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Continuous Earned Value Management with Singularity Functions for Comprehensive Project Performance Tracking and Forecasting

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Abstract

Earned value management (EVM) is a classical project monitoring technique that is widely used in construction projects. Due to its simplicity, this technique suffers from limitations due to its discrete nature – activity durations, costs, and progress are gathered only at update points with no information in between. These limitations preclude EVM from being easily implemented on some project types (e.g. repetitive projects) and in conjunction with some planning techniques (e.g. linear scheduling), where information continuity is both possible and desired. Therefore, in EVM is reformulated based on singularity functions (SF). SF are a type of expressions that can be easily concatenated to model continuous inputs at the activity-level. SF are also additive so as to immediately yield project-level performance information. It is demonstrated how the complete theory of EVM is newly expressed in SF. This offers several advantages: (1) EVM metric axes can be easily swapped (allowing exact calculation of modern metrics such as Earned Schedule or the *p*-factor); (2) activity progress data can be inserted at any frequency as the available data allow; and (3) *short-term* project duration and cost forecasts are directly possible for the first time. These advantages are exemplified on a real construction project. Finally, it is discussed how the new formulation with SF produces more accurate project duration and cost estimates compared to the former discrete EVM on real construction projects.

Keywords: Earned Value Management; singularity functions; construction projects; continuity; time forecasting; cost forecasting; performance.

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List of Abbreviations

AC	=	actual cost			
ACF	=	actual cash flow	PC	=	planned cost
AD	=	actual duration	PCF	=	planned cash flow
AP	=	actual percent (of completion)	PD	=	planned duration
AQ	=	actual quantity (of work)	PES	=	planned earliest start
AT	=	actual time	PLS	=	planned latest start
BAC	=	budget at completion	PF	=	performance factor
CV	=	cost variance	PQ	=	planned quantity (of work)
CPI	=	cost performance index	PS	=	planned start date
CSI	=	cost schedule index	PV	=	planned value
EAC	=	estimate at completion	RAC	=	real cost at completion
EV	=	earned value	SF	=	singularity function
EVM	=	earned value management	SPI	=	schedule performance index
ES	=	earned schedule	SV	=	schedule variance

1. Introduction

The EVM is a deterministic technique that draws information from individual project activities. At its core, EVM combines information about costs and durations of ongoing activities and work completion percentages as essential inputs. When activities are completed, their percentage of completion (progress) reaches 100%, and their final cost and duration are registered as the actual (final) ones. From them, EVM assesses the current status of the project to answer questions like *Are we late or early? Are we spending more or less than planned?* and gives long-term forecasts *What will be the final project cost and duration? When would it end based on current progress?*

But as we will show, EVM suffers from the conceptual limitation of being discrete: Its values are known only at those updates; it remains blind in between. This renders it an ineffective instrument for short-term predictability. We will remedy this shortcoming with a novel approach that creates a continuous, additive, and extensible mathematical formulation. It will be tested for validation purposes on the activities dataset of a real-world project to demonstrate its efficacy.

2. Literature Review

Earned Value Management (EVM) is a monitoring technique that allows project managers to track the performance of projects in their time and cost dimensions relative to baseline values [1, 2]. It was devised as a financial analysis tool by the U.S. Department of Defense (DoD) in the 1960s. But until the early 1980s, it remained largely ignored by project managers. Yet three events accelerated its adoption. First, an article published in the *Public Works Magazine* by David Burstein in 1979, which described how EVM had been successfully implemented in an architecture and construction company [3]. The second event was the cancellation of the Navy A-12 Avenger II Program by the DoD, because of performance problems that were detected only after the implementation of EVM [4]. Third, the publication of an EVM industry standard by the American National Standards Institute (ANSI EIA 748-A). From that time on, EVM adoption spread very quickly, especially after this technique was included in the first Project Management Body of Knowledge (PMBOK) guide in 1987. It was then adopted by many U.S. government agencies, e.g. the National Aeronautics and Space Administration and the Department of Energy. More recently, EVM has also been standardized in regions such as Australia (AS 4817-2003 / 2006) and Europe (ISO 21508:2018). Paralleling its adoption by governments, practitioners, certification, and professional bodies, EVM has also received extensive research attention [5, 6].

2.1. Limitations

Problematically, the inputs forthis technique – progress, costs, and durations of activities – are not collected in a continuous manner. Instead, it is compiled after approximately regular time intervals commonly known as *tracking periods* [7]. In most construction projects such tracking periods are interspersed about every one or two months [8]. Gathering information in shorter time spans is possible, but often too time-consuming [9]. Hence, most project managers perform it only in conjunction with preparing the next schedule update and invoice to the client [10].

But this comes at a disadvantage. The project manager knows what the current project status is at every tracking period, but not in between. One may think that it should be relatively easy to extrapolate the project status from those activities that have experienced significant progress since the last tracking period. But this is not the case. In EVM, either all activity information is updated at the current project date (called *actual time*, AT [11]) or none can be updated. Otherwise the EVM metrics calculation, e.g. earned Value (*EV*) and Earned Schedule (*ES*), will incur errors. Take the earned schedule (*ES*) metric proposed by Lipke in 2003 [12]: Calculating it requires performing a linear interpolation between the two adjacent tracking period dates whose planned value (*PV*) is below and above the current Earned Value (*EV*). However, the costs of all ongoing activities are attributed to the succeeding tracking period. That is, if an activity was 98% complete at the end of tracking period n, the cost for the remaining 2% is distributed linearly between tracking periods n and n+1. But this ignores whether that activity will finish one day or one month after n. The EVM method here assumes that costs are incurred throughout the entire next tracking period. The same problem will affect activities that have not started at n, but do so in tracking period n+1. Overall, this lumps all metrics into interpolations over tracking periods, which causes significant accuracy losses when implemented in real-life projects with durations of tracking periods longer than a week [13]. To address this issue, reformulations of *ES* that are calculated at the activity-level have recently been proposed, e.g. the new ES_{min} and ES_{max} metrics [6]. While these enhanced metrics better interpolate the actual project progress, they still only work with *planned* activity start and finish dates, not with *actual* activity start and finish dates.

Consequently, project managers cannot know accurately their current project status, except at *tracking points*. This is especially problematic when real projects experience cost and / or time overruns and the project manager must take immediate corrective actions, but will have to wait until the next tracking point to know if the action has actually worked. By then, it may be too late to adjust said action, as many ongoing activities may already have finished or are nearing finish.

Hence, in this paper we propose a new formulation that generates a fully continuous EVM model. With this model, project managers can input future activity start and end dates besides other modelling capabilities. This will eliminate the accuracy loss that is caused by the absence of updates between tracking points, which is especially important in projects with repetitive work cycles, i.e. repetitive projects. In these, the same activities are performed multiple times and the interpolation errors between tracking points are prone to be mistaken for underperformance. By formulating a continuous model, we will also be able to implement EVM in linear schedules, a continuous planning and monitoring technique. Our model will employ Singularity Functions (SF), which are a type of highly flexible functions that can be concatenated to describe complex composite curve patterns. SF are described in detail in a following section and will provide the necessary modelling capabilities to remedy the existing flaws of EVM with our new formulation.

EVM is a straightforward method that compares the budgeted cost of work performed against either the budgeted cost of work scheduled to evaluate the project time performance, or against the actual cost of work performed to evaluate the project cost performance. The budgeted cost of work performed is often called *earned value* (*EV*); the budgeted cost of work scheduled is shortened to *planned value* (*PV*); and the actual cost of work performed is the *actual cost* (*AC*).

A major limitation of EVM is that it does not differentiate the actual status of individual work units. All activity contributions to the overall project performance measurement are assumed as proportional to their cost, regardless of whether those activities belong to paths that are ahead or delayed [14, 15]. Thus by definition EVM fails to provide an accurate view of how a project is performing in the time dimension. Another limitation of EVM is the impossibility of considering activity duration and cost variability in the planned value curve (i.e. the cumulative cash flow curve that represents the project performance baseline) [16]. This results in the overall project duration frequently being underestimated (see [17] for a more exhaustive description of this phenomenon). These limitations could be partially overcome by combining EVM with other approaches. In this vein, EVM has received many extensions in the last 20 years [6]. It has been combined with Monte Carlo simulation [18, 19], neural networks [20, 21], fuzzy logic [22, 23], Bayesian inference [24], machine learning methods [25-27], and Kalman filter algorithms [24, 28], to cite a few. But none of them have improved its fundamentally flawed basic calculations.

Similarly EVM has also been combined with planning techniques to produce more accurate project duration and / or cost forecasts. Examples are combinations of EVM with PERT [29-35] or Critical Chain [36]. More recently Chang *et al.* [37] and Ballesteros-Pérez *et al.* [9] studied how the accuracy of EVM forecasts at tracking points could be improved. But they could not solve the complete lack of in-between information, because their approach remained discrete.

Perhaps the most notable addition to EVM in recent years has been the creation of the earned schedule (*ES*) metric [38]. Until then, EVM counterintuitively defined monetary units [*sic*!] to express its time deviations in absolute terms. But *ES* allowed measuring time deviations in time units, among other advantages [39]. It also opened the door to a partial reformulation of EVM to what has become known as Earned Duration Management (EDM) [40]. EDM is similar to EVM, but only applies to project time performance, not cost. It compares actual versus planned activity progress and durations (instead of activity costs as in EVM). Yet both remain merely discrete.

2.2. Need

Overall the lack of continuity in EVM and EDM renders them inaccurate in project performance measurement between tracking points. This is especially important for short-term forecasting. While recently many studies have tried to improve long-term forecasting capabilities [13, 38, 40-49], short-term forecasting has been almost completely ignored. By short-term forecasting we mean anticipating how the project time and cost performance will change within the next few workdays. This is a vital feature, because one can only know if corrective actions may actually work when seeing their short-term impact in the EVM metrics (particularly *EV* and *ES*). But any recent project data are only processed at the next tracking point. Until then, even if some of the activities have already finished and some others have already started, the overall project performance does not reflect these updates. This major problem at the core of EVM itself can only be solved by adopting a continuous model. By gaining such a model, project managers could at any time not just add input information on activity starts, finishes, or any duration and cost intermediate values without having to wait until the next 'official' *tracking point*, but more importantly, would immediately receive current feedback on the evolution of the project performance metrics. This is especially useful for projects that exhibit repetitive activities within their construction process, as is commonly found in practice. Repetitions should be tracked to fine-tune their progress. Another advantage would be the ability to integrate such model and its metrics directly with the already-continuous linear scheduling method. With it one could finally study activity and project performance at the fullest level of detail that the available data allow.

3. Methodology

3.1. Goal and Objectives

The goal of this research is to develop the complete theory for a continuous EVM model. To achieve this, three objectives will be addressed in turn, as is explained in the following sections.

1. Create a continuous model with singularity functions and convert existing EVM metrics to it;

2. Validate the model on a real-world example by drawing observations from its performance;

3. Explore the *short-term* forecasting capability from activity-level updates to the project-level.

3.2. Singularity Functions

Singularity functions are continuous expressions that generalize regular polynomial expressions. They were independently conceived by Föppl [50] and Macaulay [51] over a century ago for use in structural engineering. There they efficiently modeled varying loads over the length of beams. Due to their flexible nature, SF have also been used in the scheduling domain for work over time, resources over time, and cost over time [52]. Regardless of their area of application, SF perform a case distinction in their operator in Equation 1 whether its term is not yet active, i.e. zero – at *x*-values before the activation *a* – or active from there onward for a given phenomenon.

$$s \cdot \langle x-a \rangle^n = \begin{cases} 0 & if \quad x < a \\ s \cdot (x-a)^n & if \quad x \geq a \end{cases} \qquad \textbf{Eq. 1}$$

Singularity functions are named after their key feature, the singularity at *a* itself. Note that it does not mean discontinuity – the function is continuously defined and integratable and differentiable like any traditional one. Each function is the algebraic sum of *N* basic terms *i* as per Equation 2.

$$y(x)_{sng\_fnc} = \sum_{i=1}^{N} \left(s_i \cdot \langle x-a_i \rangle^{n_i} \right) \qquad \textbf{Eq. 2}$$

Multiple terms with the same *a* and *n* are shortened by adding their strengths *s*. Modeling growth will typically have a linear behavior with a slope term of $n = 1$, while $n = 0$ indicates a step term, in other words a sudden jump or drop in the function output value.

A complete SF contains an initial *s*-value plus change terms $\Delta y/\Delta x$ that update its value at later activations. In our context the activities will be represented by a SF whose *s*-value corresponds to the activity's (planned, actual, or earned) unit cost (per time unit), with an activation point *a* when the activity starts, and a second (de)activation point when the same activity is completed.

4. Model Development

Our continuous model will be developed on the four-activity network schedule of Figure 1. Its activity nodes A to D list their planned duration (*PD*) and planned value (*PV*). Each definition or transformation will be explained by deriving a formula and inserting values to demonstrate its practical application. We will manually solve all equations for transparency to allow independent verification of the results [29]. Of course, computer implementation could handle calculations for real projects with a larger number of activities. We will develop first the basic EVM metrics (*PV*, *AC*, *EV*), then other more recent metrics and performance indices (e.g. *ES*, ES_{min}, *SV*(*t*), *SPI*(*t*)). For clarity some mathematical details are found in **Appendices** (Supplemental online material).

4.1. Planned Value

Using the additive nature of SF, the planned value (*PV*) at the project-level is the sum of the activity *PV* values. Each can be written per Equation 3, which is activated at the planned start (*PS*) and is later deactivated at the planned finish (*PS*) after its planned duration *PD* = *PF* - *PS*.

$$y(x)_{\mathrm{PV\_activity}} = PV/PD \cdot [\langle x\text{-}PS\rangle^1 - \langle x\text{-}PF\rangle^1] \qquad \textbf{Eq. 3}$$

For example, activity A has *PV* = \$210 and *PD* = 2 days for $y(x)_{\mathrm{PV\_A}} = \$105/1\mathrm{d} \cdot [\langle x\text{-}0\rangle^1 - \langle x\text{-}2\rangle^1]$. This means activity A begins causing cost from its start (which coincides with the project start at 0 days); will grow linearly at a rate of \$105 per day until day 2; and then remains constant at *PV* = \$210 until the project ends. **Appendix 1-A** provides all values and equations for activities A to D as well as a bar chart with planned cash flow (*PCF*) over actual time (*AT*). *AT* is the standard term for project elapsed time in EVM, which in our equations coincides with the horizontal variable *x*. The sum of Equations 3 composes the project *PV* in Equation 4.

$$y(x)_{\mathrm{PV\_project}} = \Sigma y(x)_{\mathrm{PV\_activity}} \qquad \textbf{Eq. 4}$$

The project *PV* thus is $y(x)_{\mathrm{PV\_project}} = \$105/1\mathrm{d} \cdot \langle x\text{-}0\rangle^1 + (-\$105 + \$235)/1\mathrm{d} \cdot \langle x\text{-}2\rangle^1 + (-\$235 + \$75)/1\mathrm{d} \cdot \langle x\text{-}5\rangle^1 - \$75/1\mathrm{d} \cdot \langle x\text{-}9\rangle^1$ After merging terms with the same cutoff, this simplifies to $y(x)_{\mathrm{PV\_project}} = \$105/1\mathrm{d} \cdot \langle x\text{-}0\rangle^1 + \$130/1\mathrm{d} \cdot \langle x\text{-}2\rangle^1 - \$160/1\mathrm{d} \cdot \langle x\text{-}5\rangle^1 - \$75/1\mathrm{d} \cdot \langle x\text{-}9\rangle^1$. This cumulative expression is the baseline for the project and plotted with a dashed curve in Figure 2.

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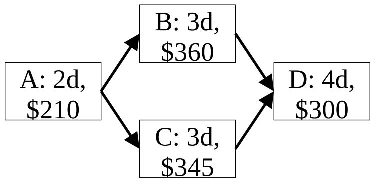

Figure 1: Development Example

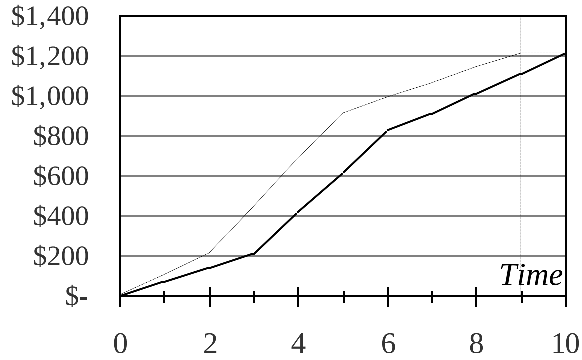

Figure 2: Project Planned Value (Dashed) and Earned Value (Solid) Curves

The project planned duration of 9 days is the maximum of activity planned finishes per Equation 5. Here the dashed curve in Figure 2 flattens, because its cost growth has been removed. This serves as a checksum, because at the project end all activity cost slopes should now add to zero.

$PD_{\text{project}} = \max\{PF_{\text{activity}}\}$ **Eq. 5**

Evaluating Equation 4 at $x = PD_{\text{project}}$ gives the project budget at completion (*BAC*, Equation 6).

$BAC_{\text{project}} = y(PD_{\text{project}})_{\text{PV\_project}}$ **Eq. 6**

This illustrates how SF are evaluated: $BAC_{\text{project}} = y(9\text{d})_{\text{PV\_project}}$ = \$105/1d · $\langle 9\text{-}0\rangle^1$ + \$130/1d · $\langle 9\text{-}2\rangle^1$ - \$160/1d · $\langle 9\text{-}5\rangle^1$ - \$75/1d · $\langle 9\text{-}9\rangle^1$ = \$945 + \$910 - \$640 - \$0 = \$1215, which is the correct sum of the activity *PV* from Figure 1. Note that the last term at PD_{project} can often be skipped, as it has only just been activated and -\$75/1d · $\langle 9\text{-}9\rangle^1$ is still \$0. This reduces the evaluation effort.

4.2. Actual Cost

Progress is measured as actual costs (*AC*) and activities' actual starts and finishes (*AS*, *AF*) to calculate their actual duration (*AD*). This information is only available as the project is carried out. Imagine that for A to D they are *AC* = {\$210, \$390, \$300, \$240} and *AD* = {3d, 3d, 4d, 3d}. This delays the *AS* of D by one day. **Appendix 1-B** provides all equations and the actual cash flow (*ACF*) over actual time (*AT*). In analogy to Equation 3, activity *AC* is written as Equation 7.

$y(x)_{\text{AC\_activity}} = AC/AD \cdot [\langle x\text{-}AS\rangle^1 - \langle x\text{-}AF\rangle^1]$ **Eq. 7**

Here activity A has *AC* = \$210 and *AD* = 3 days for the actual cost slope in $y(x)_{\text{AC\_A}}$ = \$210/3d · $[\langle x\text{-}0\rangle^1\text{-}\langle x\text{-}3\rangle^1]$. As for *PV*, it is commonly assumed that *AC* is incurred linearly during execution. Its value grows at that slope from *AS* until the current tracking period. Of course, one cannot know *AF* before it occurs – the model explicitly does *not* require writing it in the equation until the activity finishes and the slope is removed then. If this linearity assumption is not suitable, one can write a step function at the actual time of incurring any such a single expense per Equation 8.

