Assessment of the Performance of a Modified USBR Type II Stilling Basin by a Validated CFD Model

Juan Francisco Macián-Pérez 1, Francisco José Vallés-Morán 2, Rafael Garcia-Bartual 3

1JUAN FRANCISCO MACIÁN-PÉREZ, Research Engineer, Research Institute of Water and Environmental Engineering, Universitat Politècnica de València, Camí de Vera, s/n, 46022 Valencia, Spain. ORCID iD: 0000-0002-4457-9507
Email: juamapre@cam.upv.es (corresponding author)

2FRANCISCO JOSÉ VALLÉS-MORÁN, Professor, Research Institute of Water and Environmental Engineering, Universitat Politècnica de València, Camí de Vera, s/n, 46022 Valencia, Spain. ORCID iD: 0000-0001-6335-1746
Email: fvalmo@hma.upv.es

3RAFAEL GARCÍA-BARTUAL, Professor, Research Institute of Water and Environmental Engineering, Universitat Politècnica de València, Camí de Vera, s/n, 46022 Valencia, Spain. ORCID iD: 0000-0002-1811-8264
Email: rgarciab@hma.upv.es

ABSTRACT

The adaptation of existing dams is of paramount importance to face the challenge posed by climate change and new legal frameworks. Thus, it is crucial to optimise the design of stilling basins so that a reduction in the hydraulic jump dimensions is achieved without jeopardising the energy dissipation in the structure. A numerical model was developed to simulate a United States Bureau of Reclamation Type-II basin. The model was validated with a specifically-designed physical model and then was used to simulate and test the performance of the basin after adding a second chute blocks row. The results showed a reduction in the hydraulic jump dimensions in terms of the sequent depths ratio and the roller length, which were respectively 2.5% and 1.4% lower in the modified design. These results would allow an estimated increase of the discharge in the basin close to the 10%. Furthermore, this new
design showed a higher efficiency, with a 1.2% increase in this parameter. Consequently, the modifications proposed for the basin design suggest an improved performance of the structure. The issue of the hydraulic jump length estimation is also discussed, introducing and comparing different approaches. These methods follow a structured and systematic procedure and show consistent results for the developed models.

**Keywords:** Flow-structure interactions; hydraulic jumps; hydraulic structure design & management; RANS models; USBR Type II stilling basin; chute blocks.

**INTRODUCTION**

Adaptation to new scenarios posed by climate change effects and increasing society demands regarding hydraulic structures security leads to consider higher potential discharges in the design of large dams (Carrillo et al., 2020; Macián-Pérez, García-Bartual, et al., 2020). These more demanding conditions are in line with new legal frameworks and design guidelines recently developed (Ministerio para la Transición Ecológica y el Reto Demográfico, 2018). In such context, adaptation of already existing hydraulic structures becomes a key issue of enormous engineering importance. There is particular interest in the adaptation process of large dams to higher discharges than those originally considered, in which case the energy dissipation structure constitutes the most challenging part both from a technical and an economic perspective (Fernández-Bono & Vallés-Morán, 2006).

The use of typified stilling basins for energy dissipation purposes in large dams is widely spread. The design of a majority of these basins dates back many decades and has remained unaltered since then (Hager, 1992). The more demanding requirements previously mentioned, together with the development of new modelling approaches (Valero et al., 2019; Viti et al., 2019), justify the in-depth study of the flow taking place in energy dissipation structures. This will guide future design modifications, in order to optimise their hydraulic performance.

A better understanding of the flow in typified stilling basins requires a deep insight into the hydraulic jump phenomenon. The hydraulic jump is defined as the abrupt transition from supercritical to subcritical flow in open channels. This phenomenon involves strong velocity and pressure fluctuations, intense air entrainment and significant energy dissipation. It is precisely the latter feature what motivates their use in stilling basins. The so called Classical Hydraulic Jump (CHJ) is the one occurring in a horizontal, rectangular, prismatic, smooth channel. It has been investigated for almost
two centuries (Chanson & Gualtieri, 2008; Hager, 1992). First studies focused on basic characteristics like the sequent depths ratio or the free surface profile (Bakhmeteff & Matzke, 1936; Bélanger, 1841). Later, internal features of the hydraulic jump such as the pressure field or the velocity distribution were approached (McCorquodale & Khalifa, 1983; Rajaratnam, 1965). During the last decades, the turbulent characteristics of the phenomenon were brought into the spotlight (Wang & Chanson, 2015; Jesudhas et al., 2018; Toso & Bowers, 1988), together with the study of the aeration (Chanson & Brattberg, 2000; Chanson & Gualtieri, 2008; Murzyn et al., 2005). The efforts devoted to the study of the CHJ have significantly contributed to an increased knowledge of the phenomenon, which is constantly growing. However, the inherent complexity of the hydraulic jump requires further research to achieve a full understanding of the phenomenon. In particular, the study of the hydraulic jump developed in stilling basins has not received as much attention as the CHJ, despite of its practical interest (Valero et al., 2019).

In spite of the traditional experimental approach to the study of the hydraulic jump, Computational Fluid Dynamics (CFD) techniques also constitute a useful tool with undoubtedly increasing potential, as the computational power becomes larger. In fact, the complementary nature of both techniques enables a desirable double modelling approach for hydraulic engineering problems. CFD techniques have been used to successfully simulate CHJ in terms of its internal characteristics and the aeration features (Bayon et al., 2016; Macián-Pérez, Bayón, et al., 2020; Witt et al., 2015). Furthermore, some numerical studies have addressed the hydraulic jump developed in stilling basins, analysing its characteristics and the influence of the energy dissipation devices (Carvalho et al., 2008; Macián-Pérez, García-Bartual, et al., 2020; Valero et al., 2018). Nevertheless, CFD techniques still present some limitations when simulating complex hydraulic phenomena (Blocken & Gualtieri, 2012; Bombardelli, 2012). Therefore, the support of physical modelling to provide validated numerical models remains of paramount importance (Valero et al., 2019).

