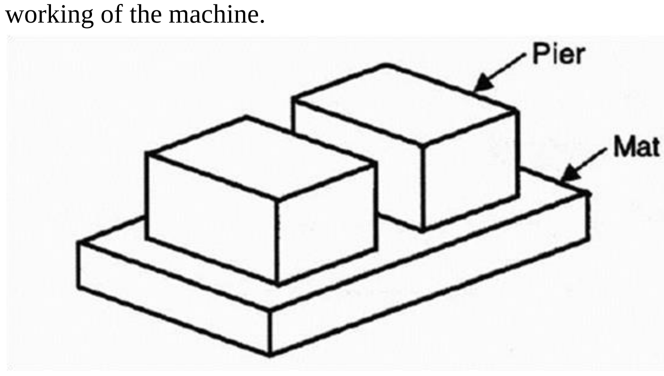

Fig. 2. Block foundations for rotative machine.

Dynamics loads applied depends on the weight of the machine and the weight of the shaft according to the ACI NORM-351.3R-04. It proposed:

$$F_e = \frac{m_e e \omega^2{}_m S_f}{1000} \; Newton \qquad (1)$$

In which:

F_e Dynamic load in N.
m_e Mass of the shaft in k
Sf Service factor. Usually 2.
ω_m Speed of the machine in rad/s
e Eccentricity of the shaft in mm.

The soil was modelled fixing its characteristics and its impedances.

Speed of the machine during the transient mode changed from zero to nominal speed. The sloping of the ramp-up was a function where variables were selected by the algorithm to achieve the displacement expected by the constrains during the transient mode.

A rump-up was described as

$$\omega(t) = \omega_{\max}[1-(1-(\frac{t}{t_m})^p]^{\frac{1}{q}} \qquad (2)$$

The D'Alambert equations differentials of the movement of the tree solids are solved using a step by step integration method (Fig. 3). The method selected was the Newmark's β method (Chowdhury & Dasgupta, 2009; Newmark, 1959). The matrixial equations are applied to the vertical, the rocking and the sliding displacement. 9 variables for the movement of the ensemble (Fig. 4). Rocking and horizontal displacement are considered coupled. All the impedances and equations were considered in the elastic zone.

Model included additional materials: springs, isolation pads and shock absorbers, to control the vibrations of the machine, inertial block and foundation that increases the number of the variables of the model and to be used as impedances.

Displacement, speed and acceleration were the objectives to calculate and to be limited according to the limit operation fixed by the Norms as constraints.

A function "COST" of the foundation was introduced to analyse the possible combination of the variables proposed and analyse the set of solutions. A random algorithm implemented in MATLAB generates solutions with the RANDOM WALK technique. Constraints were checked for each candidate solution to validate the feasible solutions.

II. Experimental model

A. Foundation Model

The foundation model selected was for an industrial turbine that makes electricity. Turbines are rotative machines very sensible to vibrations (Rodriguez et al., 2010) and produce vibrations during the starting and during the permanent mode of working that transmitted to the foundations.

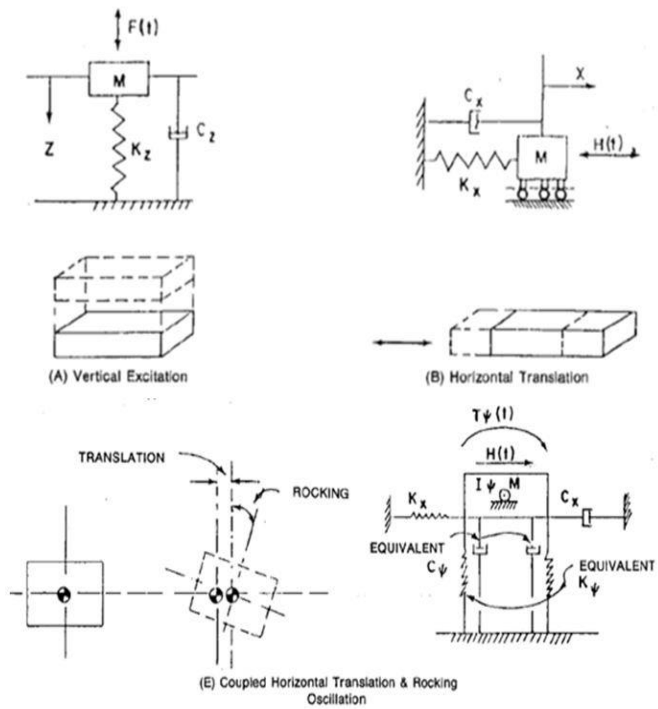

Fig. 3. Models of oscillation.

The model analysed the displacement and dynamics loads supported by the foundations:

- Vertical vibrations in the vertical axis -Z
- Horizontal vibrations in the axis -X (sliding).
- Rocking vibrations in a vertical axis crossing the vertical axe in the centre of gravity of the ensemble and in the base of each mass.