$y(x)_{\text{AC\_activity\_step}} = AC \cdot \langle x\text{-}AT\rangle^0$ **Eq. 8**

Summing all Equations 7 gives the project *AC* per Equation 9 as $y(x)_{\text{AC\_project}}$ = \$210/3d · $[\langle x\text{-}0\rangle^1 - \langle x\text{-}3\rangle^1]$ + \$390/3d · $[\langle x\text{-}3\rangle^1 - \langle x\text{-}6\rangle^1]$ + \$300/4d · $[\langle x\text{-}3\rangle^1 - \langle x\text{-}7\rangle^1]$ + \$240/3d · $[\langle x\text{-}7\rangle^1 - \langle x\text{-}10\rangle^1]$ =

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$\$70/1\text{d} \cdot \langle x\text{-}0\rangle^1 + \$135/1\text{d} \cdot \langle x\text{-}3\rangle^1 - \$130/1\text{d} \cdot \langle x\text{-}6\rangle^1 + \$5/1\text{d} \cdot \langle x\text{-}7\rangle^1 - \$80/1\text{d} \cdot \langle x\text{-}10\rangle^1$. This can now be evaluated at any x-value to give the spending up to then. Yet its cumulative actual cost slopes also offer an automatic ability to forecast project cost in the short-term future. Namely Equation 9 extrapolates it until the next *AF* of any ongoing activity or *AS* of any newly started one is recorded, which the updates its slope with the newest information as it becomes available.

$$y(x)_{\text{AC\_project}} = \Sigma y(x)_{\text{AC\_activity}} \qquad \textbf{Eq. 9}$$

Finally, for the sake of completeness, the project *AD* is the maximum of activity *AF* per Equation 10, here 10 days. In analogy to Equation 6, the real cost at completion (*RAC*) is gained by inserting AD_{project} into Equation 9 per Equation 11, which is $RAC_{\text{project}} = y(10\text{d})_{\text{AC\_project}} = \$1140$.

$$AD_{\text{project}} = \max\{AF_{\text{activity}}\} \qquad \textbf{Eq. 10}$$

$$RAC_{\text{project}} = y(AD_{\text{project}})_{\text{AC\_project}} \qquad \textbf{Eq. 11}$$

4.3. Earned Value

By definition, *EV* is the actual percent *AP* (of completion) of an activity multiplied by its planned cost *PC*. In other words, $EV = AP \cdot PC$. This *AP* per Equation 12 is not defined via *PV* or *PD*, but for the actual quantity *AQ* in work units performed so far (e.g. cubic meters of concrete) relative to the planned quantity *PQ*, which is known from the design (taken off plan drawings). Hence:

$$y(x)_{\text{AP\_activity}} = AQ/PQ \cdot \langle x\text{-}AS\rangle^1 \qquad \textbf{Eq. 12}$$

Appendix 1-C lists all *AP* values for our example, which in practice come from the same records as *AS* and *AF* in **Appendix B**. Equation 13 multiplies *PC* onto each activity *AP* to determine *EV*.

$$y(x)_{\text{EV\_activity}} = PC \cdot y(x)_{\text{AP\_activity}} \qquad \textbf{Eq. 13}$$

To write the SF, **Appendix 1-D** extracts differences of *AP* as the slope changes at *AS* and *AF* for the *AP* equations. Summing Equations 13 (**Appendix 1-E**) gives the project *EV* in Equation 14.

$$y(x)_{\text{EV\_project}} = \Sigma y(x)_{\text{EV\_activity}} \qquad \textbf{Eq. 14}$$

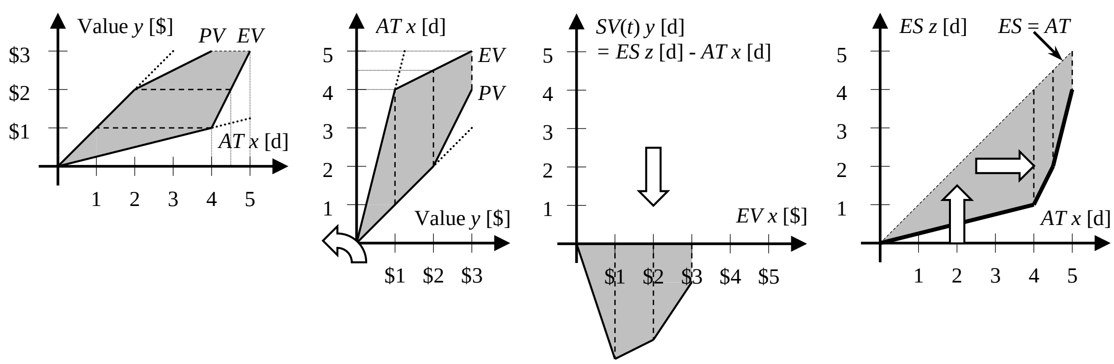
For our example the short form of Equation 14 is $y(x)_{\text{EV\_project}} = \$70/1\text{d} \cdot \langle x\text{-}0\rangle^1 + 136.25/1\text{d} \cdot \langle x\text{-}3\rangle^1 - \$120/1\text{d} \cdot \langle x\text{-}6\rangle^1 + \$13.75/1\text{d} \cdot \langle x\text{-}7\rangle^1 - \$100/1\text{d} \cdot \langle x\text{-}10\rangle^1$. It is represented by the solid line in Figure 2. As real-world projects will have longer equations, they would of course be handled by a computer implementation. Yet a strength of SF is that they can always easily be evaluated manually for verification purposes or in educational settings with smaller examples similar to this one. At the project end $AD = 10$ days it correctly gives \$1215, which equals the output of Equation 6 at $AT = 9$ days (the project planned value at its planned duration, i.e. the *BAC*).

4.4. Earned Schedule

The earned schedule (*ES*) [13] is a conceptually straightforward and well-known project-level metric. It compares the *PV* and *EV* curves in Figure 2: *At the current time, we have a certain EV. At what (earlier or later) time did the PV curve have that same value?* Essentially, this analysis moves through a *PV-EV* chart in three steps [53]: First from the current time (or tracking period) on the horizontal axis vertically up to the *EV* curve. Then horizontally over to the intersection with the *PV* curve. And finally back down to the time x-axis, where we read how much time the ongoing work has really 'earned' so far. If *ES* is to the left of the actual (current) time *AT*, then the project is behind schedule. If it lays to the right of *AT*, then the project is ahead of schedule.

Yet to develop the SF of *ES*, an intermediate step is needed. *ES* is a metric whose input is a time and its output is also in time units. This differs from SF of the three previous basic metrics (*PV*, *AC*, and *EV*) that were expressed as money over time. To compose such a new expression, we illustrate this approach with Figure 3. Conceptually speaking, we derive its steps as follows:

1. Figure 3a plot both *PV* and *EV* over time *AT*. Their difference along the horizontal time *x*-axis indicates how far the project is ahead or behind relative to perfect performance. This difference will later be called *SV*(*t*);
2. Yet taking a difference between *PV* and *EV* (Equations 4 and 14, respectively) acts in the vertical direction. To measure their horizontal (time) difference, we will switch (transpose) both axes. This operation simply rotates the chart by 90 degrees which results in Figure 3b;
3. Their (time) difference after rotation – but still expressed as a function of *EV* (money) units on the *x* axis – is represented in Figure 3c;
4. Finally, the *x*-axis in money units is replaced with *AT*. In this operation, what are the correct cutoffs on this new *AT* axis for our *ES* values? All those in which either the original *PV* or *EV* curves had a slope change. And since perfect project performance means *ES* = *AT* (the bisector or diagonal line), said difference is drawn relative to said *diagonal* to give the *ES* curve. This last step is represented in Figure 3d with a bold curve.



3a: Initial **3b: Transposed** **3c: Difference** **3d: Diagonal, Final**

Figure 3: Calculation steps of *ES* from the *PV* and *EV* curves

In summary, for every tracking period *AT*, we must find the time when *PV* equals the current *EV*. Yet this is only possible if (a) *PV* and *EV* are continuous expressions; and (b) we can obtain the exact value of *PV* (dependent variable) from *EV* (independent variable). Note that due to EVM's inherent discrete nature, conditions (a) and (b) would remain unfulfilled unless we employ SF. Such an exact calculation of the *ES* metric has been impossible in traditional EVM.

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To fulfill condition (b), both *PV* and *EV* equations must be transposed. Axis transposition in SF has already been implemented for resource leveling [54] in construction scheduling, resulting in Figure 3b. Such transposition converts all slopes from $\Delta y/\Delta x = (y_2\text{-}y_1)/(x_2\text{-}x_1)$ to $\Delta x/\Delta y = (x_2\text{-}x_1)/(y_2\text{-}y_1)$ – recall that SF contain change terms after an initial slope. It also updates their original *x*-cutoffs to *y*-values by evaluating the original SF at all *x*-cutoffs. Thus instead of first summing all of the activities and transposing the resulting project curves, it would also equivalently be possible to first invert activities and then adding them, which is the commutative property of SF.

Now the difference SF has the shape of Figure 3c, yet still needs to be modified to a dual time axis. This is accomplished by adjusting its cutoffs to the known singularities in the *PV* and *EV* curves to stretch the gray shape along the *x*-axis (Figure 3d). Taking perfect performance as the diagonal [55], each day of actual time earns one day. From this optimal line, the gray shape is subtracted to gain the final *ES* curve over *AT*, which is marked with a thick line in Figure 3d.

Appendix 1-F lists slope change terms for transposing *PV* and *EV* as $x(y)_{\mathrm{PV\_project}} = 1\mathrm{d}/\$105 \cdot \langle y\text{-}\$0\rangle^1 - 26\mathrm{d}/\$4935 \cdot \langle y\text{-}\$210\rangle^1 + 32\mathrm{d}/\$3525 \cdot \langle y\text{-}\$915\rangle^1 - 1\mathrm{d}/\$75 \cdot \langle y\text{-}\$1215\rangle^1$ and $x(y)_{\mathrm{EV\_project}} = 1\mathrm{d}/\$70 \cdot \langle y\text{-}\$0\rangle^1 - 109\mathrm{d}/\$11550 \cdot \langle y\text{-}\$210\rangle^1 + 128\mathrm{d}/\$18975 \cdot \langle y\text{-}\$828.75\rangle^1 - 11\mathrm{d}/\$6900 \cdot \langle y\text{-}\$915\rangle^1 - 1\mathrm{d}/\$100 \cdot \langle y\text{-}\$1215\rangle^1$. **Appendix G** compiles the cutoffs from both *PV* and *EV* and calculates the slope changes of *ES*. Equation 15 casts this entire intermediate step for *ES* into a compact SF.

$$z(x)_{\mathrm{ES\_project}} = 1\mathrm{d}/1\mathrm{d} \cdot \langle x\text{-}0\rangle^1 - \Sigma[\Delta(z(x(y)_{\mathrm{EV}}\text{-}x(y)_{\mathrm{PV}})/\Delta x(y_{\mathrm{i}})\,) \cdot \langle x\text{-}x(y_{\mathrm{i}})\rangle^1] \qquad \textbf{Eq. 15}$$

Its square bracket is the schedule variance in time units $y(x)_{\mathrm{SV(t)\_project}}$ that is defined in the next section. The intermediate step gives $z(x)_{\mathrm{ES}} = 2/3 \cdot \langle x\text{-}0\mathrm{d}\rangle^1 + 119/564 \cdot \langle x\text{-}3\mathrm{d}\rangle^1 - 24/47 \cdot \langle x\text{-}6\mathrm{d}\rangle^1 + 545/564 \cdot \langle x\text{-}7\mathrm{d}\rangle^1 - 1/3 \cdot \langle x\text{-}10\mathrm{d}\rangle^1$. Evaluating it gives $z(10\mathrm{d})_{\mathrm{ES}} = 20/3 + 833/564 - 96/47 + 545/188 + 0 = 9$. This means that the project has ended 1 day late.

4.5. *Earned Schedule Minimum and Maximum*

Many EVM metrics and extensions were proposed in the last 20 years [6]. It is beyond the scope of this paper to review them all. So far, we have focused on classical ones (*PV*, *AC* and *EV*) plus a more recent significant metric, *ES*. To demonstrate that upgrading the EVM framework into SF is not just feasible, but will gain real calculation advantages, we review other significant ones.

Building on the previous *ES* metric, Ballesteros-Pérez *et al.* [13] introduced the earned schedule min-max (ES_{min}, ES_{max}) metrics at the activity-level, which are also expressed in time units. They measure project progress by the progress of its most delayed path (ES_{min}) or its more ahead path (ES_{max}). This refines *ES* (see [6] for details). Per Equations 16-17, ES_{min} and ES_{max} are measured from the planned earliest start date (*PES*) and planned latest start dates (*PLS*) of said activity:

$$z(x)_{\mathrm{ES\_activity\_min}} = PLS_{\mathrm{activity}} + PD_{\mathrm{activity}} \cdot y(x)_{\mathrm{AP\_activity}} \qquad \textbf{Eq. 16}$$

$$z(x)_{\mathrm{ES\_activity\_max}} = PES_{\mathrm{activity}} + PD_{\mathrm{activity}} \cdot y(x)_{\mathrm{AP\_activity}} \qquad \textbf{Eq. 17}$$

Note that the project-level values per Equations 18-19 use cropped datasets: For $ES_{\mathrm{project\_min}}$ only activities with $AP < 100\%$ are included; for $ES_{\mathrm{project\_max}}$ only activities with $AP > 0\%$. Therefore each activity equation is multiplied by a selection operator $s(y_{\mathrm{AP\_activity}})_{\mathrm{min}} = [1\text{-}\langle 0\text{-}y_{\mathrm{AP\_activity}}\rangle^0]_{\mathrm{min}}$ or $s(y_{\mathrm{AP\_activity}})_{\mathrm{max}} = [1 - \langle y(x)_{\mathrm{AP\_activity}}\text{-}1\rangle^0]_{\mathrm{max}}$ that returns 1 for all valid values or 0 otherwise. These case distinctions, which select from the current data, are straightforward by employing SF:

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$$z(x)_{ES\_project\_min} = \Sigma\{[1 - \langle y(x)_{AP\_activity} - 1\rangle^0]_{min} \cdot z(x)_{ES\_activity\_min}\} \quad \textbf{Eq. 18}$$

$$z(x)_{ES\_project\_max} = \Sigma\{[1 - \langle 0 - y(x)_{AP\_activity}\rangle^0]_{max} \cdot z(x)_{ES\_activity\_max}\} \quad \textbf{Eq. 19}$$

Appendix 1-H tabulates the values and selections of Equations 18-19, and **Appendix 1-I** lists all of the activity-level ES_{min} and ES_{max} equations for completeness. Note that due to the symmetric structure of our development example, all activities happen to be critical and simply *PES* = *PLS*.

4.6. Variances and Indices

The cost and schedule variances (*CV* and *SV*) are differences of metrics per Equations 20 and 21.

$$y(x)_{CV\_project} = y(x)_{EV\_project} - y(x)_{AC\_project} \quad \textbf{Eq. 20}$$

$$y(x)_{SV\_project} = y(x)_{EV\_project} - y(x)_{PV\_project} \quad \textbf{Eq. 21}$$

Writing them with values gives $y(x)_{CV\_project} = + \$1.25/1d \cdot \langle x-3\rangle^1 + \$10/1d \cdot \langle x-6\rangle^1 + \$8.75/1d \cdot \langle x-7\rangle^1 - \$20/1d \cdot \langle x-10\rangle^1$, which evaluates as $y(10d)_{CV\_project} = \$8.75 + \$40 + \$26.25 - \$0 = \$75$. And inserting values gives $y(x)_{SV\_project} = - \$35/1d \cdot \langle x-0\rangle^1 - \$130/1d \cdot \langle x-2\rangle^1 + \$136.25/1d \cdot \langle x-3\rangle^1 + \$160/1d \cdot \langle x-5\rangle^1 - \$120/1d \cdot \langle x-6\rangle^1 + \$13.75/1d \cdot \langle x-7\rangle^1 + \$75/1d \cdot \langle x-9\rangle^1 - \$100/1d \cdot \langle x-10\rangle^1$, which evaluates as $y(10d)_{SV\_project} = - \$350 - \$1040 + \$953.75 + \$800 - \$480 + \$41.25 + \$75 - \$0 = \$0$. By day 10 the project was under (!) budget and finished. Yet the *SV* metric unfortunately is biased towards the project end (Equation 21 tends to 0 when we approach the project planned duration, no matter the project *AD*). Schedule variance in days is obtained from Equation 22.

$$y(x)_{SV(t)\_project} = z(x)_{ES\_project} - x \quad \textbf{Eq. 22}$$

Here $y(10d)_{SV(t)\_project} = -1$ days; the project was late. Besides absolute values, relative values are obtained as the cost and schedule performance indicators (*CPI* and *SPI*) per Equations 23-24:

$$y(x)_{CPI\_project} = y(x)_{EV\_project} / y(x)_{AC\_project} \quad \textbf{Eq. 23}$$

$$y(x)_{SPI\_project} = y(x)_{EV\_project} / y(x)_{PV\_project} \quad \textbf{Eq. 24}$$

Evaluating these indices yields $y(10d)_{CPI\_project} = \$1215/\$1140 \approx 106.6\%$ and $y(10d)_{SPI\_project} = \$1215/1215 = 100\%$ (like *SV*, the *SPI* also approaches 100% towards the project planned end). When using the *ES* metric, the *SPI* can be reformulated as the *SPI*(*t*), which is calculated as:

$$y(x)_{SPI(t)\_project} = z(x)_{ES\_project} / x \quad \textbf{Eq. 25}$$

Here, $y(10d)_{SPI(t)\_project} = 9d/10d = 90\%$; the project was late. Combined cost-schedule indices *CSI* and *CSI*(*t*), called *critical ratios*, are easy to express as products per Equations 25-27 as well:

$$y(x)_{CSI\_project} = y(x)_{CPI\_project} \times y(x)_{SPI\_project} \quad \textbf{Eq. 26}$$

$$y(x)_{CSI(t)\_project} = y(x)_{CPI\_project} \times y(x)_{SPI(t)\_project} \quad \textbf{Eq. 27}$$

Their respective values at *AT* = 10 days are 106.58% × 100% ≈ 106.6%, and 106.6% × 90% ≈ 95.9%. For the project status, its percent complete and percent money spent are Equations 28-29:

$$\textit{Percent Complete}\ (x)_{project} = y(x)_{EV\_project} / BAC \quad \textbf{Eq. 28}$$

$$\textit{Percent Money Spent}\ (x)_{project} = y(x)_{AC\_project} / BAC \quad \textbf{Eq. 29}$$

,Equations 28 and 29 give $1215/$1215 = 100% and $1140/$1215 ≈ 93.8% for our example. Variances and indices are tabulated in **Appendix 1-J**.