The present study approaches the performance of a United States Bureau of Reclamation (USBR) Type-II stilling basin using a CFD numerical model. This model was validated with the experimental data collected in a physical model designed for this purpose (Macián-Pérez, Vallés-Morán, et al., 2020). According to Hager (1992), stilling basins provide an enhanced energy dissipation and shorter and more stable hydraulic jumps, when compared to CHJ. Consequently, the performance of the basin was tested focusing on basic geometrical features, such as the sequent depths ratio and the
roller and hydraulic jump lengths, as well as the energy dissipation efficiency. These characteristics were compared with those obtained for CHJ in previous studies (Bayon et al., 2016; Hager, 1992; Hager et al., 1990; Hager & Bremen, 1989; Schulz et al., 2015). In order to deepen in the analysis of the model developed, literature results regarding stilling basin studies were also included (Macián-Pérez, García-Bartual, et al., 2020; Padulano et al., 2017; Peterka, 1978). Despite this research focuses on geometrical characteristics of the hydraulic jump, it is important to highlight that there are some other relevant features to analyse the performance of the basin, such as the bottom pressure, that have been addressed by recent studies (Stojnic et al., 2021; 2020). Finally, a modified design of the stilling basin was proposed and included in the analysis to assess a possible optimisation of the structure performance.

NUMERICAL MODEL

The CFD numerical models presented in this study were developed using the commercial software FLOW-3D®, version 11. The results provided by this code are based on the Navier-Stokes equations written in their form for incompressible fluids:

\[ \nabla \mathbf{u} = 0 \]  

\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}_b \]  

where \( \mathbf{u} \) is the velocity, \( t \) is the time, \( \rho \) is the fluid density, \( p \) the pressure and \( \nu \) the fluid kinematic viscosity. Finally, \( \mathbf{f}_b \) accounts for the body forces. In particular, FLOW-3D® uses the Finite Volume Method (FVM) (McDonald, 1971) for the spatial discretisation of the conservation laws. In regards with the time-step, its size is automatically adjusted, using a Courant-type stability criterion to minimise numerical divergence risk.

Turbulence modelling

The flow governing equations were numerically solved through a Reynolds Averaged Navier-Stokes (RANS) approach. This approach has proved to be efficient regarding computational times and resources for real-life engineering applications, as compared to other methods such as the Direct Numerical Simulation (DNS) or the Large Eddy Simulation (LES) (Bayon et al., 2016; Viti et al.,
Nevertheless, the averaging process bound to the RANS approach leads to the well-known Closure Problem. Accordingly, a turbulence model is required to estimate the eddy viscosity that results from the approach.

To this end, the RNG $k$-$\varepsilon$ model (Yakhot et al., 1992) was employed. This two-equation turbulence model addresses the transport of the turbulent kinetic energy ($k$) and its dissipation rate ($\varepsilon$). Statistical methods allow deriving the averaged equations for the turbulence quantities, in contrast with the traditional $k$-$\varepsilon$ model. The equations used to model the transport of $k$ and $\varepsilon$ are:

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho ku_i)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k - \rho \varepsilon \quad (3)$$

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon u_i)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} P_k - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \quad (4)$$

where $x_i$ and $x_j$ are the coordinates in the $i$ and $j$ axes respectively, $\mu$ is the fluid dynamic viscosity and $\mu_t$ the turbulent dynamic viscosity, whereas $P_k$ is the production of turbulent kinetic energy. Finally the terms $\sigma_k$, $\sigma_\varepsilon$, $C_{1\varepsilon}$ and $C_{2\varepsilon}$ are parameters whose values are reported in Yakhot et al. (1992). The RNG $k$-$\varepsilon$ turbulence model has proved its efficiency for hydraulic engineering applications. Its performance has been compared with other turbulence models when analysing both, a CHJ (Bayon et al., 2019) and the hydraulic jump developed in a stilling basin (Macián-Pérez et al., 2019).

**Free surface modelling**

The modelling and tracking of the free surface was approached on the basis of the Volume Of Fluid (VOF) method (Hirt & Nichols, 1981). Accordingly, the variable Fraction of Fluid ($F$) was used to determine the fractional volume of water in each cell. This variable reaches a value of 1 when the corresponding cell is completely full and a value of 0 when it is empty. The free surface is then lying along cells with values of $F$ between 0 and 1. In these terms, the free surface elevation is computed as the coordinate of the free surface in the topmost fluid element in a vertical column. The evolution of $F$ throughout the domain is solved through the equation:

$$\frac{\partial F}{\partial t} + \nabla \cdot (u F) = 0 \quad (5)$$

The case study here presented was addressed under a one-fluid approach for the resolution of the flow
equations, as recommended by FLOW-3D® for problems involving a free surface between water and air. This approach implies that the boundary conditions were applied to the free surface, so that the equations could be solved only for the water phase. In contrast, the air phase was assumed to have negligible inertia and only applied normal pressure to the free surface, allowing a significant reduction on computing times (Bombardelli et al., 2011). Regarding the refinement of the free surface, a mechanism that creates small negative divergences in internal fluid cells was used to close up partial voids and add interface sharpening.