Vertical displacement was uncoupled of sliding and rocking. Both two, sliding and rocking, were considered coupled (Prakash & Vijay, 1976).

Nominal speed of the machine was 3.000 r.p.m.

D'Alambert equations in the time domain were considered for each free solid of the model: the machine, the inertial block and the foundation block.

B. Vertical displacement in the –Z axis

The free solid diagram was as can be seen in figure 4.

Matrix equations for the free solid:

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \end{bmatrix} + \begin{pmatrix} c_{z1}+c_{z2} & -c_{z2} & 0 \\ -k_{z2} & k_{z2}+k_{z3} & -k_{z3} \\ 0 & -k_{z3} & k_{z3} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F_z(t) \end{pmatrix} \qquad (3)$$

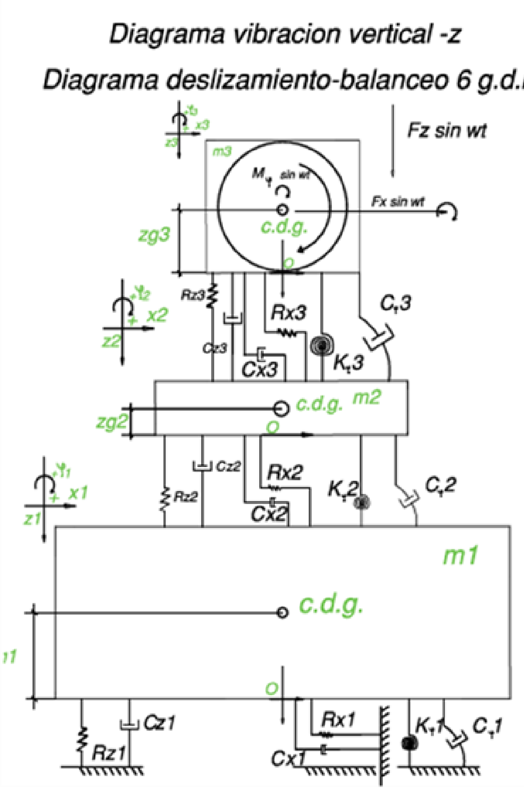

Fig. 4. 9 degrees freedom diagram.

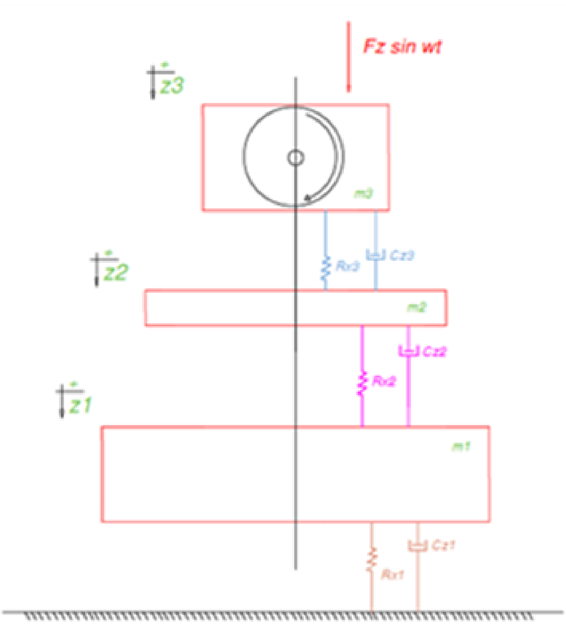

Fig. 5. Free solid diagram for –Z displacement.

$$[M][\ddot{Z}] + [C][\dot{Z}] + [K][Z] = [F] \qquad (4)$$

In which:

- $[M]$was the masses matrix
- $[\ddot{Z}], [\dot{Z}], [Z]$ were the acceleration, speed and displacement matrix
- $[C], [K]$ were the dynamic impedance and the stiffness matrix
- $[F]$ was the load matrix

Sliding and rocking

The free solid diagram was as can be seen in figure 5.