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4.7. *Forecasts*

While for this example the actual values at completion are assumed as known, this is not the case in practice until the project ends [56]. Vanhoucke [7] listed 6 ways to forecast project cost, called estimate at completion (EAC(\$), and 9 for project duration, called estimate at completion of time $EAC(t)$. Ballesteros-Pérez *et al.* [13] reviewed these and other forecasting indices for EVM. All are cast into SF as Equations 30 to 44. Numerical outputs for each are listed in **Appendix 1-K**.

$$EAC(\$)_{PF=1\_project} = y(x)_{AC\_project} + (BAC - y(x)_{EV\_project}) / 1 \quad \textbf{Eq. 30}$$

$$EAC(\$)_{PF=CPI\_project} = y(x)_{AC\_project} + (BAC - y(x)_{EV\_project}) / y(x)_{CPI\_project} \quad \textbf{Eq. 31}$$

$$EAC(\$)_{PF=SPI\_project} = y(x)_{AC\_project} + (BAC - y(x)_{EV\_project}) / y(x)_{SPI\_project} \quad \textbf{Eq. 32}$$

$$EAC(\$)_{PF=SPI(t)\_project} = y(x)_{AC\_project} + (BAC - y(x)_{EV\_project}) / y(x)_{SPI(t)\_project} \quad \textbf{Eq. 33}$$

$$EAC(\$)_{PF=CSI\_project} = y(x)_{AC\_project} + (BAC - y(x)_{EV\_project}) / y(x)_{CSI\_project} \quad \textbf{Eq. 34}$$

$$EAC(\$)_{PF=CSI(t)\_project} = y(x)_{AC\_project} + (BAC - y(x)_{EV\_project}) / y(x)_{CSI(t)\_project} \quad \textbf{Eq. 35}$$

$$EAC(t)\ (x)_{PV1\_project} = PD - (PD / BAC) \cdot y(x)_{SV\_project} \quad \textbf{Eq. 36}$$

$$EAC(t)\ (x)_{PV2\_project} = PD / y(x)_{SPI\_project} \quad \textbf{Eq. 37}$$

$$EAC(t)\ (x)_{PV3\_project} = PD / y(x)_{CSI\_project} \quad \textbf{Eq. 38}$$

$$EAC(t)\ (x)_{ED1\_project} = \max(PD, x) + AT \cdot (1 - y(x)_{SPI\_project}) \quad \textbf{Eq. 39}$$

$$EAC(t)\ (x)_{ED2\_project} = \max(PD, x) / y(x)_{SPI\_project} \quad \textbf{Eq. 40}$$

$$EAC(t)\ (x)_{ED3\_project} = \max(PD, x) / y(x)_{CSI\_project} + x \cdot (1 - 1 / y(x)_{CPI\_project}) \quad \textbf{Eq. 41}$$

$$EAC(t)\ (x)_{ES1\_project} = x + PD - y(x)_{ES\_project} \quad \textbf{Eq. 42}$$

$$EAC(t)\ (x)_{ES2\_project} = x + (PD - y(x)_{ES\_project}) / y(x)_{SPI(t)\_project} \quad \textbf{Eq. 43}$$

$$EAC(t)\ (x)_{ES3\_project} = x + (PD - y(x)_{ES\_project}) / y(x)_{CSI(t)\_project} \quad \textbf{Eq. 44}$$

These equations only use information until a specific date; the remaining project (cost or time) progress is estimated by assuming that its trend continues by a project factor (*PF*). The key difference among these EAC(\$) and $EAC(t)$ formulas lies in chosing *PF* by whether it is expected that the remaining work rate, cost trend, time trend, both, or a mixture of both will be dominant.

5. Validation

We analyze a real-world project involving the installation and implementation of a modern telecommunication system in Agnes located in Egmond aan Zee (The Netherlands). It can be found in the datafile 'C2011-05 Telecom System Agnes' from the Empirical Project Data listings of the Operations Research and Scheduling Research Group at Ghent University [57]. Broader details about this empirical dataset of projects are described in [1] and [2]. The Agnes project was selected due to the availability of its as-planned baseline schedule plus several as-built updates, including *PV*, *AC*, percent complete, and *EV*. In this paper a single example provides a proper validation, because we contrast the discrete model with our continuous one. By showing

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how short-term forecasting capabilities are incorporated, we are able to prove how a continuous EVM model with SF offers significant advantages that make it worth adopting in other projects.

Appendix 2 lists inputs, calculations, and outputs in the same order as **Appendix 1**. Its 21 activities generate a project planned duration of 43 worksdays at eight work hours daily from 5/18/2011 to 7/15/2011 without holidays. Project planned cost was €180,485, the sum of resource-specific plus fixed activity costs. The longest activities were 9, 15, and 16 (delivery, assembly, installation). The project was eventually completed in 53 workdays without incurring a cost overrun. Updates existed on five unevenly spaced dates, 5/24, 6/20, 6/27, 7/12, and 7/29 (planned workdays **5, 24, 29, 40, and 53**).

Traditional EVM employs tracking periods (here irregularly timed) after whom a snapshot of the progress until that update is captured. Thus by definition EVM is blind between updates [58]. It must assume straight-line planned or actual progress within tracking periods as bold lines show in Figure 4. This is extremely inaccurate, as project-level *PV* or *EV* change whenever an activity starts or finishes at that time. Yet this is correctly expressed by the SF as thin lines in Figure 4. Curves for *AC* are omitted in Figure 4, since according to its tracking data, this real project did not incur any cost overruns, so that *AC* and *EV* happened to coincide throughout its duration.

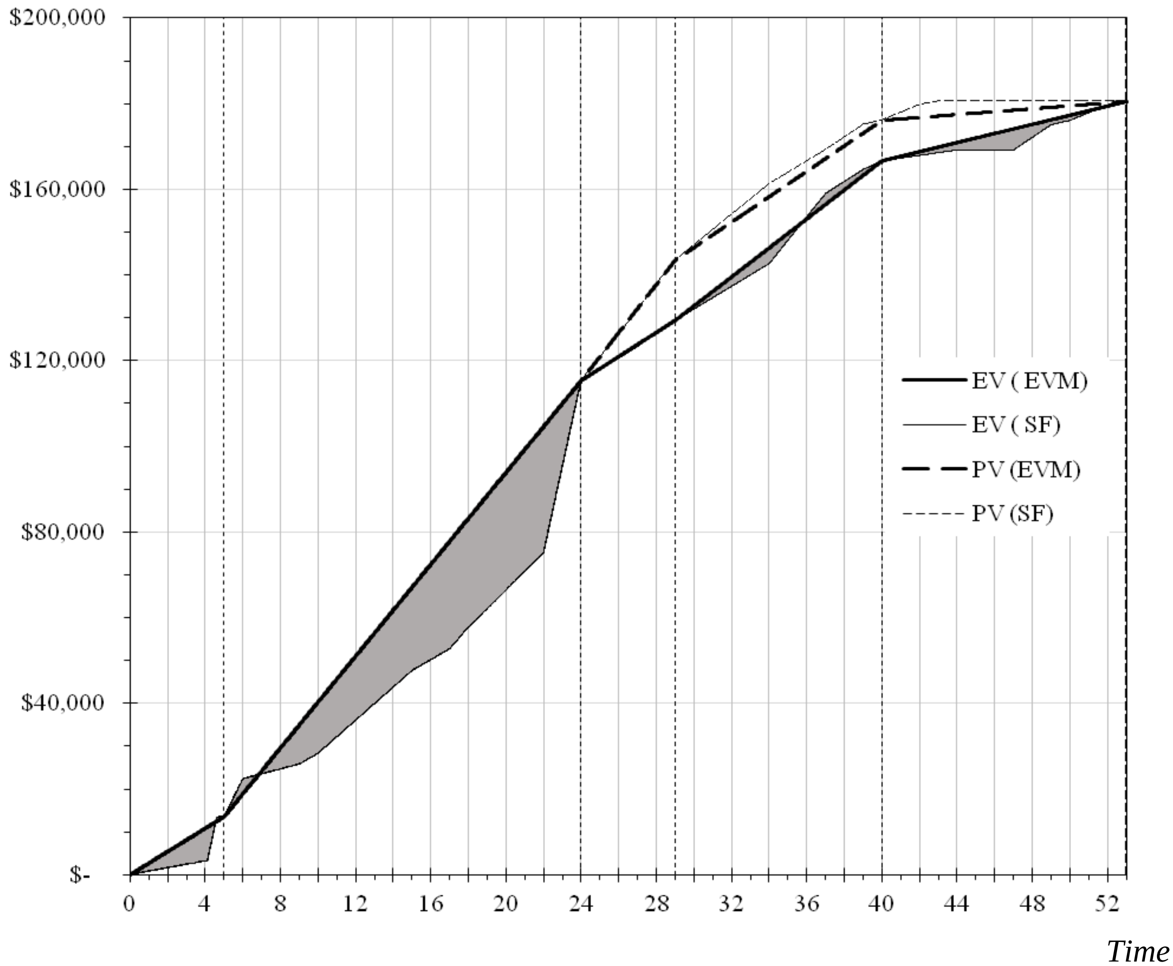

Figure 4: Project Planned Value (Dashed) and Earned Value (Solid) Curves for Traditional Earned Value Method (Thick) and Singularity Functions (Thin)

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In the tracking periods until workday 24, the discrete EVM grossly overestimates values, as the true ones are lower and more detailed with SF (difference marked gray). Only after workday 24 does EVM detect deviations: *EV* is significantly lower than *PV* for its remaining duration, which was planned at 43 workdays, but actually became 53. Yet its piecemeal approach causes inaccuracies in *EV* after workday 24, whereas the thin *EV* line with SF tracks all data precisely. Only the SF model visualizes the plateau in *PV* after the project planned finish of day 43, which discrete EVM ignores entirely. Worse, it failed to obtain an update at this project planned finish date. Worst, it was unable to incorporate the known holiday, which the *EV* with SF has captured.

Figure 4 also illustrates the short-term forecasting abilities of the new continuous SF for *EV*: Whenever such SF receives new information – activity starts and finishes or performance records from the field – its slope is updated by a change term, which then remains active until newer data are added. Thus the SF always provides a forecast that explicitly incorporates all data until that time. Note especially that times can be fractional, as all SF equations are continous for all inputs.

To demonstrate the *short-term* prediction ability of SF, consider the questions *On day 15 what is the forecasted EV for day 20; on day 29 what is the forecasted EV for day 33; or on day 40 what it the EV forecast for day 47?* The answers are calculated by evaluating $y(x)_{\text{EV\_project}}$ using only all terms that are known until the date on which the forecast is made. For the first question it is $y(20)_{\text{EV\_project on day 15}}$ = €800/d · 20d - €352/1d · 16d + €19764/1d · 15.875d - €19772/1d · 15.375d + €8387/1d · 15d - €7674.22/1d · 14d + €1376.50/1d · 11d + €1332/1d · 10d - €1332/1d · 5d = \$60,294.42. This value is impossible to calculate with discrete EVM until the ongoing tracking period ends. With day 17 and 18 events recorded, the accuracy of this forecast can be compared to $y(20)_{\text{EV\_project}}$, which differs only in two newly known terms + €2588.50/1d · 3d - €800/1d · 2d = \$6165.50, by which the short-term forecast underestimated the achieved *EV*.

Analogously, $y(33)_{\text{EV\_project on day 29}}$ = \$140,004.64 (details are left to the reader) is as accurate as the data allow, since no further activities start or finish from days 29 to 33. Forecasting on day 40 should incorporate the known 3-day-long holiday break by subtracting the cumulative slope (\$640/1d) when the break starts on day 44 (and re-adding it when the break ends again). This is $y(47)_{\text{EV\_project on day 40}}$ = \$169,180.04, which again is extremely close to the actual value. This has demonstrated that short-term progress can be forecasted while incorporating upcoming events.

To demonstrate the *long-term* prediction ability of SF, we study *What is the day 15, day 30, and (after the project exceeded its deadline already) day 47 forecast for the project finish?* These are calculated with the earned schedule SF (listed in **Appendix 2-G**), using only terms up to the cutoff 15 (or 30 or 47) and evaluating them for the *x*-value of 43. The forecasted project duration $z(x)$ that is expected at any *AT* (from all data that are known until then) is derived as follows. The horizontal axis is $x = AT$ when the forecast is made. Cutoffs are identical to the cutoffs in $z(x)_{\text{ES}}$. The forecasted project duration – is calculated as *AT* plus the remaining *ES* (until $PV = 43$) that will be produced at the current cumulative slope. For Equation 30, these cumulative slopes are found in **Appendix 2-G**. Whenever a slope change occurs in $z(x)_{\text{ES}}$, a step is added in $z(x)_{\text{forecasted project duration}}$. Such step has the oppositive sign as the slope, because if the slope (i.e. productivity) is updated to be lower, then the newly forecasted project duration will be higher, and *vice versa*. This is accomplished by the '1 -'. Step height is a difference of slopes. This elegantly shortens the expression for pairwise slope differences, because their cumulative sum cancels all duplicate terms (with opposite signs) except for just the first (index 0) and last one (index *i*) single terms.

$$z(x)_{\text{forecasted project duration}} = x + (PV - z(x)_{\text{ES}}) \,/\, (1 - \{[\Delta z(y_i)/\Delta x(y_i)_{\text{EV\_project}}] - [\Delta z(y_0)/\Delta x(y_0)_{\text{EV\_project}}]\}) \quad \textbf{Eq. 45}$$

Thus $z(15)_{\text{forecasted project duration}}$ = 15 + (43-15) / (1 - {0/2 - 0} = 43 days. On day 15 the project is still expected to finish on time. But this forecast changes as more information is created from the actual progress. By day 37 more terms are added into the *ES* equation, so $z(37)_{\text{forecasted project duration}}$ =37 + (43-33.3786) / (1 - {0.2119-0}) = 49.21 days. The project has been falling behind. The terms incorporate productivity increases and decreases that have been recorded until then. Directly after the holiday break on day 47, $z(47)_{\text{forecasted project duration}}$ = 53 days per **Appendix 2-G**.

Overall, SF offer a unique new forecasting capability – for the first time it is possible to plot exactly how the forecasted end date of the project evolves at any point of time as in Figure 5.

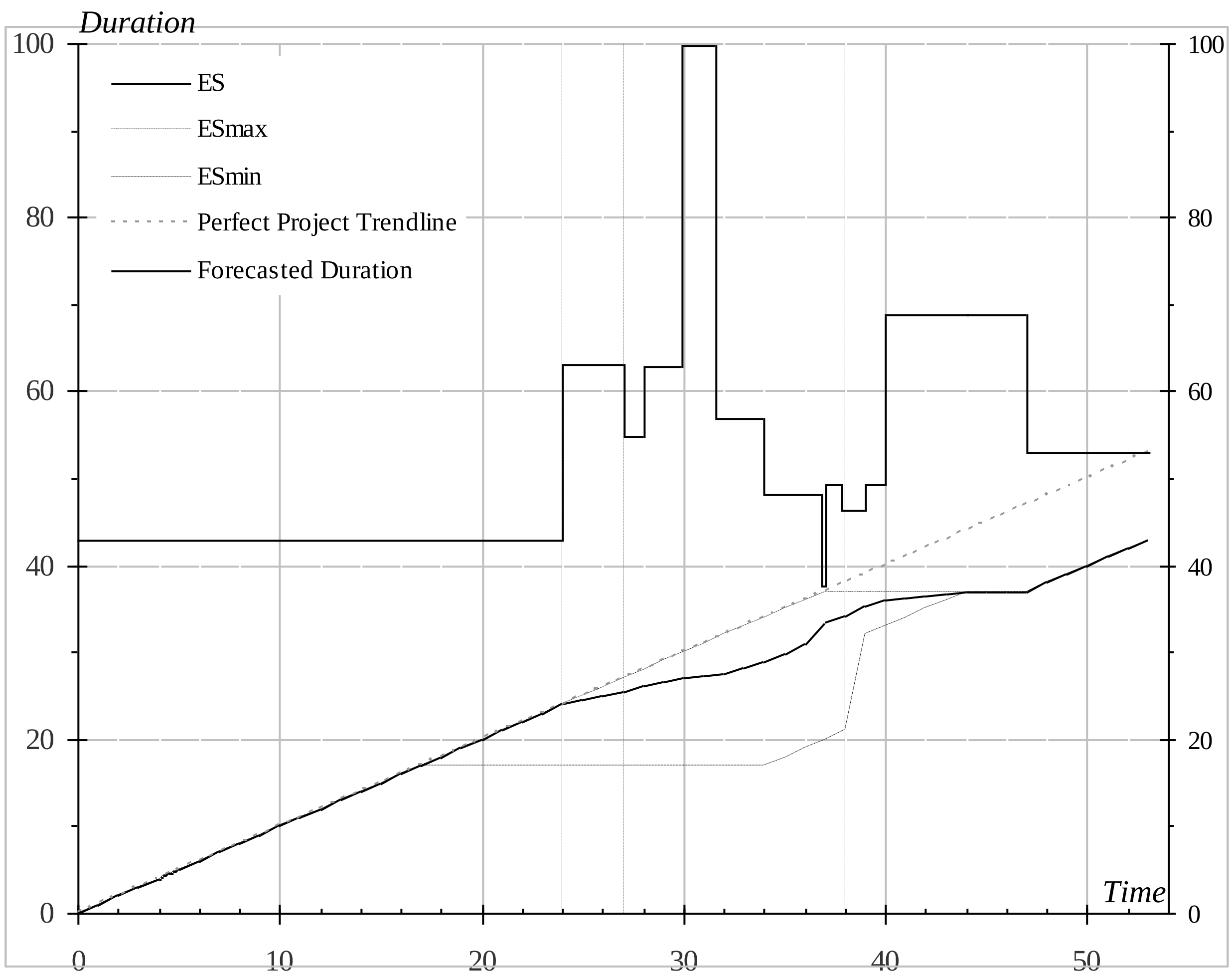

Figure 5: Earned Schedule and Project Forecasted Duration over Actual Time

Figure 5 shows at what times the project is acutely behind of schedule and how effectively the corrective actions that are taken may bring it back on track. This new continuous model remedies prior discrete inaccuracies by automatically incorporating all known data into the SF. This means the project manager will not have to wait until the contractor issues a new invoice to the client to inform the EVM model with project progress information gathered so far. In other words, its resolution will always be as fine as the data allow and there is no need to be tied to pre-specified tracking periods to extract information from the proposed continuous EVM model.

6. Conclusions

This paper contributes to the body of knowledge in performance measurement of construction projects in three ways. First, the new continuous model for the full range of EVM metrics creates the opportunity to track a project over its entire lifespan, rather than only at occasional tracking periods. Project managers can finally know their project status as accurately as is possible, rather than navigating blindly between formal updates. Second, data can be added at any fractional time, and queries can be made equally freely. Its efficiency is evidenced by the compact expressions, notwithstanding that SF should be handled by a computer implementation for real projects, especially if they encompass many activities and SF expressions will become long. Third, the continuous formulation of the model has the theoretical advantage that an entire equation can be transposed, which allows *exact calculation* of metrics like *ES*. Its insightfulness has been tested on a real-world example, which has demonstrated the new *short-term duration and cost forecasting capability* of the continuous model.

Applying the continuous EVM model will work on any project type. It is particularly interesting for projects whose work resources need to be frequently reallocated to optimize production. This is the case in repetitive projects, which are characterized by repeating relatively similar units of construction (e.g. buildings with several floors, long pipelines or ducts, or precast elements in prefabricated plants). Keeping their resources balanced and busy is necessary so that all parts of the project can progress at a rather uniform pace. But this can only be achieved if the project manager gets instant feedback about the project and its activities' (time and cost) productivities [59]. This is difficult with discrete distant tracking periods, and time-consuming with tracking periods that are close to each other (daily or weekly, for example). The continuous EVM model in this paper overcomes this limitation by showing how project progress can be monitored in real time as activities start and finish. As a result, the project manager can take action immediately, if need be. Further exploration of integrating this model with scheduling methods (e.g. the Critical Path Method or the Linear Scheduling Method) and its implications is left for future research.

While this paper has created new theory for the complete set of EVM metrics and forecasting with its continuous model, its limitations are both conceptual and integrational in nature. First, its approach could be expanded still further to develop novel analytical capabilities that do not yet even exist in EVM. The continuity, accuracy, flexibility, and integratability and differentiability of SF enable more uses toward construction projects. Second, for length limitations the scope of this paper has had to exclude integrating the new model with various scheduling methods. Such methods could benefit in novel ways from EVM-style time and cost performance measurements being directly embedded into them. Promising items that future research should address thus are:

- Since SF are integratable and differentiable, it could be investigated what beneficial trend metrics or forecasts could be derived from such rates of change.
- Adapt the *AT-ES* plot (where perfect performance is the diagonal) to EDM that uses activity durations as 'money' rather than cost.
- Decision-makers can benefit from the ability for direct *what-if* analyses by changing a single parameter within a SF, or perform sensitivity studies towards multi-objective optimization.
- Explore merging risking management with optimization by retaining time and cost variables in the equations (especially activity dates), and adding resource variables into productivities.