Air entrainment modelling

The air entrainment process was modelled through a balance between stabilising (gravity and surface tension) and destabilising (turbulent kinetic energy) forces. When destabilising forces overcome stabilising ones, air enters the flow at a rate that can be modelled as:

\[
\delta V = \begin{cases} 
  k_{air} A_S \left( \frac{2(P_t - P_d)}{\rho} \right)^{1/2} & \text{if } P_t > P_d \\
  0 & \text{if } P_t < P_d 
\end{cases}
\]  

(6)

\[ P_t = \rho k; P_d = \rho g L_T + \frac{\sigma}{L_T} \]  

(7)

\[ L_T = C_{\mu}^{3/4} k^{3/2} \]  

(8)

where \( \delta V \) is the volume of entrained air rate and \( P_t \) and \( P_d \) are respectively the destabilising and stabilising forces. In addition, \( k_{air} \) is a coefficient calibrated for each particular case and \( A_S \) is the free surface area for each cell. The calculation of the stabilising forces involves the gravity component normal to the free surface (\( g \)), the surface tension coefficient (\( \sigma \)) and the turbulent length scale (\( L_T \)). The latter is calculated with the parameter \( C_{\mu} \). For the RNG \( k-\varepsilon \) turbulence model the value of this parameter is 0.085 (Bayon et al., 2016). The varying density in the flow, resulting from the entrained air, is accounted for in terms of a fluid mixture density (\( \rho_m \)) defined as:

\[ \rho_m = F \rho_w + (1 - F) \rho_a \]  

(9)

where \( \rho_w \) and \( \rho_a \) are respectively the water and air densities. Besides, the drag force produced by
bubbles upon the carrier phase is also considered. The modelling process to obtain the drag per unit volume and the relative velocity between phases using FLOW-3D® is developed in Brethour and Hirt (2009).

**Meshing and boundary conditions**

The spatial domain of the case study was meshed with a three-dimensional structured mesh formed by regular hexahedral cells. Structured meshes are usually associated with the existence of regular connectivity, generally providing a good level of accuracy (Biswas & Strawn, 1998; Hirsch, 2007). Moreover, structured meshes lead to low latency during simulations (Keyes et al., 2000) and to a reduced numerical diffusivity for free surface modelling (Bayon & Lopez-Jimenez, 2015).

Two different blocks were employed in the meshing process to save computing time. Firstly, a refined block was used to mesh the stilling basin, covering the main area of interest, where higher flow gradients are to be expected. Secondly, a coarser mesh block downstream the stilling basin was linked to the refined one. The coarser one was used to mesh the remaining spatial domain with cells doubling the size of those forming the refined block.

Regarding the boundary conditions set for the spatial domain, a supercritical flow was set upstream the stilling basin by imposing the corresponding discharge and fluid elevation. Downstream the basin, an outflow condition was used, allowing the flow to leave the domain. Additionally, an atmospheric pressure condition was imposed in the upper boundary of the domain. For the solid contours a wall non-slip boundary condition was set, assuming a law-of-the-wall velocity profile in the vicinities of these boundaries.

**Mesh convergence analysis**

The cell sizes used in the meshing process were chosen after a mesh convergence analysis, in order to ensure a low degree of uncertainty in the results. The analysis was carried out following the American Society of Mechanical Engineers (ASME) procedure (Celik et al., 2008). Accordingly, three different meshes were tested and for each of these meshes 11 basic variables (i.e. averaged velocities along the hydraulic jump longitudinal axis) were used as indicators. Table 1 shows the cell sizes for each of the meshes assessed. These sizes were chosen considering the minimum recommended refinement ratio of 1.3 established by Celik et al. (2008). Once this is done, the apparent order was calculated, following
the ASME procedure, as an indicator to assess grid convergence.

The results of the convergence analysis for the finest of the meshes tested consisted in a mesh apparent order of 2.09, close to the numerical model’s formal order of 2, which means a good indication of the grids being in the asymptotic range (Celik et al., 2008). In addition, the resulting grid convergence index was 14.46%, in line with previous research involving numerical models of complex hydraulic phenomena like the one here presented (Bayon et al., 2016; Valero et al., 2018). Finally, Celik et al. (2008) recommends reporting the percentage occurrence of oscillatory convergence. For this particular case the percentage is 9%. On the basis of this analysis, the finest mesh was chosen for the study, meaning that the smallest chute block dimension was covered by at least four cells.

MODEL VALIDATION

The numerical model was developed reproducing the exact conditions of the USBR Type-II stilling basin physical model available in the Hydraulics Laboratory of the Department of Hydraulic Engineering and Environment at the Universitat Politècnica de València (UPV, Spain). According to Valero et al. (2019), it is highly recommendable to assume the same geometry of the experimental device in the numerical model for calibration and validation purposes. The design of the physical model followed the guidelines of the USBR for typified stilling basins (Peterka, 1978). In addition, the recommendations posed by Heller (2011) to avoid significant scale effects in the experimental device were considered. The flow conditions chosen in the design of the case study (Table 2) led to an inflow Froude number ($F_r$) of 9, which provides an adequate energy dissipation for the analysed basin (Peterka, 1978). Figure 1 shows the experimental device in which the campaign was conducted. The details of this physical model can be found in Macián-Pérez, Vallés-Morán, et al. (2020).

The mean free surface longitudinal profile and the maximum forward averaged velocity decay were measured in the physical model. A time-of-flight camera using light detection and ranging (LIDAR) techniques was employed to determine the free surface, whereas velocity measurements were taken with a Pitot tube. The measured features were used to validate the numerical model. To do so, a comparison of the experimental data with the simulated results was carried out, using dimensionless metrics (Fig. 2):

$$X = \frac{X - x_0}{L_r}$$

(10)
where $x_0$ is the jump toe position, which corresponds with the beginning of the basin for both models. 