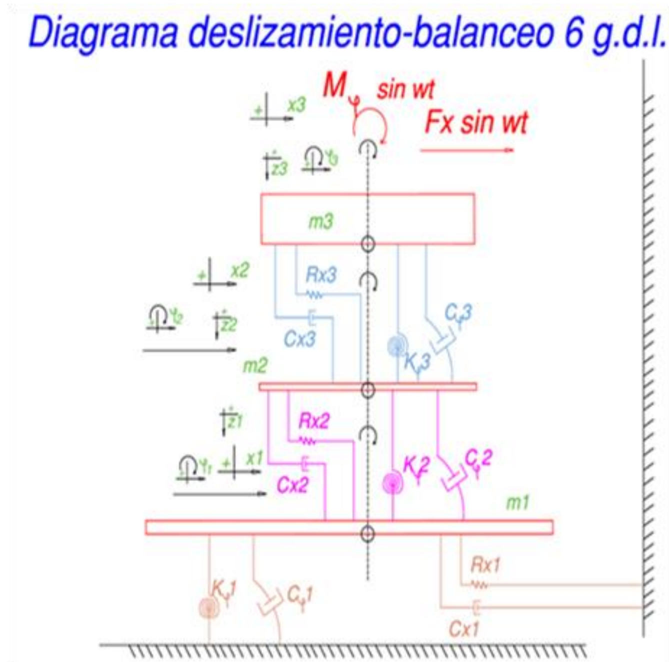

Fig. 6. Free solid diagram for sliding and rocking.

The six equations were written in matrix mode as:

$$\begin{bmatrix} & M & \end{bmatrix}\begin{bmatrix}\ddot{z}_1\\ \ddot{z}_2\\ \ddot{z}_3\end{bmatrix} + \begin{pmatrix} & C & \end{pmatrix}\begin{pmatrix}\dot{v}\end{pmatrix} + \begin{bmatrix} & K & \end{bmatrix}\begin{bmatrix}x\end{bmatrix} = \begin{pmatrix}F\end{pmatrix} \qquad (5)$$

- Mass matrix [M] and accelerations:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0\\ 0 & m_2 & 0 & 0 & 0 & 0\\ 0 & 0 & m_3 & 0 & 0 & 0\\ 0 & 0 & 0 & j_1 & 0 & 0\\ 0 & 0 & 0 & 0 & j_2 & 0\\ 0 & 0 & 0 & 0 & 0 & j_3 \end{bmatrix}\begin{bmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\\ \ddot{\phi}_1\\ \ddot{\phi}_2\\ \ddot{\phi}_3\end{bmatrix} \qquad (6)$$

- Dumping matrix [C] and speeds:

$$\begin{bmatrix} (c_{x1}+c_{x2}) & -c_{x2} & 0 & -(c_{x1}z_{g1}+c_{x2}z_{g2}) & c_{x2}z_{g2} & 0 \\ -c_{x2} & (c_{x2}+c_{x3}) & -c_{x3} & c_{x2}z_{g2} & -(c_{x2}z_{g2}+c_{x3}z_{g3}) & c_{x3}z_{g3} \\ 0 & -c_{x3} & c_{x3} & 0 & c_{x3}z_{g3}[\ ] & -c_{x3}z_{g3} \\ -(c_{x1}z_{g1}+c_{x2}z_{g2}) & c_{x2}z_{g2} & 0 & (c_{\phi 1}+c_{x1}z_{g1}^2-w_1z_{g1}+c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}) & -(c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}) & 0 \\ c_{x2}z_{g2} & -(c_{x2}z_{g2}+c_{x3}z_{g3}) & c_{x3}z_{g3} & -(c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}) & -(c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}+c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) & -(c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) \\ 0 & c_{x3}z_{g3} & -c_{x3}z_{g3} & 0 & -(c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) & (c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix}$$

$$\begin{bmatrix} (c_{x1}+c_{x2}) & -c_{x2} & 0 & -(c_{x1}z_{g1}+c_{x2}z_{g2}) & c_{x2}z_{g2} & 0 \\ -c_{x2} & (c_{x2}+c_{x3}) & -c_{x3} & c_{x2}z_{g2} & -(c_{x2}z_{g2}+c_{x3}z_{g3}) & c_{x3}z_{g3} \\ 0 & -c_{x3} & c_{x3} & 0 & c_{x3}z_{g3}[\ ] & -c_{x3}z_{g3} \\ -(c_{x1}z_{g1}+c_{x2}z_{g2}) & c_{x2}z_{g2} & 0 & (c_{\phi 1}+c_{x1}z_{g1}^2-w_1z_{g1}+c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}) & -(c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}) & 0 \\ c_{x2}z_{g2} & -(c_{x2}z_{g2}+c_{x3}z_{g3}) & c_{x3}z_{g3} & -(c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}) & -(c_{\phi 2}+c_{x2}z_{g2}^2-w_2z_{g2}+c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) & -(c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) \\ 0 & c_{x3}z_{g3} & -c_{x3}z_{g3} & 0 & -(c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) & (c_{\phi 3}+c_{x3}z_{g3}^2-w_3z_{g3}) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \end{bmatrix}$$

- Stiffness matrix [K] and displacement:
- The strength and momentum matrix [F]:

$$\begin{bmatrix} 0 \\ 0 \\ F_x \sin(\omega t) \\ 0 \\ 0 \\ M_x \sin(\omega t) \end{bmatrix} \tag{7}$$

The equations were integrated in the time domain using the Newmark's β methodology (Chowdhury & Dasgupta, 2009; Newmark, 1959) with:

- $\beta = 1/4$
- $\Delta t = 0.001$ s.