- Merge the continuous EVM with linear scheduling, resource management, and cash flows that have already been expressed as SF to enable an integrated approach for optimization.

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Appendix Part 1: Development Example

Appendix 1-A.1: Activity Planned Value

Activity	***PC***	***PS***	***PD***	**Ratio**	**Output**
A	\$210	0	2d	\$210/2d	$y(x)_{PV\_A} = \$105/1d \cdot [\langle x\text{-}0\rangle^1 - \langle x\text{-}2\rangle^1]$
B	\$360	2	3d	\$360/3d	$y(x)_{PV\_B} = \$120/1d \cdot [\langle x\text{-}2\rangle^1 - \langle x\text{-}5\rangle^1]$
C	\$345	2	3d	\$345/3d	$y(x)_{PV\_C} = \$115/1d \cdot [\langle x\text{-}2\rangle^1 - \langle x\text{-}5\rangle^1]$
D	\$300	5	4d	\$300/4d	$y(x)_{PV\_D} = \$75/1d \cdot [\langle x\text{-}5\rangle^1 - \langle x\text{-}9\rangle^1]$

Appendix 1-A.2: Planned Cash Flow

Planned Schedule			***AT→***									***PD***	
Activity	***PD***	***PC***	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	2d	\$210	0	105	105								
B	3d	\$360				120	120	120					
C	3d	\$345				115	115	115					
D	4d	\$300							75	75	75	75	
Planned Cash Flow (PCF)			0	105	105	235	235	235	75	75	75	75	0
PV (cumulative PCF)			0	105	210	445	680	915	990	1065	1140	1215	1215
												BAC	

Appendix 1-B.1: Activity Actual Cost

Activity	***AC***	***AD***	***AS***	**Output**
A	\$210	3d	0	$y(x)_{AC\_A} = \$210/3d \cdot [\langle x\text{-}0\rangle^1 - \langle x\text{-}3\rangle^1]$
B	\$390	3d	3	$y(x)_{AC\_B} = \$390/3d \cdot [\langle x\text{-}3\rangle^1 - \langle x\text{-}6\rangle^1]$
C	\$300	4d	3	$y(x)_{AC\_C} = \$300/4d \cdot [\langle x\text{-}3\rangle^1 - \langle x\text{-}7\rangle^1]$
D	\$240	3d	7	$y(x)_{AC\_D} = \$240/3d \cdot [\langle x\text{-}7\rangle^1 - \langle x\text{-}10\rangle^1]$

Appendix 1-B.2: Actual Cash Flow

Actual Schedule			***AT→***										***RD***
Activity	***AD***	***AC***	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	3d	\$210	0	70	70	70							
B	3d	\$390					130	130	130				
C	4d	\$300					75	75	75	75			
D	3d	\$240									80	80	80
Actual Cash Flow (*ACF*)			0	70	70	70	205	205	205	75	80	80	80
***AC* (cumulative *ACF*)**			0	70	140	210	415	620	825	900	980	1060	1140
													RAC

Appendix 1-C: Activity Actual Percent Table

Activity	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A		1/3	2/3	1	1	1	1	1	1	1	1
B					1/3	2/3	1	1	1	1	1
C					1/4	2/4	3/4	1	1	1	1
D									1/3	2/3	1

Appendix 1-D: Activity Actual Percent Differences Table

https://doi.org/10.1016/j.autcon.2022.104583

Activity	0	1	2	3	4	5	6	7	8	9	10
A	1/3	.	.	-1/3							
B				1/3	.	.	-1/3				
C				1/4	.	.	.	-1/4			
D								1/3	.	.	-1/3

Appendix 1-E.1: Activity Actual Percent and Earned Value

Activity	Actual Percent (*AP*)	*PC*	Earned Value (*EV*)
A	$y(x)_{AP\_A} = {}^{1}/_{3}/d \cdot \langle x\text{-}0\rangle^1 - {}^{1}/_{3}/d \cdot \langle x\text{-}3\rangle^1$	\$210	$y(x)_{EV\_1} = \$70/d \cdot \langle x\text{-}0\rangle^1 - 70/d \cdot \langle x\text{-}3\rangle^1$
B	$y(x)_{AP\_B} = {}^{1}/_{3}/d \cdot \langle x\text{-}3\rangle^1 - {}^{1}/_{3}/d \cdot \langle x\text{-}6\rangle^1$	\$360	$y(x)_{EV\_2} = \$120/d \cdot \langle x\text{-}3\rangle^1 - \$120/d \cdot \langle x\text{-}6\rangle^1$
C	$y(x)_{AP\_C} = {}^{1}/_{4}/d \cdot \langle x\text{-}3\rangle^1 - {}^{1}/_{4}/d \cdot \langle x\text{-}7\rangle^1$	\$345	$y(x)_{EV\_3} = \$86.25/d \cdot \langle x\text{-}3\rangle^1 - \$86.25/d \cdot \langle x\text{-}7\rangle^1$
D	$y(x)_{AP\_D} = {}^{1}/_{3}/d \cdot \langle x\text{-}7\rangle^1 - {}^{1}/_{3}/d \cdot \langle x\text{-}10\rangle^1$	\$300	$y(x)_{EV\_4} = \$100/d \cdot \langle x\text{-}7\rangle^1 - \$100/d \cdot \langle x\text{-}10\rangle^1$

Appendix 1-E.2: Earned Value Table

			AT→										
Activity	***AD***	***PC***	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	3d	\$210	0	70	140	210	210	210	210	210	210	210	210
B	3d	\$360					120	240	360	360	360	360	360
C	4d	\$345					86.25	172.50	258.75	345	345	345	345
D	3d	\$300									100	200	300
Sum	**N/A**	**EV**	0	70	140	210	416.25	622.5	828.75	915	1015	1115	1215

Appendix 1-F.1: Differences for Transposition (Planned Value)

Time Cutoff	0d	∑	2d	∑	5d	∑	9d	∑
Slope Change	+\$105/d		+\$130d		-\$160/d		-\$75/d	
Total Slope		\$105/d		\$235/d		\$75/d		0
Cost Cutoff	\$0		\$210		\$915		\$1215	
Inverted Slope		1d/\$105		1d/\$235		1d/\$75		0
Slope Change	+1d/\$105		+1/235-1/105 = -130/24675 = -26d/\$4935		1/75-1/235 = 160/17625 = 32d/\$3525		-1d/\$75	

Non-Cumulative, Cumulative

$x(y)_{PV\_project} = 1d/\$105 \cdot \langle y\text{-}\$0\rangle^1 - 26d/\$4935 \cdot \langle y\text{-}\$210\rangle^1 + 32d/\$3525 \cdot \langle y\text{-}\$915\rangle^1 - 1d/\$75 \cdot \langle y\text{-}\$1215\rangle^1$

Appendix 1-F.2: Differences for Transposition (Earned Value)

Time Cutoff	0d	∑	3d	∑	6d	∑	7d	∑	10d	∑
Slope Change	+\$70/d		+\$136.25/d		-\$120/d		+\$13.75/d		-\$100/d	
Total Slope		\$70/d		\$206.25/d		\$86.25/d		\$100/d		0
Cost Cutoff	\$0		\$210		\$828.75		\$915		\$1215	
Inverted Slope		1d/\$70		1d/\$206.25 = 4d/\$825		1d/\$86.25 = 4d/\$345		1d/\$100		0

Slope Change	+1d/$70		+4/825-1/70 = -545/57750 = -109d/$11550		+4/345-4/825 = 1920/284625 = 128d/$18975		+1/100-1/86.25 = -13.75/8625 = -11d/$6900		-1d/$100	

Non-Cumulative, Cumulative

$x(y)_{\text{EV\_project}} = 1\text{d}/\$70 \cdot \langle y\text{-}\$0\rangle^1 - 109\text{d}/\$11550 \cdot \langle y\text{-}\$210\rangle^1 + 128\text{d}/\$18975 \cdot \langle y\text{-}\$828.75\rangle^1 - 11\text{d}/\$6900 \cdot \langle y\text{-}\$915\rangle^1 - 1\text{d}/\$100 \cdot \langle y\text{-}\$1215\rangle^1$

Appendix 1-G: Earned Schedule Derivation

Get rises $z_i(y_i)$ from transposed difference $z(y) = x(y)_{\text{EV\_project}} - x(y)_{\text{PV\_project}} = 1\text{d}/\$210 \cdot \langle y\text{-}\$0\rangle^1 - 2263\text{d}/\$542850 \cdot \langle y\text{-}\$210\rangle^1 + 128\text{d}/\$18975 \cdot \langle y\text{-}\$828.75\rangle^1 - 3461\text{d}/\$324300 \cdot \langle y\text{-}\$915\rangle^1 + 1\text{d}/\$300 \cdot \langle y\text{-}\$1215\rangle^1$

Evaluate it at each cutoff to get its absolute height z there in days. And evaluate $x(y)_{\text{EV\_project}}$ at any y_i-cutoffs from both *EV* and *PV* as new cutoffs for $z(x)_{\text{ES\_project}}$

y_i	$z_i(y_i)$	$x(y_i)_{\text{EV\_project}}$
$0	0d	0d
$210	1d	210/70 = 3d
$828.75	221/56 - 6789/2632 = 257/188 ≈ 1.3670d	663/56 - 327/56 = 6d
$915	61/14 - 2263/770 + 32/55 = 1540/770 = 2d	183/14 - 5123/770 + 32/55 = 5390/770 = 7d
$1215	81/14 - 151621/36190 + 3296/1265 - 3461/1081 = 832370/832370 = 1d	243/14 - 7303/770 + 3296/1265 - 11/23 = 7700/770 = 10d

Determine runs $\Delta x(y_i)$ as distances between new cutoffs and compose updated slopes

Cutoff	**Run**	**Verticals**	**Rise**	**Slope**	**ΔSlope**	**Diagonal**	**New Slope**
$x(y_i)_{\text{EV\_project}}$	$\Delta x(y_i)_{\text{EV\_project}}$	$z_i(y_i)$	$\Delta z(y_i)$	**Rise/Run**	$y(x_i)_{\text{SV(t)}}$	**Linear**	$z(x_i)_{\text{ES}}$
0d	+3d	0d	+1d	1/3d/d	+1/3d/d	1d/d	2/3d/d
3d	+3d	1d	+69/188d	23/188d/d	-119/564d/d	1d/d	119/564d/d
6d	+1d	257/188d	+119/188d	119/188d/d	+24/47d/d	1d/d	-24/47d/d
7d	+3d	2d	-1d	-1/3d/d	-545/564d/d	1d/d	545/564d/d
10d	N/A	1d	N/A	0d/d	1/3d/d	1d/d	-1/3d/d
Checksum	**10d**	**N/A**	**1d**	**N/A**	**0d/d**	**N/A**	**1d/d**

This gives schedule variance in time units $y(x)_{\text{SV(t)}} = 1/3 \cdot \langle x\text{-}0\rangle^1 - 119/564 \cdot \langle x\text{-}3\text{d}\rangle^1 + 24/47 \cdot \langle x\text{-}6\text{d}\rangle^1 - 545/564 \cdot \langle x\text{-}7\text{d}\rangle^1 + 1/3 \cdot \langle x\text{-}10\text{d}\rangle^1$

Write $z(x)_{\text{ES}}$ as diagonal $1\text{d}/1\text{d} \cdot \langle x\text{-}0\text{d}\rangle^1$ minus $y(x)_{\text{SV(t)}}$ as earned schedule $z(x)_{\text{ES}} = 2/3 \cdot \langle x\text{-}0\text{d}\rangle^1 + 119/564 \cdot \langle x\text{-}3\text{d}\rangle^1 - 24/47 \cdot \langle x\text{-}6\text{d}\rangle^1 + 545/564 \cdot \langle x\text{-}7\text{d}\rangle^1 - 1/3 \cdot \langle x\text{-}10\text{d}\rangle^1$

AT	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ES	0	2/3 ≈ 0.67	4/3 ≈ 1.33	2	1623/564 ≈ 2.88	2118/564 ≈ 3.76	2613/564 ≈ 4.63	5	19/3 ≈ 6.33	23/3 ≈ 7.67	9

Appendix 1-H.1: Activity Earned Schedule Minimum Table (AP < 100%)

Activity	***PES***	***PLS***	***PD***	***AD***	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	0	0	2d	3d	**0**	**2/3**	**4/3**		.	.	.	.	.	.	.
B	2	2	3d	3d	2	2	2	**2**	3	4		.	.	.	.
C	2	2	3d	4d	2	2	2	2	**11/4**	**14/4**	**17/4**		.	.	.
D	5	5	4d	3d	5	5	5	5	5	5	5	**15/3**	**19/3**	**23/3**	
Minimum	N/A	N/A	N/A	N/A	0	0.67	1.33	2	2.75	3.5	4.25	5	6.33	7.67	

*Minima are **bold**. Asterisk is excluded value.*

Appendix 1-H.2: Activity Earned Schedule Maximum Table (AP > 0%)

Activity	***PES***	***PLS***	***PD***	***AD***	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	0	0	2d	3d		**2/3**	**4/3**	**2**	2	2	2	2	2	2	2
B	2	2	3d	3d	.	.	.		**3**	**4**	**5**	5	5	5	5
C	2	2	3d	4d	.	.	.		11/4	14/4	17/4	**5**	5	5	5
D	5	5	4d	3d	.	.	.	.	.	.	.		**19/3**	**23/3**	**27/3**
Maximum	N/A	N/A	N/A	N/A		0.67	1.33	2	3	4	5	5	6.33	7.67	9

*Maxima are **bold**. Asterisk is excluded value.*

Appendix 1-I.1: Activity Earned Schedule Minimum

Activity	**Output**
A	$z(x)_{ES\_A\_min} = [1 - \langle 0 - {}^{1}/{}_{3}/d\cdot\langle x\text{-}0\rangle^{1} + {}^{1}/{}_{3}/d\cdot\langle x\text{-}3\rangle^{1} - 1\rangle^{0}]_{min} \cdot [0 + {}^{2}/{}_{3}/d\cdot\langle x\text{-}0\rangle^{1} - {}^{2}/{}_{3}/d\cdot\langle x\text{-}3\rangle^{1}]$
B	$z(x)_{ES\_B\_min} = [1 - \langle 0 - {}^{1}/{}_{3}/d\cdot\langle x\text{-}3\rangle^{1} + {}^{1}/{}_{3}/d\cdot\langle x\text{-}6\rangle^{1} - 1\rangle^{0}]_{min} \cdot [2 + 1/d\cdot\langle x\text{-}3\rangle^{1} - 1/d\cdot\langle x\text{-}6\rangle^{1}]$
C	$z(x)_{ES\_C\_min} = [1 - \langle 0 - {}^{1}/{}_{4}/d\cdot\langle x\text{-}3\rangle^{1} + {}^{1}/{}_{4}/d\cdot\langle x\text{-}7\rangle^{1} - 1\rangle^{0}]_{min} \cdot [2 + {}^{3}/{}_{4}/d\cdot\langle x\text{-}3\rangle^{1} - {}^{3}/{}_{4}/d\cdot\langle x\text{-}7\rangle^{1}]$
D	$z(x)_{ES\_D\_min} = [1 - \langle 0 - {}^{1}/{}_{3}/d\cdot\langle x\text{-}7\rangle^{1} + {}^{1}/{}_{3}/d\cdot\langle x\text{-}10\rangle^{1} - 1\rangle^{0}]_{min} \cdot [5 + {}^{4}/{}_{3}/d\cdot\langle x\text{-}7\rangle^{1} - {}^{4}/{}_{3}/d\cdot\langle x\text{-}10\rangle^{1}]$

Appendix 1-I.2: Activity Earned Schedule Maximum

Activity	**Output**
A	$z(x)_{ES\_A\_max} = [1 - \langle {}^{1}/{}_{3}/d\cdot\langle x\text{-}0\rangle^{1} - {}^{1}/{}_{3}/d\cdot\langle x\text{-}3\rangle^{1}\rangle^{0}]_{min} \cdot [0 + {}^{2}/{}_{3}/d\cdot\langle x\text{-}0\rangle^{1} - {}^{2}/{}_{3}/d\cdot\langle x\text{-}3\rangle^{1}]$
B	$z(x)_{ES\_B\_max} = [1 - \langle {}^{1}/{}_{3}/d\cdot\langle x\text{-}3\rangle^{1} - {}^{1}/{}_{3}/d\cdot\langle x\text{-}6\rangle^{1}\rangle^{0}]_{min} \cdot [2 + 1/d\cdot\langle x\text{-}3\rangle^{1} - 1/d\cdot\langle x\text{-}6\rangle^{1}]$
C	$z(x)_{ES\_C\_max} = [1 - \langle {}^{1}/{}_{4}/d\cdot\langle x\text{-}3\rangle^{1} - {}^{1}/{}_{4}/d\cdot\langle x\text{-}7\rangle^{1}\rangle^{0}]_{min} \cdot [2 + {}^{3}/{}_{4}/d\cdot\langle x\text{-}3\rangle^{1} - {}^{3}/{}_{4}/d\cdot\langle x\text{-}7\rangle^{1}]$
D	$z(x)_{ES\_D\_max} = [1 - \langle {}^{1}/{}_{3}/d\cdot\langle x\text{-}7\rangle^{1} - {}^{1}/{}_{3}/d\cdot\langle x\text{-}10\rangle^{1}\rangle^{0}]_{min} \cdot [5 + {}^{4}/{}_{3}/d\cdot\langle x\text{-}7\rangle^{1} - {}^{4}/{}_{3}/d\cdot\langle x\text{-}10\rangle^{1}]$

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Appendix 1-J: Variances and Indices

Index	Unit	0	1	2	3	4	5	6	7	8	9	10
SV	$	0	35	70	235	263.75	292.50	161.25	150	125	1000	0
SV(t)	d	0	-1/3 ≈ -0.33	-2/3 ≈ -0.67	-1	-211/188 ≈ -1.12	-117/94 ≈ -1.24	-257/188 ≈ -1.37	-2	-5/3 ≈ -1.67	-4/3 ≈ -1.33	-1
CV	$	0	0	0	0	1.25	2.50	3.75	15	35	55	75
SPI	-	1	2/3 ≈ 0.67	2/3 ≈ 0.67	42/89 ≈ 0.47	333/544 ≈ 0.61	83/122 ≈ 0.68	221/264 ≈ 0.84	61/71 ≈ 0.86	203/228 ≈ 0.89	223/243 ≈ 0.92	1
SPI(t)	-	1	2/3 ≈ 0.67	2/3 ≈ 0.67	2/3 ≈ 0.67	541/752 ≈ 0.72	353/470 ≈ 0.75	871/1128 ≈ 0.77	5/7 ≈ 0.71	19/24 ≈ 0.79	23/27 ≈ 0.85	9/10
CPI	-	1	1	1	1	333/332 ≈ 1.00	249/248 ≈ 1.00	221/220 ≈ 1.00	61/60 ≈ 1.02	29/28 ≈ 1.04	223/212 ≈ 1.05	81/76 ≈ 1.07
CSI	-	1	2/3 ≈ 0.67	2/3 ≈ 0.67	42/89 ≈ 0.47	110889/180608 ≈ 0.61	20667/30256 ≈ 0.68	48841/58080 ≈ 0.84	3721/4260 ≈ 0.87	841/912 ≈ 0.92	49729/51516 ≈ 0.96	81/76 ≈ 1.07
CSI(t)	-	1	2/3 ≈ 0.67	2/3 ≈ 0.67	2/3 ≈ 0.67	180153/249664 ≈ 0.72	87897/116560 ≈ 0.75	192491/248160 ≈ 0.78	61/84 ≈ 0.73	551/672 ≈ 0.82	5129/5724 ≈ 0.90	729/760 ≈ 0.96
Percent Complete	%	0	5.76%	11.52%	17.28%	34.26%	51.23%	68.21%	75.31%	83.54%	91.77%	100%
Percent Money Spent	%	0	5.76%	11.52%	17.28%	34.16%	51.03%	67.90%	74.07%	80.66%	87.24%	93.83%