$L_r$ is the hydraulic jump roller length estimation (see section 5.2). Besides, $y_1$ and $y_2$ are respectively the supercritical and subcritical flow depths upstream and downstream of the hydraulic jump. In regards with Eq. (12), $u_{\text{max}}$ is the maximum forward averaged velocity for each vertical profile measured in the roller and $u_1$ and $u_2$ are respectively the supercritical and subcritical mean flow velocities.

Figure 2 shows a good level of agreement between the numerical and the experimental model, both for the free surface profile and for the maximum velocity decay. In particular, the accuracy of the numerical model can be assessed using the coefficient of determination ($R^2$) (Bennett et al., 2013). The values of this coefficient achieved by the model were 0.992 for the free surface profile and 0.964 for the maximum forward velocity decay, indicating a successful validation of the numerical model.

STILLING BASIN MODIFIED DESIGN

A modification of the original USBR Type-II stilling basin design was tested with the validated numerical model. The main purpose of this structure is to enhance energy dissipation, generally involving shorter and more stable hydraulic jumps (Hager, 1992). Any improvement in the design should therefore yield to a reduction in the dimensions of the hydraulic jump without losing energy dissipation benefits. The improved performance of a typified basin could not only reduce the economic cost of new structures, but more importantly, contribute to the adaptation of existing structures to more demanding operational conditions.

Previous attempts to optimise the performance of stilling basins were made focusing on both the design and the flow conditions. On the one hand, Valero et al. (2015) tested different sizes for the USBR Type-II stilling basin chute blocks. These devices lift a portion of the inflow water jet, leading to an increased number of dissipating eddies, which in turn results in shorter jump lengths. In addition, the chute blocks help to stabilise the hydraulic jump in the basin under adverse tailwater conditions (Peterka, 1978; Valero et al., 2015). These authors found that the original design dimensions of the chute blocks led to the optimal performance of the basin, since they achieved better submergences and
Soori et al. (2017) also tested different sizes for the chute blocks in a USBR Type-II stilling basin, changing the end sill for a series of steps too. The results presented by these authors also point out to an optimal dimension of the chute blocks in line with the original design.

On the other hand, Montano and Felder (2019) conducted an experimental research to optimise the performance of stilling basins without energy dissipation devices by changing the hydraulic jump type in regards with the tailwater conditions. These authors found similar energy dissipation and higher stabilities for hydraulic jumps partially developed in the slope upstream the basin. Thus, an improved performance compared to the case of traditional stilling basins, for which the hydraulic jump strictly takes place downstream the slope, was found. Nevertheless, the experimental campaign only covered inlet slopes up to 5° and values of $F_1$ up to 4.6. Such slopes are quite reduced for prototype dam cases. Besides, the tested $F_1$ values only covered the lowest part of the range for adequate energy dissipation in stilling basins, as established by the USBR (Peterka, 1978). Montano and Felder (2019) found that some of the benefits of changing the tailwater conditions decreased with increasing slopes, suggesting the need for further research involving steeper slopes and higher $F_1$.

In line with these previous considerations, the present research proposes a modification of the USBR Type-II stilling basin original design. It consists in the addition of a second row of chute blocks right upstream the original row (Fig. 3). These additional chute blocks are located in chessboard order so that immediately downstream of one of the new blocks, there exists a gap between two of the original blocks. The modification aims at reducing the hydraulic jump dimensions, without affecting the energy dissipation performance of the structure.

The modified stilling basin was simulated using FLOW-3D®, with the numerical setup developed for the validated model. Thus, similar flow conditions ($F_1 = 9$, $q = 0.147 \text{ m}^2 \text{ s}$), meshing and modelling parameters were employed. As stated by Schulz et al. (2015), numerical models can be used to test geometrical modifications in hydraulic structures, if the resulting design is sufficiently close to the experimental background. Figure 4 shows an example of the results obtained from the simulations of both basin designs.

RESULTS AND DISCUSSION

An appropriate design for energy dissipation stilling basins must consider the dimensions of the hydraulic jump developed within the structure. According to Schulz et al. (2015), the most relevant
Free surface profile

The study of the hydraulic jump longitudinal free surface profile provides an insight into the dimensions of the phenomenon, which can be used to assess the effect of the stilling basin design. Firstly, the sequent depths ratio ($y_2 / y_1$) was analysed. To this end, the values obtained in the numerical simulations were compared with the theoretical expression proposed by Hager and Bremen (1989) for CHJ:

$$\frac{y_2}{y_1} = \frac{1}{2} \left( \left[ 1 + 8 F_1^2 \right]^{1/2} - 1 \right) \cdot \left[ 1 - 0.7 \left( \log R_e \right)^{5/2} \right] \cdot \left[ 1 - \frac{3.25 y_1}{b^5 R_e} \left( \log R_e \right)^{3} \right]$$

where $b$ is the width of the hydraulic jump or the stilling basin and $R_e$ is the Reynolds number. The analysis also includes bibliographic values of the sequent depths ratio for hydraulic jumps with similar $F_1$ occurring in different types of stilling (Table 3).

Table 3 shows a significant variability of the results depending on the source. These differences suggest that the sequent depths ratio is not only influenced by the inflow Froude number but also by the other factors included in Eq. (13). In these terms, in spite of the similar $F_1$ values, the reason behind the differences observed could be the particular inflow depth, inflow Reynolds number or unit discharge of the sources under analysis. Focusing on the stilling basin cases modelled in this research, which share the same value for the previously mentioned factors ($F_1$, $R_e$, $y_1$, $b$), the hydraulic jump in the modified design shows a lower sequent depths ratio than the one in the original USBR Type-II basin. In particular, the sequent depths ratio is 2.5% lower in this modified design. This suggests a reduction in the dimensions of the hydraulic jump with the addition of the second row of chute blocks, leading to an optimisation in the structure design.