C. Operation parameters and variables. Sloping ramp-up.

A rump-up to start the operation of the machine was described as

$$\omega(t) = \omega_{\max}[(1-(1-(\frac{t}{t_m})^p]^{\frac{1}{q}} \tag{8}$$

In which the parameters were two:

Parameter	Description	Value
ω_{max}	Nominal speed in rad/s	100 rad/s

The three operation variables were:

Variable	Description	Values
p	Variable of the equation	1,2,3,4
q	Variable of the equation	1,2,3
tm	Time to get the machine the nominal speed.	max 3.000 s.

D. Parameters of the machine and the foundation model

A total of 7 geometric variables and 14 parameters of the soil and machine were described as:

TABLE I
GEOMETRIC VARIABLES

Parameter	Description	Value/Units
a_2	Length of the inertial block	m.
b_2	Wide of the inertial block	m.
c_2	Height of the inertial block	m.
a_1	Length of the foundation block	m.
b_1	Wide of the inertial block	m.
c_1	Height of the foundation block	m.
e_b	Embedded depth of the foundation block	%

TABLE II
MACHINE AND SOIL PARAMETERS

Parameter	Description	Value/Units
m_3	Machine mass	178.170 kg.
m_e	Mass of the shaft	16.365 kg.
ω	Operation speed	3.000 r.p.m.
a_3	Length of the machine	16 m.
b_3	Wide of the machine	8 m.
c_3	Height of the machine	3 m.
Sf	Service factor	2
e	Eccentricity of the shaft	5.00 10-02 mm.
G	Soil shear modulus	81.549 kN/m^2
ν	Soil Poisson's ratio	0.33
ρgs	Soil mass density	20 kN/m^3
βm	Dumping material ratio	5%
σs	Static bearing of the soil	26 kN/m^2
γh	Weight of the steel concrete	25 kN/m^3

E. Variables of stiffness and dumping impedances

The impedance of stiffness and dumping between the machine and the inertial block, the inertial block and the block foundation and the soil-block foundation were designed with spring, isolation pads and shock absorbers. These elements were commercial and real, mixed and selected the number of necessary units in a discrete way to get the necessary impedance for the objective displacement, speed, acceleration of the machine and strength on the soil. Isolation pads were considered in percentage over the surface. There was a total of 16 variables.

TABLE III
SPRING AND SHOCK ABSORBERS IMPEDANCESVARIABLES

Variable	Description	Units
n_{rz3}	Number of springs between inertial block and foundation block in the -Z axis.	u.
n_{rz2}	Number of springs between the machine and the inertial block in the -Z axis.	u.
n_{dz3}	Number of shock absorbers between the inertial block and the foundation block in the -Z axis.	u.
n_{dz2}	Number of shock absorbers between the machine and the inertial blocks in the -X axis.	u.
n_{rx3}	Number of springs between the inertial block and the foundation block in the -X axis.	u.
n_{rx2}	Number of springs between the inertial block and the foundation block in the -X axis.	u.
n_{rx1}	Number of springs in the block foundation in the -X axis.	u.
n_{dx3}	Number of shock absorbers between the machine and the inertial block in the -X axis.	u.
n_{dx2}	Number of shock absorbers between the inertial block and the block foundation in the -X axis.	u.
n_{dx1}	Number of shock absorbers in the block foundation in the -X axis	u.

TABLE IV
PAD MATTRESS IMPEDANCESVARIABLES

Variable	Description	Units-Interval
pbix1	Percentage of the surface with isolation pad in the walls of the foundation block in the -X axis.	% - 0-100%
pbix2	Percentage of the surface with isolation pad in the inertial block in the -X axis	% - 0-100%
pbix3	Percentage of the surface with isolation pad in the machine support in the -X axis	% - 0-100%
pbiz1	Percentage of the surface with isolation pad under the foundation block in the -Z axis	% - 0-100%
pbiz2	Percentage of the surface with isolation pad in the inertial block in the -Z axis	% - 0-100%
pbiz3	Percentage of the surface with isolation pad in the machine support in the -Z axis	% - 0-100%
pbix1	Percentage of the surface with isolation pad in the walls of the foundation block in the -X axis.	% - 0-100%
pbix2	Percentage of the surface with isolation pad in the inertial block in the -X axis	% - 0-100%
pbix3	Percentage of the surface with isolation pad in the machine support in the -X axis	% - 0-100%

