A VIBROACOUSTIC MODEL OF THE STATIONARY RAILWAY WHEEL FOR SOUND RADIATION PREDICTION THROUGH AN AXISYMMETRIC APPROACH

V. T. Andrés*, J. Martínez-Casas, J. Carballeira and F. D. Denia

* Instituto de Ingeniería Mecánica y Biomecánica (I2MB)
Universitat Politècnica de València
Valencia, Spain
e-mail: vicanrui@etsid.upv.es, jomarc12@mcm.upv.es, jacarmo@mcm.upv.es, fdenia@mcm.upv.es

Key words: Stationary railway wheel dynamics, wheel axisymmetry, two-dimensional sound radiation model, computational efficiency.

Abstract: *In the literature, different dynamic models of the railway wheel have been developed to predict its sound radiation; however, there are still certain aspects that can be improved. Specifically, the high computational cost of these models, either because they solve the fluid-structure interaction or because they solve the dynamics and acoustics of the three-dimensional wheel, makes it difficult to carry out numerous simulations with the aim of achieving quieter designs. In the present work, a vibroacoustic model of the stationary wheel is developed through an axisymmetric approach, yielding an efficient and comprehensive acoustic prediction tool. The calculation methodology consists of, firstly, adopting an axisymmetric approach to solve the vibratory dynamics of the wheel from its cross-section, using finite element techniques; subsequently, the acoustic radiation of the three-dimensional wheel is calculated from the dynamics of the aforementioned section through an analytical formulation. Finally, the vibroacoustic model developed is validated via comparison with commercial software that solves the fluid-structure interaction, showing the aforementioned computational advantages that the former has over the latter.*

1 INTRODUCTION

Wheel/rail interaction generates a dynamic contact force due to the roughness of their surfaces. This excites the wheel causing a vibrational response which, in turn, leads to a sound radiation known as rolling noise. It is considered an important source of noise from railway activities [1], especially in urban areas where the vehicle velocity is relatively low [2]. The frequency range of interest for rolling noise radiation is approximately up to 6 kHz.

The interest in predicting the noise radiated by the railway wheel has resulted in the development of vibroacoustic models [3]; in general, the sound radiation is evaluated through the vibrational field of the wheel boundary. The wheel dynamic behaviour is commonly reproduced by the Finite Element Method (FEM) [4], which allows considering the flexibility of the body. Given the wheel geometry axisymmetry, a Fourier series expansion is feasible [5], solving analytically the vibrational response in the circumferential direction and therefore reducing the associated computational cost.

In this work, a vibroacoustic model of the axisymmetric wheel is presented. The description of the dynamic response of the wheel along the circumferential direction by means of Fourier series establishes a similar distribution of its modal properties. By adopting a modal approach, analytical relations between the vibrational field on the wheel boundary and on the wheel cross-section are found. This allows computing the acoustic problem also in a two-dimensional frame, with the computational advantages that it entails.

The mathematical formulation of the vibroacoustic model is presented in Section 2. The

https://doi.org/10.4995/YIC2021.2021.12584

results of this model are compared with results from a commercial software in Section 3. Finally, in Section 4 some conclusions are summarized.

2 VIBROACOUSTIC MODEL

2.1 Dynamics

Considering a cylindrical reference system, the displacement field of an axisymmetric wheel due to its flexible behaviour is expanded along the circumferential direction using Fourier series as follows [5]:

$$\begin{aligned} u_r =& u_{r,0} + \sum_{n>0} \left(u_{r,n} \cos(n\theta) - \overline{u}_{r,n} \sin(n\theta) \right), \\ u_\theta =& -\overline{u}_{\theta,0} + \sum_{n>0} \left(u_{\theta,n} \sin(n\theta) - \overline{u}_{\theta,n} \cos(n\theta) \right), \\ u_z =& u_{z,0} + \sum_{n>0} \left(u_{z,n} \cos(n\theta) - \overline{u}_{z,n} \sin(n\theta) \right), \end{aligned} \tag{1}$$

where subscripts r, θ and z indicate radial, tangential and axial direction, respectively. In this expansion, harmonic amplitudes without bar represent symmetric displacements about $\theta = 0$ and those with a bar represent antisymmetric displacements about $\theta = 0$, θ being the circumferential coordinate; all harmonic amplitudes are function of the coordinates r and z. Variable n symbolises each Fourier term. Similarly, the external forces applied on the flexible wheel can be expanded as Fourier series.