The mean free surface profile along the longitudinal axis of the hydraulic jump was also measured in the numerical models (Fig. 5). Figure 5a shows that the hydraulic jump free surface profile is quite similar in the original stilling basin and in the modified design. However, some relevant differences should be pointed out. The additional row of chute blocks, placed in the modified design
immediately upstream of the original row, leads to higher flow depths at the beginning of the basin. These higher flow depths disappear for downstream positions. Thus, for $x$ values greater than 0.5 m until the end of the basin, the hydraulic jump profile in the modified structure is placed slightly below the one developed in the original design (around 2.5% lower flow depth values in the modified design).

Regarding the dimensionless values shown in Fig. 5b, all of the represented profiles follow similar trends. Nevertheless, a particular difference between the CHJ and the stilling basin should be pointed out. For the basin profiles, the subcritical flow depth is reached for $X$ values around 1, whereas the CHJ profiles keep increasing for a longer distance, so that the subcritical depth is reached downstream (around $X = 1.5$). This result indicates a shortened hydraulic jump both for the original and the modified basins, in comparison with the CHJ.

**Hydraulic jump roller length**

The hydraulic jump roller length ($L_r$) is of paramount importance for the design of stilling basins since it constitutes a geometrical feature strictly linked to the structure dimensions. The roller region determines the boundary between backward and forward flow, starting at the toe of the jump and ending at the surface stagnation point (Hager & Bremen, 1989). Besides, the most intense energy dissipation within the hydraulic jump is enclosed in this region. The estimation of this feature was conducted following the stagnation point criterion for both numerical models (Fig. 6). Accordingly, a series of streamwise velocity vertical profiles were characterised along the hydraulic jump longitudinal axis. For each of these profiles, the stagnation point (i.e. point where velocity tends to zero) was identified. Finally, the intersection between the line joining all the stagnation points and the mean free surface profile marks the end section for the roller region (Hager et al., 1990). The extrapolation done to meet the free surface consisted in an exponential adjustment with $R^2$ values above 0.95.

Following this procedure, the estimated dimensionless roller lengths ($L_r/y_1$) were respectively 48.00 and 47.33 for the original and for the modified basin numerical models, which means a 1.4% reduction for this dimensionless value. For comparison purposes, the expression proposed by Hager et al. (1990) to determine the hydraulic jump roller length was employed:

$$L_r = y_1 \left[ -12 + 160 \tanh \left( \frac{F_1}{20} \right) \right] \text{ for } y_1/b < 0.10$$

$$L_r = y_1 \left[ -12 + 100 \tanh \left( \frac{F_1}{12.5} \right) \right] \text{ for } 0.10 < y_1/b < 0.70$$

(14)
This expression, originally thought for CHI, provides a dimensionless length of 55.67. Therefore, the basin objective of shortening the space in which energy dissipation occurs (Hager, 1992) is successfully accomplished. In terms of the basin design, the roller region for the hydraulic jump developed in the modified basin was slightly shorter, which could lead to a reduction in the dimensions of the structure. Figure 6b includes the roller boundary for the original USBR II model, showing this reduction in the roller length. The comparison also shows that the roller region in the modified design is lifted up from its original position.

Hydraulic jump length

From an engineering perspective, the hydraulic jump length \(L_j\) can be identified as the distance in which bottom protection against erosion is needed for the design of stilling basins (Hager, 1992). However, there is no a clear or unique theoretical definition for this dimension. According to Valero et al. (2019), the estimation of the hydraulic jump length usually implies an important degree of uncertainty. In fact, traditional approaches are based on the visual determination of this feature. For instance, the hydraulic jump end section has been previously identified with the section where the hydraulic jump is fully deaerated or where the free surface is essentially horizontal (Hager et al., 1990; Kramer & Valero, 2020).

This study aims at shedding light on the determination of the hydraulic jump length and thus, different methods were tested. Overall, the objective was to achieve a reliable estimation of this parameter to assess the influence of the stilling basin design. Some recent studies shared this objective and developed physical criteria, less based on subjective interpretation (Stojnic et al., 2021). For this particular research, two different procedures were assessed. On the one hand, the streamwise averaged velocity vertical profiles were analysed. In these terms, Hager (1992) referred to the hydraulic jump end section as the section where gradually varied flow conditions reappear, whereas Bayon et al. (2016) pointed to the study of the velocity profile to identify the hydraulic jump end section. On this basis, streamwise averaged velocity vertical profiles along the basin longitudinal axis were obtained for both numerical models. These profiles were compared with the expression proposed by Kirkgöz and Ardiçlioğlu (1997) for open channel flow:

\[
\frac{u_{\text{max}} - u}{u^*} = -2.44 \ln \left( \frac{z}{y_2} \right) + 0.488 \left[ \cos \left( \frac{\pi z}{2y_2} \right) \right]^2
\]  

(15)
where $z$ is the vertical position in the profile and $u^*$ is the shear velocity, that can be obtained as:

$$u^* = \left( \frac{\tau_0}{\rho} \right)^{1/2}$$  \hspace{1cm} (16)$$

$$\tau_0 = \gamma R_H I$$  \hspace{1cm} (17)$$

where $\tau_0$ is the wall shear stress, $\gamma$ is the specific weight and $R_H$ the hydraulic radius. In addition, $I$ is the energy line slope which can be estimated from Manning equation. The comparison between the modelled profiles and Eq. (15) was then used to assess where the open channel flow conditions were reached.