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pbix2	Percentage of the surface with isolation pad in the inertial block in the -X axis	% - 0-100%
pbix3	Percentage of the surface with isolation pad in the machine support in the -X axis	% - 0-100%
pbiz1	Percentage of the surface with isolation pad under the foundation block in the -Z axis	% - 0-100%
pbiz2	Percentage of the surface with isolation pad in the inertial block in the -Z axis	% - 0-100%
pbiz3	Percentage of the surface with isolation pad in the machine support in the -Z axis	% - 0-100%
pbix1	Percentage of the surface with isolation pad in the walls of the foundation block in the -X axis.	% - 0-100%
pbix2	Percentage of the surface with isolation pad in the inertial block in the -X axis	% - 0-100%
pbix3	Percentage of the surface with isolation pad in the machine support in the -X axis	% - 0-100%

F. Material type selection impedance variables

There were 11 variables considered to select the type of elastomeric pad mattress and the size of the shock absorber. For the pad mattress and the shock absorbers were three types of each one and the value of variables could be 1,2,3 or 4.

TABLE V
PAD MATERIAL TYPE SELECTION IMPEDANCES VARIABLE

Variable	Description	Value
b_{iz3}	Type of isolation pad between the machine and the inertial block in the axis -Z	1,2,3,4
b_{iz2}	Type of isolation pad between the inertial block and the block foundation in the axis -Z	1,2,3,4
b_{iz1}	Type of isolation pad between the inertial block foundation and soil in the axis -Z	1,2,3,4
b_{ix3}	Type of isolation pad between the machine and the inertial block in the axis -X	1,2,3,4
b_{ix2}	Type of isolation pad between the inertial block and the block foundation in the axis -X	1,2,3,4
b_{ix1}	Type of isolation pad between the inertial block foundation and soil in the axis -X	1,2,3,4
airpz3	Type of shock absorber between the machine and the inertial block in the axis -Z	1,2,3
airpz2	Type of shock absorber between the inertial block and the block foundation in the axis -Z	1,2,3
airpx3	Type of shock absorber between the machine and the inertial block in the axis -X	1,2,3
airpx2	Type of shock absorber between the inertial block and the block foundation in the axis -X	1,2,3
airpx1	Type of shock absorber between the inertial block foundation and soil in the axis -X	1,2,3

G. Constraints

For each one of the variables constrains had been fixed to make a real foundation. There were three types of constraints:

1) Constraints of the interval of values of the variable.
2) Geometric constraints.
3) Mechanical constraints related with the physical operation and the construction of the foundation

TABLE VI
CONSTRAINTS RELATED TO VARIABLE INTERVAL. NUMBER 1 TO 15

Nº	VAR.	Description	Units	Max value	Min value
1	a_2	Length of the inertial block	m.	19.2	16
2	b_2	Wide of the inertial block	m.	9.6	8
3	c_2	Height of the inertial block	m.	2	1
4	a_1	Length of the block foundation	m.	28.8	16
5	b_1	Wide of the block foundation	m.	14.4	8
6	c_1	Height of the block foundation	m.	3	1
7	e_b	Embedded depth of the foundation block	%	100%	40%
8	p	Parámetro característico de la curva de arranque	-----	4	1
9	q	Parámetro característico de la curva de arranque	-----	3	1
10	nr_{z3}	Number of springs between inertial block and foundation block in the -Z axis.	-----	110	60
11	n_{rz2}	Number of springs between the machine and the inertial block in the -Z axis.	-----	110	60
12	n_{dz3}	Number of shock absorbers between the inertial block and the foundation block in the -Z axis.	-----	90	40
13	n_{dz2}	Number of shock absorbers between the machine and the inertial blocks in the -X axis.	-----	90	40
14	n_{rx3}	Number of springs between the inertial block and the foundation block in the -X axis.	-----	90	40
15	n_{rx2}	Number of springs between the inertial block and the foundation block in the -X axis.	-----	90	40

TABLE VII
CONSTRAINTS RELATED TO VARIABLE INTERVAL. NUMBER 16 TO 20

Nº	VAR.	Description	Units	Max value	Min value
16	nrx1	Number of springs in the block foundation in the -X axis.	-----	90	40
17	ndx3	Number of shock absorbers between the machine and the inertial block in the -X axis.	-----	90	40
18	ndx2	Number of shock absorbers between the inertial block and the block foundation in the -X axis.	-----	90	40
19	ndx1	Number of shock absorbers in the block foundation in the -X axis	-----	90	20
20	tr	Time from starting to nominal operation mode.	s.	3600	1800