Appendix 1-K: Forecasts

Index	Unit	0	1	2	3	4	5	6	7	8	9	10
$EAC(\$)_{PF=1\_project}$	$	1215	1215	1215	1215	1213.75	1212.5	1211.25	1200	1180	1160	1140
$EAC(\$)_{PF=CPI\_project}$	$	1215	1215	1215	1215	1211.35	1210.12	1209.50	1195.08	1173.10	1155.07	1140
$EAC(\$)_{PF=SPI\_project}$	$	1215	1787.5	1752.5	2339.64	1719.86	1490.90	1286.40	1249.18	1204.63	1168.97	1140
$EAC(\$)_{PF=SPI(t)\_project}$	$	1215	1787.5	1752.5	1717.50	1525.28	1408.88	1325.22	1320.00	1232.63	1177.39	1140
$EAC(\$)_{PF=CSI\_project}$	$	1215	1787.5	1752.5	2339.64	1715.95	1487.41	1284.31	1243.46	1196.88	1163.59	1140
$EAC(\$)_{PF=CSI(t)\_project}$	$	1215	1787.5	1752.5	1717.50	1521.94	1405.71	1322.95	1313.11	1223.92	1171.60	1140
$EAC(t)\ (x)_{PV1\_project}$	d	9	9.26	9.52	10.74	10.95	11.17	10.19	10.11	9.93	9.74	9
$EAC(t)\ (x)_{PV2\_project}$	d	9	13.50	13.50	19.07	14.70	13.23	10.75	10.48	10.11	9.81	9
$EAC(t)\ (x)_{PV3\_project}$	d	9	13.50	13.50	19.07	14.66	13.18	10.70	10.30	9.76	9.32	8.44
$EAC(t)\ (x)_{ED1\_project}$	d	9	9.33	9.67	10.58	10.55	10.60	9.98	9.99	9.88	9.74	10
$EAC(t)\ (x)_{ED2\_project}$	d	9	13.50	13.50	19.07	14.70	13.23	10.75	10.48	10.11	9.81	10
$EAC(t)\ (x)_{ED3\_project}$	d	9	13.50	13.50	19.07	14.67	13.20	10.73	10.42	10.04	9.77	10
$EAC(t)\ (x)_{ES1\_project}$	d	9	9.33	9.67	10.00	10.12	10.24	10.37	11.00	10.67	10.33	10

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***EAC(t) (x)*$_{ES2\_project}$**	d	9	13.50	13.50	13.50	12.51	11.98	11.66	12.60	11.37	10.57	10
***EAC(t) (x)*$_{ES3\_project}$**	d	9	13.50	13.50	13.50	12.48	11.95	11.63	12.51	11.25	10.49	10

Appendix Part 2: Validation Example (Telecom Agnes)

Appendix 2-A: Activity Planned Value

Activity	*PC*	*PS*	*PD*	Ratio	Output
1	€2400	0	3d	€2400/3d	$y(x)_{PV\_1} = €800/1d \cdot [\langle x-0d\rangle^{1} - \langle x-3d\rangle^{1}]$
2	€800	3	1d	€800/1d	$y(x)_{PV\_2} = €800/1d \cdot [\langle x-3d\rangle^{1} - \langle x-4d\rangle^{1}]$
3	€440	4	1d	€440/1d	$y(x)_{PV\_3} = €440/1d \cdot [\langle x-4d\rangle^{1} - \langle x-5d\rangle^{1}]$
4	€800	17	1d	€800/1d	$y(x)_{PV\_4} = €800/1d \cdot [\langle x-17d\rangle^{1} - \langle x-18d\rangle^{1}]$
5	€1	4	0.125d	€8/1d	$y(x)_{PV\_5} = €8/1d \cdot [\langle x-4d\rangle^{1} - \langle x-4.125d\rangle^{1}]$
6	€800	27	1d	€800/1d	$y(x)_{PV\_6} = €800/1d \cdot [\langle x-27d\rangle^{1} - \langle x-28d\rangle^{1}]$
7	€9886	4.125	0.5d	€19772/1d	$y(x)_{PV\_7} = €19772/1d \cdot [\langle x-4.125d\rangle^{1} - \langle x-4.625d\rangle^{1}]$
8	€8827	5	1d	€8827/1d	$y(x)_{PV\_8} = €8827/1d \cdot [\langle x-5d\rangle^{1} \cdot \langle x-6d\rangle^{1}]$
9	€20750	6	18d	€10375/9d	$y(x)_{PV\_9} = €10375/9d \cdot [\langle x-6d\rangle^{1} - \langle x-24d\rangle^{1]}$
10	€11012	9	8d	€11012/8d	$y(x)_{PV\_10} = €1376.50/1d \cdot [\langle x-9d\rangle^{1} - \langle x-17d\rangle^{1}]$
11	€6660	10	5d	€6660/5d	$y(x)_{PV\_11} = €1332/1d \cdot [\langle x-10d\rangle^{1} - \langle x-15d\rangle^{1}]$
12	€14027	24	5d	€14027/5d	$y(x)_{PV\_12} = €2805.40/1d \cdot [\langle x-24d\rangle^{1} - \langle x-29d\rangle^{1}]$
13	€4550	29	5d	€4550/5d	$y(x)_{PV\_13} = €910/1d \cdot [\langle x-29d\rangle^{1} \cdot \langle x-34d\rangle^{1}]$
14	€15825	17	5d	€15825/5d	$y(x)_{PV\_14} = €3165/1d \cdot [\langle x-17d\rangle^{1} - \langle x-22d\rangle^{1}]$
15	€42642	22	15d	€42642/15d	$y(x)_{PV\_15} = €2842.80/1d \cdot [\langle x-22d\rangle^{1} - \langle x-37d\rangle^{1}]$
16	€29760	22	15d	€29760/15d	$y(x)_{PV\_16} = €1984/1d \cdot [\langle x-22d\rangle^{1} - \langle x-37d\rangle^{1}]$
17	€5881	37	2d	€5881/2d	$y(x)_{PV\_17} = €2940.50/1d \cdot [\langle x-37d\rangle^{1} - \langle x-39d\rangle^{1}]$
18	€1008	39	1d	€1008/1d	$y(x)_{PV\_18} = €1008/1d \cdot [\langle x-39d\rangle^{1} - \langle x-40d\rangle^{1}]$
19	€1280	40	2d	€1280/2d	$y(x)_{PV\_19} = €640/1d \cdot [\langle x-40d\rangle^{1} \cdot \langle x-42d\rangle^{1}]$
20	€2336	40	2d	€2336/2d	$y(x)_{PV\_20} = €1168/1d \cdot [\langle x-40d\rangle^{1} - \langle x-42d\rangle^{1}]$
21	€800	42	1d	€800/1d	$y(x)_{PV\_21} = €800/1d \cdot [\langle x-42d\rangle^{1} \cdot \langle x-43d\rangle^{1}]$

Appendix 2-B: Activity Actual Cost

Activity	*AC*	*AD*	*AS*	Output
1	€2400	3d	0	$y(x)_{AC\_1} = €800/1d \cdot [\langle x-0d\rangle^{1} - \langle x-3d\rangle^{1}]$

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Activity	***AC***	***AD***	***AS***	**Output**
2	€800	1d	3	$y(x)_{AC\_2} = €800/1d \cdot [\langle x\text{-}3d\rangle^1 - \langle x\text{-}4d\rangle^1]$
3	€440	1d	4	$y(x)_{AC\_3} = €440/1d \cdot [\langle x\text{-}4d\rangle^1 - \langle x\text{-}5d\rangle^1]$
4	€800	1d	17	$y(x)_{AC\_4} = €800/1d \cdot [\langle x\text{-}17d\rangle^1 - \langle x\text{-}18d\rangle^1]$
5	€1	0.125d	4	$y(x)_{AC\_5} = €8/1d \cdot [\langle x\text{-}4d\rangle^1 - \langle x\text{-}4.125d\rangle^1]$
6	€800	1d	27	$y(x)_{AC\_6} = €800/1d \cdot [\langle x\text{-}27d\rangle^1 - \langle x\text{-}28d\rangle^1]$
7	€9886	0.5d	4.125	$y(x)_{AC\_7} = €19772\ /1d \cdot [\langle x\text{-}4.125d\rangle^1 - \langle x\text{-}4.625d\rangle^1]$
8	€8827	1d	5	$y(x)_{AC\_8} = €8827/1d \cdot [\langle x\text{-}5d\rangle^1 - \langle x\text{-}6d\rangle^1]$
9	€20750	18d	6	$y(x)_{AC\_9} = €1152.78/1d \cdot [\langle x\text{-}6d\rangle^1 - \langle x\text{-}24d\rangle^1]$
10	€11012	8d	9	$y(x)_{AC\_10} = €1376.50/1d \cdot [\langle x\text{-}9d\rangle^1 - \langle x\text{-}17d\rangle^1]$
11	€6660	5d	10	$y(x)_{AC\_11} = €1332/1d \cdot [\langle x\text{-}10d\rangle^1 - \langle x\text{-}15d\rangle^1]$
12	€14027	5d	34	$y(x)_{AC\_12} = €2805.46/1d \cdot [\langle x\text{-}34d\rangle^1 - \langle x\text{-}39d\rangle^1]$
13	€4550	5d	32	$y(x)_{AC\_13} = €1990/1d \cdot \langle x\text{-}39d\rangle^1 - €1350/1d \cdot \langle x\text{-}40d\rangle^1 - €640/1d \cdot \langle x\text{-}44d\rangle^1$
14	€15825	5d	17	$y(x)_{AC\_14} = €3165/1d \cdot [\langle x\text{-}17d\rangle^1 - \langle x\text{-}22d\rangle^1]$
15	€42642	15d	22	$y(x)_{AC\_15} = €12709.80 \cdot \langle x\text{-}22d\rangle^1 - €11385/1d \cdot \langle x\text{-}24d\rangle^1 - €1324.8/1d \cdot \langle x\text{-}37d\rangle^1$
16	€29760	15d	22	$y(x)_{AC\_16} = €6268.80 \cdot \langle x\text{-}22d\rangle^1 - €4944/1d \cdot \langle x\text{-}24d\rangle^1 - €1324.80/1d \cdot \langle x\text{-}37d\rangle^1$
17	€5881	2d	47	$y(x)_{AC\_17} = €2940.50/1d \cdot [\langle x\text{-}47d\rangle^1 - \langle x\text{-}49d\rangle^1]$
18	€1008	1d	49	$y(x)_{AC\_18} = €1008/1d \cdot [\langle x\text{-}49d\rangle^1 - \langle x\text{-}50d\rangle^1]$
19	€1280	2d	50	$y(x)_{AC\_19} = €640/1d \cdot [\langle x\text{-}50d\rangle^1 - \langle x\text{-}52d\rangle^1]$
20	€2336	2d	50	$y(x)_{AC\_20} = €1168/1d \cdot [\langle x\text{-}50d\rangle^1 - \langle x\text{-}52d\rangle^1]$
21	€800	1d	52	$y(x)_{AC\_21} = €800/1d \cdot [\langle x\text{-}52d\rangle^1 - \langle x\text{-}53d\rangle^1]$

Appendix 2-C: Activity Actual Percent Table

Activity	**0**	**1**	**2**	**3**	**4**	**4.125**	**4.25**	**4.375**	**4.5**	**4.625**	**4.75**	**4.875**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
1	0	1/3	2/3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2				0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3					0	1/8	2/8	3/8	4/8	5/8	6/8	7/8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4																										1	1	1	1	1
6					0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5																														
7						0	1/4	2/4	3/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8													0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9														0	1/18	2/18	3/18	4/18	5/18	6/18	7/18	8/18	9/18	10/18	11/18	12/18	13/18	14/18	15/18	16/17

10	0	1/8	2/8	3/8	4/8	5/8	6/8	7/8	1	1	1	1	1	1
11		0	1/5	2/5	3/5	4/5	1	1	1	1	1	1	1	1
12														
13														
14									0	1/5	2/5	3/5	4/5	1
15														0
16														0
17														
18														
19														
20														
21														

(*Continues*)

23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
				0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17/18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
											0	1/5	2/5	3/5	4/5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
																0	1/5	2/5	3/5	4/5	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13/200	13/100	99/500	133/500	167/500	201/500	47/100	429/800	241/400	107/160	147/200	641/800	347/400	747/800	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13/200	13/100	99/500	133/500	167/500	201/500	47/100	429/800	241/400	107/160	147/200	641/800	347/400	747/800	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
																								0	½	1	1	1	1	1
																										0	1	1	1	1
																											0	½	1	1
																											0	½	1	1
																													0	1

Appendix 2-D: Activity Actual Percent Differences Table

Activity	**0**	**1**	**2**	**3**	**4**	**4.125**	**4.25**	**4.375**	**4.50**	**4.625**	**4.75**	**4.875**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
1	**1/3**	**.**	**.**	**-1/3**																										

2	**1/1 -1/1**
3	**1/8 . . . . -1/8**
4	**1/1 -1/1**
6	**1/1 -1/1**
5	
7	**1/4 . . . -1/4**
8	**1/1 -1/1**
9	**1/18 . . . .**
10	**1/8 . . . . -1/8**
11	**1/5 . . . . -1/5**
12	
13	
14	**1/5 . . . . -1/5**
15	**13/200**
16	**13/200**
17	
18	
19	
20	
21	

(*Continues*)

23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
				1/1	-1/1																									
.	-1/18																													
											1/5	.	.	.	.	-1/5														
																1/5	.	.	.	.	-1/5									
.	3/1000	.	.	.	.	-7/4000	.	.	.	.	.	.	.	-53/800																
.	3/1000	.	.	.	.	-7/4000	.	.	.	.	.	.	.	-53/800																
																								1/2	.	-1/2				

	1/1	**-1/1**			
		1/2	**.**	**-1/2**	
		1/2	**.**	**-1/2**	
				1/1	**-1/1**

Appendix 2-E.1: Activity Actual Percent and Earned Value

Activity	Actual Percent (*AP*)	*PC*	Earned Value (*EV*)
1	$y(x)_{AP\_1} = {}^{1}/_{3}/d \cdot [\langle x-0\rangle^1 - \langle x-3\rangle^1]$	€2400	$y(x)_{EV\_1}$ = €800/d · [$\langle x-0d\rangle^1 - \langle x-3d\rangle^1$]
2	$y(x)_{AP\_2} = {}^{1}/_{1}/d \cdot [\langle x-3\rangle^1 - \langle x-4\rangle^1]$	€800	$y(x)_{EV\_2}$ = €800/d · [$\langle x-3d\rangle^1 - \langle x-4d\rangle^1$]
3	$y(x)_{AP\_3} = {}^{1}/_{8}/d \cdot [\langle x-4\rangle^1 - \langle x-5\rangle^1]$	€440	$y(x)_{EV\_3}$ = €440/d · [$\langle x-4d\rangle^1 - \langle x-5d\rangle^1$]
4	$y(x)_{AP\_4} = {}^{1}/_{1}/d \cdot [\langle x-17\rangle^1 - \langle x-18\rangle^1]$	€800	$y(x)_{EV\_4}$ = €800/d · [$\langle x-17d\rangle^1 - \langle x-18d\rangle^1$]
6	$y(x)_{AP\_6} = {}^{1}/_{1}/d \cdot [\langle x-4\rangle^1 - \langle x-4.125\rangle^1]$	€1	$y(x)_{EV\_6}$ =€8/d · [$\langle x-4d\rangle^1 - \langle x-4.125d\rangle^1$]
5	$y(x)_{AP\_5} = {}^{1}/_{1}/d \cdot [\langle x-27\rangle^1 - \langle x-28\rangle^1]$	€800	$y(x)_{EV\_5}$ = €800/d · [$\langle x-27d\rangle^1 - \langle x-28d\rangle^1$]
7	$y(x)_{AP\_7} = {}^{1}/_{4}/d \cdot [\langle x-4.125\rangle^1 - \langle x-4.625\rangle^1]$	€9886	$y(x)_{EV\_7}$ = €19772/d · [$\langle x-4.125d\rangle^1 - \langle x-4.625d\rangle^1$]
8	$y(x)_{AP\_8} = {}^{1}/_{1}/d \cdot [\langle x-5\rangle^1 - \langle x-6\rangle^1]$	€8827	$y(x)_{EV\_8}$ = €8827/d · [$\langle x-5d\rangle^1 - \langle x-6d\rangle^1$]
9	$y(x)_{AP\_9} = {}^{1}/_{18}/d \cdot [\langle x-6\rangle^1 - \langle x-24\rangle^1]$	€20750	$y(x)_{EV\_9}$ = €1152.78/d · [$\langle x-6d\rangle^1 - \langle x-24d\rangle^1$]
10	$y(x)_{AP\_10} = {}^{1}/_{8}/d \cdot [\langle x-9\rangle^1 - \langle x-17\rangle^1]$	€11012	$y(x)_{EV\_10}$ = €1376.50/d · [$\langle x-9d\rangle^1 - \langle x-17d\rangle^1$]
11	$y(x)_{AP\_11} = {}^{1}/_{5}/d \cdot [\langle x-10\rangle^1 - \langle x-15\rangle^1]$	€6660	$y(x)_{EV\_11}$ = €1332/d · [$\langle x-10d\rangle^1 - \langle x-15d\rangle^1$]
12	$y(x)_{AP\_12} = {}^{1}/_{5}/d \cdot [\langle x-34\rangle^1 - \langle x-39\rangle^1]$	€14027	$y(x)_{EV\_12}$ = €2805.40/d · [$\langle x-34d\rangle^1 - \langle x-39d\rangle^1$]
13	$y(x)_{AP\_13} = {}^{1}/_{5}/d \cdot [\langle x-39\rangle^1 - \langle x-44\rangle^1]$	€4550	$y(x)_{EV\_13}$ = €1990/d · $\langle x-39d\rangle^1$ - €1350/d · $\langle x-40d\rangle^1$ - €640/d · $\langle x-44d\rangle^1$
14	$y(x)_{AP\_14} = {}^{1}/_{5}/d \cdot [\langle x-17\rangle^1 - \langle x-22\rangle^1]$	€15825	$y(x)_{EV\_14}$ = €3165/d · [$\langle x-17d\rangle^1 - \langle x-22d\rangle^1$]
15	$y(x)_{AP\_15} = {}^{13}/_{200}/d \cdot \langle x-22\rangle^1 + {}^{3}/_{1000}/d \cdot \langle x-24\rangle^1 - {}^{7}/_{4000}/d \cdot \langle x-28\rangle^1 - {}^{53}/_{800}/d \cdot \langle x-37\rangle^1$	€42642	$y(x)_{EV\_15}$ = €12709.80/d · $\langle x-22d\rangle^1$ - €11385/d · $\langle x-24d\rangle^1$ - €1324.80/d · $\langle x-37d\rangle^1$
16	$y(x)_{AP\_16} = {}^{13}/_{200}/d \cdot \langle x-22\rangle^1 + {}^{3}/_{1000}/d \cdot \langle x-24\rangle^1 - {}^{7}/_{4000}/d \cdot \langle x-28\rangle^1 - {}^{53}/_{800}/d \cdot \langle x-37\rangle^1$	€29760	$y(x)_{EV\_16}$ = €6268.80/d · $\langle x-22d\rangle^1$ - €4944/d · $\langle x-24d\rangle^1$ - €1324.80/d · $\langle x-37d\rangle^1$
17	$y(x)_{AP\_17} = {}^{1}/_{2}/d \cdot [\langle x-47\rangle^1 - \langle x-49\rangle^1]$	€5881	$y(x)_{EV\_17}$ = €2940.50/d · [$\langle x-47d\rangle^1 - \langle x-49d\rangle^1$]
18	$y(x)_{AP\_18} = {}^{1}/_{1}/d \cdot [\langle x-49\rangle^1 - \langle x-50\rangle^1]$	€1008	$y(x)_{EV\_18}$ = €1008/d · [$\langle x-49d\rangle^1 - \langle x-50d\rangle^1$]
19	$y(x)_{AP\_19} = {}^{1}/_{2}/d \cdot [\langle x-50\rangle^1 - \langle x-52\rangle^1]$	€1280	$y(x)_{EV\_19}$ = €640/d · [$\langle x-50d\rangle^1 - \langle x-52d\rangle^1$]
20	$y(x)_{AP\_20} = {}^{1}/_{2}/d \cdot [\langle x-50\rangle^1 - \langle x-52\rangle^1]$	€2336	$y(x)_{EV\_20}$ = €1168/d · [$\langle x-50d\rangle^1 - \langle x-52d\rangle^1$]
21	$y(x)_{AP\_21} = {}^{1}/_{1}/d \cdot [\langle x-52\rangle^1 - \langle x-53\rangle^1]$	€800	$y(x)_{EV\_21}$ = €800/d · [$\langle x-52d\rangle^1 - \langle x-53d\rangle^1$]