Making use of the expansion in Eq. (1), the kinetic energy of the flexible wheel E_k is analytically integrated over the circumferential direction. After that, it can be proved that the kinetic energy can be divided into the contribution of each motion associated with a Fourier harmonic and this, in turn, into the symmetric and antisymmetric displacements about $\theta = 0$. Thus, the kinetic energy can expressed as follows:

$$E_k = E_{k,0} + \overline{E}_{k,0} + \sum_{n>0} E_{k,n} + \sum_{n>0} \overline{E}_{k,n}. \tag{2}$$

Similarly, the strain energy of the wheel accomplishes the following expression:

$$E_p = E_{p,0} + \overline{E}_{p,0} + \sum_{n>0} E_{p,n} + \sum_{n>0} \overline{E}_{p,n}. \tag{3}$$

Applying the Lagrange Equations, a set of Equations of Motion (EoM) are obtained; each of these describes the motion associated with a Fourier term and a type of motion (symmetric or antisymmetric). The set of EoM for $n = 0$, considering a FE approach for the wheel cross-section, is given by:

$$\begin{aligned} \mathbf{M}_0 \ddot{\mathbf{u}}_0 + \mathbf{K}_0 \mathbf{u}_0 &= \mathbf{F}_0, \\ \overline{\mathbf{M}}_0 \ddot{\overline{\mathbf{u}}}_0 + \overline{\mathbf{K}}_0 \overline{\mathbf{u}}_0 &= \overline{\mathbf{F}}_0, \end{aligned} \tag{4}$$

where $\mathbf{u}_0$ contains the amplitudes $u_{r,0}$ and $u_{z,0}$ for each node of the wheel cross-section mesh while $\overline{\mathbf{u}}_0$ contains the amplitudes $\overline{u}_{\theta,0}$. The force vectors come from expanding the wheel/rail interaction force; $\mathbf{F}_0$ represents the even Fourier coefficients and $\overline{\mathbf{F}}_0$ the odd coefficients, both defined for $n = 0$. Similarly, the EoM for motion with $n > 0$ are given by:

$$\begin{aligned} \mathbf{M}_n \ddot{\mathbf{u}}_n + \mathbf{K}_n \mathbf{u}_n &= \mathbf{F}_n, \\ \overline{\mathbf{M}}_n \ddot{\overline{\mathbf{u}}}_n + \overline{\mathbf{K}}_n \overline{\mathbf{u}}_n &= \overline{\mathbf{F}}_n, \end{aligned} \tag{5}$$

where $\mathbf{u}_n$ contains the symmetric amplitudes $u_{r,n}$, $u_{\theta,n}$ and $u_{z,n}$ for each node of the wheel cross-section while $\overline{\mathbf{u}}_n$ contains the antisymmetric amplitudes $\overline{u}_{r,n}$, $\overline{u}_{\theta,n}$ and $\overline{u}_{z,n}$. Matrices accomplish the following relations:

$$\begin{aligned} \overline{\mathbf{M}}_n &= \mathbf{M}_n, \\ \overline{\mathbf{K}}_n &= \mathbf{K}_{-n}. \end{aligned} \tag{6}$$

Matrix $\mathbf{M}_n$ is indeed independent of n whereas $\mathbf{K}_n$ is defined for each n.