TABLE VIII
GEOMETRICAL CONSTRAINTS G1 TO G11

Ref.	Constrain	Description
G1	a2< 1,2 * a3	Length of the inertial block is not bigger than 20% length of the machine
G2	b2< 1,2 * b3	Wide of the inertial block is not bigger than 20% of the wide of the machine
G3	a1< 1,5 * a2	Length of the foundation block is not bigger than 50% of the length of the inertial block
G4	b1< 1,5 * b2	Wide of foundation block is not bigger than the 50% of the wide of the inertial block
G5	a1/b1<= 2	Ratio between length and wide of the block foundation must be less than 2, to apply the calculation of the impedance for a circular foundation equal with radius R [1]
G6	c1<=b1/2	Height of block foundation is equal or less than half of its wide
G7	c2<=b2/2	Height of the inertial block is equal or less than half of its wide
G8	c2<=c1	Height of the inertial block is equal is less than the height of the block foundation
G9	a3<=a2	Length of the machine is less or equal then length inertial block
G10	a2<=a1	Length of the inertial block is equal or less than the length of the block foundation
G11	b3<=b2	Wide of the block foundation is equal or bigger than the wide of the inertial block

TABLE IX
GEOMETRICAL CONSTRAINTS G12 TO G16

Ref.	Constrain	Description
G12	b2<=b1	Wide of the block foundation is equal or bigger then wide of the inertial block
G13	c2<=b2/2	Height of the inertial block equal or less than the half of the wide of the inertial block
G14	supx3<=supx res3	Surface occupied by the spring and shock absorbers of the machine is less or equal to the Surface free in the -X axis
G15	supx2<=supx res2	Surface occupied by the spring and shock absorbers of the inertial block is less or equal to the Surface free in the -X axis
G16	supx1<=supx res1	Surface occupied by the spring and shock absorbers of the foundation block is less or equal to the Surface free in the -X axis

TABLE X
MECHANICAL CONSTRAINS M1 TO M9

Ref.	Constrain	Description
M1	wg3+fz3<nrz3*wrz3 + pbiz3*biz3	Statics loads supported by springs and isolation pad between the machine and the inertial block calculated were less than addition of maximum load of each spring and isolation pad in the -Z axis.
M2	wg3+fz3+wg2<nrz2 *wrz2 + pbiz2*biz2	Statics loads supported by springs and isolation pad between the inertial block and the block foundation calculated were less than addition of maximum load of each spring and isolation pad in the -Z axis.
M3	wg3+fz3+wg2+wg1 <pbiz1*biz1	Statics loads supported by isolation pad between the block foundation and the soil calculated were less than addition of maximum load of the isolation pad in the -Z axis
M4	Fx<nrx3 *wrx3+ pbix3*bix3	Static sliding load supported by the machine is less than the addition of each spring and isolation pad
M5	Fx<nrx2 *wrx2 + pbix2*bix2	Static sliding load supported by the inertial block is less than the addition of the maximum load of each spring and isolation pad
M6	Fx<nrx1 *wrx1 + pbix1*bix1	Static sliding load supported by the soil is less than the maximum load supported by isolation pad
M7	$v < 0.88 \sqrt[4]{\frac{4\,E\,I}{k_c b_1}}$	Foundation is rigid and the value of the flight of the footing is less than the equation proposed [17]
M8	σsmed<σs	Media stress produced by the dynamic and static loads is less than the maximum stress of the soil
M9	σspunta< 1,25 * σs	Maximum stress by the dynamic and static loads is less than 1,25 times the maximum stress of the soil

TABLE XI
DATA TABLE 11 CONSTRAINTS OF OPERATION THE MACHINE.
LIMITS OF DISPLACEMENT, SPEED AND ACCELERATION IN -X AND -Z AXIS

Variable	Norma Applied	Nominal operation		Transitory operation
x_3	VDI Norm 2056 * cos 45º	105 10-3 mm	x 100	10.5 mm.
$\dot{x}_3$	ISO Norm 10816-1995	4.5 mm/s	x 100	0.45 m/s
$\ddot{x}_3$	Blake chart,1964 [7]	0.1 g	x 100	10 g
z_3	VDI Norm 2056 * sin 45º	105 10-3 mm	x 100	10.5 mm.
$\dot{z}_3$	ISO Norm 10816-1995	4.5 mm/s	x 100	0.45 m/s
$\ddot{z}_3$	Blake Chart, 1964[7]	0.1 g	x 100	10 g

For the transitory operation mode there is not any norm or reference that propose any limit to the displacement, speed or acceleration. Only the authors Rodriguez et al. (2010) in their paper commented that sensors of the turbine were disconnected in the transitory operation. Displacement of 10 mm. are eventually accepted by the manufacturers and operators of the turbines in the transitory mode operation. The constrains for the transitory operation mode were proposed by the authors of this paper to limit the possible cross of the resonance frequency during the starting to the permanent operation

H. Cost Function

To analyse the set of solutions of the foundation a cost function was defined. Cost function is a no continuous function, a discrete function, and so, analytic methodology was not possible to be used to generate solutions or optimization.