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Appendix 2-E.2: Earned Value Table

Activity	0	1	2	3	4	4.125	4.25	4.375	4.50	4.625	4.75	4.875	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1		800	1600	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400
2					800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800
3						55	110	165	220	275	330	385	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440
4																										800	800	800
6						1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5																												
7						0	2471.50	4943	7414.50	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886
8														8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827
9														0	1152.78	2305.56	3458.33	4611.11	5763.89	6916.67	8069.44	9222.22	10375	11527.78	12680.56	13833.33	14986.11	16138.89
10																	0	1376.50	2753	4129.50	5506	6882.50	8259	9635.50	11012	11012	11012	11012
11																		0	1332	2664	3996	5328	6660	6660	6660	6660	6660	6660
12																												
13																												
14																									0	3165	6330	9495
15																												
16																												
17																												
18																												
19																												
20																												
21																												
EV	0	800	1600	2400	3200	3256	5782.50	8309	10835.50	13362	13417	13472	13527	22354	23506.78	24659.56	25812.33	28341.61	32202.89	36064.17	39925.44	43786.72	47648	50177.28	52706.56	57824.33	62142.11	66459.89

(Continues)

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400
800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800
440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440	440
800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800	800
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
							800	800	800	800	800	800	800	800	800	800	800	800	800	800	800
9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886
8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827
17291.67	18444.44	19597.22	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750
11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012
6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660
													0	2805.40	5610.80	8416.20	11221.60	14027	14027	14027	14027
																		0	1990	2630	3270
12660	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825
	0	12709.80	25419.60	26744.40	28069.20	29394	30718.80	32043.60	33368.40	34693.20	36018	37342.80	38667.60	39992.40	41317.20	42642	42642	42642	42642	42642	42642
	0	6268.80	12537.60	13862.40	15187.20	16512	17836.80	19161.60	20486.40	21811.20	23136	24460.80	25785.60	27110.40	28435.20	29760	29760	29760	29760	29760	29760
70777.67	75095.44	95226.82	115358.20	118007.80	120657.40	123307	126756.60	129406.20	132055.80	134705.40	137355	140004.60	142654.20	148109.20	153564.20	159019.20	161824.60	164630	166620	167260	167900

(Continues)

43	44	45	46	47	48	49	50	51	52	53
2400	2400	2400	2400	2400	2400	2400	2400	2400	2400	2400
800	800	800	800	800	800	800	800	800	800	800
440	440	440	440	440	440	440	440	440	440	440
800	800	800	800	800	800	800	800	800	800	800

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1	1	1	1	1	1	1	1	1	1	1
800	800	800	800	800	800	800	800	800	800	800
9886	9886	9886	9886	9886	9886	9886	9886	9886	9886	9886
8827	8827	8827	8827	8827	8827	8827	8827	8827	8827	8827
20750	20750	20750	20750	20750	20750	20750	20750	20750	20750	20750
11012	11012	11012	11012	11012	11012	11012	11012	11012	11012	11012
6660	6660	6660	6660	6660	6660	6660	6660	6660	6660	6660
14027	14027	14027	14027	14027	14027	14027	14027	14027	14027	14027
3910	4550	4550	4550	4550	4550	4550	4550	4550	4550	4550
15825	15825	15825	15825	15825	15825	15825	15825	15825	15825	15825
42642	42642	42642	42642	42642	42642	42642	42642	42642	42642	42642
29760	29760	29760	29760	29760	29760	29760	29760	29760	29760	29760
				0	2940.50	5881	5881	5881	5881	5881
							1008	1008	1008	1008
							0	640	1280	1280
							0	1168	2336	2336
										800
168540	169180	169180	169180	169180	172120.50	175061	176069	177877	179685	180485

$$y(x)_{PV\_project} = €800/1d \cdot \langle x\text{-}0d\rangle^1 - €352/1d \cdot \langle x\text{-}4d\rangle^1 + €19764/1d \cdot \langle x\text{-}4.125d\rangle^1 - €19772/1d \cdot \langle x\text{-}4.625d\rangle^1 + €8387/1d \cdot \langle x\text{-}5d\rangle^1 - €7674.22/1d \cdot \langle x\text{-}6d\rangle^1 + €1376.5/1d \cdot \langle x\text{-}9d\rangle^1 + €1332/1d \cdot \langle x\text{-}10d\rangle^1 - €1332/1d \cdot \langle x\text{-}15d\rangle^1 + €2588.50/1d \cdot \langle x\text{-}17d\rangle^1 - €800/1d \cdot \langle x\text{-}18d\rangle^1 + €15813.60/1d \cdot \langle x\text{-}22d\rangle^1 - €14676.3778/1d \cdot \langle x\text{-}24d\rangle^1 + €800/1d \cdot \langle x\text{-}27d\rangle^1 - €800/1d \cdot \langle x\text{-}28d\rangle^1 - €1895.40/1d \cdot \langle x\text{-}29d\rangle^1 - €910/1d \cdot \langle x\text{-}34d\rangle^1 + €290.90/1d \cdot \langle x\text{-}37d\rangle^1 - €1932.50/1d \cdot \langle x\text{-}39d\rangle^1 + €800/1d \cdot \langle x\text{-}40d\rangle^1 - €1008/1d \cdot \langle x\text{-}42d\rangle^1 - €800/1d \cdot \langle x\text{-}43d\rangle^1$$

$$y(x)_{EV\_project} = €800/d \cdot \langle x\text{-}0d\rangle^1 - €352/1d \cdot \langle x\text{-}4d\rangle^1 + €19764/1d \cdot \langle x\text{-}4.125d\rangle^1 - €19772/1d \cdot \langle x\text{-}4.625d\rangle^1 + €8387/1d \cdot \langle x\text{-}5d\rangle^1 - €7674.22/1d \cdot \langle x\text{-}6d\rangle^1 + €1376.50/1d \cdot \langle x\text{-}9d\rangle^1 + €1332/1d \cdot \langle x\text{-}10d\rangle^1 - €1332/1d \cdot \langle x\text{-}15d\rangle^1 + €2588.50/1d \cdot \langle x\text{-}17d\rangle^1 - €800/1d \cdot \langle x\text{-}18d\rangle^1 + €15813.60/1d \cdot \langle x\text{-}22d\rangle^1 - €17481.78/1d \cdot \langle x\text{-}24d\rangle^1 + €800/1d \cdot \langle x\text{-}27d\rangle^1 - €800/1d \cdot \langle x\text{-}28d\rangle^1 + €2805.40/1d \cdot \langle x\text{-}34d\rangle^1 - €2649.60/1d \cdot \langle x\text{-}37d\rangle^1 - €815.40/1d \cdot \langle x\text{-}39d\rangle^1 - €1350/1d \cdot \langle x\text{-}40d\rangle^1 - €640/1d \cdot \langle x\text{-}44d\rangle^1 + €2940.50/1d \cdot \langle x\text{-}47d\rangle^1 - €1932.50/1d \cdot \langle x\text{-}49d\rangle^1 + €800/1d \cdot \langle x\text{-}50d\rangle^1 - €1008/1d \cdot \langle x\text{-}52d\rangle^1 - €800/1d \cdot \langle x\text{-}53d\rangle^1$$

Appendix 2-F.1: Differences for Transposition (Planned Value)

Time Cutoff	0d	Σ	3d	Σ	4d	Σ	4.125d	Σ	4.625d	Σ	5d	Σ
Slope Change	+€800/d		+€0/d		-€352/d		+€19764/1d		-€19772/1d		+€8387/d	
Total Slope		€800/d		€800/d		€448/d		€20212/d		€440/d		€8827/d
Cost Cutoff	€0		€2400		€3200		€3256		€13362		€13257	
Inverted Slope		1d/€800		1d/€800		1d/€448		1d/€20212		1d/€440		1d/€8827
Slope Change	+1d/€800		=1d/€800-1d/€800 =0d/€1		=1d/€448-1d/€800 =11d/€11200		=1d/€20212-1d/€448 =-4941d/€2263744		=1d/€440-1d/€20212 =4943d/€222320		=1d/€8827-1d/€440 =-8387d/€3883880	

(Continues)

6d	Σ	9d	Σ	10d	Σ	15d	Σ	17d	Σ	18d	Σ

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-€69068/9d		+€2753/2d		+€1332/d		-€1332/1d		+€5177/2d		-€800/1d	
	€10375/9d		€45527/18d		€69503/18d		€45527/18d		€46060/9d		€38860/9d
€22354		€25812.33		€28341.61		€47648		€52706.556		€57824.33	
	9d/€10375		18d/€45527		18d/€69503		18d/€45527		9d/€46060		9d/€38860
=9d/€10375-1d/€8827 =205d/€271818		=18d/€45527-9d/€10375 =-272d/€576149		=18d/€69503-18d/€45527 =-199d/€1459071		=18d/€45527-18d/€69503 =199d/€1459071		=9d/€46060-18d/€45527 =-1126d/€5630775		=9d/€38860-9d/46060 =162d/€4474729	

(Continues)

22d	Σ	24d	Σ	34d	Σ	27d	Σ	28d	Σ	29d	Σ
+€79068/5d		-€660437/45d		-€910/1d		+€800/1d		-€800/1d		-€9477/5d	
	€905912/45d		€5455/1d		€13248/5d		€6255/1d		€5455/1d		€17798/5d
€75095.44		€115358.20		€161231.20		€131723.20		€137978.20		€143433.20	
	3d/60394		1d/€5455		5d/€13248		1d/€6255		1d/€5455		5d/€17798
=3d/€60394-9d/€38860 =-1333d/€7327118		=1d/€5455-3d/€60394 =680d/€5088131		=5d/€13248-5d/€17798 =-547d/€5669274		=1d/€6255-1d/€5455 =-32d/€1364841		=1d/€5455-1d/€6255 =32d/€1364841		=5d/€17798-1d/€5455 =600d/€6146761	

(Continues)

37d	Σ	39d	Σ	40d	Σ	42d	Σ	43d	Σ
+€2909/10d		-€3865/2d		+€800/1d		-€1008/1d		-€800/1d	
	€5881/2d		€1008/1d		€1808/d		€800/1d		€0/1d
€169180		€175061		€176069		€179685		€180485	
	2d/€5881		1d/€1008		1d/€1808		1d/€800		0d/€1
=2d/€5881-5d/€13248 =-146d/€3910305		=1d/€1008-2d/€5881 =3865d/€5928048		=1d/€1808-1d/€1008 =-25d/€56952		=1d/€800-1d/€1808 =63d/€90400		=0d/€1-1d/€800 =-1d/€800	

Non-Cumulative, Cumulative

$$x(y)_{PV\_project} = 1d/€800 \cdot \langle y\text{-€}0\rangle^1 + 11d/€11200 \cdot \langle y\text{-€}3200\rangle^1 - 4941d/€2263744 \cdot \langle y\text{-€}3256\rangle^1 + 4943d/€2223320 \cdot \langle y\text{-€}13362\rangle^1 - 8387d/€3883880 \cdot \langle y\text{-€}13527\rangle^1 + 205d/€271818 \cdot \langle y\text{-€}22354\rangle^1 - 272d/€576149 \cdot \langle y\text{-€}25812.33\rangle^1 - 199d/€1459071 \cdot \langle y\text{-€}28341.61\rangle^1 + 199d/€1459071 \cdot \langle y\text{-€}47648\rangle^1 - 1126d/€5630775 \cdot \langle y\text{-€}52706.56\rangle^1 + 162d/€4474729 \cdot \langle y\text{-€}57824.33\rangle^1 - 1333d/€7327118 \cdot \langle y\text{-€}75095.44\rangle^1 + 680d/€5088131 \cdot \langle y\text{-€}115358.20\rangle^1 - 32d/€1364841 \cdot \langle y\text{-€}131723.20\rangle^1 + 32d/€1364841 \cdot \langle y\text{-€}137978\rangle^1 + 600d/€6146761 \cdot \langle y\text{-€}143433.20\rangle^1 + 547d/€5669274 \cdot \langle y\text{-€}161231\rangle^1 - 146d/€3910305 \cdot \langle y\text{-€}169180\rangle^1 + 3865d/€5928048 \cdot \langle y\text{-€}175061\rangle^1 - 25d/€56952 \cdot \langle y\text{-€}176069\rangle^1 + 63d/€90400 \cdot \langle y\text{-€}179685\rangle^1 - 1d/€800 \cdot \langle y\text{-€}180485\rangle^1$$

Appendix 2-F.2: Differences for Transposition (Earned Value)

Time Cutoff	0d	Σ	3d	Σ	4d	Σ	4.125d	Σ	4.625d	Σ	5d	Σ
Slope Change	+€800/d		+€0/d		-€352/d		+€19764/d		-€19772/d		+€8387/1d	
Total Slope		€800/d		€800/d		€448/d		€20212/d		€440/1d		€8827/1d
Cost Cutoff	€0		€2400		€3200		€3256		€13362		€13527	
Inverted Slope		1d/€800		1d/€800		1d/€448		1d/€20212		1d/€440		1d/€8827
Slope Change	1d/€800		1/800-1/800 =0d/€1		1/448-1/800 =11d/€11200		1/20212-1/448 =-4941d/€2263744		1/440-1/20212 =4943d/€2223320		1/8827-1/440 =-8387d/€3883880	

(Continues)

6d	Σ	9d	Σ	10d	Σ	15d	Σ	17d	Σ	18d	Σ
-€7674.22/1d		+€1376.50/1d		+€1332/1d		-€1332/d		+€2588.50/d		-€800/d	
	€10375/9d		€45527/18d		€69503/18d		€45527/18d		€46060/9d		€38860/9d
€22354		€25812.33		€28341.61		€47648		€52706.56		€57824.33	
	9d/€10375		18d/€45527		18d/€69503		18d/€45527		9d/€46060		9d/€38860
9/10375-1/8827 =205d/€271818		18/45527-9/10375 =-272d/€576149		18/69503-18/45527 =-199d/€1459071		18/45527-18/69503 =199d/€1459071		9/46060-18/45527 =-1126d/€5630775		9/38860-9/46060 =162d/€4474729	

(Continues)

22d	Σ	24d	Σ	27d	Σ	28d	Σ	34d	Σ	37d	Σ
+€15813.60/d		-€17481.78/d		+€800/1d		-€800/1d		+€2805.40/1d		-€2649.60/1d	
	€905912/45d		€13248/5d		€17248/5d		€13248/5d		€5455/1d		€14027/5d
€75095.44		€115358.20		€123307		€126756.60		€142654.20		€159019.20	
	45d/€905912		5d/€13248		5d/€17248		5d/€13248		1d/€5455		5d/€14027
45/905912-9/38860 =-1333d/€7327118		5/13248-45/905912 =1089d/€3322738		5/17248-5/13248 =-625d/€7140672		5/13248-5/17248 =625d/€7140672		1/5455-5/13248 =-324d/€1669265		5/14027-1/5455 =289d/€1669195	

(Continues)

39d	Σ	40d	Σ	44d	Σ	47d	Σ	49d	Σ	50d	Σ	52d	Σ	53d	Σ
-€815.40/d		-€1350/d		-€640/d		+€2940.50/d		-€1932.50/d		+€800/1d		-€1008/1d		-€800/1d	
	€1990/1d		€640/1d		€0/1d		€5881/2d		€1008/1d		€1808/1d		€800/1d		€0/1d
€164630		€166620		€169180		€169180		€175061		€176069		€179685		€180485	
	1d/€1990		1d/€640		0d/€1		2d/€5881		1d/€1008		1d/€1808		1d/€800		0d/€1
1/1990-5/14027 =-115d/€787363		1/640-1/1990 =27d/€25472		0/1-1/640 =-1d/€640		2/5881-0/1 =2d/€5881		1/1008-2/5881 =3865d/€5928048		1/1808–1/1008 =-25d/€56952		1/800-1/1808 =63d/€90400		0/1-1/800 =-1/800	

Non-Cumulative, Cumulative

$x(y)_{\text{EV\_project}} = 1\text{d}/\text{€}800 \cdot \langle y\text{-€}0\rangle^1 + 11\text{d}/\text{€}11200 \cdot \langle y\text{-€}3200\rangle^1 - 4941\text{d}/\text{€}2263744 \cdot \langle y\text{-€}3256\rangle^1 + 4943\text{d}/\text{€}2223320 \cdot \langle y\text{-€}13362\rangle^1 - 8387\text{d}/\text{€}3883880 \cdot \langle y\text{-€}13527\rangle^1 + 205\text{d}/\text{€}271818 \cdot \langle y\text{-€}22354\rangle^1 - 272\text{d}/\text{€}576149 \cdot \langle y\text{-€}25812.33\rangle^1 - 199\text{d}/\text{€}1459071 \cdot \langle y\text{-€}28341.61\rangle^1 + 199\text{d}/\text{€}1459071 \cdot \langle y\text{-€}47648\rangle^1 - 1126\text{d}/\text{€}5630775 \cdot \langle y\text{-€}52706.56\rangle^1 + 162\text{d}/\text{€}4474729 \cdot \langle y\text{-€}57824.33\rangle^1 - 1333\text{d}/\text{€}7327118 \cdot \langle y\text{-€}75095.44\rangle^1 + 1089\text{d}/\text{€}3322738 \cdot \langle y\text{-€}115358.20\rangle^1 - 625\text{d}/\text{€}7140672 \cdot \langle y\text{-€}123307\rangle^1 + 625\text{d}/\text{€}7140672 \cdot \langle y\text{-€}126756.60\rangle^1 - 324\text{d}/\text{€}166915 \cdot \langle y\text{-€}142654.20\rangle^1 + 289\text{d}/\text{€}1669195 \cdot \langle y\text{-€}159019.20\rangle^1 + 115\text{d}/\text{€}787363 \cdot \langle y\text{-€}164630\rangle^1 + 27\text{d}/\text{€}25472 \cdot \langle y\text{-€}166620\rangle^1 - 1\text{d}/\text{€}640 \cdot \langle y\text{-€}169180\rangle^1 + 2\text{d}/\text{€}5881 \cdot \langle y\text{-€}169180\rangle^1 + \mathbf{3d \cdot \langle y\text{-€}169180\rangle^0} + 3865\text{d}/\text{€}5928048 \cdot \langle y\text{-€}175061\rangle^1 - 25\text{d}/\text{€}56952 \cdot \langle y\text{-€}176069\rangle^1 + 63\text{d}/\text{€}90400 \cdot \langle y\text{-€}179685\rangle^1 - 1\text{d}/\text{€}800 \cdot \langle y\text{-€}180485\rangle^1$