A modal approach is adopted in order to solve the dynamics of the flexible wheel. A set of modes coming from the EoM defined for a certain n is described in the literature as modes with n nodal diameters [2]. The eigenproblem of the EoM for $n = 0$ gives as a result a set of radial and axial modes with zero nodal diameters for the case of symmetric motion and a set of circumferential modes with zero nodal diameters for the case of antisymmetric motion. When considering the EoM for $n > 0$, for each vibration mode coming from the symmetric EoM, an analogous mode is obtained from the antisymmetric EoM, both being in quadrature of phase and with the same natural frequency. The wheel modeshapes can be also decomposed into harmonic functions with the angular coordinate similar to Eq. (1).

After solving the modal problem, the dynamic response of the wheel due to the contact force from the wheel/rail interaction is solved by modal superposition. Details of the interaction model can be found in [6]; in this work, the radial and axial directions are solved in the interaction problem. Although damping matrix is not considered in the EoM, spectral damping is introduced in the model as proposed by Thompson in [2], where it is suggested that modes with $n = 0$ have $\xi = 10^{-3}$, modes with $n = 1$ have $\xi = 10^{-2}$ and modes with $n \geq 2$ have $\xi = 10^{-4}$. Thus, the velocity of a point of the wheel with coordinates (r, θ, z) formulated in the frequency domain ω is given by:

$$v_j(r,\theta,z,\omega) = \sum_{p=1}^{m} A^p(\omega)\phi_j^p(r,\theta,z)\left(\sum_{k=r,z} F_k(\omega)\phi_{k,c}^p\right), \quad j = r,\theta,z, \tag{7}$$

where superscript p represents the pth vibration mode, m is the number of modes considered as a basis of the response, ϕ_j^p is the pth modeshape particularized in the jth Degree of Freedom (DoF) of the point, F_k is the kth component of the interaction force, $\phi_{k,c}^p$ is the pth modeshape particularized in the kth DoF of the wheel contact point and A^p is defined as:

$$A^p(\omega) = \frac{\mathrm{i}\omega}{\omega_p^2 - \omega^2 + 2\mathrm{i}\xi_p\omega_p\omega}, \tag{8}$$

with ω_p and ξ_p being the natural frequency and damping ratio, respectively, of the pth vibration mode.

2.2 Sound radiation

In this work, the acoustic model developed by Thompson [3] is employed. The sound radiation of the wheel is evaluated by postprocessing the vibrational field on its surface. Particularly, this model states that the acoustic power is the sum of the power associated with each set of modes with the same number of nodal diameters n. Thus, the sound power of the wheel W is given by:

$$W(\omega) = \rho c \sum_n \left(\sigma_{z,n}(\omega) S_z \overline{\widetilde{v}_{z,n}^2}(\omega) + \sigma_{r,n}(\omega) S_r \overline{\widetilde{v}_{r,n}^2}(\omega)\right), \tag{9}$$

where ρ is the density of air and c is the speed of sound. Functions σ are the radiation ratios and a set of fitting expressions for them is proposed in [3]. Surfaces S_z and S_r are the projected surfaces of the wheel normal to the axial and radial direction, respectively. Squared velocities $v_{z,n}^2$ and $v_{r,n}^2$ are the projected velocities in the axial and radial direction, respectively, and they are averaged over time ($\widetilde{\ }$) and over the wheel surface ($\overline{\ \ }$). The former is evaluated in the frequency domain as the Root Mean Square (RMS) value of the velocity amplitude v whereas the latter is computed as an integral given by:

$$\overline{\widetilde{v}_{j,n}^2} = \frac{1}{S_j} \int_S \widetilde{v}_{j,n}^2 dS_j, \quad j = r, z, \tag{10}$$

where S is the wheel surface. Note that $\widetilde{v}_{j,n}$ is the contribution to the velocity of a set of modes with n nodal diameters, including both symmetric and antisymmetric ones, and it can be computed through the modal superposition approach presented in Eq. (7), where m is replaced by m_n, the last being the number of modes with n nodal diameters. The integral in Eq. (10) can be divided into an integral over the circumferential direction and an integral over the wheel cross-section boundary. The former can be evaluated analytically by means of Eq. (7) and the circumferential expansion of the displacements in Eq. (1). Furthermore, by developing this over the wheel modeshapes, some relations are found between the vibrational field of the three-dimensional wheel and the response of the wheel cross-section. Finally, a numerical approach based on the FEM for the cross-section boundary is performed to complete the evaluation of the integral in Eq. (10).