Figure 7 shows that there is an evolution of the velocity profiles downstream of the stilling basin ($x = 1.76$ m). Thus, the shape of modelled profiles tends to the open channel flow profile as the distance from the hydraulic jump toe increases. In both numerical models, the profile for $x = 2.4$ m shows a good agreement ($R^2 \geq 0.9$) with Eq. (15).

On the other hand, the turbulent kinetic energy ($k$) decay was also employed to figure out the hydraulic jump length, as proposed by Bayon et al. (2016). The turbulent kinetic energy can be defined as half the sum of the variances of the spatial velocity components:

$$k = \frac{1}{2} \left[ \left( u_x' \right)^2 + \left( u_y' \right)^2 + \left( u_z' \right)^2 \right]$$  \hspace{1cm} (18)$$

The aforementioned authors established a 95% decay of the maximum turbulent kinetic energy as an approximate threshold to determine the hydraulic jump end section in a numerical model. Hence, the values of $k$ along the hydraulic jump longitudinal axis were obtained in both numerical models and compared in Fig. 8.

The criterion developed by Bayon et al. (2016) provides a hydraulic jump length of 3.2 m for both numerical models. Nevertheless, if a 90% $k$ decay threshold is taken as reference, the hydraulic jump length would be 2.4 m, in perfect agreement with the previously presented method. Considering that this procedure was developed by Bayon et al. (2016) for CHJ, the proposed threshold could be varied by the stilling basin design. The chute blocks immediately upstream the jump toe provide additional energy dissipation, so that the maximum $k$ is lower than the one obtained in a CHJ with the same conditions. Hence, a lower decay of the maximum $k$ would be needed to achieve the subcritical
flow conditions. It is also important to remark that the turbulent kinetic energy for the modified basin model is constantly below that of the original model (Fig. 8), especially in the vicinity of the jump toe.

Peterka (1978) established an experimental relationship between the hydraulic jump length and the $F_1$ value for a variety of typified energy dissipation structures. Furthermore, Hager (1992) proposed the following expression to estimate the jump length in CHJ with $F_1$ values between 4 and 12:

$$L_j = y_1 220 \tanh \left( \frac{(F_1 - 1)/22}{2} \right)$$

(Movahed et al. 2018) argued that the accuracy in the estimation of the hydraulic jump length can be improved by considering the Froude number downstream of the hydraulic jump ($F_2$) and provided a semi-analytical equation to obtain $L_j$:

$$L_j = y_1 \left( 3.7 + 3/F_2 \right)$$

The different hydraulic jump length dimensionless values obtained are displayed in Table 4 for convenient comparison. In summary, it can be stated that the methods presented in this research for the estimation of the hydraulic jump length provide different results, being the one based on the velocity profiles closer to the bibliographic data. However, if the previously mentioned variation in the $k$ decay threshold is assumed, both methods show a perfect agreement. The agreement between these two methods contributes to the consistency in the estimation of a parameter such as the $L_j$, usually surrounded by a high degree of uncertainty.

Past studies generally provided lower $L_j$ values than the presented models. These differences could be explained by several factors. On the one hand, different methods were employed to obtain the hydraulic jump length, which undoubtedly affects the results. In these terms, the data collected by Peterka (1978) clearly underestimates this parameter in the USBR Type-II stilling basin, as previously observed by Movahed et al. (2018). On the other hand, Habibzadeh et al. (2019) found an increase in $L_j$ in the presence of energy dissipation blocks for hydraulic jumps with $F_1$ greater than 5. These authors based their estimation on the water surface fluctuations and found that large-scale turbulence structures created by the blocks in the form of surface fluctuations persisted for longer distances.
Hydraulic jump efficiency

Any modification made in the design of a stilling basin to reduce the hydraulic jump dimensions, must also account for the energy dissipation purpose of the structure. Accordingly, improving or maintaining appropriate hydraulic jump efficiencies remains an indispensable condition to optimise the performance of a stilling basin. The hydraulic jump efficiency ($\eta$), based upon differences in the specific head upstream and downstream the hydraulic jump, gives a measure of the dissipated energy:

$$\eta = \frac{H_{01} - H_{02}}{H_{01}} \quad (21)$$

where $H_{01}$ and $H_{02}$ are respectively the specific energy heads upstream and downstream of the hydraulic jump. Padulano et al. (2017) and Macián-Pérez, Vallés-Morán, et al. (2020) obtained the hydraulic jump efficiency in a USBR Type-II basin for $F_1$ values around 9. Besides, using literature expressions (Eqs. 13 and 21), the efficiency in a CHJ with the studied inflow conditions can be estimated. Table 5 shows the modelled and bibliographic $\eta$ values for comparison purposes.

As it should be expected, these values are in line with the results shown in Table 3, due to the strong correlation between the sequent depths ratio and the hydraulic jump efficiency. Thus, the numerical models here presented provided a higher sequent depths ratio than the bibliographic results, leading to a lower efficiency. Focusing on the comparison between the original and the modified USBR II design, there is a slight increase (around 1.2%) in the efficiency caused by the additional chute blocks row, which also represents an improved performance of the stilling basin.