Appendix 2-G: Earned Schedule Derivation

Get rises $z_i(y_i)$ from transposed difference equation $z(y) = x(y)_{\text{EV\_project}} - x(y)_{\text{PV\_project}} = (1\text{d}/\text{€}800-1\text{d}/\text{€}800) \cdot \langle y\text{-€}0\rangle^1 + (0\text{d}/\text{€}1-0\text{d}/\text{€}1) \cdot \langle y\text{-€}2400\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}3200\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}3256\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}13362\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}13527\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}22354\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}25812.33\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}28341.61\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}47648\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}52706.56\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}57824.33\rangle^1 + 0\text{d}/\text{€}1 \cdot \langle y\text{-€}75095.44\rangle^1 + 324\text{d}/\text{€}1669265 \cdot \langle y\text{-€}115358.20\rangle^1 - 625\text{d}/\text{€}7140672 \cdot \langle y\text{-€}123307\rangle^1 + 625\text{d}/\text{€}7140672 \cdot \langle y\text{-€}126756.60\rangle^1 + 32\text{d}/\text{€}1364841 \cdot \langle y\text{-€}131723.20\rangle^1 - 32\text{d}/\text{€}1364841 \cdot \langle y\text{-}$

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€137978.20⟩^{1} - 324d/€1669265 · ⟨*y*-142654.20⟩^{1} - 600d/€6146761 · ⟨*y*-143433.20⟩^{1} + 289d/€1669195 · ⟨*y*-159019.20⟩^{1} - 547d/€5669274 · ⟨*y*-161231.20⟩^{1} + 115d/€787363 · ⟨*y*-164630⟩^{1} + 27d/€25472 · ⟨*y*-166620⟩^{1} - 2571d/€8242399 · ⟨*y*-169180⟩^{1} + 2d/€5881 · ⟨*y*-169180⟩^{1} + **3d/€1 · ⟨*y*-€169180⟩^{0}** + 0d/€1 · ⟨*y*-€175061⟩^{1} + 0d/€1 · ⟨*y*-€176069⟩^{1} + 0d/€1 · ⟨*y*-€179685⟩^{1} + 0d/€1 · ⟨*y*-€180485⟩^{1}

Evaluate it at each cutoff to get its absolute height *z* there in days. And evaluate $x(y)_{EV\_project}$ at any y_i-cutoffs from both *EV* and *PV* as new cutoffs for $z(x)_{ES\_project}$

y_i	$z_i(y_i)$	$x(y_i)_{EV\_project}$
€0.00	0d	0d
€2,400.00	0d	3d
€3,200.00	0d	4d
€3,256.00	0d	4.125d
€13,362.00	0d	4.625d
€13,527.00	0d	5d
€22,354.00	0d	6d
€25,812.33	0d	9d
€28,341.61	0d	10d
€47,648.00	0d	15d
€52,706.56	0d	17d
€57,824.33	0d	18d
€75,095.44	0d	22d
€115,358.20	0d	24d
€123,307.00	1.5428d	27d
€126,756.60	1.9105d	28d
€131,723.20	2.8745d	29.8745d
€137,978.20	4.2352d	31.6393d
€142,654.20	5.1428d	34d
€143,433.20	5.1428d	36.8572d
€159,019.20	3.6214d	37d
€161,231.20	3.7885d	37.7885d
€164,630.00	3.7172d	39d
€166,620.00	3.9662d	40d
€169,180.00	7d	44d
€169,180.00	10d	47d
€175,061.00	10d	49d
€176,069.00	10d	50d
€179,685.00	10d	52d
€180,485.00	10d	53d

Determine runs $\Delta x(y_i)$ as distances between new cutoffs and compose updated slopes.

Cutoff	Run	Verticals	Rise	Slope	ΔSlope	Diagonal	New Slope
$x(y_i)_{EV\_project}$	$\Delta x(y_i)_{EV\_project}$	$z_i(y_i)$	$\Delta z(y_i)$	Rise/Run	$y(x_i)_{SV(t)}$	Linear	$z(x_i)_{ES}$
0	3d	0	0	0	0	1d/d	1
3	1d	0	0	0	0	0	0
4	0.125d	0	0	0	0	0	0
4.125	0.5d	0	0	0	0	0	0
4.625	0.375d	0	0	0	0	0	0
5	1d	0	0	0	0	0	0
6	3d	0	0	0	0	0	0
9	1d	0	0	0	0	0	0
10	5d	0	0	0	0	0	0
15	2d	0	0	0	0	0	0
17	1d	0	0	0	0	0	0
18	4d	0	0	0	0	0	0
22	2d	0	0	0	0	0	0
24	3d	0	-1.5428d	-0.5143d/d	0.5143d/d	0	-0.5143d/d

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27	1d	1.5428d	-0.3676d	-0.3676d/d	-0.1467d/d	0	0.1467d/d
28	1.8745d	1.9105d	-0.9640d	-0.5143d/d	0.1467d/d	0	-0.1467d/d
29.8745	1.7648d	2.8745d	-1.3607d	-0.7710d/d	0.2568d/d	0	-0.2568d/d
31.6393	2.3607d	4.2352d	-0.9076d	-0.3845d/d	-0.3866d/d	0	0.3866d/d
34	2.8572d	5.1428d	0	0	-0.3845d/d	0	0.3845d/d
36.8572	0.1428d	5.1428d	1.5214d	10.6536d/d	-10.6536d/d	0	10.6536d/d
37	0.7885d	3.6214d	-0.1671d	-0.2119d/d	10.8655d/d	0	-10.8655d/d
37.7885	1.2115d	3.7885d	0.0712d	0.0588d/d	-0.2707d/d	0	0.2707d/d
39	1d	3.7172d	-0.2489d	-0.2489d/d	0.3077d/d	0	-0.3077d/d
40	4d	3.9662d	-3.0338d	-0.7585d/d	0.5095d/d	0	-0.5095d/d
44	3d	7d	-3d	-1d/d	0.2415d/d	0	-0.2415d/d
47	2d	10d	0	0	-1d/d	0	1d/d
49	1d	10d	0	0	0	0	0
50	2d	10d	0	0	0	0	0
52	1d	10d	0	0	0	0	0
53	0d	10d	N/A	0	N/A	0	N/A
Checksum	**53d**	**N/A**	**-10d**	**N/A**	**0d/d**	**N/A**	**1d/d**

This gives schedule variance in time units $y(x)_{SV(t)} = 0.5143/1 \cdot \langle x\text{-}24\rangle^1 - 0.1467/1 \cdot \langle x\text{-}27\rangle^1 + 0.1467/1 \cdot \langle x\text{-}28\rangle^1 + 0.2568/1 \cdot \langle x\text{-}29.8745\rangle^1 - 0.3866/1 \cdot \langle x\text{-}31.6393\rangle^1 - 0.3845/1 \cdot \langle x\text{-}34\rangle^1 -10.6536/1 \cdot \langle x\text{-}36.8572\rangle^1 + 10.8655/1 \cdot \langle x\text{-}37\rangle^1 - 0.2707/1 \cdot \langle x\text{-}37.7885\rangle^1 + 0.3077/1 \cdot \langle x\text{-}39\rangle^1 + 0.5095/1 \cdot \langle x\text{-}40\rangle^1 + 0.2415/1 \cdot \langle x\text{-}44\rangle^1 - 1/1 \cdot \langle x\text{-}47\rangle^1$

Write $z(x)_{ES}$ as diagonal 1d/1d $\cdot \langle x\text{-}0\text{d}\rangle^1$ minus terms of updated slopes at new cutoffs.

Earned schedule $z(x)_{ES} = 1/1 \cdot \langle x\text{-}0\rangle^1 - 0.5143/1 \cdot \langle x\text{-}24\rangle^1 + 0.1467/1 \cdot \langle x\text{-}27\rangle^1 - 0.1467/1 \cdot \langle x\text{-}28\rangle^1 - 0.2568/1 \cdot \langle x\text{-}29.8745\rangle^1 + 0.3866/1 \cdot \langle x\text{-}31.6393\rangle^1 + 0.3845/1 \cdot \langle x\text{-}34\rangle^1 +10.6536/1 \cdot \langle x\text{-}36.8572\rangle^1 - 10.8655/1 \cdot \langle x\text{-}37\rangle^1 + 0.2707/1 \cdot \langle x\text{-}37.7885\rangle^1 - 0.3077/1 \cdot \langle x\text{-}39\rangle^1 - 0.5095/1 \cdot \langle x\text{-}40\rangle^1 - 0.2415/1 \cdot \langle x\text{-}44\rangle^1 + 1/1 \cdot \langle x\text{-}47\rangle^1$

Evaluating this equation for growing *AT* gives the following tabulated values.

AT x	***ES z(x)***	***AT x***	***ES z(x)***	***AT x***	***ES z(x)***	***AT x***	***ES z(x)***	***AT x***	***ES z(x)***	***AT x***	***ES z(x)***
1d	1d	10d	10d	19d	19d	28d	26.09d	37d	33.38d	46d	37d
2d	2d	11d	11d	20d	20d	29d	26.5779d	38d	34.2222d	47d	37d
3d	3d	12d	12d	21d	21d	30d	27.0333d	39d	35.28d	48d	38d
4d	4d	13d	13d	22d	22d	31d	27.2626d	40d	36.03d	49d	39d
5d	5d	14d	14d	23d	23d	32d	27.6309d	41d	36.2725d	50d	40d
6d	6d	15d	15d	24d	24d	33d	28.2455d	42d	36.5150d	51d	41d
7d	7d	16d	16d	25d	24.4867d	34d	28.86d	43d	36.7575d	52d	42d

8d	8d	17d	17d	26d	24.9733d	35d	29.86d	44d	37d	53d	43d
9d	9d	18d	18d	27d	25.46d	36d	30.86d	45d	37d	.	.

Using the approach described in the paper allows deriving the equation for the forecasted project duration, which has these values.

Cutoff	**Run**	**Rise**	**Slope**	**Value**
$x(y_i)_{EV\_project}$	$\Delta x(y_i)_{EV\_project}$	$\Delta z(y_i)$	**Rise/Run**	$z(x)_{forecast}$
0d	24d	0d	0	43d
24d	3d	1.5428d	0.5143	63.1172d
27d	1d	0.3676d	0.3676	54.7412d
28d	1.8745d	0.9640d	0.5143	62.8153d
29.8745d	1.7648d	1.3607d	0.7710	99.8798d
31.6393d	2.3607d	0.9076d	0.3845	56.8313d
34d	2.8572d	0d	0	48.1428d
36.8572d	0.1428d	-1.5214d	-10.6536	37.7137d
37d	0.7885d	0.1671d	0.2119	49.2080d
37.7885d	1.2115d	-0.0712d	-0.0588	46.2460d
39d	1d	0.2489d	0.2489	49.2752d
40d	4d	3.0338d	0.7585	68.8400d
44d	3d	3d	1	68.8400d
47d	2d	0d	0	53d
49d	1d	0d	0	53d
50d	2d	0d	0	53d
52d	1d	0d	0	53d
53d	0d	0d	0	53d
Checksum	**53d**	**10**	**N/A**	**N/A**

Appendix 2-H.1: Activity Earned Schedule Minimum Table (AP < 100%)

Activity	***PES***	***PLS***	***PD***	***AD***	**0**	**1**	**2**	**3**	**4**	**4.13**	**4.25**	**4.38**	**4.50**	**4.63**	**4.75**	**4.88**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0	0	3	3	**0**	**1**	**2**																									
2	3	3	1	1	3	3	3	**3**																								
3	4	4	1	1	4	4	4	4	**4**	**4.125**	**4.25**	**4.375**	**4.5**	**4.625**	**4.75**	**4.875**																
4	17	30	1	1	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30			
6	4	4.375	0.125	0.125	4.375	4.375	4.375	4.375	4.375																							
5	27	31	1	1	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31

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7	4.125	4.5	0.5	0.5	4.5	4.5	4.5	4.5	4.5	4.5	4.625	4.75	4.875																			
8	5	5	1	1	5	5	5	5	5	5	5	5	5	5	5	5	**5**															
9	6	6	18	18	6	6	6	6	6	6	6	6	6	6	6	6	6	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	18	19	20
10	9	9	8	8	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**				
11	10	12	5	5	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	13	14	15	16						
12	24	17	5	5	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	**17**	**17**	**17**	**17**
13	29	32	5	5	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
14	17	17	5	5	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	**17**	18	19	20
15	22	22	15	15	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
16	22	22	15	15	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
17	37	37	2	2	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37
18	39	39	1	1	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39
19	40	40	2	2	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
20	40	40	2	2	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
21	42	42	1	1	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
Minimum	**N/A**	**N/A**	**N/A**	**N/A**	0	1	2	3	4	4.125	4.25	4.375	4.5	4.625	4.75	4.875	5	6	7	8	9	10	11	12	13	14	15	16	17	17	17	17

880 *(Continues)*

21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**
31	31	31	31	31	31	**31**																										
21	22	23																														
17	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**17**	**18**	**19**	**20**	**21**															
32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32	**32**	**33**	**34**	**35**	**36**										
21																																
22	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36																	
22	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36																	
37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	37	**37**	**37**	**37**	**37**	**38**					
39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	39	**39**				
40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	**40**	**41**		
40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	**40**	**41**		
42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	**42**	
17	17	17	17	17	17	17	17	17	17	17	17	17	17	18	19	20	21	32	33	34	35	36	37	37	37	37	38	39	40	41	42	

881 *Minima are **bold**. Asterisk is excluded value.*

Appendix 2-H.2: Activity Earned Schedule Maximum Table (AP > 0%)

Activity	*PES*	*PLS*	*PD*	*AD*	0	1	2	3	4	4.13	4.25	4.38	4.50	4.63	4.75	4.88	5	6	7	8	9	10	11	12	13	14	15
1	0	0	30	30		**1**	**2**	**3**	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
2	3	3	10	10					**4**	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
3	4	4	10	10						**4.125**	**4.25**	**4.375**	**4.5**	**4.625**	**4.75**	**4.875**	**5**	5	5	5	5	5	5	5	5	5	5
4	17	30	10	10																							
6	4	4.375	0.125	0.125						**4.125**	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125
5	27	31	10	10																							
7	4.125	4.5	0.500	0.500							**4.25**	**4.375**	**4.5**	**4.625**	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625
8	5	5	10	10														**6**	6	6	6	6	6	6	6	6	6
9	6	6	180	180															**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
10	9	9	80	80																		**10**	**11**	**12**	**13**	**14**	**15**
11	10	12	50	50																			**11**	**12**	**13**	**14**	**15**
12	24	17	50	50																							
13	29	32	50	50																							
14	17	17	50	50																							
15	22	22	150	150																							
16	22	22	150	150																							
17	37	37	20	20																							
18	39	39	10	10																							
19	40	40	20	20																							
20	40	40	20	20																							
21	42	42	10	10																							
Maximum	**N/A**	**N/A**	**N/A**	**N/A**	0	1	2	3	4	4.125	4.25	4.375	4.5	4.625	4.75	4.875	5	6	7	8	9	10	11	12	13	14	15

(Continues)

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
		18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125
												28	28	28	28	28	28	28	28	28	28	28	28	28	28
4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24
16	**17**	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
																			25	26	27	28	29	29	29

																								30	31
		18	**19**	**20**	**21**	**22**	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
							23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**37**	**37**	**37**	**37**
							23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**37**	**37**	**37**	**37**
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	37	37	37	37

(Continues)

42	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**
3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5
18	18	18	18	18	18	18	18	18	18	18	18
4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125	4.125
28	28	28	28	28	28	28	28	28	28	28	28
4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625	4.625
6	6	6	6	6	6	6	6	6	6	6	6
24	24	24	24	24	24	24	24	24	24	24	24
17	17	17	17	17	17	17	17	17	17	17	17
15	15	15	15	15	15	15	15	15	15	15	15
29	29	29	29	29	29	29	29	29	29	29	29
32	33	34	34	34	34	34	34	34	34	34	34
22	22	22	22	22	22	22	22	22	22	22	22
37	**37**	**37**	**37**	**37**	**37**	37	37	37	37	37	37
37	**37**	**37**	**37**	**37**	**37**	37	37	37	37	37	37
						38	**39**	39	39	39	39
								40	40	40	40
									41	**42**	42
									41	**42**	42
											43
37	37	37	37	37	37	38	39	40	41	42	43

*Maxima are **bold**. Asterisk is excluded value.*

Appendix 2-I.1: Activity Earned Schedule Minimum

Activity	**Output**
1	$z(x)_{ES\_1\_min} = [1 - \langle 0 - {}^{1}/_{3}/d \cdot \langle x\text{-}0\rangle^{1} + {}^{1}/_{3}/d \cdot \langle x\text{-}3\rangle^{1}\rangle^{0}]_{min} \cdot [0 + {}^{1}/_{3}/d \cdot \langle x\text{-}0\rangle^{1} - {}^{1}/_{3}/d \cdot \langle x\text{-}3\rangle^{1}]$