3 RESULTS

The vibroacoustic model presented in Section 2 is compared with the commercial software Ansys. To perform this comparison, the Frequency Response Function (FRF) of the contact point and the Sound poWer Level (SWL) of the wheel are evaluated with both approaches and the results are shown in this section. The fluid-structure interaction model available in Ansys computes the acoustic pressure field in the air surrounding the wheel, which requires a high computational cost as the number of DoF increases. In this work, a straight web wheel with a diameter of 900 mm and a S1002 profile [7] is considered, as well as a load per wheel of 50 kN. The receptances of the wheel contact point, computed with Ansys and with the proposed model, are shown in Figure 1.

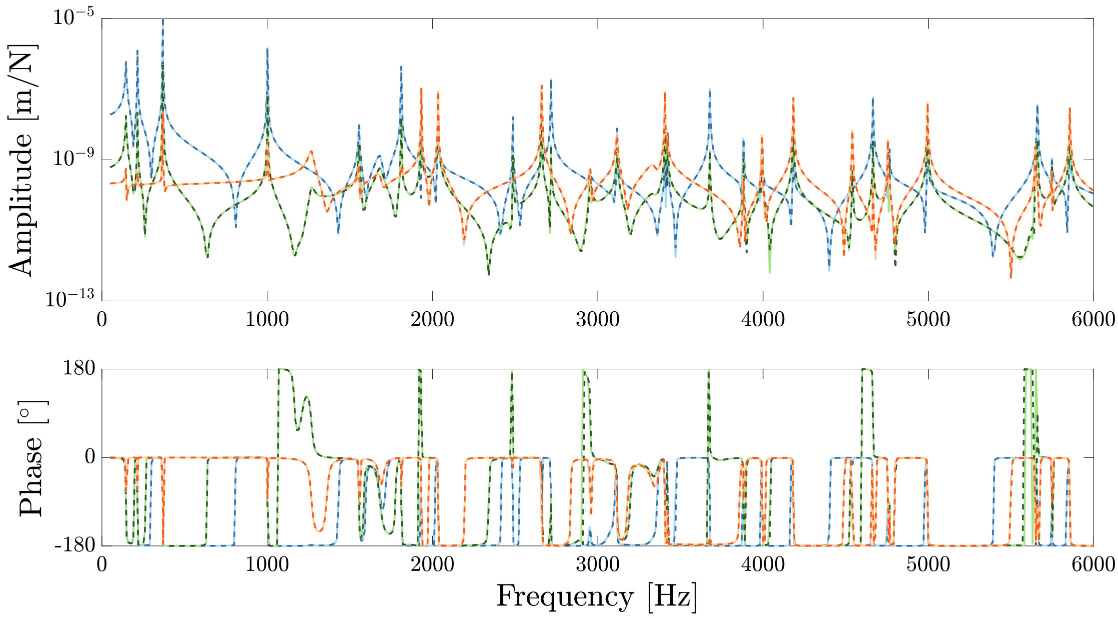

Figure 1: Receptances in the wheel contact point from Ansys (——) and axisymmetric approach (- - -): direct axial/axial (blue), cross axial/radial (green) and direct radial/radial (orange).

For the purpose of comparing the acoustic results from both approaches, the sound power radiated by the wheel is evaluated considering unit roughness excitation, the result being therefore a transfer function. The SWL of the wheel is shown in Figure 2. The greater differences between the proposed model and Ansys software appear at low and medium frequencies, where the sound power levels are low and the radiation ratios influence is important; at high frequencies, where the radiated levels are greater, the presented vibroacoustic model predicts the SWL accurately. The proposed model needs approximately 15 seconds for solving the vibroacoustic problem while Ansys software requires more than 24 hours, using a PC running with an ®Intel i7-9700 processor with 64 GB RAM.