Let S(X) a set of solutions,
Where:
X: Was a vector 1x 37that € S(x)

X: Is a solution

Cost(X) was defined as:

$$Cost(X) = \sum_{i=1}^{20} c_i(x_i) \tag{9}$$

Cost function was a sum of 20 addend depending on 37 variables X_i:

C1	Cost of the excavation	(a1*b1*eb+0.2) * OC(1)
C2	Concrete base	(a1*b1)* OC(2)
C3	Framework of the inertial block and foundation	((2*a1*c1+2*b1*c1) + (2*a2*c2+2*b2*c2)) * OC(3)
C4	Steel concrete cost	((a1*b1*c1)+(a2*b2 *c2)) * OC(4)
C5	Cost of the insolation pad between soil and foundation block in the -Z axis	(a1*b1)*pbiz1*biz1
C6	Cost of the insolation pad between the foundation block and the inertial block in the -Z axis	(a2*b2)*pbiz2*biz2
C7	Cost of the insolation pad between the inertial block and the machine in the -Z axis	(a3*b3)*pbiz3*biz3
C8	Cost of the insolation pad between soil and foundation block in the -X axis	(b1*c1*eb*2)*pbix1 *bix1
C9	Cost of the isolation pad on the inertial block in the -X axis.	(b2*c2)*pbix2*bix2
C10	Cost of the isolation pad on the machine in the -X axis.	(b3*hr)*pbix3*bix3
C11	Cost of the spring installed between the foundation and the inertial block in the -Z axis.	nrz2*ctr
C12	Cost of the springs installed between the inertial block and the machine in the -Z axis	nrz3*ctr
C13	Cost of the springs installed on the foundation block in the -X axis.	nrx1 * ctr
C14	Cost of the spring installed on the inertial block in the -X axis	nrx2 * ctr
C15	Cost of the spring installed on the machine in the -X axis.	nrx3 * ctr
C16	Cost of the shock absorbers between the foundation and the inertial block in the -Z axis	ndz2 * ctd
C17	Cost of the shock absorbers installed between the inertial block and the machine in the -Z axis	ndz3 * ctd
C18	Cost of the shock absorbers installed on the foundation block in the -X axis.	ndx1 * ctd
C19	Cost of the shock absorbers installed on the inertial block in the -X axis.	ndx2 * ctd
C20	Cost of the shock absorbers installed on the machine in the -X axis.	ndx3 * ctd

Finally parameters OC(1), OC(2), OC(3), OC(4), pbix1, pbix2,pbix3, ctr and ctd were the unit cost for the civil works, isolation pad, springs and shock absorbers.

III. RESULTS

The RANDOM WALK (Martinez & Francisco, 2007) technique had as target to get information about the set of solutions. Candidate solutions were randomly generated and checked all the constrains to define is the solution is accepted. This statistical analysis permitted to know the tendency of each variable and the performance of the cost function.

A "MATLAB" program had been developed to generate the solutions of the cost function. The program is based in the next flux diagram of figure 7.

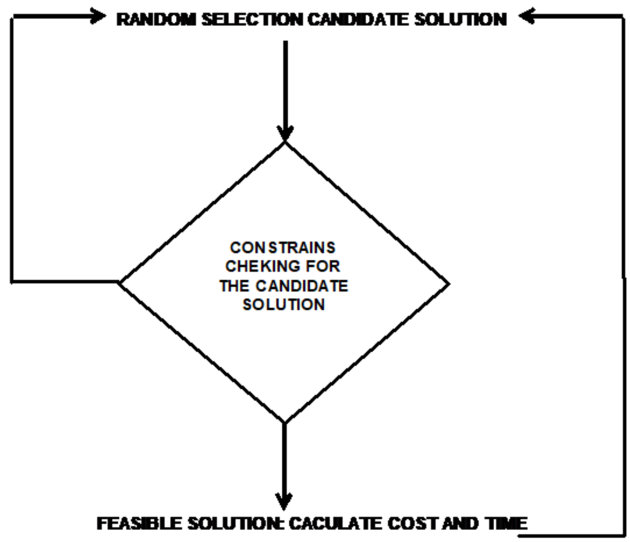

Fig. 7. Flux diagram.

Algorithm follows the next steps:

1) Randomly 37 values for the variables are selected between the intervals fixed for each variable. A candidate solution is generated.
2) Program checks geometrical, mechanical and operation constrains.
3) If the candidate achieved all the constrains is accepted and its cost was calculated. Time to get the solution is stored.
4) If the solution didn't achieve the constrains it was rejected.
5) A new candidate solution is generated in both cases.

The algorithm stopped when 2.000 feasible solutions were generated.

The computer used for the realization of the experiences was:

- 4 GB RAM
- Fourth nucleus CPU Intel Core i5
- Operation System Windows 7 - 64 bits
- Program MATLAB R-2015a

For finding 2.000 feasibly solutions had been necessary to generate 1.357.985 candidate solutions.The probability of finding a feasible solution from a candidate solution was:

$$Prob.\ feasible\ solution = \frac{2.000}{1.357.985} = 1.47\ 10^{-3}$$
$$= 0.00147 = 0.147\ \%$$

That meant that the probability of finding a feasible solution is 0.15% and its needed to reject unless 680 candidate solutions to get one feasible.