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2	$z(x)_{ES\_2\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-3 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1} \rangle^{0}]_{min} \cdot [3 + {}^{1}/_{1}/d \cdot \langle x-3 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1}]$
3	$z(x)_{ES\_3\_min} = [1 - \langle 0 - {}^{1}/_{8}/d \cdot \langle x-4 \rangle^{1} + {}^{1}/_{8}/d \cdot \langle x-5 \rangle^{1} \rangle^{0}]_{min} \cdot [4 + {}^{1}/_{8}/d \cdot \langle x-4 \rangle^{1} - {}^{1}/_{8}/d \cdot \langle x-5 \rangle^{1}]$
4	$z(x)_{ES\_4\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-17 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-18 \rangle^{1} \rangle^{0}]_{min} \cdot [30 + {}^{1}/_{1}/d \cdot \langle x-17 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-18 \rangle^{1}]$
6	$z(x)_{ES\_6\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-4.125 \rangle^{1} \rangle^{0}]_{min} \cdot [4.375 + {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-4.125 \rangle^{1}]$
5	$z(x)_{ES\_5\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-27 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-28 \rangle^{1} \rangle^{0}]_{min} \cdot [31 + {}^{1}/_{1}/d \cdot \langle x-27 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-28 \rangle^{1}]$
7	$z(x)_{ES\_7\_min} = [1 - \langle 0 - {}^{1}/_{4}/d \cdot \langle x-4.125 \rangle^{1} + {}^{1}/_{4}/d \cdot \langle x-4.625 \rangle^{1} \rangle^{0}]_{min} \cdot [4.5 + {}^{1}/_{4}/d \cdot \langle x-4.125 \rangle^{1} - {}^{1}/_{4}/d \cdot \langle x-4.625 \rangle^{1}]$
8	$z(x)_{ES\_8\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-5 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-6 \rangle^{1} \rangle^{0}]_{min} \cdot [5 + {}^{1}/_{1}/d \cdot \langle x-5 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-6 \rangle^{1}]$
9	$z(x)_{ES\_9\_min} = [1 - \langle 0 - {}^{1}/_{18}/d \cdot \langle x-6 \rangle^{1} + {}^{1}/_{18}/d \cdot \langle x-24 \rangle^{1} \rangle^{0}]_{min} \cdot [6 + {}^{1}/_{18}/d \cdot \langle x-6 \rangle^{1} - {}^{1}/_{18}/d \cdot \langle x-24 \rangle^{1}]$
10	$z(x)_{ES\_10\_min} = [1 - \langle 0 - {}^{1}/_{8}/d \cdot \langle x-9 \rangle^{1} + {}^{1}/_{8}/d \cdot \langle x-17 \rangle^{1} \rangle^{0}]_{min} \cdot [9 + {}^{1}/_{8}/d \cdot \langle x-9 \rangle^{1} - {}^{1}/_{8}/d \cdot \langle x-17 \rangle^{1}]$
11	$z(x)_{ES\_11\_min} = [1 - \langle 0 - {}^{1}/_{5}/d \cdot \langle x-10 \rangle^{1} + {}^{1}/_{5}/d \cdot \langle x-15 \rangle^{1} \rangle^{0}]_{min} \cdot [12 + {}^{1}/_{5}/d \cdot \langle x-10 \rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-15 \rangle^{1}]$
12	$z(x)_{ES\_12\_min} = [1 - \langle 0 - {}^{1}/_{5}/d \cdot \langle x-34 \rangle^{1} + {}^{1}/_{5}/d \cdot \langle x-39 \rangle^{1} \rangle^{0}]_{min} \cdot [17 + {}^{1}/_{5}/d \cdot \langle x-34 \rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-39 \rangle^{1}]$
13	$z(x)_{ES\_13\_min} = [1 - \langle 0 - {}^{1}/_{5}/d \cdot \langle x-39 \rangle^{1} + {}^{1}/_{5}/d \cdot \langle x-44 \rangle^{1} \rangle^{0}]_{min} \cdot [32 + {}^{1}/_{5}/d \cdot \langle x-39 \rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-44 \rangle^{1}]$
14	$z(x)_{ES\_14\_min} = [1 - \langle 0 - {}^{1}/_{5}/d \cdot \langle x-17 \rangle^{1} + {}^{1}/_{5}/d \cdot \langle x-22 \rangle^{1} \rangle^{0}]_{min} \cdot [17 + {}^{1}/_{5}/d \cdot \langle x-17 \rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-22 \rangle^{1}]$
15	$z(x)_{ES\_15\_min} = [1 - \langle 0 - {}^{1}/_{15}/d \cdot \langle x-22 \rangle^{1} + {}^{1}/_{15}/d \cdot \langle x-37 \rangle^{1} \rangle^{0}]_{min} \cdot [22 + {}^{1}/_{15}/d \cdot \langle x-22 \rangle^{1} - {}^{1}/_{15}/d \cdot \langle x-37 \rangle^{1}]$
16	$z(x)_{ES\_16\_min} = [1 - \langle 0 - {}^{1}/_{15}/d \cdot \langle x-22 \rangle^{1} + {}^{1}/_{15}/d \cdot \langle x-37 \rangle^{1} \rangle^{0}]_{min} \cdot [22 + {}^{1}/_{15}/d \cdot \langle x-22 \rangle^{1} - {}^{1}/_{15}/d \cdot \langle x-37 \rangle^{1}]$
17	$z(x)_{ES\_17\_min} = [1 - \langle 0 - {}^{1}/_{2}/d \cdot \langle x-47 \rangle^{1} + {}^{1}/_{2}/d \cdot \langle x-49 \rangle^{1} \rangle^{0}]_{min} \cdot [37 + {}^{1}/_{2}/d \cdot \langle x-47 \rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-49 \rangle^{1}]$
18	$z(x)_{ES\_18\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-49 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-50 \rangle^{1} \rangle^{0}]_{min} \cdot [39 + {}^{1}/_{1}/d \cdot \langle x-49 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-50 \rangle^{1}]$
19	$z(x)_{ES\_19\_min} = [1 - \langle 0 - {}^{1}/_{2}/d \cdot \langle x-50 \rangle^{1} + {}^{1}/_{2}/d \cdot \langle x-52 \rangle^{1} \rangle^{0}]_{min} \cdot [40 + {}^{1}/_{2}/d \cdot \langle x-50 \rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-52 \rangle^{1}]$
20	$z(x)_{ES\_20\_min} = [1 - \langle 0 - {}^{1}/_{2}/d \cdot \langle x-50 \rangle^{1} + {}^{1}/_{2}/d \cdot \langle x-52 \rangle^{1} \rangle^{0}]_{min} \cdot [40 + {}^{1}/_{2}/d \cdot \langle x-50 \rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-52 \rangle^{1}]$
21	$z(x)_{ES\_21\_min} = [1 - \langle 0 - {}^{1}/_{1}/d \cdot \langle x-52 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-53 \rangle^{1} \rangle^{0}]_{min} \cdot [42 + {}^{1}/_{1}/d \cdot \langle x-52 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-53 \rangle^{1}]$

Appendix 2-I.2: Activity Earned Schedule Maximum

Activity	**Output**
1	$z(x)_{ES\_1\_max} = [1 - \langle {}^{1}/_{3}/d \cdot \langle x-0 \rangle^{1} - {}^{1}/_{3}/d \cdot \langle x-3 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [0 + {}^{1}/_{1}/d \cdot \langle x-0 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-3 \rangle^{1}]$
2	$z(x)_{ES\_2\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-3 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [3 + {}^{1}/_{1}/d \cdot \langle x-3 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1}]$
3	$z(x)_{ES\_3\_max} = [1 - \langle {}^{1}/_{8}/d \cdot \langle x-4 \rangle^{1} - {}^{1}/_{8}/d \cdot \langle x-5 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [4 + {}^{1}/_{8}/d \cdot \langle x-4 \rangle^{1} - {}^{1}/_{8}/d \cdot \langle x-5 \rangle^{1}]$
4	$z(x)_{ES\_4\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-17 \rangle^{1} + {}^{1}/_{1}/d \cdot \langle x-18 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [17 + {}^{1}/_{8}/d \cdot \langle x-17 \rangle^{1} - \tfrac{1}{4}/d \cdot \langle x-18 \rangle^{1}]$
6	$z(x)_{ES\_6\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-4.125 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [4 + {}^{1}/_{1}/d \cdot \langle x-4 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-4.125 \rangle^{1}]$
5	$z(x)_{ES\_5\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-27 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-28 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [27 + {}^{1}/_{1}/d \cdot \langle x-27 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-28 \rangle^{1}]$
7	$z(x)_{ES\_7\_max} = [1 - \langle {}^{1}/_{4}/d \cdot \langle x-4.125 \rangle^{1} - {}^{1}/_{4}/d \cdot \langle x-4.625 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [4.125 + {}^{1}/_{4}/d \cdot \langle x-4.125 \rangle^{1} - {}^{1}/_{4}/d \cdot \langle x-4.625 \rangle^{1}]$
8	$z(x)_{ES\_8\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-5 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-6 \rangle^{1} - 1 \rangle^{0}]_{max} \cdot [5 + {}^{1}/_{1}/d \cdot \langle x-5 \rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-6 \rangle^{1}]$

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9	$z(x)_{ES\_9\_max} = [1 - \langle {}^{1}/_{18}/d \cdot \langle x-6\rangle^{1} - {}^{1}/_{18}/d \cdot \langle x-24\rangle^{1} - 1\rangle^{0}]_{max} \cdot [6 + {}^{1}/_{18}/d \cdot \langle x-6\rangle^{1} - {}^{1}/_{18}/d \cdot \langle x-24\rangle^{1}]$
10	$z(x)_{ES\_10\_max} = [1 - \langle {}^{1}/_{8}/d \cdot \langle x-9\rangle^{1} - {}^{1}/_{8}/d \cdot \langle x-17\rangle^{1} - 1\rangle^{0}]_{max} \cdot [9 + {}^{1}/_{8}/d \cdot \langle x-9\rangle^{1} - {}^{1}/_{8}/d \cdot \langle x-17\rangle^{1}]$
11	$z(x)_{ES\_11\_max} = [1 - \langle {}^{1}/_{5}/d \cdot \langle x-10\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-15\rangle^{1} - 1\rangle^{0}]_{max} \cdot [10 + {}^{1}/_{5}/d \cdot \langle x-10\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-15\rangle^{1}]$
12	$z(x)_{ES\_12\_max} = [1 - \langle {}^{1}/_{5}/d \cdot \langle x-34\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-39\rangle^{1} - 1\rangle^{0}]_{max} \cdot [24 + {}^{1}/_{5}/d \cdot \langle x-34\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-39\rangle^{1}]$
13	$z(x)_{ES\_13\_max} = [1 - \langle {}^{1}/_{5}/d \cdot \langle x-39\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-44\rangle^{1} - 1\rangle^{0}]_{max} \cdot [29 + {}^{1}/_{5}/d \cdot \langle x-39\rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-44\rangle^{1}]$
14	$z(x)_{ES\_14\_max} = [1 - \langle {}^{1}/_{5}/d \cdot \langle x-17\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-22\rangle^{1} - 1\rangle^{0}]_{max} \cdot [17 + {}^{1}/_{5}/d \cdot \langle x-17\rangle^{1} - {}^{1}/_{5}/d \cdot \langle x-22\rangle^{1}]$
15	$z(x)_{ES\_15\_max} = [1 - \langle {}^{1}/_{15}/d \cdot \langle x-22\rangle^{1} - {}^{1}/_{15}/d \cdot \langle x-37\rangle^{1} - 1\rangle^{0}]_{max} \cdot [22 + {}^{1}/_{15}/d \cdot \langle x-22\rangle^{1} - {}^{1}/_{15}/d \cdot \langle x-37\rangle^{1}]$
16	$z(x)_{ES\_16\_max} = [1 - \langle {}^{1}/_{15}/d \cdot \langle x-22\rangle^{1} - {}^{1}/_{15}/d \cdot \langle x-37\rangle^{1} - 1\rangle^{0}]_{max} \cdot [22 + {}^{1}/_{15}/d \cdot \langle x-22\rangle^{1} - {}^{1}/_{15}/d \cdot \langle x-37\rangle^{1}]$
17	$z(x)_{ES\_17\_max} = [1 - \langle {}^{1}/_{2}/d \cdot \langle x-47\rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-49\rangle^{1} - 1\rangle^{0}]_{max} \cdot [37 + {}^{1}/_{2}/d \cdot \langle x-47\rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-49\rangle^{1}]$
18	$z(x)_{ES\_18\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-49\rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-50\rangle^{1} - 1\rangle^{0}]_{max} \cdot [39 + {}^{1}/_{1}/d \cdot \langle x-49\rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-50\rangle^{1}]$
19	$z(x)_{ES\_19\_max} = [1 - \langle {}^{1}/_{2}/d \cdot \langle x-50\rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-52\rangle^{1} - 1\rangle^{0}]_{max} \cdot [40 + {}^{1}/_{2}/d \cdot \langle x-50\rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-52\rangle^{1}]$
20	$z(x)_{ES\_20\_max} = [1 - \langle {}^{1}/_{2}/d \cdot \langle x-50\rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-52\rangle^{1} - 1\rangle^{0}]_{max} \cdot [40 + {}^{1}/_{2}/d \cdot \langle x-50\rangle^{1} - {}^{1}/_{2}/d \cdot \langle x-52\rangle^{1}]$
21	$z(x)_{ES\_21\_max} = [1 - \langle {}^{1}/_{1}/d \cdot \langle x-52\rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-53\rangle^{1} - 1\rangle^{0}]_{max} \cdot [42 + {}^{1}/_{1}/d \cdot \langle x-52\rangle^{1} - {}^{1}/_{1}/d \cdot \langle x-53\rangle^{1}]$

Appendix 2-J: Variances and Indices

Index	Unit	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
SV	Money	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
SV(t)	Time	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CV	Money	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
SPI	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SPI(t)	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CPI	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CSI	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CSI(t)	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Percent Complete	%	0	0.44	0.89	1.33	1.77	7.49	12.39	13.02	13.66	14.30	15.70	17.84	19.98	22.12	24.26	26.40	27.80	29.20	32.04	34.43	36.82
Percent Money Spent	%	0	0.44	0.89	1.33	1.77	7.49	12.39	13.02	13.66	14.30	15.70	17.84	19.98	22.12	24.26	26.40	27.80	29.20	32.04	34.43	36.82

(Continues)

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
$0	$0	$0	$0	-$2,805.40	-$5,610.80	-$8,416.20	-$11,221.60	-$14,027	-$14,937	-$15,847	-$16,757	-$17,667	-$18,577	-$15,771.60
0	0	0	0	-0.51	-1.03	-1.54	-1.91	-2.42	-2.97	-3.74	-4.37	-4.75	-5.14	-5.14
$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
1	1	1	1	0.98	0.96	0.94	0.92	0.90	0.90	0.89	0.89	0.89	0.88	0.90
1	1	1	1	0.98	0.96	0.94	0.93	0.92	0.90	0.88	0.86	0.86	0.85	0.85

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0.98	0.96	0.94	0.92	0.90	0.90	0.89	0.89	0.89	0.88	0.90
1	1	1	1	0.98	0.96	0.94	0.92	0.90	0.90	0.89	0.89	0.89	0.88	0.90
39.22	41.61	52.76	63.92	65.38	66.85	68.32	70.23	71.70	73.17	74.64	76.10	77.57	79.04	82.06
39.22	41.61	52.76	63.92	65.38	66.85	68.32	70.23	71.70	73.17	74.64	76.10	77.57	79.04	82.06

(Continues)

36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
-$12,966.20	-$10,160.80	-$10,295.90	-$10,431	-$9,449	-$10,617	-$11,785	-$11,945	-$11,305	-$11,305	-$11,305	-$11,305	-$8,364.50	-$5,424	-$4,416
-5.14	-3.62	-3.78	-3.72	-3.97	-4.73	-5.49	-6.24	-7	-8	-9	-10	-10	-10	-10
$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
0.92	0.94	0.94	0.94	0.95	0.94	0.93	0.93	0.94	0.94	0.94	0.94	0.95	0.97	0.98
0.86	0.90	0.90	0.90	0.90	0.88	0.87	0.85	0.84	0.82	0.80	0.79	0.79	0.80	0.80
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.92	0.94	0.94	0.94	0.95	0.94	0.93	0.93	0.94	0.94	0.94	0.94	0.95	0.97	0.98
0.92	0.94	0.94	0.94	0.95	0.94	0.93	0.93	0.94	0.94	0.94	0.94	0.95	0.97	0.98
85.08	88.11	89.66	91.22	92.32	92.67	93.03	93.38	93.74	93.74	93.74	93.74	95.37	96.99	97.55
85.08	88.11	89.66	91.22	92.32	92.67	93.03	93.38	93.74	93.74	93.74	93.74	95.37	96.99	97.55

(Continues)

51	52	53
-$2,608	-$800	$0
-10	-10	-10
$0	$0	$0
0.99	1	1
0.80	0.81	0.81
1	1	1
0.99	1	1
0.99	1	1
98.56	99.56	100
98.56	99.56	100

Appendix 2-K: Forecasts

$y(47d)_{CPI\_project} = y(47d)_{EV\_project} / y(47d)_{AC\_project} = €169180/€169180 = 100\%$

$y(47d)_{SPI\_project} = y(47d)_{EV\_project} / y(47d)_{PV\_project} = €169180/€180485 = 93.74\%$

$y(47d)_{SPI(t)\_project} = y(47d)_{ES\_project} / 47d = 37d/47d = 78.72\%$

$y(47d)_{CSI\_project} = y(47d)_{CPI\_project} \times y(47d)_{SPI\_project} = 100\% \times 93.74\% = 93.74\%$

$y(47d)_{CSI(t)\_project} = y(47d)_{CPI\_project} \times y(47d)_{SPI(t)\_project} = 100\% \times 78.72\% = 78.72\%$

Forecast	Calculation	Result
$EAC(\$)\ (47d)_{PF=1\_project}$	$y(47d)_{AC\_project} + (BAC - y(47d)_{EV\_project}) / 1 = €169180 + €11305 / 1$	€180485.00

$EAC(\$)$ (47d)$_{PF=CPI\_project}$	$y(47d)_{AC\_project} + (BAC - y(47d)_{EV\_project}) / y(47)_{CPI\_project}$ = €169180 + €11305 / 100%	€180485.00
$EAC(\$)$ (47d)$_{PF=SPI\_project}$	$y(47d)_{AC\_project} + (BAC - y(47d)_{EV\_project}) / y(47)_{SPI\_project}$ = €169180 + €11305 / 93.74%	€181240.43
$EAC(\$)$ (47d)$_{PF=SPI(t)\_project}$	$y(47d)_{AC\_project} + (BAC - y(47d)_{EV\_project}) / y(47)_{SPI(t)\_project}$ = €169179.97 + €11305 / 78.72%	€183540.41
$EAC(\$)$ (47d)$_{PF=CSI\_project}$	$y(47d)_{AC\_project} + (BAC - y(47d)_{EV\_project}) / y(47)_{CSI\_project}$ = €169179.97 + €11305 / 93.74%	€183540.43
$EAC(\$)$ (47d)$_{PF=CSI(t)\_project}$	$y(47d)_{AC\_project} + (BAC - y(47d)_{EV\_project}) / y(47)_{CSI(t)\_project}$ = €169179.97 + €11305 / 78.72%	€183540.41

$y(47d)_{CV\_project} = y(47d)_{EV\_project} - y(47d)_{AC\_project}$ = €169180 - €169180 = €0 The project incurs no cost overruns.

$y(47d)_{SV\_project} = y(47d)_{EV\_project} - y(47d)_{PV\_project}$ = €169180 - €180485 = -€11305

Forecast	Calculation	Result
$EAC(t)$ (47d)$_{PV1\_project}$	43d - (43d / €180485) × $y(47d)_{SV\_project}$ = 43d - (43d / €180485) × -€11305	45.6934d
$EAC(t)$ $(x)_{PV2\_project}$	43d / $y(47d)_{SPI\_project}$ = 43d / 93.74%	45.8734d
$EAC(t)$ $(x)_{PV3\_project}$	43d / $y(47d)_{CSI\_project}$ = 43d / 93.74%	45.8734d
$EAC(t)$ $(x)_{ED1\_project}$	43d + 47d × (1 - $y(47d)_{SPI\_project}$) = 43d + 47d × (1 -93.74%)	45.9439d
$EAC(t)$ $(x)_{ED2\_project}$	43d / $y(47d)_{SPI\_project}$ = 43d / 93.74%	45.8734d
$EAC(t)$ $(x)_{ED3\_project}$	43d / $y(47d)_{CSI\_project}$ + 47d × (1 - 1 / $y(47d)_{CPI\_project}$) = 43d / 93.74% + 47d × (1 - 1 / 100%)	45.8734d
$EAC(t)$ $(x)_{ES1\_project}$	47d + 43d - $y(47d)_{ES\_project}$ = 47d + 43d - 37	53.0000d
$EAC(t)$ $(x)_{ES2\_project}$	47d + (43d - $y(47d)_{ES\_project}$) / $y(47d)_{SPI(t)\_project}$ = 47d + (43d - 37d) / 78.72%	54.6200d
$EAC(t)$ $(x)_{ES3\_project}$	47d + (43d - $y(47d)_{ES\_project}$) / $y(47d)_{CSI(t)\_project}$ = 47d + (43d - 37d) / 78.72%	54.6200d