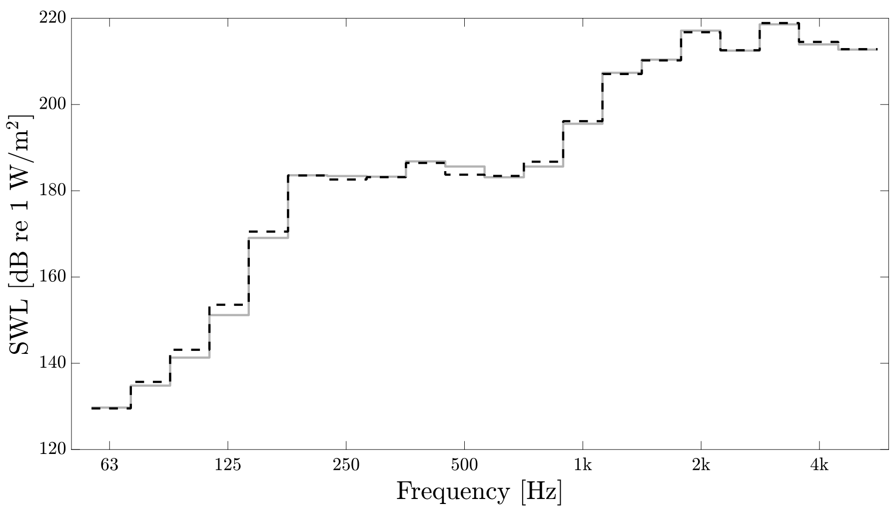

Figure 2: Wheel sound power level from Ansys (——) and axisymmetric approach (- - -).

4 CONCLUSIONS

A vibroacoustic model for a stationary and axisymmetric railway wheel is presented in this work, in which the displacement field is expanded using Fourier series. This allows solving analytically the dynamics and acoustics of the wheel along the circumferential direction, reducing the computational cost associated with numerical calculations. Also, the formulation developed leads to some relations between the dynamic response of the three-dimensional wheel and the response of the wheel cross-section, making it possible to employ the proposed acoustic methodology in combination with alternative three-dimensional dynamic models instead of that presented here. The vibroacoustic model is compared with the commercial FE package Ansys, which solves the fluid-structure interaction problem. The dynamic response and sound radiation results of both approaches show an excellent agreement, the computational cost of the presented methodology being much lower.

5 ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of Agencia Estatal de Investigación and European Regional Development Fund (grant FPU18/03999, project TRA2017-84701-R and project PID2020-112886RA-I00).

REFERENCES

[1] X. Zhang, D. J. Thompson, E. Quaranta and G. Squicciarini. An engineering model for the prediction of the sound radiation from a railway track. *Journal of Sound and Vibration*, Vol. **461**, 114921, (2019). DOI: https://doi.org/10.1016/j.jsv.2019.114921.

[2] D. J. Thompson. *Railway Noise and Vibration. Mechanisms, Modelling and Means of Control*. Elsevier, 2009. ISBN: 978-0-08-045147-3.

[3] D. J. Thompson and C. J. C. Jones. Sound radiation from a vibrating railway wheel. *Journal of Sound and Vibration*, Vol. **253**(2), pp. 401-–419, (2002). DOI: https://doi.org/10.1006/jsvi.2001.4061.

[4] D. J. Thompson. Wheel-rail noise generation, part II: Wheel vibration. *Journal of Sound and Vibration*, Vol. **161**(3), pp. 401-–419, (1993). DOI: https://doi.org/10.1006/jsvi.1993.1083.

[5] M. Petyt. *Introduction to Finite Element Vibration Analysis*. Cambridge University Press, 2nd Edition, 2010. ISBN: 9780521191609.

[6] D. J. Thompson. Wheel-rail noise generation, part I: Introduction and interaction model. *Journal of Sound and Vibration*, Vol. **161**(3), pp. 387-–400, (1993). DOI: https://doi.org/10.1006/jsvi.1993.1082.

[7] Railway applications - Wheelsets and bogies - Wheels - Tread profile. EN 13715:2020. European Committee for Standardization, 2020.