CONCLUSIONS

This research presents a detailed numerical model of a USBR Type-II stilling basin. The model was developed on the basis of CFD techniques and validated with a specifically-designed physical model. A modified design resulting from the addition of a second row of chute blocks was also implemented and tested. The comparison between both designs was carried out in terms of the hydraulic jump dimensions and the energy dissipation, in order to assess the performance of the basin. The results obtained were generally quite similar and in good agreement with literature studies. However, the modified design showed a reduction in the dimensions of the hydraulic jump. This conclusion was obtained after evaluation of the resulting sequent depths ratios, roller lengths, and hydraulic jump
efficiencies. In particular, the sequent depths ratio and the dimensionless roller length were respectively 2.5% and 1.4% lower in the modified design, whereas the energy dissipation efficiency increased a 1.2%. Consequently, the additional chute blocks row seems to enhance the performance of the basin. This modification constitutes an interesting novelty as it tends to reduce the expected dimension of the hydraulic jump, and thus, might be of interest in order to reduce the cost of the structure. It is difficult to quantify the potential discharge increase allowed by the reduction in the hydraulic jump dimensions provided by the modified design. However, a first estimation can be done. Using the graphs provided by the USBR for the Type-II stilling basin (Peterka, 1978) that establish a relationship between the hydraulic jump dimensions and the inflow conditions, the decrease observed in the sequent depths ratio and the dimensionless roller length is associated with a potential discharge increase around the 10%. Nevertheless, it is important to highlight that these results were obtained in the application range of the simulations and must be confirmed with further research, testing more demanding discharges. The inherent uncertainties around the definition and evaluation of the hydraulic jump length were also investigated. Two methods were tested and compared. The presented methods follow a structured and systematic procedure, based on quantitative information, and show consistent results for the developed models. Therefore, they might be useful for future studies in which the jump length needs to be determined.

The results here presented constitute a first step towards an optimised design of the USBR II stilling basin. In these terms, the proposed solution establishes a simple and straightforward modification starting from the traditional USBR Type-II design that can be used by engineers to reduce the dimensions of the hydraulic jump and still preserve the energy dissipation in the corresponding basin. These results must be confirmed by future research on the topic, testing different and more demanding inflow conditions ($F_1$, $Re$, $y_1$, $q$) and alternative modifications in the basin design. Furthermore, some other crucial features for the performance assessment of the basin such as the dynamic bottom pressures and the void fraction distribution need to be analysed. Overall, the numerical model developed was used to simulate a modified design of the USBR II stilling basin providing an optimised energy dissipation performance with reductions in the hydraulic jump dimensions. This information could be useful not only for the design of new energy dissipation structures, but also for the adaptation of existing basins to more demanding conditions posed by climate change and flood protection requirements.
DATA AVAILABILITY STATEMENT

All data, models and code generated or used during the study appear in the submitted article.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the collaboration of the Hydraulics Laboratory of the Department of Hydraulic Engineering and Environment from Universitat Politècnica de València (UPV) and their technicians Juan Carlos Edo and Joaquín Oliver in the construction of the experimental device used for the numerical model setup and validation. The work was supported by the research project: ‘La aireación del flujo y su implementación en prototipo para la mejora de la disipación de energía de la lámina vertiente por resalto hidráulico en distintos tipos de presas’ (BIA2017-85412-C2-1-R), funded by the Spanish Agencia Estatal de Investigación and FEDER.

NOTATION

The following symbols are used in this paper:

\[ A_s = \text{free surface area for each cell (m}^2\text{)} \]
\[ b = \text{hydraulic jump width (m)} \]
\[ C_\mu = \text{parameter for the turbulent length scale (-)} \]
\[ C_{1\varepsilon}, C_{2\varepsilon} = \text{turbulence model parameters (-)} \]
\[ f_b = \text{body forces (N)} \]
\[ F = \text{fraction of fluid (-)} \]
\[ F = \text{Froude number (-)} \]
\[ g = \text{gravity acceleration (m s}^{-2}\text{)} \]
\[ H_0 = \text{specific head (m)} \]
\[ I = \text{linear hydraulic head loss (-)} \]
\[ k = \text{turbulent kinetic energy (J kg}^{-1}\text{)} \]
\[ k_{\text{air}} = \text{air entrainment coefficient (-)} \]
\[ L_j = \text{hydraulic jump length (m)} \]
\[ L_r = \text{hydraulic jump roller length (m)} \]
\[ L_T = \text{turbulent length (m)} \]
\[ P_d = \text{stabilising forces (N m}^{-2}\text{)} \]
\[ P_k = \text{production of turbulent kinetic energy (kg m}^{-1}\text{ s}^{-2}\text{)} \]
\[ P_t = \text{destabilising forces (N m}^{-2}\text{)} \]
\[ p = \text{pressure (Pa)} \]
\[ q = \text{unit discharge (m}^2\text{ s)} \]
\[ R_e = \text{Reynolds number (-)} \]
\[ R_H = \text{hydraulic radius (m)} \]
\( t = \) time (s)  
\( \nu = \) velocity (m s\(^{-1}\))  
\( \nu^* = \) shear velocity (m s\(^{-1}\))  
\( x = \) distance along the basin longitudinal axis (m)  
\( x_0 = \) hydraulic jump toe position (m)  
\( y = \) flow depth (m)  
\( \gamma = \) specific weight (N m\(^3\))  
\( \delta V = \) volume of entrained air rate (m\(^3\) s\(^{-1}\))  
\( \varepsilon = \) turbulent kinetic energy dissipation rate (J kg\(^{-1}\) s\(^{-1}\))  
\( \eta = \) hydraulic jump efficiency (-)  
\( \mu = \) dynamic viscosity (Pa s)  
\( \nu = \) kinematic viscosity (m\(^2\) s\(^{-1}\))  
\( \rho = \) density (kg m\(^{-3}\))  
\( \sigma = \) surface tension coefficient (N m\(^{-1}\))  
\( \sigma_k, \sigma_\varepsilon = \) turbulence model parameters (-)  
\( \tau_0 = \) wall shear stress (Pa)  

REFERENCES


Fernández-Bono, J. F., & Vallés-Morán, F. J. 2006. “Criterios metodológicos de adaptación del diseño de cuencos de disipación de energía a pie de presa con resalto hidráulico, a caudales superiores a los de diseño” [Methodological criteria for the stilling basin design adaption to larger discharges than those considered in the design]. Proceedings of the 22nd Congreso Latinoamericano de Hidráulica.