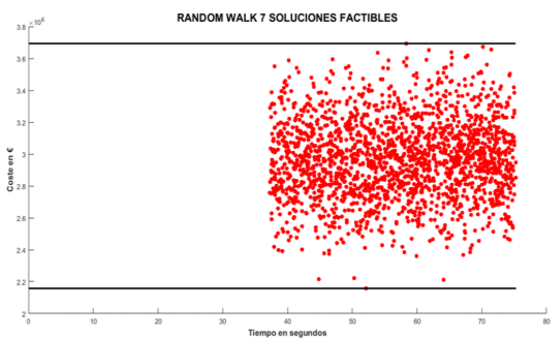

Fig. 8. Set of feasible solutions.

Figure 7 showed the set of solutions of the cost function between the less value 2.158.976,13 € y and the 3.694.036,22 €. Both values are represented in the graphic by two black lines. In the -x axis time to get the feasible solution was represented for each feasible solution.

In figure 8 was represented rejected solutions versus feasible solutions. A straight line with slope m:

$$m = \frac{1}{0.00147} = 680$$

Fig. 9. Rejected solutions versus feasible solutions.

Cost function versus number of solutions is represented in figure 9. There was a big plateau of solutions between 2,8 y 3,2 million of euros.

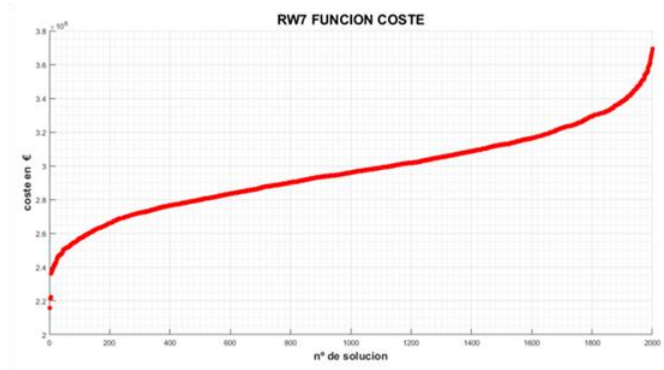

Fig. 10. Cost function vs number of solutions.

The statistical parameters for the set of solutions obtained was:

TABLE XII
STATISTICAL PARAMETERS FOR THE SET OF SOLUTIONS

	Program Time	Solution Cost
Mean Value	56,835 s	2.966.733,34 €
Minimum value	37,258 s	2.158.976,13 €
Maximum value	75,265 s	3.694.036,22 €
Standard deviation	10,866 s	242.283,91 €
Mode	37,258 s	2.960.532,43 €

Next, the histogram of the set of solutions was represented. Most frequency of solutions were around the 3.000.000 € value.

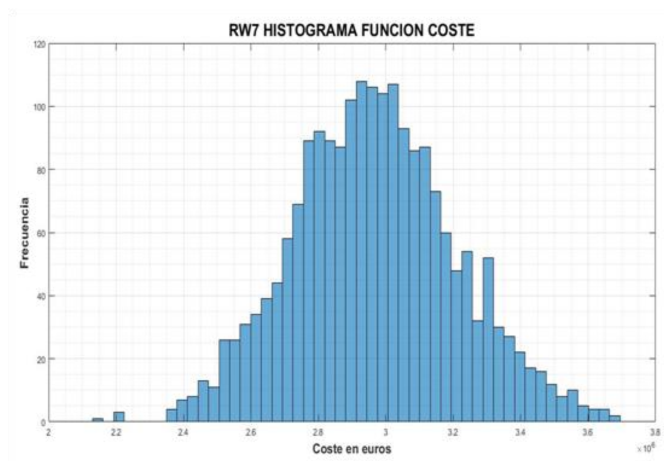

Fig. 11. Histogram of the set of solutions cost function.

IV. Discussion

The methodology proposed permitted to design the foundation and all the elements necessary to limit the vibrations of the machine and its transmission to soil in the time domain.

The model of foundation selected, ensemble of machine-inertial block-block foundation, worked correctly for a rotatory machine.

Control by limitation of the displacement during the transient operation mode permitted cross the resonance frequency to get the operation speed of the machine.

A rump-up designed with the foundation guarantees the limit of the vibration in the transient mode.

Limits of the normal operation mode are not valid in the transient mode. There are not feasible solutions for that limits in the transient mode.

Random Walk was a powerful technique to get information about the set of solutions.

The definition of a “cost function” to study the set of possible solutions got ready the way to apply heuristics to optimize the foundation.

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