Hirsch, C. 2007. Numerical computation of internal and external flows: The fundamentals of
computational fluid dynamics. John Wiley & Sons.


equation to estimate hydraulic jump length.” *Periodica Polytechnica Civil Engineering*, 62(4), 1001–1006.


**LIST OF FIGURES**

**Fig. 1.** Experimental set-up in the Hydraulics Laboratory of the Department of Hydraulic Engineering and Environment at the Universitat Politècnica de València (UPV, Spain)

**Fig. 2.** Validation of the numerical model: (a) mean free surface longitudinal profile, (b) maximum forward velocity decay

**Fig. 3.** Modelled stilling basins: (a) typified USBR II stilling basin, (b) modified design with an additional chute blocks row

**Fig. 4.** Velocity field obtained in the numerical model: (a) typified USBR II stilling basin, (b) modified design with an additional chute blocks row

**Fig. 5.** Hydraulic jump free surface profile: (a) mean longitudinal profile in the typified USBR II stilling basin and in the modified design, (b) dimensionless comparison with bibliographic data for CHJ

**Fig. 6.** Roller length estimation following the stagnation point criterion: (a) typified USBR II stilling basin, (b) modified design

**Fig. 7.** Subcritical velocity profiles: (a) bibliographic profile in open channel flow, (b) modelled profiles downstream the typified USBR II basin, (c) modelled profiles downstream the modified USBR II basin

**Fig. 8.** Turbulent kinetic energy decay along the hydraulic jump longitudinal axis
Table 1. Cell sizes tested in the mesh convergence analysis

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Refined mesh block</th>
<th>Coarse mesh block</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.014 m</td>
<td>0.028 m</td>
</tr>
<tr>
<td>2</td>
<td>0.010 m</td>
<td>0.020 m</td>
</tr>
<tr>
<td>3</td>
<td>0.007 m</td>
<td>0.014 m</td>
</tr>
</tbody>
</table>

Table 2. Case study flow conditions

<table>
<thead>
<tr>
<th>Q (m³/s)</th>
<th>q (m²/s)</th>
<th>y₁ (m)</th>
<th>u₁ (m/s)</th>
<th>F₁</th>
<th>Re₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.113</td>
<td>0.147</td>
<td>0.03</td>
<td>4.89</td>
<td>9</td>
<td>146,753</td>
</tr>
</tbody>
</table>

Table 3. Sequent depths ratio for the numerical models and for bibliographic studies

<table>
<thead>
<tr>
<th>Case</th>
<th>Hydraulic jump conditions</th>
<th>y₂/y₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical model</td>
<td>Typified USBR II stilling basin</td>
<td>14.43</td>
</tr>
<tr>
<td>Numerical model</td>
<td>Modified USBR II stilling basin</td>
<td>14.06</td>
</tr>
<tr>
<td>(Hager &amp; Bremen, 1989)</td>
<td>CHJ</td>
<td>12.17</td>
</tr>
<tr>
<td>(Schulz et al., 2015)</td>
<td>CHJ</td>
<td>11.73</td>
</tr>
<tr>
<td>(Macián-Pérez, Vallés-Morán, et al., 2020)</td>
<td>Typified USBR II stilling basin</td>
<td>12.00</td>
</tr>
<tr>
<td>(Padulano et al., 2017)</td>
<td>Typified USBR II stilling basin</td>
<td>10.18</td>
</tr>
</tbody>
</table>

Table 4. Hydraulic jump length for the numerical models and for bibliographic studies

<table>
<thead>
<tr>
<th>Case</th>
<th>Hydraulic jump conditions</th>
<th>Methodology</th>
<th>Lj/y₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical model</td>
<td>Typified USBR II stilling basin</td>
<td>Velocity profiles</td>
<td>80.0</td>
</tr>
<tr>
<td>Numerical model</td>
<td>Modified USBR II stilling basin</td>
<td>Velocity profiles</td>
<td>80.0</td>
</tr>
<tr>
<td>(Hager, 2013)</td>
<td>CHJ</td>
<td>Empirical expression</td>
<td>76.7</td>
</tr>
<tr>
<td>(Movahed et al., 2018)</td>
<td>CHJ</td>
<td>Semi-analytical expression</td>
<td>71.3</td>
</tr>
<tr>
<td>(Peterka, 1978)</td>
<td>Typified USBR II stilling basin</td>
<td>Collected data</td>
<td>58.7</td>
</tr>
</tbody>
</table>

Table 5. Hydraulic jump efficiency for the numerical models and for bibliographic studies

<table>
<thead>
<tr>
<th>Case</th>
<th>Hydraulic jump conditions</th>
<th>η</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical model</td>
<td>Typified USBR II stilling basin</td>
<td>0.648</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Numerical model</td>
<td>Modified USBR II stilling basin</td>
<td>0.656</td>
</tr>
<tr>
<td>(Hager &amp; Bremen, 1989)</td>
<td>CHJ</td>
<td>0.701</td>
</tr>
<tr>
<td>(Macián-Pérez, Vallés-Morán, et al., 2020)</td>
<td>Typified USBR II stilling basin</td>
<td>0.705</td>
</tr>
<tr>
<td>(Padulano et al., 2017)</td>
<td>Typified USBR II stilling basin</td>
<td>0.720</td>
</tr>
</tbody>
</table>